Structural Estimation HW8

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Part 1

Problem 1

First, look at the Figure given in the assignment sheet and consider a cost minimization, where the cost function here is $G_t(x)$. Without the fixed cost K, we can ensure that $G_t(x)$ is convex and find the minimum point. However, the presence of K does not guarantee that $G_t(x)$ is convex. Thus, we relax the convexity condition by introducing K-convexity. Intuitively, it ensures that we do not order if our cost is less than $G_t(x) + K$, where S is such that $G_t(S) = \min_x G_t(x)$. Moreover, we can find s s.t. for all $l < s$, $G_l > G_S + K$, which means that we would order; in particular, we would order up to S.

Problem 2

In his paper, Aguirregabiria states that the decision rules depend on the marginal conditions of optimality and optimal discrete choices, but the estimation in the paper relies on the latter only, not the former, thus not using insights that can be obtained from Euler equations. However, the author mentions that estimating Euler equations gave imprecise results and that the optimality of an (s, S) policy is not so much affected due to the endogeneity, perhaps the loss of information is mild.

Problem 3

We think so. So, we do not solve the entire dynamic programming or implement the policy iteration until convergence here; rather, we implement the policy implementation only once while using the "previous" CCP. Moreover, we can use the appropriate value function approximation and get the correct direction (similarly as drawn in the lecture video) instead of the nonparametric kernel and probability transition matrix originally used in the paper.

Part 2

Problem 1

$$\begin{aligned}
\tilde{V}_i &= \mathbb{E}_s\left[\bar{V}_{i,s}\right] \\
&= \mathbb{E}_s\left[\mathbb{E}_\epsilon V_{i,s,\epsilon}\right] \\
&= \mathbb{E}_s\left[\mathbb{E}_\epsilon\left[\max_q U_{i,s}(q) + \epsilon_q + \beta V_Q\right]\right] \\
&= \mathbb{E}_s\left[\mathbb{E}_\epsilon\left[U_{i,s}(q) + \epsilon_q - \log P(s|i) + \beta\tilde{V}_Q\right]\right] \\
&= \mathbb{E}_s\left[U_{i,s}(q) - \log P(s|i) + \beta\tilde{V}_Q\right] \\
&= \mathbb{E}_s\left[U_{i,s}(1) - \log P_{i,s}(1) + \beta\tilde{V}_Q\right] \\
&= f_i + \beta\tilde{V}_Q.
\end{aligned}$$

We can also add γ from the fourth equality from the last.

Problem 2

If we do decide to order,

$$\begin{aligned}
\bar{v}_{i,s}(1) &= U_{i,s}(1) + \beta\tilde{V}_Q \\
&= U_{i,s}(1) + \beta f_Q + \beta^2\tilde{V}_Q
\end{aligned}$$

Now, if we do not,

$$\begin{aligned}
\bar{v}_{i,s}(0) &= U_{i,s}(0) + \beta\tilde{V}_{i-s} \\
&= U_{i,s}(0) + \beta f_{i-s} + \beta^2\tilde{V}_Q.
\end{aligned}$$

So, we can write $P_{i,s}$ as follows:

$$\begin{aligned}
P_{i,s} &= \frac{\exp(v_{i,s}(1))}{\sum_q \exp(v_{i,s}(q))} \\
&= \frac{\exp(U_{i,s}(1) + \beta f_Q + \beta^2\tilde{V}_Q)}{\exp(U_{i,s}(1) + \beta f_Q + \beta^2\tilde{V}_Q + U_{i,s}(0) + \beta f_{i-s} + \beta^2\tilde{V}_Q)}.
\end{aligned}$$

Problem 3

Note that we can observe the inventory and demand, meaning that we can obtain an empirical distribution of the demand. Using MLE, we can estimate the parameters from which we can find the critical fractile with the aforementioned empirical distribution. Then, we can proceed to estimate the fixed costs (shipping costs here) by assuming that the agents order the optimal quantity and observing the inventory level and sales, which are observable to us.