3.14［15 分］

（1）$f(t)=\frac{\sin(2\pi(t-2))}{\pi(t-2)}=2Sa(2\pi(t-2))$

$\because \varepsilon\left(t+\frac{\tau}{2}\right)-\varepsilon\left(t-\frac{\tau}{2}\right)\leftrightarrow \tau Sa(\frac{\tau\omega}{2})$

$\therefore$ 当$\tau=4\pi$，$\varepsilon(t+2\pi)-\varepsilon(t-2\pi)\leftrightarrow 4\pi Sa(2\pi\omega)$

由对称特性$F(t)\leftrightarrow 2\pi f(-\omega)$，且为实偶函数 $\therefore F(t)\leftrightarrow 2\pi f(\omega)$

$\therefore 4\pi Sa(2\pi t)\ \leftrightarrow 2\pi[\varepsilon(\omega+2\pi)-\varepsilon(\omega-2\pi)]$

$2Sa(2\pi t)\ \leftrightarrow \varepsilon(\omega+2\pi)-\varepsilon(\omega-2\pi)$

由延时特性$f(t-t_0)\leftrightarrow F(j\omega)e^{-j\omega t_0}$

$\therefore 2Sa(2\pi(t-2))\ \leftrightarrow [\varepsilon(\omega+2\pi)-\varepsilon(\omega-2\pi)]\ e^{-2j\omega}$

（2）$f(t)=\frac{2a}{a^2+\omega^2}$

$\because e^{-a|t|}\leftrightarrow \frac{2a}{a^2+\omega^2}$，由对称特性$F(t)\leftrightarrow 2\pi f(-\omega)$，且为实偶函数 $\therefore F(t)\leftrightarrow 2\pi f(\omega)$

$\therefore\ \frac{2a}{a^2+t^2}\leftrightarrow 2\pi e^{-a|\omega|}$

（3）$f(t)=\left[\frac{\sin(2\pi t)}{2\pi t}\right]^2=Sa^2(2\pi t)$

法一:

$\varepsilon\left(t+\frac{\tau}{2}\right)-\varepsilon\left(t-\frac{\tau}{2}\right)\leftrightarrow \tau Sa(\frac{\tau\omega}{2})$

当$\tau=4\pi$时，$\varepsilon(t+2\pi)-\varepsilon(t-2\pi)\leftrightarrow 4\pi Sa(2\pi\omega)$

由对称特性$F(t)\leftrightarrow 2\pi f(-\omega)$，且为实偶函数 $\therefore F(t)\leftrightarrow 2\pi f(\omega)$

可得$4\pi Sa(2\pi t)\leftrightarrow 2\pi[\varepsilon(\omega+2\pi)-\varepsilon(\omega-2\pi)]\ \therefore Sa(2\pi t)\leftrightarrow \frac{1}{2}[\varepsilon(\omega+2\pi)-\varepsilon(\omega-2\pi)]$

由频域卷积定理$f_1(t)f_2(t)\leftrightarrow \frac{1}{2\pi}[F_1(j\omega)*F_2(j\omega)]$

$Sa^2(2\pi t)\leftrightarrow \frac{1}{8\pi}[\varepsilon(\omega+2\pi)-\varepsilon(\omega-2\pi)]*[\varepsilon(\omega+2\pi)-\varepsilon(\omega-2\pi)]$

$=\frac{1}{8\pi}[\varepsilon(\omega+2\pi)*\varepsilon(\omega+2\pi)-2\varepsilon(\omega+2\pi)*\varepsilon(\omega-2\pi)+\varepsilon(\omega-2\pi)*\varepsilon(\omega-2\pi)]$

$=\frac{1}{8\pi}[(\omega-4\pi)\varepsilon(\omega-4\pi)-2\omega\varepsilon(\omega)+(\omega+4\pi)\varepsilon(\omega+4\pi)]$

当$\omega\le-4\pi$时，上式=0

当$-4\pi<\omega\le 0$时，上式$=\frac{1}{8\pi}(\omega+4\pi)=\frac{1}{2}-\frac{|\omega|}{8\pi}$

当$0<\omega\le 4\pi$时，上式$=\frac{1}{8\pi}(\omega+4\pi-2\omega)=\frac{1}{2}-\frac{|\omega|}{8\pi}$

当$4\pi<\omega$时，上式$=\frac{1}{8\pi}(\omega-4\pi-2\omega+\omega+4\pi)=0$

综上$[Sa(2\pi t)]^2\leftrightarrow\begin{cases}\frac{1}{2}-\frac{|\omega|}{8\pi}) & |\omega|\le\ 4\pi\\ 0 & |\omega|>4\pi\end{cases}$

法二:

$\because g(t)=\begin{cases}1-\frac{|t|}{\tau} & |t|\le \tau \\ 0 & |t|>\tau\end{cases} \leftrightarrow \tau\left[Sa\left(\frac{\omega\tau}{2}\right)\right]^2$

当$\tau=4\pi$时，$g(t)=\begin{cases}1-\frac{|t|}{4\pi} & |t|\le 4\pi \\ 0 & |t|>4\pi\end{cases} \leftrightarrow 4\pi[Sa(2\pi\omega)]^2$

由对称特性$F(t)\leftrightarrow 2\pi f(-\omega)$，且为实偶函数 $\therefore F(t)\leftrightarrow 2\pi f(\omega)$

$\therefore\ 4\pi[Sa(2\pi t)]^2 \leftrightarrow 2\pi g(\omega)=\begin{cases}2\pi(1-\frac{|\omega|}{4\pi}) & |\omega|\le 4\pi \\ 0 & |\omega|>4\pi\end{cases}$

$\therefore\ [Sa(2\pi t)]^2 \leftrightarrow \frac{1}{2}g(\omega)=\begin{cases}\frac{1}{2}-\frac{|\omega|}{8\pi}) & |\omega|\le 4\pi \\ 0 & |\omega|>4\pi\end{cases}$

3.16a［15 分］

解：(a)图

$G_2(t)=\varepsilon(t+1)-\varepsilon(t-1)\quad G_2(t)\leftrightarrow 2Sa(\omega)$

$\therefore f(t)=\ G_2(t-2)+G_2(t+2)$

(1) 由延时特性$f(t-t_0)\leftrightarrow F(j\omega)e^{-j\omega t_0}$

$G_2(t-2)\leftrightarrow 2Sa(\omega)\,e^{-2j\omega}\qquad G_2(t+2)\leftrightarrow 2Sa(\omega)\,e^{2j\omega}$

$\therefore f(t)\leftrightarrow\ 2Sa(\omega)e^{-2j\omega}+2Sa(\omega)e^{2j\omega}$

根据欧拉公式$e^{jx}=cosx+jsinx$

可得$f(t)\leftrightarrow 4Sa(\omega)cos2\omega$

(2)$f(t)=G_2(t-2)+G_2(t+2)=\ \varepsilon(t-1)-\varepsilon(t-3)+\varepsilon(t+3)-\varepsilon(t+1)$

$f'(t)=\delta(t-1)-\delta(t-3)+\delta(t+3)-\delta(t+1)$

$\delta(t)\leftrightarrow 1$，由延时特性$f(t-t_0)\leftrightarrow F(j\omega)e^{-j\omega t_0}, \delta(t-1)\leftrightarrow e^{-j\omega}$

$\therefore f'(t)\leftrightarrow e^{-j\omega}-e^{-3j\omega}+e^{3j\omega}\text{-}e^{j\omega}$

根据欧拉公式$e^{jx}=cosx+jsinx$, $f'(t)\leftrightarrow\ 2j(sin3\omega-sin\omega)$

由积分特性, $f(t)=\int_{-\infty}^{t}f'(\tau)d\tau\leftrightarrow\frac{2}{\omega}(sin3\omega-sin\omega)$

3.24［20 分］

解：根据图像，可得当$0\le\ t<\frac{T}{2}$时，$i(t)=\frac{2I}{T}t$，当$\frac{T}{2}\ \le\ t<T$时，$i(t)=-\frac{2I}{T}t+2I$

（1）平均电流 $\bar{i}\ =\frac{1}{T}\int_0^T i(t)dt=\frac{1}{T}\int_0^{\frac{T}{2}}i(t)dt+\frac{1}{T}\int_{\frac{T}{2}}^{T}i(t)dt=\frac{I}{2}\mathrm{T}$

方均值 $\overline{i^2}=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}i^2(t)\,dt=\frac{2}{T}\int_0^{\frac{T}{2}}i^2(t)dt=\frac{1}{4}\int_0^4 i^2(t)dt=\frac{1}{4}\int_0^4\left(\frac{I}{4}t\right)^2dt=\frac{I^2}{3}\quad\therefore\bar{i}=\frac{\sqrt{3}}{3}I$

（2）平均功率 $\overline{P}=i^2=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}i^2(t)\,dt=\frac{I^2}{3}$

直流功率 $P_1=\left(\frac{a_0}{2}\right)^2=\left(\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}i(t)\,dt\right)^2=\frac{I^2}{4}$

交流功率 $P_2=\frac{1}{2}\sum_{n=1}^{+\infty}A_n^2=\frac{1}{2}\sum_{n=1}^{+\infty}(a_n^2+b_n^2)$

由于该电流为偶函数，所以 $b_n=0$

$P_2=\frac{1}{2}\sum_{n=1}^{+\infty}\left(\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}i(t)\,cosn\omega tdt\right)^2=\frac{1}{2}\sum_{n=1}^{+\infty}\left(\frac{4}{T}\int_0^{\frac{T}{2}}\frac{2I}{T}t\,cosn\omega tdt\right)^2$

$=\frac{32I^2}{T^4}\sum_{n=1}^{+\infty}\left(\int_0^{\frac{T}{2}}t\,cos\frac{2n\pi}{T}\ tdt\right)^2=\frac{1}{12}I^2$

$\because\overline{P}=P_1+P_2\ \therefore$符合帕塞瓦尔定理

4.1［30 分］

解：当$0 \le t \le 2$ 时，$e(t) = \begin{cases} Asin(\pi t), & 0 \le t < 1 \\ -Asin(\pi t), & 1 \le t \le 2 \end{cases}$，$T = 2$，　$\Omega = \frac{2\pi}{T} = \pi$

$e(t) = \frac{a_0}{2} + \sum_n^{+\infty}(a_n cosn\pi t + b_n sinn\pi t)$

$\because e(t)$为偶函数，　$\therefore b_n = 0$

$a_0 = \frac{2}{T}\int_0^T e(t)dt = \int_0^1 Asin(\pi t)dt + \int_1^2 -Asin(\pi t)dt = \frac{4A}{\pi}$

$a_n = \frac{2}{T}\int_0^T e(t)cosn\pi tdt = 2\int_0^1 Asin\pi tcosn\pi tdt$

使用积化和差公式，可算出$a_n = \begin{cases} \frac{4A}{(1-n^2)\pi}, n\text{为偶数} \\ 0, \quad n\text{为奇数} \end{cases}$

$\therefore e(t) = \frac{2A}{\pi} - \frac{4A}{3\pi}cos2\pi t - \frac{4A}{15\pi}cos4\pi t \ldots\ldots$

$H(n\Omega) = H(j\omega) = \frac{\frac{1}{jn\pi C}}{\frac{1}{jn\pi C}+R} = \frac{\frac{1}{jn\pi}}{\frac{1}{jn\pi}+1} = \frac{1}{1+n\pi j} = \frac{1-n\pi j}{1+n^2\pi^2} = \frac{1}{\sqrt{1+n^2\pi^2}}\angle \arctan(-n\pi)$

即幅度为$\frac{1}{\sqrt{1+n^2\pi^2}}$，相位为 $\arctan(-n\pi)$

输出直流分量$R_0=\frac{2A}{\pi}H(0)=\frac{2A}{\pi}$

输出 n 次谐波分量 $R_n = a_n H(n\Omega) = \frac{1}{\sqrt{1+n^2\pi^2}}\frac{4A}{(1-n^2)\pi}\angle n\pi t - \arctan(n\pi)$。

$r(t) = \frac{2A}{\pi} - \sum_n^{+\infty}\frac{1}{\sqrt{1+n^2\pi^2}}\frac{4A}{(1-n^2)\pi}\cos(n\pi t - \arctan(n\pi))$

不为零的前三个分量：

直流分量 $\frac{2A}{\pi}$

基波分量 $\frac{-4A}{3\pi\sqrt{1+4\pi^2}}\cos(2\pi t - \arctan(2\pi))$

二次谐波分量 $\frac{-4A}{15\pi\sqrt{1+16\pi^2}}\cos(4\pi t - \arctan(4\pi))$

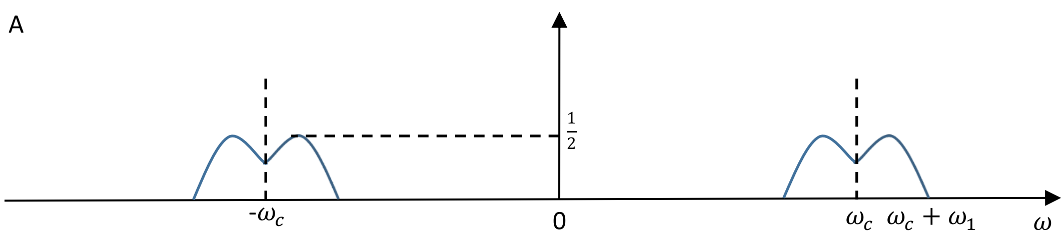
4.6［20 分］

解：

1、$e(t) \times cos\omega_c t$

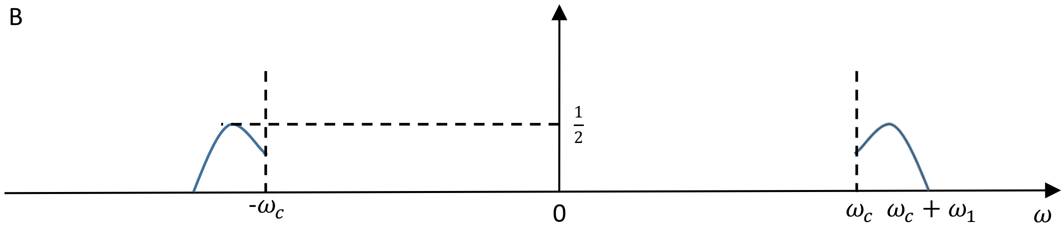
根据频域卷积定理$f_1(t)f_2(t) \leftrightarrow \frac{1}{2\pi}[F_1(j\omega) * F_2(j\omega)]$可知，

$$e(t) \times cos\omega_c t \leftrightarrow \frac{1}{2\pi}\{E(j\omega) * \pi[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)]\} = \frac{1}{2}[E(j(\omega+\omega_c)) + E(j(\omega-\omega_c)]$$



2、理想高通滤波器的截止频率为ω_c，因此只有$|\omega| > \omega_c$才能通过

B



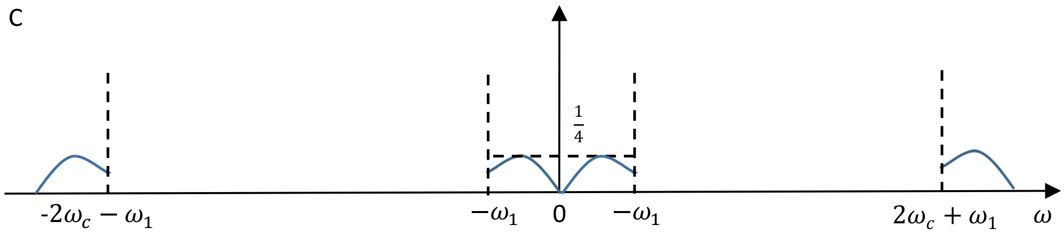
3、$e_b(t) \times \cos[(\omega_c + \omega_1)t]$

根据频域卷积定理$f_1(t)f_2(t) \leftrightarrow \frac{1}{2\pi}[F_1(j\omega) * F_2(j\omega)]$可知，

$$e_b(t) \times \cos[(\omega_c + \omega_1)t] \leftrightarrow \frac{1}{2\pi}\{E_b(j\omega) * \pi[\delta(\omega + \omega_c + \omega_1) + \delta(\omega - \omega_c - \omega_1)]\}$$

$$= \frac{1}{2}[E_b\big(j(\omega + \omega_c + \omega_1)\big) + E_b(j(\omega - \omega_c - \omega_1)]$$

C



4、理想低通滤波器的截止频率为ω_c，因此只有$|\omega| < \omega_c$才能通过

D