0.1 Preliminaries

Multisets

A **Multiset** M with **underlying set** S is a set of ordered pairs

$$M = \{(s_i, n_i) | s_i \in S, n_i \in \mathbb{Z}^+, s_i \neq s_j \forall i \neq j\}$$

Matrices

$\mathcal{M}_{m,n}(F)$: the set of $m \times n$ matrices.

Properties of transpose:

1. $\left(A^T\right)^T = A$
2. $\left(A + B\right)^T = A^T + B^T$
3. $\left(rA\right)^T = rA^T \ \forall r \in F$
4. $\left(AB\right)^T = B^T A^T$ provided that AB is defined.
5. $\det\left(A^T\right) = \det\left(A\right)$

Partitions/Block Matrices

Actually there is another expression:

$$\begin{bmatrix} A_1 & A_2 & \cdots & A_k \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_k \end{bmatrix} = \sum_{i=1}^{k} A_i B_i$$

provided that $A_i B_i$ is defined.

Determinants

1. $\det\left(AB\right) = \det\left(A\right) \det\left(B\right)$
2. If

$$M = \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & B_n \end{bmatrix}$$

then $\det\left(M\right) = \prod \det(B_i)$.

Polynomials

More on it later. It's basically drawing analogies between polynomials and integers.

Function

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