

2.7 [20 分]

抽样性质: 函数 $f(t)$ 在 $t = t_1$ 是连续的, 则 $\int_{-\infty}^{\infty} f(t)\delta(t - t_1)dt = f(t_1)$

$$(1) \int_{-\infty}^{\infty} \delta(t - 2) \sin t dt$$

$f(t) = \sin t$ 在 $t=2$ 处是连续的, 所以 原式 = $\sin 2$

$$(2) \int_{-\infty}^{\infty} \frac{\sin 2t}{t} \delta(t) dt = \lim_{t \rightarrow 0} \frac{\sin 2t}{t} = 2$$

$f(t) = \frac{\sin 2t}{t}$, 在 $t=0$ 是 $f(t)$ 的一个可去间断点

$$(3) \int_{-\infty}^{\infty} \delta(t + 3) e^{-t} dt = e^3$$

$$(4) \int_{-\infty}^{\infty} (t^3 + 4)\delta(1 - t) dt$$

令 $t = -x$, 原式 = $\int_{\infty}^{-\infty} (-x^3 + 4)\delta(x + 1)d(-x) = 1 + 4 = 5$

$h(t) (CP + \frac{1}{R}) = \delta(t)$

2.11 [40 分]

解:

把电容 C 看成是一个阻值为 $\frac{1}{Cp}$ 的电阻,

$$i(t) = u_R Cp + \frac{u_R}{R} \Rightarrow u_R = \frac{\frac{1}{C}}{p + \frac{1}{RC}} i(t), \text{ 得到转移算子 } H(p) = \frac{\frac{1}{C}}{p + \frac{1}{RC}}$$

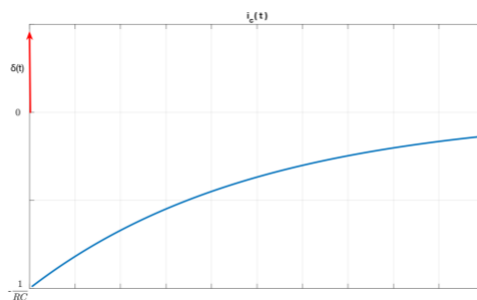
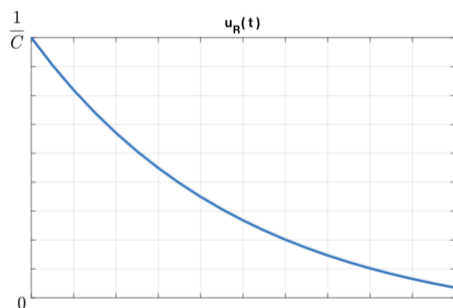
① 当 $i(t) = \delta(t)$ 时,

$$\text{冲激响应为 } h(t) = H(p)\delta(t) = \frac{\frac{1}{C}}{p + \frac{1}{RC}} \delta(t),$$

$$\text{解得 } u_R(t) = \frac{1}{C} e^{\frac{-t}{RC}} \varepsilon(t),$$

$$u(t) = h(t) i(t)$$

$$i_c(t) = C \frac{du_R}{dt} = \delta(t) - \frac{1}{RC} e^{\frac{-t}{RC}} \varepsilon(t) \quad \text{注: } t=0 \text{ 时, } i_c(t) = \delta(t), \text{ 为单位冲激响应, 如下图:}$$

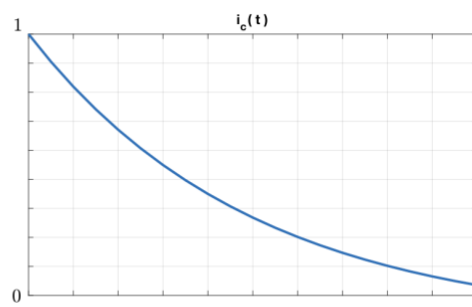
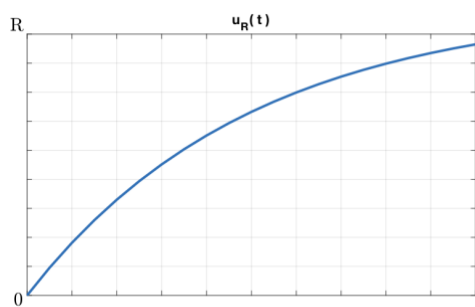


② 当 $i(t) = \varepsilon(t)$ 时,

因为阶跃响应是冲激响应的积分, 所以

$$u_R(t) = \int_{0^-}^t \frac{1}{C} e^{\frac{-\tau}{RC}} \varepsilon(\tau) d\tau = R(1 - e^{\frac{-t}{RC}}) \varepsilon(t),$$

$$i_c(t) = C \frac{du_R}{dt} = e^{\frac{-t}{RC}} \varepsilon(t)$$



2.17 [10 分]

解:

$$\begin{aligned}
 h(t) &= \delta(t) + h_1(t) + h_1(t) * h_2(t) \\
 &= \delta(t) + \delta(t-1) + \delta(t-1) * [\delta(t) - \delta(t-3)] \\
 &= \delta(t) + 2\delta(t-1) - \delta(t-4)
 \end{aligned}$$

第二章补充题 [30 分]

解:

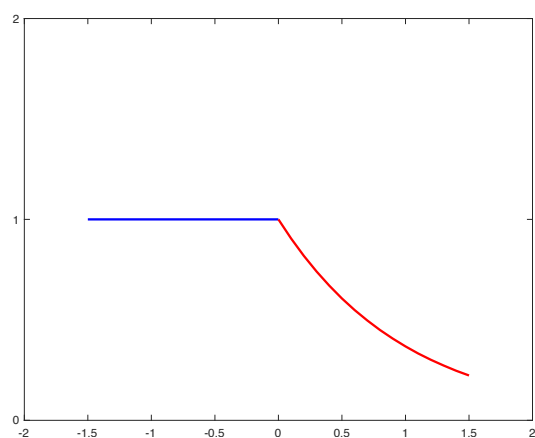
$$x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) d\tau \quad \text{①}$$

当 $t < 0$ 时, ①式 = 0

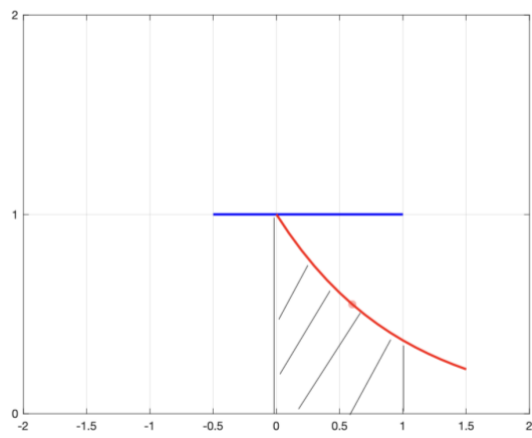
$$\text{当 } 0 \leq t < 1.5 \text{ 时, ①式} = \int_0^t h(\tau) \cdot x(t-\tau) d\tau = \int_0^t e^{-\tau} d\tau = 1 - e^{-t}$$

$$\text{当 } 1.5 \leq t < 3 \text{ 时, ①式} = \int_{t-1.5}^{1.5} h(\tau) \cdot x(t-\tau) d\tau = e^{1.5-t} - e^{-1.5}$$

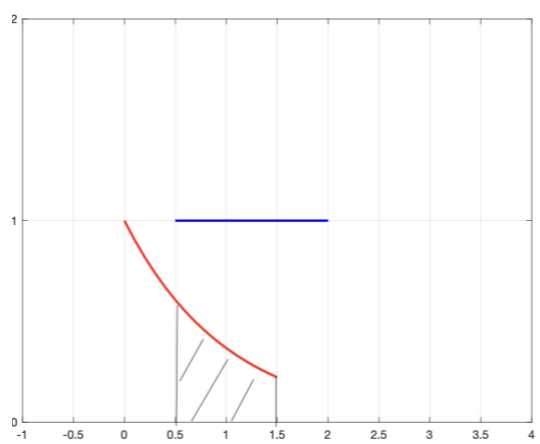
当 $t \geq 3$ 时, ①式 = 0



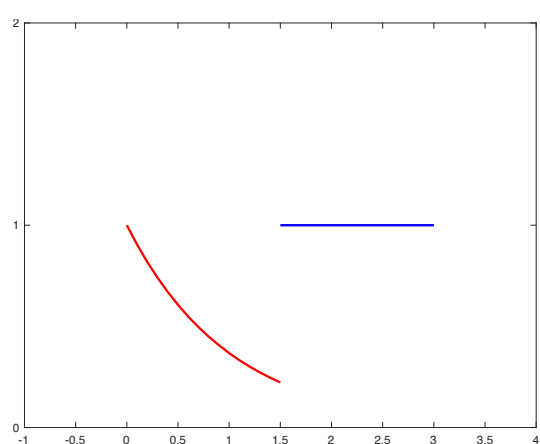
(1)



(2)



(3)



(4)

