

Homework 1: Topological manifolds

Math 761, Tullia Dymarz

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Due on gradescope Thursday September 11 at noon.

1. Prove that the Möbius strip

$$M = [0, 1] \times \left(-\frac{1}{2}, \frac{1}{2}\right) / (0, y) \sim (1, -y)$$

is a topological manifold.

2. Prove that \mathbb{RP}^2 is Hausdorff, second-countable, and compact.
3. Prove that \mathbb{RP}^2 is homeomorphic to the closed 2-disk with opposite boundary points identified:

$$\mathbb{RP}^2 \cong \bar{\mathbb{D}}^2 / (\cos \theta, \sin \theta) \sim (\cos(\theta + \pi), \sin(\theta + \pi)).$$

(use either of the definitions I gave in class).

4. (**Group discussion**) How would you define a manifold with boundary? Is the boundary is itself a manifold of one dimension lower? Give some examples. (This one won't be graded. Just draw some pictures, list some questions/concerns you might have and list who you discussed with.)