2022년 2학기
계량경제학연구
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Problem Set #2

- Due date/time: 11월 15일 23:59까지 eTL로 pdf파일 제출

교과서 연습문제

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Exercise 7.2 Take the model $Y = X'\beta + e$ with $\mathbb{E}[Xe] = 0$. Define the **ridge regression** estimator

$$\widehat{\beta} = \left(\sum_{i=1}^{n} X_i X_i' + \lambda \boldsymbol{I}_k\right)^{-1} \left(\sum_{i=1}^{n} X_i Y_i\right) \tag{7.43}$$

here $\lambda > 0$ is a fixed constant. Find the probability limit of $\widehat{\beta}$ as $n \to \infty$. Is $\widehat{\beta}$ consistent for β?

Exercise 7.7 Of the variables (Y^*, Y, X) only the pair (Y, X) are observed. In this case we say that Y^* is a **latent variable**. Suppose

$$\begin{aligned} Y^* &= X'\beta + e \\ \mathbb{E}[Xe] &= 0 \\ Y &= Y^* + u \end{aligned}$$

where u is a measurement error satisfying

$$\begin{aligned} \mathbb{E}[Xu] &= 0 \\ \mathbb{E}\left[Y^* u\right] &= 0. \end{aligned}$$

Let $\widehat{\beta}$ denote the OLS coefficient from the regression of Y on X.

(a) Is β the coefficient from the linear projection of Y on X?

(b) Is $\widehat{\beta}$ consistent for β as $n \to \infty$?

(c) Find the asymptotic distribution of $\sqrt{n}\left(\widehat{\beta} - \beta\right)$ as $n \to \infty$.

Exercise 7.17 An economist reports a set of parameter estimates, including the coefficient estimates $\widehat{\beta}_1 = 1.0$, $\widehat{\beta}_2 = 0.8$, and standard errors $s(\widehat{\beta}_1) = 0.07$ and $s(\widehat{\beta}_2) = 0.07$. The author writes "The estimates show that β_1 is larger than β_2."

(a) Write down the formula for an asymptotic 95% confidence interval for $\theta = \beta_1 - \beta_2$, expressed as a function of $\widehat{\beta}_1$, $\widehat{\beta}_2$, $s(\widehat{\beta}_1)$, $s(\widehat{\beta}_2)$ and $\widehat{\rho}$, where $\widehat{\rho}$ is the estimated correlation between $\widehat{\beta}_1$ and $\widehat{\beta}_2$.

(b) Can $\widehat{\rho}$ be calculated from the reported information?

(c) Is the author correct? Does the reported information support the author's claim?

Exercise 7.28 As in Exercise 3.26, use the `cps09mar` dataset and the subsample of white male Hispanics. Estimate the regression

$$\widehat{\log(wage)} = \beta_1\, education + \beta_2\, experience + \beta_3\, experience^2/100 + \beta_4.$$

(a) Report the coefficient estimates and robust standard errors.

(b) Let θ be the ratio of the return to one year of education to the return to one year of experience for *experience*= 10. Write θ as a function of the regression coefficients and variables. Compute $\widehat{\theta}$ from the estimated model.

(c) Write out the formula for the asymptotic standard error for $\widehat{\theta}$ as a function of the covariance matrix for $\widehat{\beta}$. Compute $s(\widehat{\theta})$ from the estimated model.

(d) Construct a 90% asymptotic confidence interval for θ from the estimated model.

(e) Compute the regression function at *education*= 12 and *experience*= 20. Compute a 95% confidence interval for the regression function at this point.

(f) Consider an out-of-sample individual with 16 years of education and 5 years experience. Construct an 80% forecast interval for their log wage and wage. [To obtain the forecast interval for the wage, apply the exponential function to both endpoints.]

Exercise 9.3 Let T be a t-statistic for $\mathbb{H}_0 : \theta = 0$ versus $\mathbb{H}_1 : \theta \neq 0$. Since $|T| \to_d |Z|$ under $\mathbb{H}_0$, someone suggests the test "Reject $\mathbb{H}_0$ if $|T| < c_1$ or $|T| > c_2$, where c_1 is the $\alpha/2$ quantile of $|Z|$ and c_2 is the $1-\alpha/2$ quantile of $|Z|$.

(a) Show that the asymptotic size of the test is α.

(b) Is this a good test of $\mathbb{H}_0$ versus $\mathbb{H}_1$? Why or why not?

Exercise 9.9 Suppose a researcher uses one dataset to test a specific hypothesis $\mathbb{H}_0$ against $\mathbb{H}_1$ and finds that he can reject $\mathbb{H}_0$. A second researcher gathers a similar but independent dataset, uses similar methods and finds that she cannot reject $\mathbb{H}_0$. How should we (as interested professionals) interpret these mixed results?

Exercise 9.25 The data set `Invest1993` on the textbook website contains data on 1962 U.S. firms extracted from Compustat, assembled by Bronwyn Hall, and used in Hall and Hall (1993).

The variables we use in this exercise are in the table below. The flow variables are annual sums. The stock variables are beginning of year.

	year	year of the observation
I	inva	Investment to Capital Ratio
Q	vala	Total Market Value to Asset Ratio (Tobin's Q)
C	cfa	Cash Flow to Asset Ratio
D	debta	Long Term Debt to Asset Ratio

(a) Extract the sub-sample of observations for 1987. There should be 1028 observations. Estimate a linear regression of I (investment to capital ratio) on the other variables. Calculate appropriate standard errors.

(b) Calculate asymptotic confidence intervals for the coefficients.

(c) This regression is related to Tobin's q theory of investment, which suggests that investment should be predicted solely by Q (Tobin's Q). This theory predicts that the coefficient on Q should be positive and the others should be zero. Test the joint hypothesis that the coefficients on cash flow (C) and debt (D) are zero. Test the hypothesis that the coefficient on Q is zero. Are the results consistent with the predictions of the theory?

(d) Now try a nonlinear (quadratic) specification. Regress I on Q, C, D, Q^2, C^2, D^2, $Q \times C$, $Q \times D$, $C \times D$. Test the joint hypothesis that the six interaction and quadratic coefficients are zero.

Exercise 10.30 In Exercise 7.28 you estimated a wage regression with the `cps09mar` dataset and the subsample of white Male Hispanics. Further restrict the sample to those never-married and live in the Midwest region. (This sample has 99 observations.) As in subquestion (b) let θ be the ratio of the return to one year of education to the return of one year of experience.

(a) Estimate θ and report standard errors calculated by asymptotic, jackknife and the bootstrap.

(b) Explain the discrepancy between the standard errors.

(c) Report confidence intervals for θ using the BC percentile method.