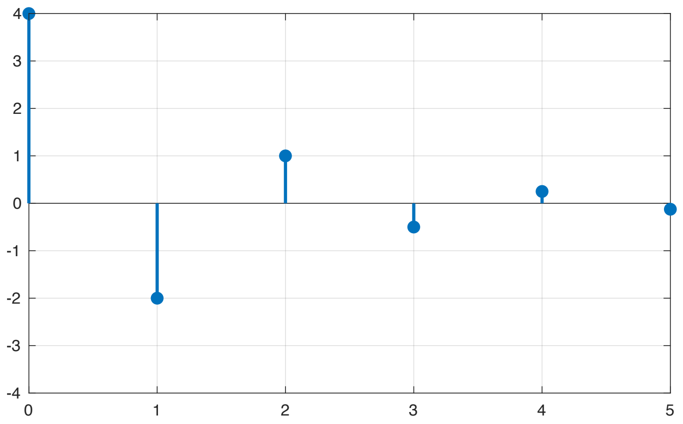
7.1 [20分]

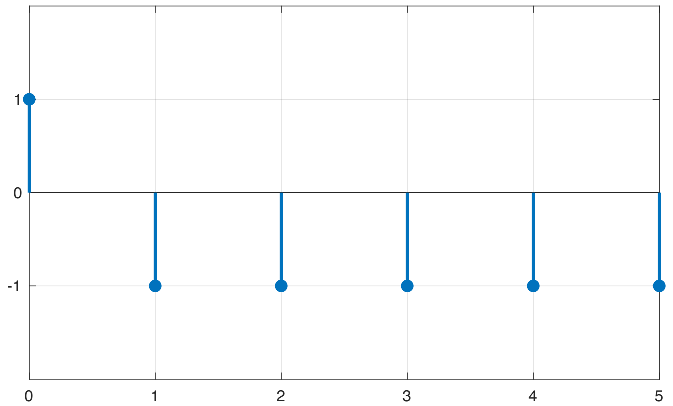
(1) $\left(-\frac{1}{2}\right)^{k-2}\varepsilon(k)$

$$\varepsilon(k)=\begin{cases}1, & k\geq 0\\ 0, & k<0\end{cases}, \quad \left(-\frac{1}{2}\right)^{k-2}\text{是一个等比数列：}$$



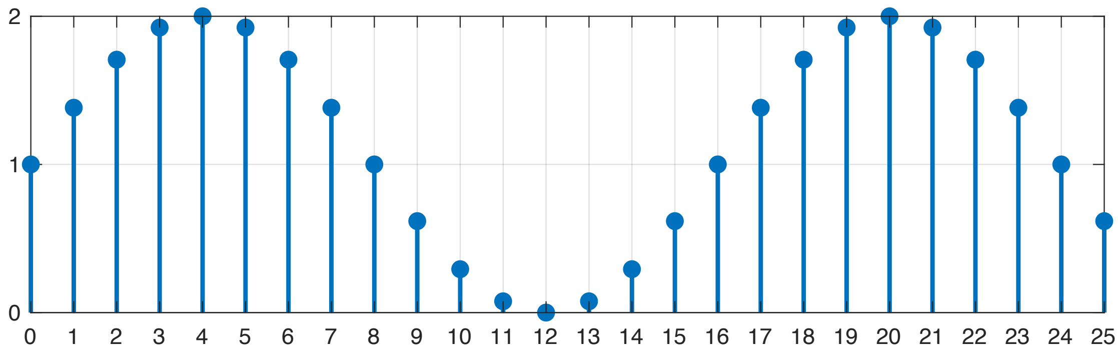
(2) $2\delta(k)-\varepsilon(k)$

$$\delta(k)=\begin{cases}1, & k=0\\ 0, & k\neq 0\end{cases}, \quad \varepsilon(k)=\begin{cases}1, & k\geq 0\\ 0, & k<0\end{cases}$$



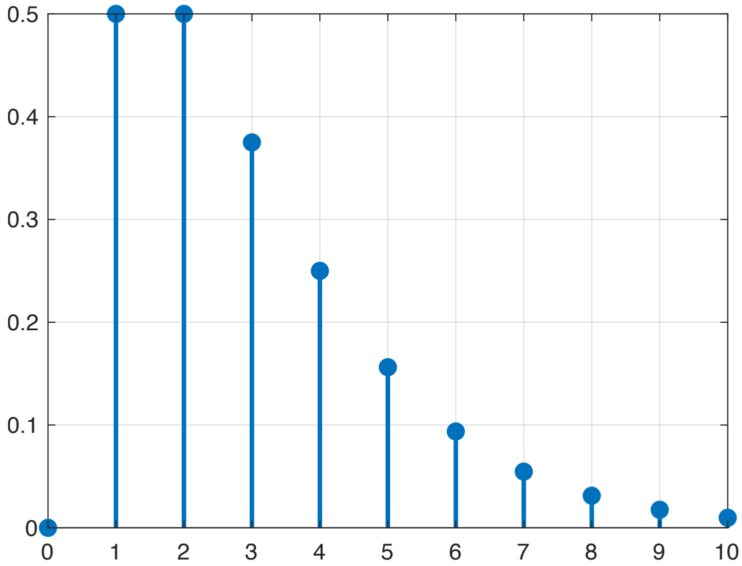
(3) $\varepsilon(k)+\sin\left[\frac{k\pi}{8}\varepsilon(k)\right]$

$$\varepsilon(k)=\begin{cases}1, & k\geq 0\\ 0, & k<0\end{cases}, \ \sin\left[\frac{k\pi}{8}\varepsilon(k)\right]\text{的周期是}16$$



（4）$k\cdot 2^{-k}\varepsilon(k)$

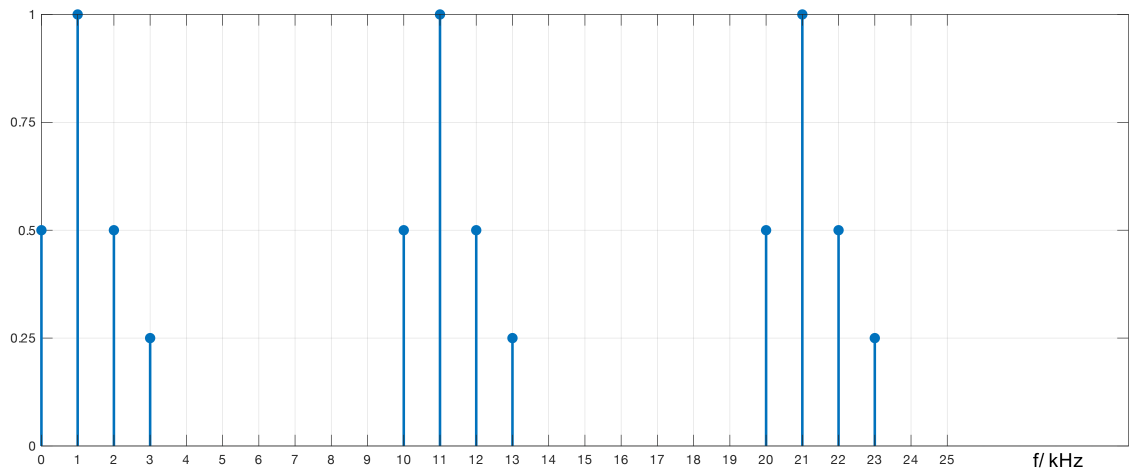
$$\varepsilon(k)=\begin{cases}1, & k\geq 0\\ 0, & k<0\end{cases}，2^{-k}是一个等比数列$$



7.7［15分］

解：

根据抽样定理，该频谱是以原连续时间信号频谱排列构成的周期函数，周期为采样频率的大小即 10kHz，所以原信号频谱在 0-25 kHz 范围内总共重复三次，如下图：



7.12［15分］

解：假设零输入响应为$y_i(k)$，当激励为$e(k)$时，系统的零状态响应为$y_s(k)$

$$\therefore y_1(k) = y_i(k) + y_s(k) = \left[\left(\frac{1}{2}\right)^k + 1\right]\varepsilon(k)$$

当激励为$-e(k)$时，$y_2(k) = y_i(k) - y_s(k) = \left[\left(-\frac{1}{2}\right)^k - 1\right]\varepsilon(k)$

联立上面两式，可解得 $y_i(k) = \left[\left(\frac{1}{2}\right)^{k+1} + \frac{1}{2}\times\left(-\frac{1}{2}\right)^k\right]\varepsilon(k)$，$y_s(k) = \left[\left(\frac{1}{2}\right)^{k+1} - \frac{1}{2}\times\left(-\frac{1}{2}\right)^k + 1\right]\varepsilon(k)$

∴ 当初始状态增加一倍，激励为$4e(k)$时，

$$y_3(k) = 2y_i(k) + 4y_s(k) = \left[\left(\frac{1}{2}\right)^k + \left(-\frac{1}{2}\right)^k\right]\varepsilon(k) + \left[2\times\left(\frac{1}{2}\right)^k - 2\times\left(-\frac{1}{2}\right)^k + 4\right]\varepsilon(k)$$

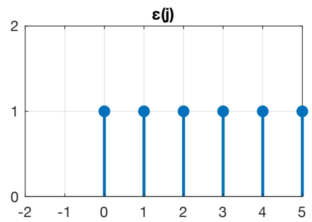
$$= \left[3\times\left(\frac{1}{2}\right)^k - \left(-\frac{1}{2}\right)^k + 4\right]\varepsilon(k)$$

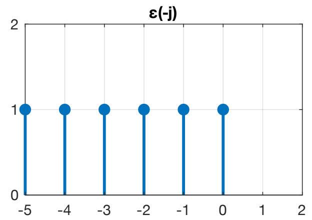
7.24 (1)(3)两小题，至少一个用图解法［15分］

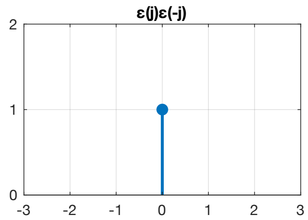
解：（1）$\varepsilon(k) * \varepsilon(k)$

法一：$\varepsilon(k) * \varepsilon(k) = \sum_{j=0}^{k}\varepsilon(j)\varepsilon(k-j) = (k+1)\varepsilon(k)$

法二 图解法：$\varepsilon(k) * \varepsilon(k) = \sum_{j=0}^{k}\varepsilon(j)\varepsilon(k-j)$

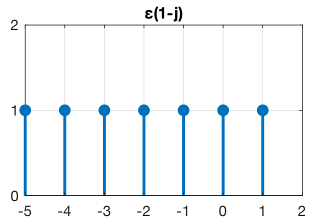


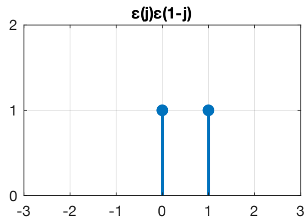




当 k=0 时，

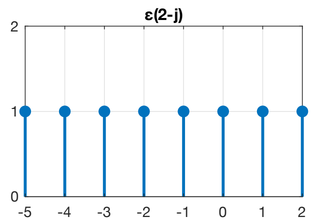
$\varepsilon(0) * \varepsilon(0) = \varepsilon(0)\varepsilon(0) = 1$

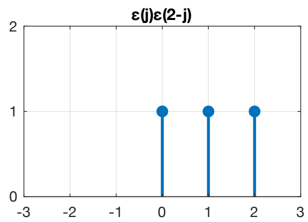




当 k=1 时，

$$\varepsilon(1) * \varepsilon(1) = \sum_{j=0}^{1}\varepsilon(j)\varepsilon(1-j) = 2$$





当 k=2 时，

$$\varepsilon(2) * \varepsilon(2) = \sum_{j=0}^{2}\varepsilon(j)\varepsilon(1-j) = 3$$

依次类推，可得$\varepsilon(k) * \varepsilon(k) = (k+1)\varepsilon(k)$

（2） $2^k\varepsilon(k)*3^k\varepsilon(k)$

法一： $2^k\varepsilon(k)*3^k\varepsilon(k)$

$$=\sum_{j=0}^{k}2^{k-j}\varepsilon(k-j)\cdot 3^j\varepsilon(j)$$

$$=(3^0\cdot 2^k+3\cdot 2^{k-1}+3^2\cdot 2^{k-2}+\ldots+3^k\cdot 2^0)\varepsilon(k)$$

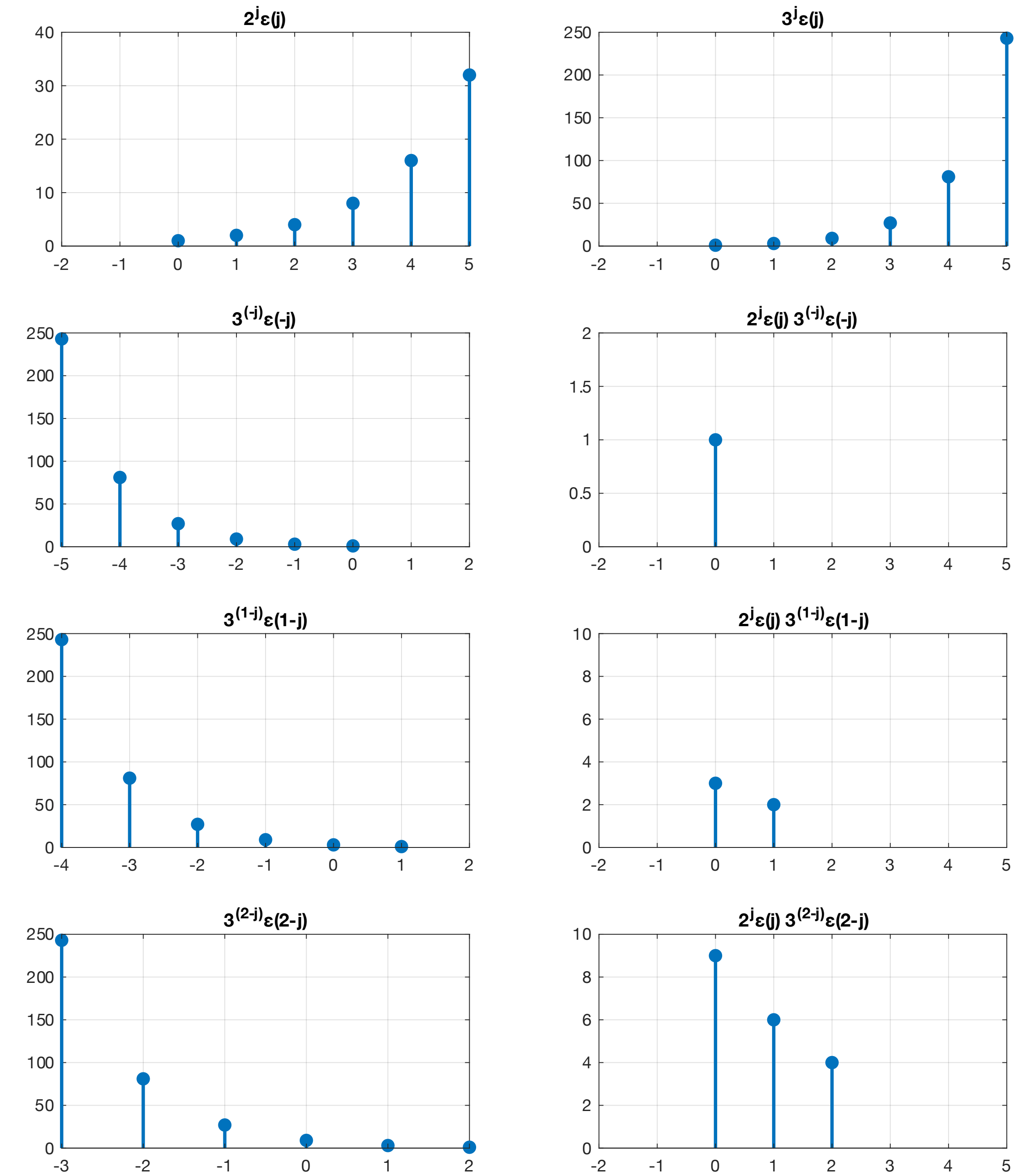
令 $x=3^0\cdot 2^k+3\cdot 2^{k-1}+3^2\cdot 2^{k-2}+\ldots+3^k\cdot 2^0$，则 $3x=3^1\cdot 2^k+3^2\cdot 2^{k-1}+\ldots+3^{k+1}\cdot 2^0$

$$2x=3^{k+1}-2^k+(3^1\cdot 2^{k-1}+3^2\cdot 2^{k-2}+\ldots+3^k\cdot 2^0)=3^{k+1}-2^k+(x-2^k)$$

解得 $x=3^{k+1}-2^{k+1}$

$$\therefore 2^k\varepsilon(k)*3^k\varepsilon(k)=(3^{k+1}-2^{k+1})\varepsilon(k)$$

法二 图解法：$2^k\varepsilon(k)*3^k\varepsilon(k)=\sum_{j=0}^{k}2^j\varepsilon(j)3^{k-j}\varepsilon(k-j)$



当 k=0 时，$2^0\varepsilon(0)*3^0\varepsilon(0)=2^0\varepsilon(0)3^0\varepsilon(0)=1$

当 k=1 时，$2^1\varepsilon(1)*3^1\varepsilon(1)=\sum_{j=0}^{1}2^j\varepsilon(j)3^{1-j}\varepsilon(1-j)=3+2=5$

当 k=2 时，$2^2\varepsilon(2)*3^2\varepsilon(2)=\sum_{j=0}^{2}2^j\varepsilon(j)3^{2-j}\varepsilon(2-j)=9+6+4=19$

观察规律，看出这是首项2^k, 公比为$\frac{3}{2}$的等比数列，前 k+1 项和为 $\frac{2^k\left[1-\left(\frac{3}{2}\right)^{k+1}\right]}{1-\frac{3}{2}}\varepsilon(k)=(3^{k+1}-2^{k+1})\varepsilon(k)$

7.28［20分］

$y(k+2)+y(k+1)+y(k)=\varepsilon(k+1)$，$y_{zi}(0)=1$，$y_{zi}(1)=2$

解：（1）由题目所给差分方程可得$(S^2+S+1)y(k)=S\varepsilon(k)$

转移算子$H(S)=\frac{S}{S^2+S+1}=\frac{1}{2}\frac{2S+1-1}{(S+\frac{1+\sqrt{3}j}{2})(S+\frac{1-\sqrt{3}j}{2})}=\frac{1}{2}[\frac{1}{S+\frac{1+\sqrt{3}j}{2}}+\frac{1}{S+\frac{1-\sqrt{3}j}{2}}-\frac{1}{\sqrt{3}j}(\frac{1}{S+\frac{1+\sqrt{3}j}{2}}-\frac{1}{S+\frac{1-\sqrt{3}j}{2}})]$

根据欧拉公式，可得$\frac{1+\sqrt{3}j}{2}=e^{\frac{\pi}{3}j},\frac{1-\sqrt{3}j}{2}=e^{-\frac{\pi}{3}j}$

$$\therefore H(S)=\frac{1}{2}\left[\frac{1}{S+e^{\frac{\pi}{3}j}}+\frac{1}{S+e^{-\frac{\pi}{3}j}}-\frac{1}{\sqrt{3}j}\left(\frac{1}{S+e^{\frac{\pi}{3}j}}-\frac{1}{S+e^{-\frac{\pi}{3}j}}\right)\right]$$

$$h(k)=\frac{1}{2}\left[\left(-e^{\frac{\pi}{3}j}\right)^{k-1}+\left(-e^{-\frac{\pi}{3}j}\right)^{k-1}\right]\varepsilon(k-1)-\frac{1}{2\sqrt{3}j}\left[\left(-e^{\frac{\pi}{3}j}\right)^{k-1}-\left(-e^{-\frac{\pi}{3}j}\right)^{k-1}\right]\varepsilon(k-1)$$

$y_{zs}(k)=h(k)*\varepsilon(k)$

$$=\frac{1}{2}\left[\left(-e^{\frac{\pi}{3}j}\right)^{k-1}+\left(-e^{-\frac{\pi}{3}j}\right)^{k-1}\right]\varepsilon(k-1)*\varepsilon(k)-\frac{1}{2\sqrt{3}j}\left[\left(-e^{\frac{\pi}{3}j}\right)^{k-1}-\left(-e^{-\frac{\pi}{3}j}\right)^{k-1}\right]\varepsilon(k-1)*\varepsilon(k)$$

$$=\frac{1}{2}\left[\frac{1-\left(-e^{\frac{\pi}{3}j}\right)^{k}}{1+e^{\frac{\pi}{3}j}}+\frac{1-\left(-e^{-\frac{\pi}{3}j}\right)^{k}}{1+e^{-\frac{\pi}{3}j}}\right]\varepsilon(k)-\frac{1}{2\sqrt{3}j}\left[\frac{1-\left(-e^{\frac{\pi}{3}j}\right)^{k}}{1+e^{\frac{\pi}{3}j}}-\frac{1-\left(-e^{-\frac{\pi}{3}j}\right)^{k}}{1+e^{-\frac{\pi}{3}j}}\right]\varepsilon(k)$$

化简可得：

$$y_{zs}(k)=\frac{\varepsilon(k)}{3}-\frac{(-1)^k\varepsilon(k)}{3}\left[cos\frac{k\pi}{3}+cos\frac{(k-1)\pi}{3}\right]-\frac{(-1)^k\varepsilon(k)}{3\sqrt{3}}[sin\frac{k\pi}{3}+sin\frac{(k-1)\pi}{3}]$$

由$H(S)$的极点可知，$y_{zi}(k)=[C_1\left(-e^{\frac{\pi}{3}j}\right)^k+C_2\left(-e^{-\frac{\pi}{3}j}\right)^k]\varepsilon(k)$

$y_{zi}(0)=C_1+C_2=1$，$y_{zi}(1)=-e^{\frac{\pi}{3}j}C_1-e^{-\frac{\pi}{3}j}C_2=2$

解得$C_1=\frac{j}{\sqrt{3}}\left(2+e^{-\frac{\pi}{3}j}\right),C_2=-\frac{j}{\sqrt{3}}\left(2+e^{\frac{\pi}{3}j}\right)$

$$\therefore y_{zi}(k)=\left[\frac{j}{\sqrt{3}}\left(2+e^{-\frac{\pi}{3}j}\right)\left(-e^{\frac{\pi}{3}j}\right)^k-\frac{j}{\sqrt{3}}\left(2+e^{\frac{\pi}{3}j}\right)\left(-e^{-\frac{\pi}{3}j}\right)^k\right]\varepsilon(k)=-\frac{2}{\sqrt{3}}(-1)^k[2sin\frac{k\pi}{3}k+sin\frac{(k-1)\pi}{3}]\varepsilon(k)$$

$y(k)=y_{zs}(k)+y_{zi}(k)$

$$=\frac{\varepsilon(k)}{3}-\frac{(-1)^k\varepsilon(k)}{3}\left[cos\frac{k\pi}{3}+cos\frac{(k-1)\pi}{3}\right]-\frac{(-1)^k\varepsilon(k)}{3\sqrt{3}}[sin\frac{k\pi}{3}k+sin\frac{(k-1)\pi}{3}]-\frac{2}{\sqrt{3}}(-1)^k[2sin\frac{k\pi}{3}k+sin\frac{(k-1)\pi}{3}]\varepsilon(k)$$

$$=\frac{\varepsilon(k)}{3}-\frac{(-1)^k\varepsilon(k)}{3}\left[cos\frac{k\pi}{3}+cos\frac{(k-1)\pi}{3}\right]-\frac{13}{3\sqrt{3}}(-1)^k sin\frac{k\pi}{3}\varepsilon(k)-\frac{7}{3\sqrt{3}}(-1)^k sin\frac{(k-1)\pi}{3}\varepsilon(k)$$

（2） $y(0)=\frac{1}{3}-\frac{1}{3}(1+\frac{1}{2})+\frac{7}{3\sqrt{3}}\times\frac{\sqrt{3}}{2}=1$=$y_{zi}(0)$，$y_{zs}(0)=0$

$y(1)=\frac{1}{3}+\frac{1}{3}(\frac{1}{2}+1)+\frac{13}{3\sqrt{3}}\times\frac{\sqrt{3}}{2}=3$，$y_{zs}(1)=1$

当 k=0，1 时全响应值和给定的初始条件值不同的是因为：

该系统的全响应分为零输入响应和零状态响应两部分，而零状态响应是指仅由外加的激励源引起的响应；而该系统的激励为单位阶跃信号，k<0 时，该激励为 0；k>0 时，激励为 1。因此当 k=0 时，零状态响应仍为 0，而 k=1 时零状态响应为 1，所以当 k=0，1 时全响应值和给定的初始条件值不同。

7. 30［15分］

设第 k 次弹起的高度为$y(k)$，则$y(0)=10$, 由题意可得差分方程:

$$y(k)=\frac{3}{4}y(k-1) => y(k)-\frac{3}{4}y(k-1)=0$$

则特征方程为$\lambda-\frac{3}{4}=0, \therefore y(k)=C\left(\frac{3}{4}\right)^k\varepsilon(k)$

由$y(0)=10$,可得$C=10$, $\therefore y(k)=10\times\left(\frac{3}{4}\right)^k\varepsilon(k)$

第 5 次弹起的高度: $y(5)=10\times\left(\frac{3}{4}\right)^5\varepsilon(5)=2.373(m)$

第 8 次弹起的高度: $y(8)=10\times\left(\frac{3}{4}\right)^8\varepsilon(8)=1.001(m)$