Structural Estimation HW6

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Part 1

Problem 1

Two reasons why $\frac{1}{1+e^{-f}}$ is easier for R to evaluate than $\frac{e^f}{e^f+1}$. First, suppose $f \geq 0$ (although it may not be necessary, actually). Then, the first fraction does not explode a sense that as $f \to \infty$, $\frac{1}{1+e^{-f}} \to 1$; in contrast, as $f \to \infty$, $\frac{e^f}{e^f+1} \to \frac{\infty}{\infty}$, which is not well-defined. Also, executing a function, namely the exp() function, once is faster than doing so twice.

Problem 2

It has to do with the numerical calculation methods of R. R can deal with a very small number with high precision, but once one adds a number of some higher order, R loses that precision due to rounding. Thus, if $p \ll 1$ ($p \approx 0$), we would like to calculate and store its complement separately to maintain the precision. Otherwise, the sum will not add up to 1.

Problem 3

We know that

$$p_1(x,s) = \frac{1}{1+\exp\left(v_2(x,s) - v_1(x,s)\right)},$$

where

$$v_j(x,s) = \begin{cases} \beta V(0,s), & j=1 \\ \theta_1 s - \theta_2 x + \beta V(x+1,s), & j=2. \end{cases}$$

Since s is unobserved, and we want $p_1(x)$, let us first try to take the smaller of $p_1(x,s)$ among all s's by redefining $s := \max\{s_1, s_2, ..., s_n\}$, so that the minimum of $p_1(x,s)$ is still larger than $1-\epsilon$. Now, since v is non-increasing in x, we get

$$\begin{aligned} p_1(x) &= \frac{1}{1+\exp\left(\theta_1 s - \theta_2 x + \beta V(x+1,s) - \beta V(0,s)\right)} \\ &= \frac{1}{1+\exp\left(\theta_1 s - \theta_2 x + \beta\left(\ln(p_1(0,s)) - \ln(p_1(x+1,s))\right)\right)} \\ &\geq \frac{1}{1+\exp\left(\theta_1 s - \theta_2 x\right)} \\ &\geq 1-\epsilon, \end{aligned}$$

where the first inequality follows from the fact that p_1 is weakly increasing in x. Rearranging the last inequality, we get

$$\begin{aligned}\theta_2 x &\geq \theta_1 s - \ln\left(\frac{1}{1-\epsilon} - 1\right), \\ x &\geq \frac{1}{\theta_2}\left(\theta_1 s - \ln\left(\frac{1}{1-\epsilon} - 1\right)\right).\end{aligned}$$