## 0.1 Global Round 21: D. Permutation Graph

### Brief

There is a permutation of $n$ long, and the indices are viewed as vertices on an imaginary graph. Two vertices are said to be bi-connected if and only if the corresponding value of each index are the respective maximum and minimum of their underlying interval on the permutation array. Literally, for $i < j$,

$$\begin{aligned} i \leftrightarrow j \iff & \\ a_i = \min\{a_i, a_{i+1}, \ldots, a_j\}, a_j = \max\{a_i, a_{i+1}, \ldots, a_j\} & \\ \text{or } a_i = \max\{a_i, a_{i+1}, \ldots, a_j\}, a_j = \min\{a_i, a_{i+1}, \ldots, a_j\} & \end{aligned}$$

Find the shortest path from 1 to $n$.

### Solution

After working on the last sample, it appears that traveling toward the right as far as possible at each step will work. Backward to the left seems uneconomical since the distance to $n$ increases, adding constraints. This approach can be proved correct, and the implementation is $O(n \log n)$ interval minimum/maximum. Nevertheless, establishing the greedy will lead to a $O(n)$ algorithm.

Suppose $a_i = 1, a_j = n$, WLOG assume $i < j$. Partition the array into three sets: $S_1 = [1, \ldots, i-1], S_2 = [i+1, \ldots, j-1], S_3 = [j+1, \ldots, n]$ (allowing intervals to be empty). We notice that there should be no edge from between these three sets, since $a_i, a_j$ are the extreme values in the permutation. Therefore for reaching $n$ from 1, one must bypass $i, j$. The jump from $i$ to $j$ takes one step, and the remainings can be solved recursively.

By the same sub-structure of the problem, the strategy of consistent heading right is obviously correct. The divide-and-conquer implementation takes $O(n)$.