Formalisation in Lean 4 sec:formalisation

The Lean 4 Proof Assistant sec:lean4-intro

Lean 4 MouraUllrich2021 is an interactive theorem prover based on dependent type theory. Its expressive type system allows natural formalisation of philosophical arguments, while its strong verification guarantees ensure logical correctness. Mechanical verification means every proof step is checked by Lean's kernel, ensuring no gaps, unstated assumptions, or logical errors exist.

Core Definitions sec:core-definitions

The formalisation comprises three files totaling 2,874 lines: Cubitt_Sugden.lean (574 lines) demonstrates the syntactic baseline where A1–A6 are axioms; Sillari_improved.lean (1,050+ lines) proves modal logic fails with verified counterexamples; and reasons_improved.lean (1,250+ lines) provides the solution via justification logic where A1 and A6 become theorems. We focus on the third file, which contains the main contribution.

Reasons and Beliefs sec:reasons-beliefs

The fundamental innovation of justification logic is making reasons explicit. Rather than saying "individual i believes φ" or "individual i knows φ", we say "reason r justifies individual i in believing φ."

definition[Justification relation] def:justification-relation The primitive relation is: rb : reason $\to$ $indiv \to Prop \to PropWereadrb\, r\, i\, \varphi$ as: "reason r justifies individual i in believing proposition φ."