

Vectors are often interpreted in two main ways.

They can be thought of as a geometric object. An arrow with a tail and a head, that points in a certain direction. A geometric vector also has a magnitude correlated with its length. Scaling it changes the magnitude, and rotating it changes the direction.

An alternative view is considering a vector to be a numerical list. This notion is more popular in a computational setting. A vector is an object that holds data in each coordinate, a setup that is convenient for calculations.

We can combine these ideas and plot vectors on a coordinate plane, where the information a vector holds directly corresponds to its length and direction.

Vector arithmetic follows two rules called the superposition principles. Vectors can be multiplied by any real number, which will scale its components. The set of all possible scalings of a vector is called its span, which is a fancy way of saying it's a line.

Vectors can also be added or subtracted to create a new vector.

Let's take a look at what these principles mean geometrically.

You'll notice that when you use these rules on two vectors that don't fall on the same line, you can construct any vector you want in 2D as long as you stretch and add them in the right way.

In nerd speak, a vector that has been created by utilizing the superposition principles is called a linear combination.

As a formality, we often define basis vectors, the most simple vectors possible in a coordinate plane, which we use to construct any other vector we want. Taking all these ideas together, we now have a vector space!

Something fun to note is that there is nothing special about the basis vectors. They can have any length or direction; so long as they aren't multiples of each other, I can construct any vector I'd like.

Let's take a look at some fun vector operations.

The dot product is computed by summing the products of the x and y components of two vectors. The dot product finds the magnitude of the vector that falls from the shadow of the first vector onto the second. We call this the projection.

You can think of it like a work integral. When calculating a work integral you consider the force that is applied in the direction of work. If you are pushing a box on a flat surface, you will move the box more if you push in line with the floor, whereas if you push up, you will move the box

nowhere. The dot product works in the same way, the smaller the angle is between two vectors, the greater the dot product. If there is a 90 degree angle between the vectors, the dot product will be zero.

The final operation is the cross product. In 2D, the cross product generates something called “the determinant”, a measurement of the area of the parallelogram generated between two vectors. In 3D, the cross product gives the components of a vector that is perpendicular to the two vectors you apply the operation to.

Before I learned Linear Algebra, I had the impression that it was a boring subject that is only useful for brute force computations. In reality, it is a field that is rich and complex, I suggest you take the time to learn more about it. A fluency in linear algebra will allow you to navigate some of the most fascinating topics, including Quantum Mechanics, Differential Equations, Classical Mechanics, and so much more. Unfortunately that’s all the time I have, thanks for watching!