

# Decision Trees

# 3

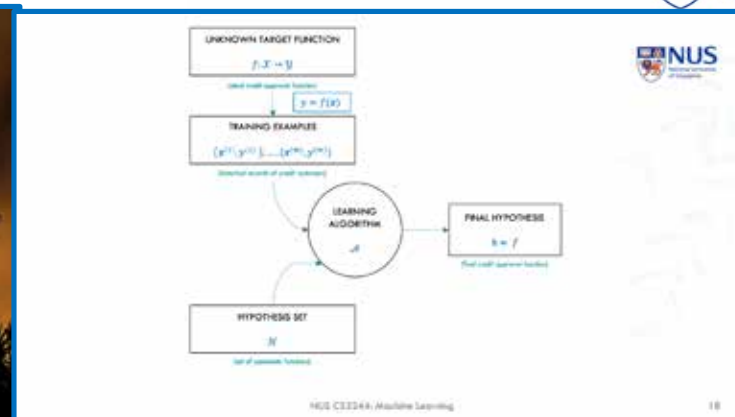
# A

**CS 3244**  
**Machine Learning**



**NUS** | Computing

# Recap from Week 02



## Futility of Bias Free Learning

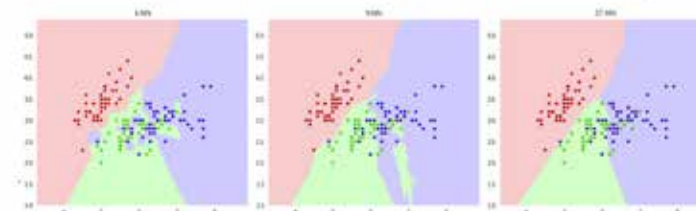
A learner that makes no *a priori* assumptions on  $f$  has *no rational basis* for classifying any unseen instances.

Making prior assumptions – **Inductive Bias** – is the only way to make learning feasible.

Each family (“tribe”) of models  $\mathcal{A}$  has a different take on its inductive bias, which forms of  $\mathcal{H}$  it can represent:  $\mathcal{H}$ .

## Effect of $k$

Smaller  $k$ : complex surface.  
Larger  $k$ : smoother surface.



# Forecast for Week 03



Learning Outcomes for this week:

- Understand and implement Decision Trees;
- Master Information Gain as a means of choosing optimal features for decisions;
- Formalize cost functions as a means of evaluating a supervised learner's performance;
- Describe simple ensembling methods that integrate multiple classifiers;
  - Understand Decision Trees as an instance of an ensemble classifier of conditional decision stamps;
- Reason about inductive bias of the decision tree algorithm

# Thursday: AMA and Datasets



Your subgroup may want to attend Thursday to get to talk to some of the TAs who know some of the curated datasets well.

We'll have breakout rooms for different curated datasets so you can check with the TAs what might be suitable projects, even before the proposal. 😎

# Python Tutorial



Our head TA Pranavan will be holding an optional

Numpy and Pandas Tutorial

Fri 26 Aug, 18:00–21:00 @ Seminar Room @ LT19

If you want to attend, please react on #tutorials. Note that it is likely physical only, not hybrid. First 45 🧑🧒 reacts get to go.





# Decision Trees

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# How do people make decisions?

Well, let's examine some recent choices you made:

*What module did you enroll in?*

*What did you eat today for lunch?*

*What did you wear today?*







Variables:

- 1, Rating and reviews of module
- 2, Urgency of clearing the module
- 3, Interest to this module
- 4, Timing of the module
- 5, Prerequisite of the module
- 6, Workload of the module
- 7, Professor of the module
- 8, Content of the module

Logic:

1. Check the suggested study plan and see which are the core modules that I need to take in this semester (top priority)
2. Look at the list of modules which can fulfil my graduation requirements and look through the module overview+reviews on NUSMods
3. Select the ones which I'm more interested in and see if there is a clash in timetable (selection of module based on variables listed above)
4. Repeat  if MogReg failed me 



Which module you enrolled in?

bool: prerequisites, time clash

num: workload, interest, teaching staff, necessity, vacancy

logic:

for module in sorted(module\_list, key=interest):

if  $(a1 * \text{workload} + a2 * \text{interest} + a3 * \text{teaching staff} + a4 * \text{necessity} + a5 * \text{vacancy}) * \text{prerequisites} * \text{time clash} >$

threshold:

enroll module

update workload & time clash





What lunch you ate today?

**Context:** I'm hungry; gotta eat. Could be at home, in school, during class etc

**Variables:** What I'm doing now, location, amount of food per order, queue/cooking duration, schedule, what I want to eat, what is available

**Logic:** I usually just choose randomly; have to save brain cells for studying/other stuff. Otherwise, usually cravings have the highest weightage provided I brought enough money.



What lunch should I eat today?

**Context:** Its lunch time and I need to make a decision about eating before its too late

**Variables:** Available stalls, price and variety at each stall, portion size, waiting time, free time available, hunger level, cravings, company

**Logic:** Use a backtracking depth-first search to find a valid solution for the above constraint solving problem based on the following order of variables - available stall -> cravings -> (free time - waiting time) -> (portion size - hunger level) -> price -> company



What lunch you ate today?

**Context and Variables:**

- Am I at home or in school?
- What do I feel like eating today?
- How convenient is it to get what I want?

**Logic:**

If I have no food preference,  
and If I am at home, I would cook something simple for myself.

Else If I am in school, I would go to the nearest canteen to eat.

Else, I would weigh how much I crave for that specific food against how convenient it is for me to get it



Which clothes you are wearing now?

Context: Will it be appropriate if I wear this clothing? Do I feel good when I am wearing it?

Variable: Weather, occasion, colour of pants, colour of shirt

Logic:

If the weather is sunny, I won't wear tops with long sleeves

If I am just going to lecture or doing informal activity, I will wear short pants at the top of the pile

Else if I am going to a formal event, I will wear any long pants at the top of the pile

Take formal tops that is on top of my stack of shirt

If the colour of the shirt fits my pants, I will wear it

Else, look at the shirt at the bottom of the previous shirt taken and loop through

Else if the weather is cold, wear shirt with long sleeves

Wear long pants on top of the pile

Take long sleeves shirt

If the colour of the shirt fits my pants, I will wear it

Else, look at the shirt at the bottom of the previous shirt taken and loop through



QN: Which clothes you are wearing now?

Variables:

- Activity I will be engaging in
- Clothing pieces and styles available based on inventory
- Convenience (time available..etc)

Logic: The event/activity I will be doing will help to determine the overall type of clothes I will wear, for instance: physical outdoor activities would call for sporty dri-fit attire, whereas, going out for a nice meal will require more formal and stylish clothes. In a similar manner, the clothing items I have available will also determine my dressing - where I can only pick outfit from pieces I already have. Additionally, convenience is another factor that must be considered, where the time I have available to curate an outfit will affect its quality! I believe that this can be best represented with the use of a decision tree.

# How do people make decisions?



In physical subgroups or Zoom breakout rooms, do:

(1 min): Introduce yourself! 🤔

(5 mins): *What's the inductive bias of our method?*

Is it like how  $k$  nearest neighbors decides?

Ask one member to write your collective answer to the [#lectures](#) thread. Upvote others that you like.

# How do people make decisions?



Would we say it's like how  $k$  nearest neighbors make a decision?

(i.e., looking at what you did for a similar item last time?)

No, not exactly. It doesn't make much sense in this instance. Instead, people often focus on just a 1 or 2 dimensions of the evidence.

Decision trees embody this paradigm.

# Programming

Imperative programming control flow also embodies this:

```
if test1:
    if test2:
        # block 1
    else:
        # block 2
else:
    if test3:
        # block 3
    else:
        # block 4
```

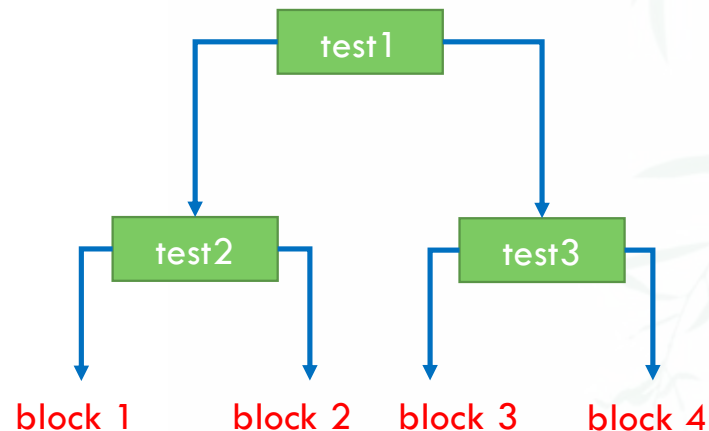


# Programming

Imperative programming control flow also embodies this

```
if test1:  
    if test2:  
        # block 1  
    else:  
        # block 2  
else:  
    if test3:  
        # block 3  
    else:  
        # block 4
```

This is a decision tree!



# Parts of a tree



## 1. Internal nodes are tests

In most systems, a node tests exactly one component of  $x$ : (a feature; e.g.,  $x_0$ )

## 2. A branch in the tree corresponds to a test result

A branch corresponds to an attribute value or a range of attribute values

## 3. Each leaf node assigns

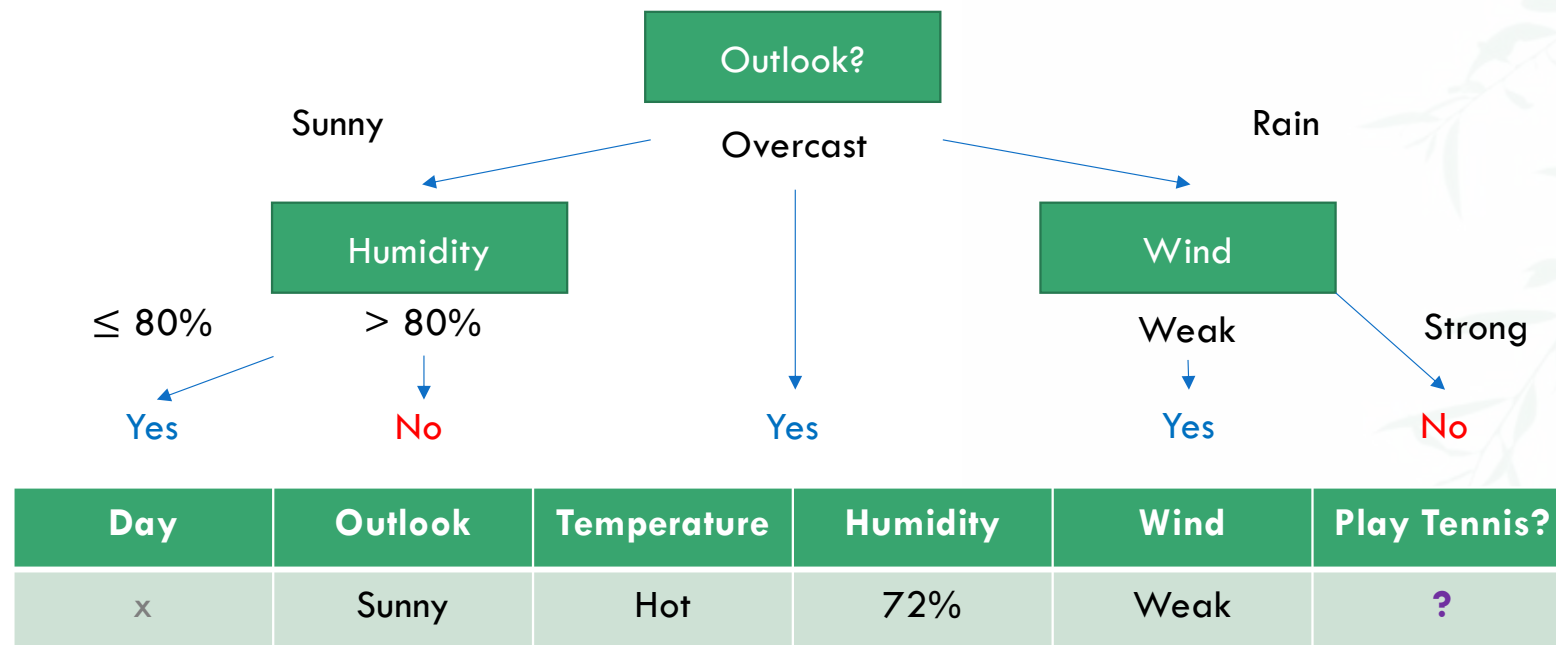
- A class  $y \in \{enum\}$ : Classification tree
- Real value  $y \in \mathbb{R}$ : Regression tree

# Shall we play tennis?

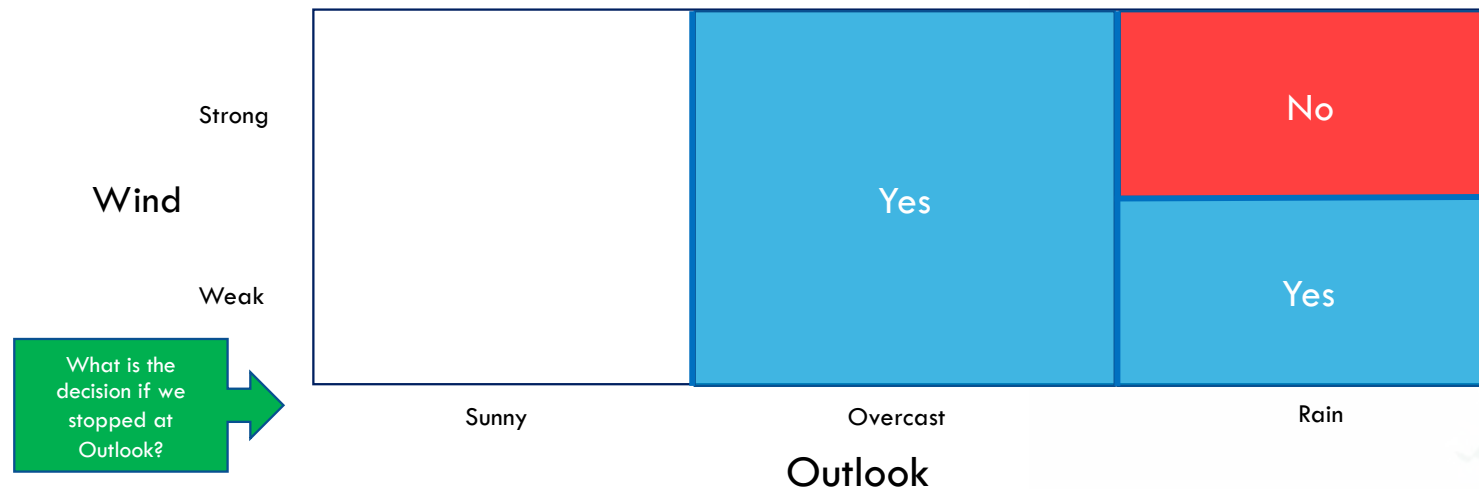


Day	Outlook	Temperature	Humidity	Wind	Play Tennis?
1	Sunny	Hot	95%	Weak	No
2	Sunny	Hot	92%	Strong	No
3	Overcast	Hot	95%	Weak	Yes
4	Rain	Mild	90%	Weak	Yes
5	Rain	Cool	60%	Weak	Yes
6	Rain	Cool	65%	Strong	No
7	Overcast	Cool	70%	Strong	Yes
8	Sunny	Cool	70%	Weak	Yes

# Shall we play tennis?



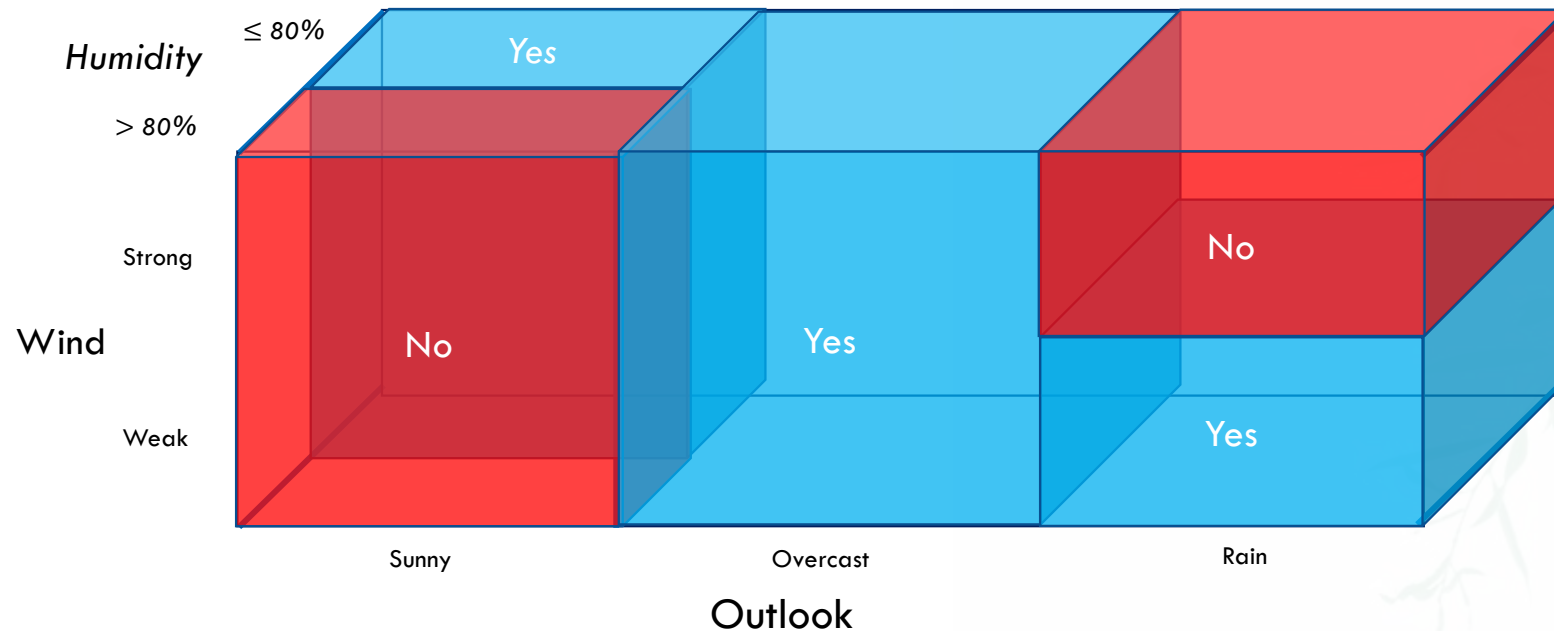
# Inductive Bias – Decision boundary



Axis perpendicular (recursive) decision boundaries



# Inductive Bias – Decision boundary



Axis perpendicular (recursive) decision boundaries

**function** **DTL** (*examples*, *attributes*, *default*) **returns** a decision tree



**if** *examples* is empty **then return** *default*

**else if** all *examples* have the same classification **then return** the classification

**else if** *attributes* is empty **then return** **MODE**(*examples*)

**else**

*best*  $\leftarrow$  **CHOOSE-ATTRIBUTE**(*attributes*, *examples*)

*tree*  $\leftarrow$  a new decision tree root test *best*

**for each** value  $v_i$  of *best* **do**

*examples<sub>i</sub>*  $\leftarrow$  {elements of *examples* with *best* =  $v_i$ }

*subtree*  $\leftarrow$  **DTL**(*examples<sub>i</sub>*, *attributes* – *best*, **MODE**(*examples*))

add a branch to *tree* with label  $v_i$ , and subtree *subtree*

**return** *tree*

# 3 Important Questions

Prefer important decisions at the top

1. How do we pick a feature to split on?
2. How do we discretize continuous features?

Prune back complete trees<sup>\*</sup>

3. How do we decide where to prune?

<sup>\*</sup> we could decide when to stop growing the tree, but in practice this doesn't work well.

# Decisions, decisions, decisions...



In physical subgroups or Zoom breakout rooms, do:

**(5 mins)**: Discuss any **one** of your three decisions as a team.

Answer: *What criteria do you use to decide which feature of your decision is the most important one to test first?*

Ask one member to write your collective answer to the [#lectures](#) thread. Upvote others that you like.

# Entropy and Information Gain

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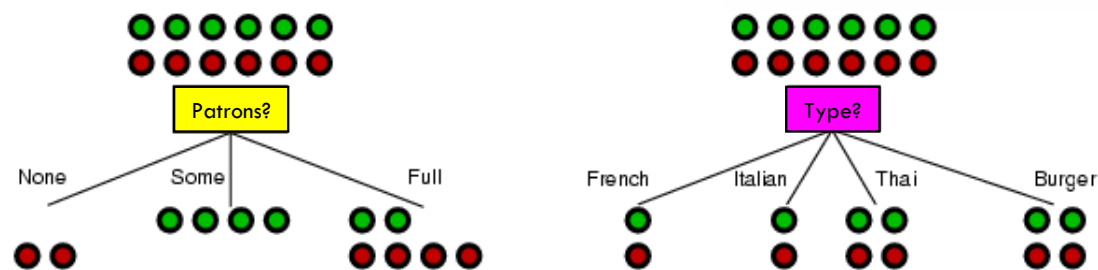
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# Choosing an attribute

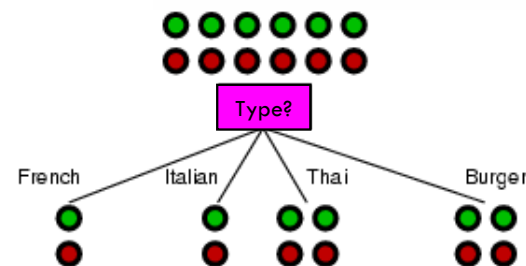
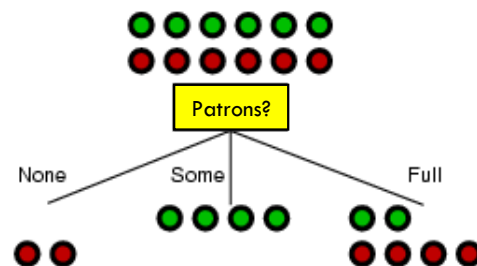
Idea: a good attribute (feature) splits the training set into subsets that are (ideally) “all positive” or “all negative”.

Let's look at a case of *What did you eat for lunch?*



# Choosing an attribute

Idea: a good attribute (feature) splits the training set into subsets that are (ideally) “all positive” or “all negative”.



**Patrons?** is a better choice. Why?

Purity of the subproblems.

# Q1. How do we pick a feature to split on?



To implement **Choose Attribute** in DTL for enumerated feature sets with  $C$  possible values, we first need a concept of purity. We'll use **Entropy**:

$$H(X) = - \sum_{i=0}^C P_i \log_2(P_i)$$

⚠  $P \equiv$  probability,  
 $p \equiv$  # of positive examples.

Example: for a training set containing  $p$  positive examples and  $n$  negative examples:

$$H\left(\frac{p}{p+n}, \frac{n}{p+n}\right) = -\frac{p}{p+n} \log_2\left(\frac{p}{p+n}\right) - \frac{n}{p+n} \log_2\left(\frac{n}{p+n}\right)$$

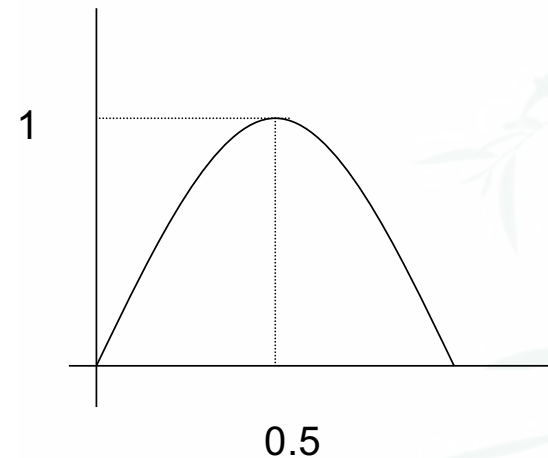
# Entropy curve

For  $\frac{p}{p+n}$ , the 2-class entropy is:

0 when  $\frac{p}{p+n}$  is 0

1 when  $\frac{p}{p+n}$  is 0.5

0 when  $\frac{p}{p+n}$  is 1



- monotonically increasing between 0 and 0.5
- monotonically decreasing between 0.5 and 1

# Information gain

A chosen feature  $x_i$  divides the example set  $S$  into subsets  $S_1, S_2, \dots, S_c$  according to the  $C_i$  distinct values for  $x_i$ .

The entropy then reduces to the entropy of the subsets  $S_1, S_2, \dots, S_c$ :

$$\text{remainder}(S, x_i) = \sum_{j=1}^{C_i} \frac{|S_j|}{|S|} H(S_j)$$

**Information Gain** (IG; “reduction in entropy”) from knowing the value of  $x_i$ .  
Choose the attribute with the largest IG:

$$\text{IG}(S, x_i) = H(S) - \text{remainder}(S, x_i)$$



# Information gain

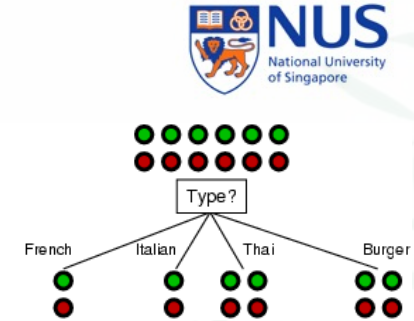
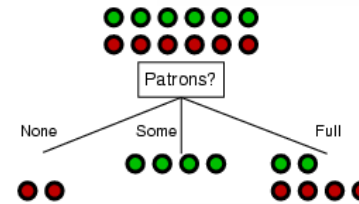
For the training set at the root,  
 $p = n = 6$ ,  $H\left(\frac{6}{12}, \frac{6}{12}\right) = 1$  bit.

Consider the attributes **Patrons** and **Type**:

$$IG(\text{Patrons}) = 1 - \left[ \frac{2}{12}H(0,1) + \frac{4}{12}H(1,0) + \frac{6}{12}H\left(\frac{2}{6}, \frac{4}{6}\right) \right] = 0.541 \text{ bits}$$

$$IG(\text{Type}) = 1 - \left[ \frac{2}{12}H\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{2}{12}H\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{4}{12}H\left(\frac{2}{4}, \frac{2}{4}\right) + \frac{4}{12}H\left(\frac{2}{4}, \frac{2}{4}\right) \right] = 0 \text{ bits}$$

*Patrons* has the highest IG, and so is chosen by DTL as the root.





# Cost Functions

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## Weighty Decisions



(3 mins): Let's do this for those of you attached virtually on Zoom! 💪💪. Go ahead and annotate the screen (be polite!) I'll also prompt you all orally in class (Don't be shy!)

*Are there particular instances of your scenario that are more important? Describe them and how much they are worth.*

# Cost functions

What does  $h_\theta \approx f$  mean?

Need a **cost** function  $L(h_\theta, f)$

← “Cost” is same “loss”. Also, you’ll see  $J(h_\theta, f)$  instead of  $L(h_\theta, f)$

This is almost always a pointwise definition:  $l(h_\theta(x), f(x))$

Simple examples:

Squared error  $(h_\theta(x) - f(x))^2$

Binary error  $[h_\theta(x) \neq f(x)]$

← Iverson bracket  
 $[x] \equiv \begin{cases} 1, & \text{if } x \text{ is true} \\ 0, & \text{if } x \text{ is false} \end{cases}$

# From pointwise to overall

Overall cost  $L(h, f) =$

average of pointwise cost  $l(h(x), f(x))$

Training cost:

$$L_{train} = \frac{1}{m} \sum_{j=1}^m l(h_{\theta}(x^{(j)}), f(x^{(j)}))$$

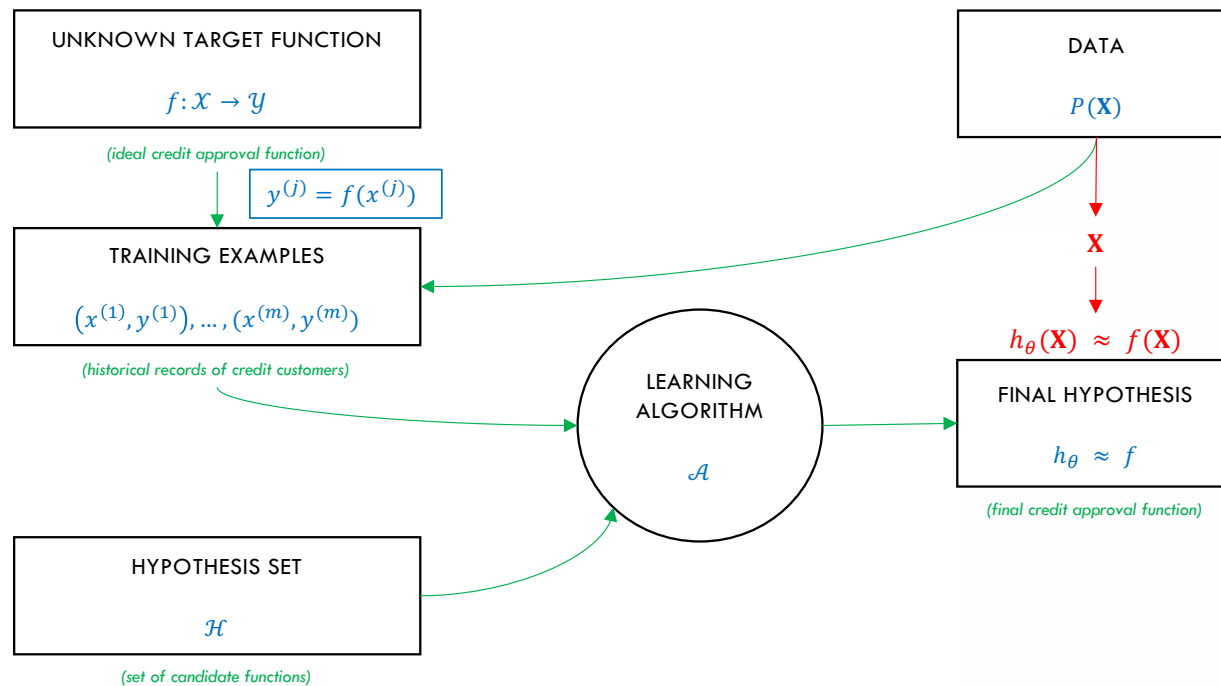
Why not a sum  
instead of an  
average?

Test cost:

$$L_{test} = \mathbb{E}_x[l(h_{\theta}(x), f(x))]$$

⚠ Expectations also use  
square brackets; doesn't  
mean T/F as in previous  
slide's Iverson notation.

# The learning diagram with testing data, pointwise error





# Choosing your cost function



Are you sick?

Two types of costs:

*false positive* (accept) or

*false negative* (reject)

How should we penalize  
for each type?

$$f \longrightarrow \begin{cases} +1 & \text{sick} \\ -1 & \text{well} \end{cases}$$

$h_\theta$

	+1	-1
+1	True Positive	False Positive
-1	False Negative	True Negative

# During your last final exam before graduation

Are you sick?

*False negative* – get better on your own, or come back to the clinic later. At least you graduate on time.

*False positive* – Take the exam next year. Possibly pay tuition fees. \$\$\$



$$f \longrightarrow \begin{cases} +1 & \text{sick} \\ -1 & \text{well} \end{cases}$$

$f$

$h_\theta$

	+1	-1
+1	0 True Positive	100 Positive
-1	1 False Negative	0 Negative

# Pre-vaccine COVID-19

Are you sick?

*False negative* highly costly!  
People and the economy dies.

*False positive* requires the  
inconvenience of quarantine.



$$f \longrightarrow \begin{cases} +1 & \text{sick} \\ -1 & \text{well} \end{cases}$$

$h_\theta$

	+1	-1
+1	0 True Positive	False Positive
-1	False Negative 10,000	0 True Negative

# Your measure matters

Where possible, we should use cost functions that fit the task, specified by the user.

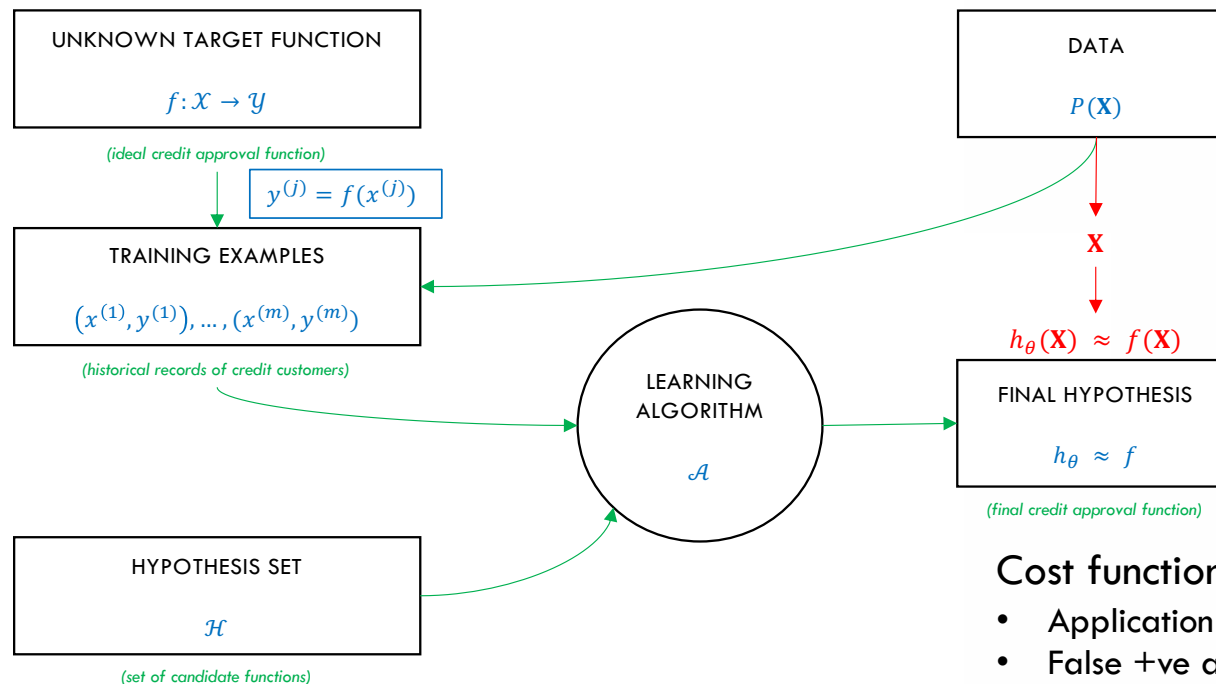
However, this isn't always possible. Then use:

**Plausible** measures: squared error  $\equiv$  Gaussian noise

**Convenient** measures: closed form solution,  
convex optimization

*The sweet spot:  
Plausible AND  
Convenient*

# Summary – the learning diagram with testing data, pointwise error



**Quick Question:**  
where does the  
cost function go?

## Cost functions

- Application specific, user should specify
- False +ve and -ve may differ in cost

# How you measure matters



(5 mins): Go through the three decisions from your pre-lecture activity as a team. Build on your examples earlier and answer:  
*What do you think the appropriate false positive and negative costs should be for each decision?*

Choose one random answer and write your collective answer to the [#lectures](#) thread. Upvote others that you like. 👍



# Learning Theory 101: No Free Lunch

Machine learning works only because we apply some *inductive bias* (modelled by  $\mathcal{A}$ ) to conclude that, once we've seen a particular pattern multiple times, we'll see that pattern again in unseen test data (induction).


But just because we saw something happen many times before, doesn't mean it will happen again.

Any outcome is possible, so no learning algorithm is better than any other given a training set and no other knowledge about the problem.

This is the essence of the No Free Lunch Theorem.



David Hume, image in public domain



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# Ensembles

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4.2

# Ensembles:

## Revisiting the Netflix Prize

Just three weeks after it began, at least 40 teams had bested the Netflix classifier.

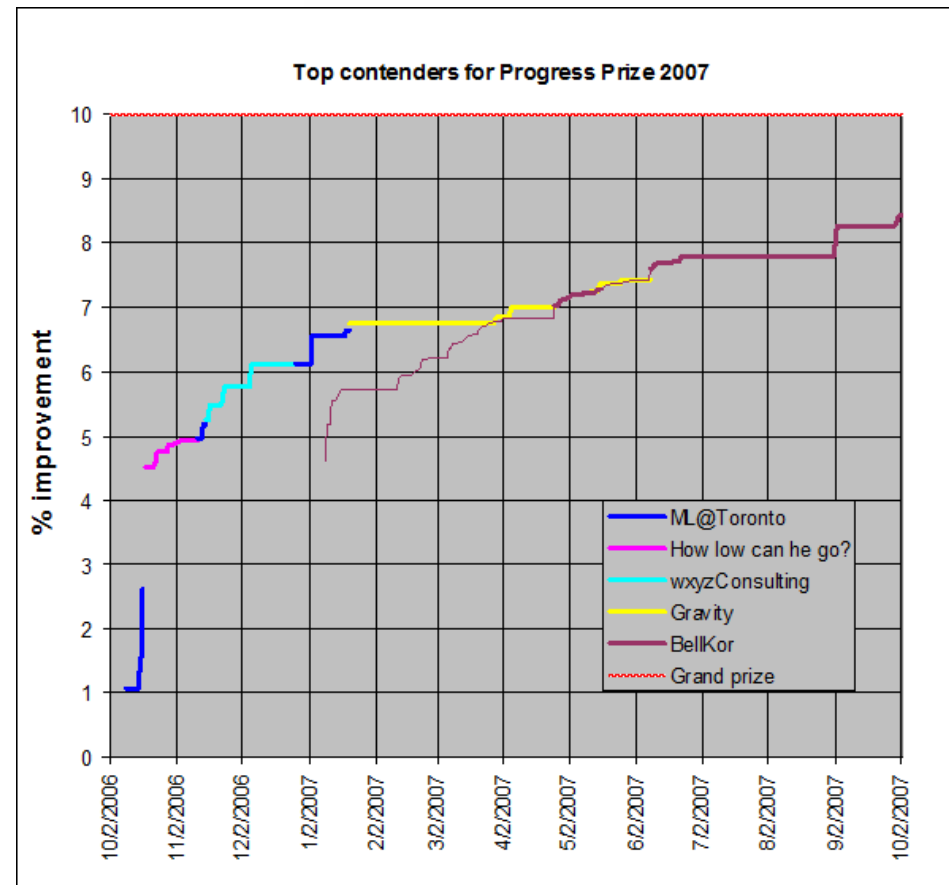
Top teams showed about 5% improvement.

Netflix Prize		
me	Rules	Leaderboard
Register	Update	Submit
Download		
Leaderboard		
Team Name	Best Score	% Improvement
No Grand Prize candidates yet	--	--
Grand Prize - RMSE <= 0.8563		
How low can he go?	0.9046	4.92
<a href="#">ML@UToronto A</a>	0.9046	4.92
<a href="#">ssorkin</a>	0.9089	4.47
<a href="#">wxyzconsulting.com</a>	0.9103	4.32
The Thought Gang	0.9113	4.21
<a href="#">NIPS Reject</a>	0.9118	4.16
<a href="#">simonfunk</a>	0.9145	3.88
Bozo_The_Clown	0.9177	3.54
Elliptic Chaos	0.9179	3.52
datcracker	0.9183	3.48
<a href="#">Foreseer</a>	0.9214	3.15
bsdfish	0.9229	3.00
Three Blind Mice	0.9234	2.94
<a href="#">Bocsimacko</a>	0.9238	2.90
Remco	0.9252	2.75
<a href="#">karmatics</a>	0.9301	2.24
Chapelator	0.9314	2.10
<a href="#">Flmod</a>	0.9325	1.99
mthrox	0.9328	1.96

However,  
improvement  
slowed...

Source:

<http://www.research.att.com/~volinsky/netflix/>



# Rookies

“Thanks to Paul Harrison's collaboration, a simple mix of our solutions improved our result from 6.31 to 6.75”

--	No Progress Prize candidates yet	--	--
Progress Prize - RMSE <= 0.8625			
1	<a href="#">BellKor</a>	0.8705	8.50
Progress Prize 2007 - RMSE = 0.8712 - Winning Team: KorBell			
2	<a href="#">KorBell</a>	0.8712	8.43
3	<a href="#">When Gravity and Dinosaurs Unite</a>	0.8717	8.38
4	<a href="#">Gravity</a>	0.8743	8.10
5	<a href="#">basho</a>	0.8746	8.07
6	<a href="#">Dinosaur Planet</a>	0.8753	8.00
7	<a href="#">ML@UToronto A</a>	0.8787	7.64
8	<a href="#">Arek Paterek</a>	0.8789	7.62
9	<a href="#">NIPS Reject</a>	0.8808	7.42
10	Just a guy in a garage	0.8834	7.15
11	<a href="#">Ensemble Experts</a>	0.8841	7.07
12	<a href="#">mathematical capital</a>	0.8844	7.04
13	<a href="#">HowLowCanHeGo2</a>	0.8847	7.01
14	<a href="#">The Thought Gang</a>	0.8849	6.99
15	<a href="#">Reel Ingenuity</a>	0.8855	6.93
16	<a href="#">strudeltamale</a>	0.8859	6.88
17	<a href="#">NIPS Submission</a>	0.8861	6.86
18	<a href="#">Three Blind Mice</a>	0.8869	6.78
19	<a href="#">TrainOnTest</a>	0.8869	6.78
20	<a href="#">Geoff Dean</a>	0.8869	6.78
21	<a href="#">Rookies</a>	0.8872	6.75
22	<a href="#">Paul Harrison</a>	0.8872	6.75
23	<a href="#">ATTEAM</a>	0.8873	6.74
24	<a href="#">xyzconsulting.com</a>	0.8874	6.73
25	<a href="#">ICMLsubmission</a>	0.8875	6.72
26	<a href="#">Efratko</a>	0.8877	6.70
27	<a href="#">Kitty</a>	0.8881	6.65
28	<a href="#">SecondaryResults</a>	0.8884	6.62
29	<a href="#">Birgit Kraft</a>	0.8885	6.61



# Arek Paterek

“My approach is to **combine the results of many methods** (also two-way interactions between them) using linear regression on the test set. The best method in my ensemble is regularized SVD with biases, post processed with kernel ridge regression”

[http://rainbow.mimuw.edu.pl/~ap/ap\\_kdd.pdf](http://rainbow.mimuw.edu.pl/~ap/ap_kdd.pdf)

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19	TrainOnTest	0.8869	6.78
20	Geoff Dean	0.8869	6.78
21	<a href="#">Rookies</a>	0.8872	6.75
22	<a href="#">Paul Harrison</a>	0.8872	6.75
23	ATTEAM	0.8873	6.74
24	<a href="#">xyzconsulting.com</a>	0.8874	6.73
25	ICMLsubmission	0.8875	6.72
26	Efratko	0.8877	6.70
27	<a href="#">Kitty</a>	0.8881	6.65
28	SecondaryResults	0.8884	6.62
29	<a href="#">Birgit Kraft</a>	0.8885	6.61



# When Gravity and Dinosaurs Unite

“Our common team blends the result of team Gravity and team Dinosaur Planet.”

Might have guessed from the name...

--	No Progress Prize candidates yet	--	--
Progress Prize - RMSE <= 0.8625			
1	<a href="#">BellKor</a>	0.8705	8.50
Progress Prize 2007 - RMSE = 0.8712 - Winning Team: KorBell			
2	<a href="#">KorBell</a>	0.8712	8.43
3	<a href="#">When Gravity and Dinosaurs Unite</a>	0.8717	8.38
4	<a href="#">Gravity</a>	0.8743	8.10
5	<a href="#">basho</a>	0.8746	8.07
6	<a href="#">Dinosaur Planet</a>	0.8753	8.00
7	<a href="#">ML@UToronto A</a>	0.8787	7.64
8	<a href="#">Arek Paterek</a>	0.8789	7.62
9	<a href="#">NIPS Reject</a>	0.8808	7.42
10	Just a guy in a garage	0.8834	7.15
11	<a href="#">Ensemble Experts</a>	0.8841	7.07
12	<a href="#">mathematical capital</a>	0.8844	7.04
13	<a href="#">HowLowCanHeGo2</a>	0.8847	7.01
14	<a href="#">The Thought Gang</a>	0.8849	6.99
15	<a href="#">Reel Ingenuity</a>	0.8855	6.93
16	<a href="#">strudelamale</a>	0.8859	6.88
17	NIPS Submission	0.8861	6.86
18	Three Blind Mice	0.8869	6.78
19	TrainOnTest	0.8869	6.78
20	Geoff Dean	0.8869	6.78
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# Ensembles, for the win



The team BellKor's Pragmatic Chaos is a combined team of BellKor, Pragmatic Theory and BigChaos. [BellKor](#) consists of Robert Bell, Yehuda Koren and Chris Volinsky. The members of [Pragmatic Theory](#) are Martin Piotte and Martin Chabbert. Andreas Töschler and Michael Jahrer form the team [BigChaos](#).

“...There were two main factors which improved the overall accuracy: The quality of the individual algorithms and [the ensemble idea](#) ... Best blending results are achieved when the whole ensemble has the right tradeoff between diversity and accuracy. ”

## Leaderboard

Display top  leaders.

Rank	Team Name	Best Score	% Improvement	Last Submit Time
1	<a href="#">BellKor's Pragmatic Chaos</a>	0.8558	10.05	2009-06-26 18:42:37
Grand Prize - RMSE <= 0.8563				
2	<a href="#">PragmaticTheory</a>	0.8582	9.80	2009-06-25 22:15:51
3	<a href="#">BellKor in BigChaos</a>	0.8590	9.71	2009-05-13 08:14:09
4	<a href="#">Grand Prize Team</a>	0.8593	9.68	2009-06-12 08:20:24
5	<a href="#">Dace</a>	0.8604	9.56	2009-04-22 05:57:03
6	<a href="#">BigChaos</a>	0.8613	9.47	2009-06-23 23:06:52

[http://www.netflixprize.com/assets/GrandPrize2009\\_BPC\\_BigChaos.pdf](http://www.netflixprize.com/assets/GrandPrize2009_BPC_BigChaos.pdf)

## Feature Film, Rating Count at least 25,000, Sci-Fi (Sorted by IMDb Rating Descending)

1-50 of 797 titles. | [Next »](#)

View Mode: [Compact](#) | [Detailed](#)

Sort by: [Popularity](#) | [Release Date](#) | [Date](#)

# Meta Learning: Ensembles



### 1. **Inception** (2010)

PG-13 | 148 min | Action, Adventure, Sci-Fi

★ **8.8** ☆ [Rate this](#)

**74** Metascore

A thief who steals corporate secrets through the use of dream-sharing technology is given the inverse task of planting an idea into the mind of a C.E.O.

Director: [Christopher Nolan](#) | Stars: [Leonardo DiCaprio](#), [Joseph Gordon-Levitt](#), [Elliot Page](#), [Ken Watanabe](#)

Votes: 2,156,755 | Gross: \$292.58M



### 2. **The Matrix** (1999)

R | 136 min | Action, Sci-Fi



Photo by [David Travis](#) on [Unsplash](#)

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Photo by [Wan San Yip](#) on [Unsplash](#)



Photo by [CDC](#) on [Unsplash](#)

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# Ensembles



We have  $T$  reports  $h_1, h_2, \dots, h_T$  predicting whether a stock will go up as  $h(x)$ .

We can:

1. Select the most trustworthy of them based on their usual performance

Training performance:  $h(x) = h_{t^*}(x)$  with  $t^* = \operatorname{argmin}_{t \in \{1, 2, \dots, T\}} L_{\text{train}}(h_t)$

2. Let each report have a vote


Uniform Vote:  $h(x) = \operatorname{sign}(\sum_{t=1}^T 1 \cdot h_t(x))$

3. Weight the reports non-uniformly

Weighted Vote:  $h(x) = \operatorname{sign}(\sum_{t=1}^T \alpha_t \cdot h_t(x))$  where  $\alpha_t \geq 0$ .

4. Combine the predictions conditionally

Given the outcome of a test, decide which learner to use.



Which is the  
decision tree?

# Stock Market – Ensemble



We have  $T$  reports  $h_1, h_2, \dots, h_T$  predicting whether a stock will go up as  $h(x)$ .

We can:

1. Select the most trustworthy of them based on their usual performance

Training performance:  $h(x) = h_{t^*}(x)$  with  $t^* = \operatorname{argmin}_{t \in \{1, 2, \dots, T\}} L_{\text{train}}(h_t)$

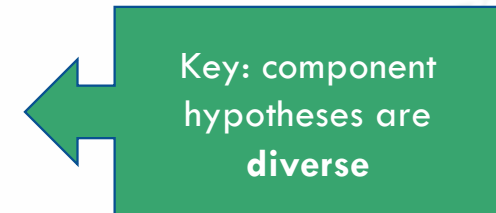
2. Let each report have a vote

Uniform Vote:  $h(x) = \operatorname{sign}(\sum_{t=1}^T 1 \cdot h_t(x))$

3. Weight the reports non-uniformly

Weighted Vote:  $h(x) = \operatorname{sign}(\sum_{t=1}^T \alpha_t \cdot h_t(x))$  where  $\alpha_t \geq 0$ .

4. Combine the predictions conditionally



# How is a decision tree an ensemble?



Conditional Ensemble: decide on a learner based on a test.

Divide and Conquer – make smaller problems (like Quicksort)



What type of learner do we use in a decision tree?

Answer: A *decision stump*



# The Decision Stump

a.k.a. 1-level decision tree:

$$h_{s,i,\theta}(x) = s \cdot \text{sign}(x_i - \theta)$$

Positive and negative rays on some feature, three parameters (feature  $i$ , threshold  $\theta$ , direction  $s$ )

Visualization: axis-perpendicular planes in  $\mathbb{R}^n$ .

Efficient to optimize:  $O(n \cdot \log m)$  time

Allows efficient minimization of  $L_{train}$ , but can be too weak to work by itself (hence called a weak learner).

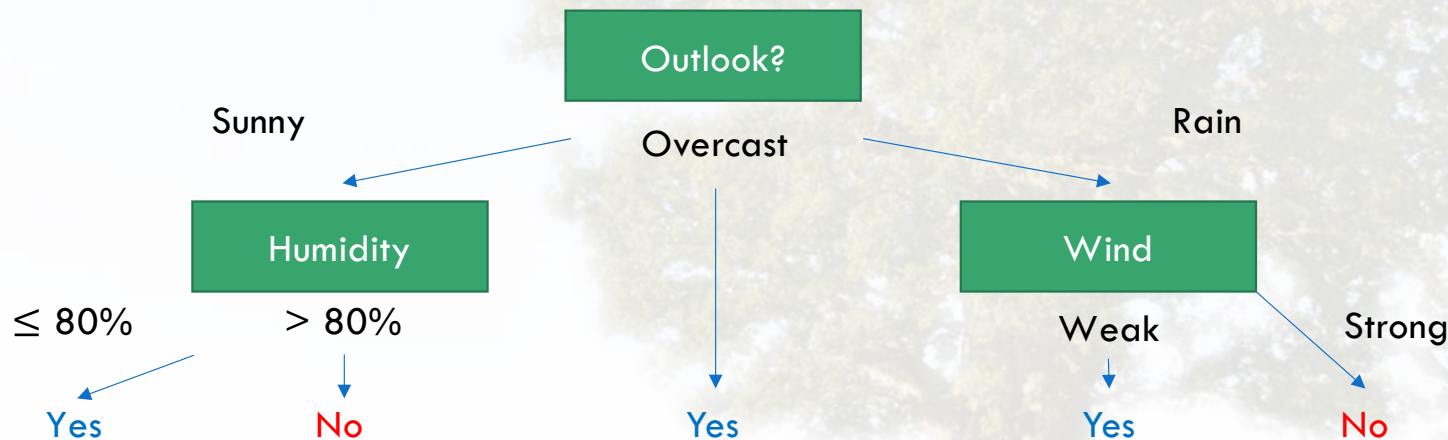





# Building a tree

Conditionally combine each decision stump to build the tree.

Entropy / Information Gain used to make the subproblems smaller and simpler than the root problem.





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# Discretizing

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## 3 Important Questions

Prefer important decisions at the top

1. How do we pick a feature to split on? (*already covered*)
2. How do we discretize continuous features?

Prune back complete trees<sup>\*</sup>

3. How do we decide where to prune?

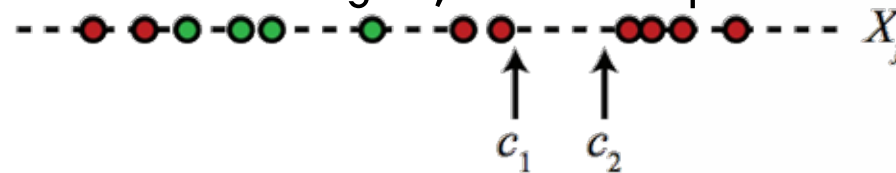
<sup>\*</sup> we could decide when to stop growing the tree, but in practice this doesn't work well.



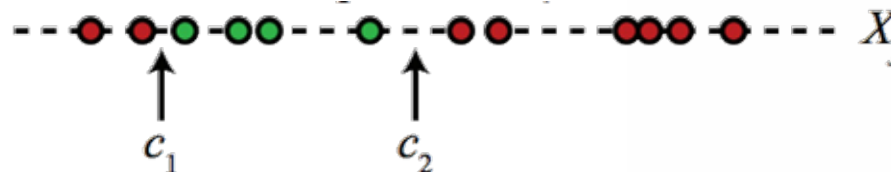
## Q2. How do we discretize continuous features?

How can we choose among an infinite number of split points for a continuous feature  $x_j$ ?

Observation 1. Moving split points between two observed values with the same label has no effect on information gain; use the midpoint.



Thus, 2. Only splitting between examples from different labels  $y$  could improve information gain.





# Pruning

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# Analyzing Decision Trees



How expressive is the  $\mathcal{H}$  of decision trees?

They can express any row  $(x_1, x_2, \dots, x_n, y)$  in the data set, trivially (how?).

You can always have  $L_{train} = 0$  for a decision tree, if there is no **noise** in  $y$ .



What's the problem with an unpruned learned decision tree, in terms of learning theory?

### Q3. How do we decide where to prune?

In Zoom breakout or physical subgroups, do:

(5 mins): Answer: *Why bother pruning? Why not build the decision tree and just use it?*

Ask one member to write your collective answer to the [#lectures](#) thread.  
Upvote others that you like.

```
function DTL(examples, attributes, default) returns a decision tree
    if examples is empty then return default
    else if all examples have the same classification then return the classification
    else if attributes is empty then return MODE(examples)
    else
        best ← CHOOSE-ATTRIBUTE(attributes, examples)
        tree ← a new decision tree root test best
        for each value  $v_i$  of best do
            examplesi ← {elements of examples with best =  $v_i$ }
            subtree ← DTL(examplesi, attributes - best, MODE(examples))
            add a branch to tree with label  $v_i$ , and subtree subtree
        return tree
```



# The need for pruning

Day	Outlook	Temperature	Humidity	Wind	Tennis?
1	Sunny	Hot	95%	Weak	No
2	Sunny	Hot	92%	Strong	No
3	Overcast	Hot	95%	Weak	Yes
4	Rain	Mild	90%	Weak	Yes
5	Rain	Cool	60%	Weak	Yes
6	Rain	Cool	65%	Strong	No
7	Overcast	Cool	70%	Strong	Yes
8	Sunny	Cool	70%	Weak	Yes

### Q3. How do we decide where to prune?

Answer:

1. Don't let leaves get too small. Require leaves have at least  $i$  training data.
2. Don't let tree grow too deep. Don't allow more than  $j$  levels of tests.
3. Use part of the training data to check performance (called **validation**)!

Remove node and replace by the **MODE** of labels  $y$  if the pruned tree performs better than original.



# Extending for Regression



*(Regression  $\equiv$  Continuous  $y$  output aka predictor variable)*

Both Decision Trees and  $k$  NN predict categorical features.



How can we extend these two algorithms to handle regression?

- Decision Trees: A test in a leaf predicts the **MEAN** of its training instances.
- $k$  NN: a test instance predicts the **MEAN** of its  $k$  NN.

*Many other options for extending to regression.  
What could you propose?*

# Summary:

## Analyzing Decision Tree Learning



### The good

Interpretable: Intuitive for data exploration.

Efficient: both in training (greedy search) and testing

Discards irrelevant feature through use of Information Gain.

### The bad

Instability: Susceptible to small fluctuations (high variance)

Hard decision boundaries: by default, no probabilistic interpretation of boundaries

Axis perpendicular decisions: doesn't capture interaction between features (feature AND bug)



# Wrapping up Week 03

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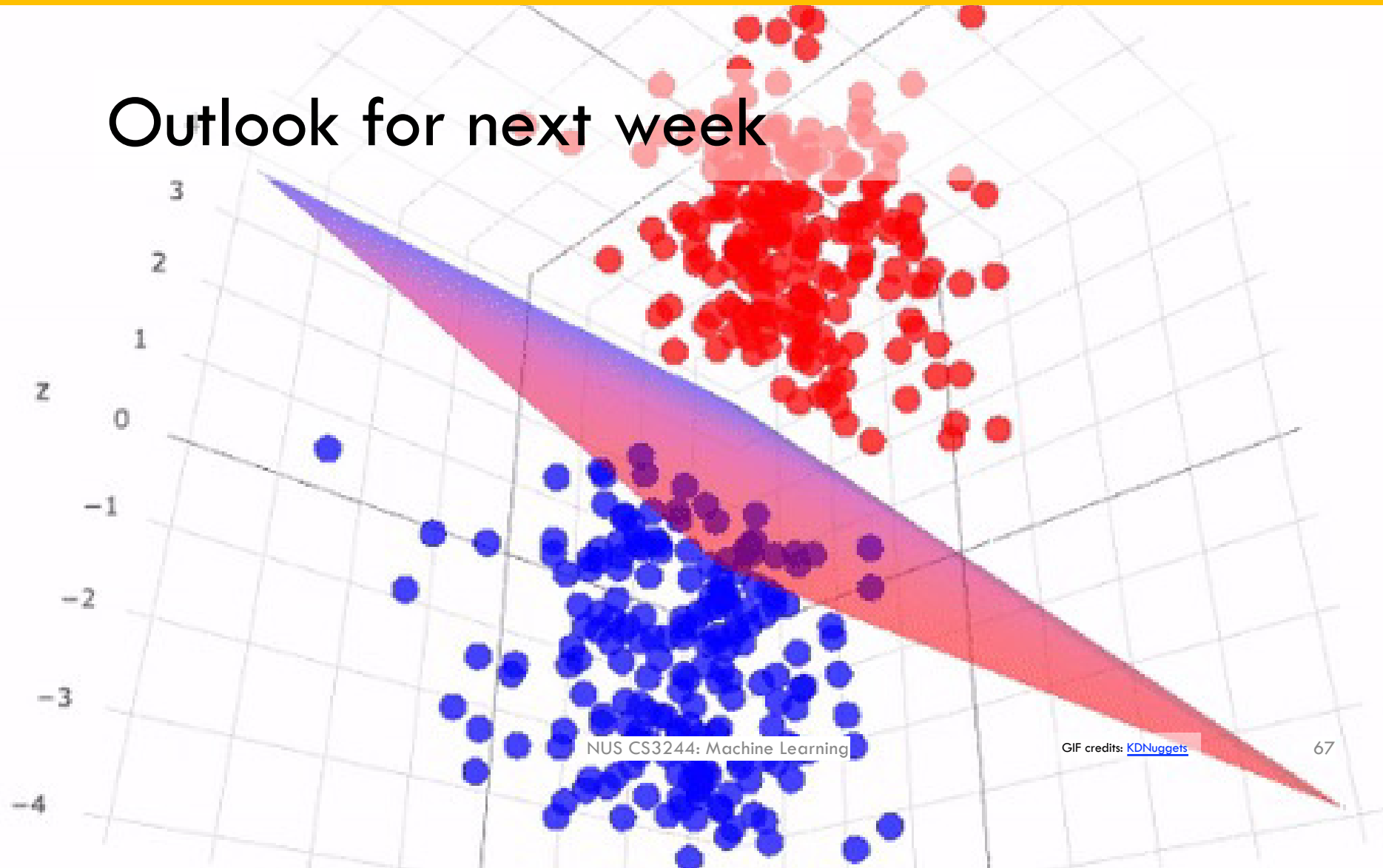


# What did we learn this week?



- Learning Outcomes for this week:
- Understand and implement Decision Trees;
- Master Information Gain as a means of choosing optimal features for decisions;
- Formalize cost functions as a means of evaluating a supervised learner's performance;
- Describe simple ensembling methods that integrate multiple classifiers;
  - Understand Decision Trees as an instance of an ensemble classifier of conditional decision stamps;
- Reason about inductive bias of the decision tree algorithm

# Outlook for next week



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GIF credits: [KDNuggets](#)

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# Assignment #1 released



Min, go over the  $k$  NN versus Decision Tree assignment:

[http://www.comp.nus.edu.sg/~cs3244/AY22S1/  
01.assignment.html](http://www.comp.nus.edu.sg/~cs3244/AY22S1/01.assignment.html)

Due this Sunday night. Start today!

## Assigned Task (due before next Mon)



Read the Quora post [Whats-the-difference-between-linear-and-non-linear-machine-learning-model](#), especially Robby's answer (5 mins)  
(You may want to use guest / incognito mode, if asked to sign up)

Post a 1–2 sentence answer to the question in tutorial group: [#tg-xx](#)

Can we model non-linearity with a linear classifier?

[Don't worry if you're not sure, we'll cover this again in Week 04.]