Assignment 5 - Epipolar Geometry

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1 Introduction

In this assignment we find the fundamental matrix and matched points. We using the Harris/Hessian Affine feature point detector. Match these using SIFT feature descriptors. From the matched points the fundamental matrix can be estimated. To acount for outliers we use RANSAC to optimize the estimation of the fundamental matrix. Finally we show the results by ploting the epipolar lines defined by the fundamental matrix.

2 Finding the Fundamental matrix

We found salient points in the two consecutive images with the Harris/Hessian Affine feature point detector. We used the given code[1]. From this code next to the locations also the descriptors are given for each point. After that we match the points in the two images with the sift matching from the vl-toolbox[2]. We use the location of the matched points between two images as input to find the fundamental Matrix F. First the points are normalized with the following matrix:

$$T = \begin{bmatrix} \frac{\sqrt{2}}{d} & 0 & -m_x \frac{\sqrt{2}}{d} \\ 0 & \frac{\sqrt{2}}{d} & -m_y \frac{\sqrt{2}}{d} \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where $m_{\text{dimension}}$ is the mean of the points in a certain dimension and d is the average distance from the mean. This matrix is a combination of translating the mean of the points to the origin and scaling them to have a average distance to the mean of $\sqrt{2}$.

After that we find the singular value decomposition of the matrix A in the eqaution $A * F = 0$ with

$$A = \begin{bmatrix} x_1x'_1 & x_1y'_1 & x_1 & y_1x'_1 & y_1y'_1 & y_1 & x'_1 & y'_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_nx'_n & x_ny'_n & x_n & y_nx'_n & y_ny'_n & y_n & x'_n & y'_n & 1 \end{bmatrix} \tag{2}$$

[1] http://www.robots.ox.ac.uk/ vgg/research/affine/detectors.html
[2] http://www.vlfeat.org/

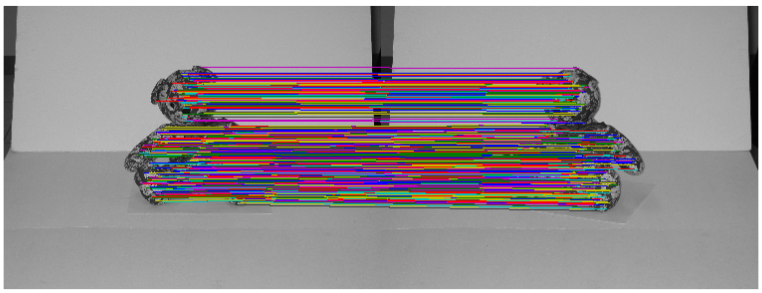

Figure 1: Matches after applying RANSAC

where (x_1, y_1) is the location of the first point in the first image and (x'_1, y'_1) in the matched point in the second image. The fundamental matrix will be the last column in the matrix V of the singular value decomposition $A = UDV^T$, which belongs to the smalles singular value in the matrix D. This matrix F is not always singular, so we do again a singular value decomposition, but this time on the matrix F. In the resulting diagonal matrix we set the smallest singular value to zero and calculate the fundamental matrix with $F = UD'V^T$, where D' is the adjusted diagonal matrix. So that the rank of F is now 2. Finally F is denormalized using T.

3 RANSAC to find best matrix

To find the correct fundamental matrix between two images, we use the RANSAC algorithm. We want to find the matrix where the most points are close to the contstraint $p1 * F * p2 = 0$.In every step of RANSAC we choose random 20 points and calculate the fundamental Matrix F. After that we calculate the Sampson distance D for every point and reject the points where $D > 1$. Finally from the inliers found by RANSAC, F is reëstimated.
The resulting matches for the first two images are shown in figure 1

4 Epipolar lines

To check the fundamental matrix estimation we use the fundamental matrix to plot the estimated epipolar lines. The epipolar line can be thought of as the projection of the line on which the point in the other image could have originated from. The fundamental matrix gives this. This is shown in the

following derivation:

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$$ax' + by' + c = 0 \tag{3}$$

The epipolar lines are plotted in figure 2 using equation 3. The figure shows that the epipolar lines of matched points coincide with the matched points.

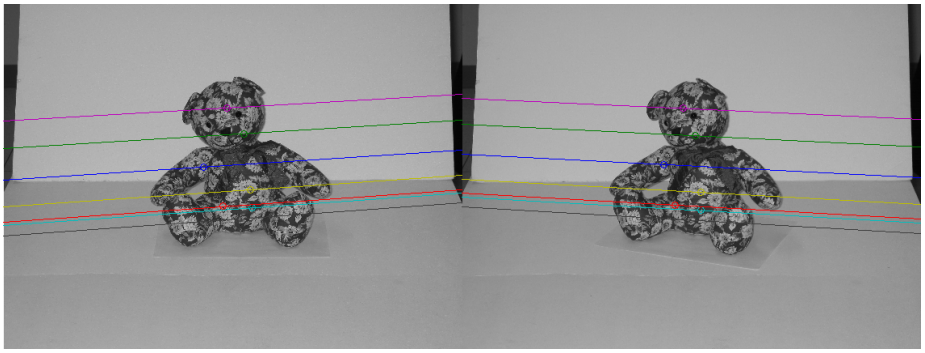

Figure 2: Seven of the matches used for estimating the fundamental matrix linked in the image along with the epipolar lines of these points