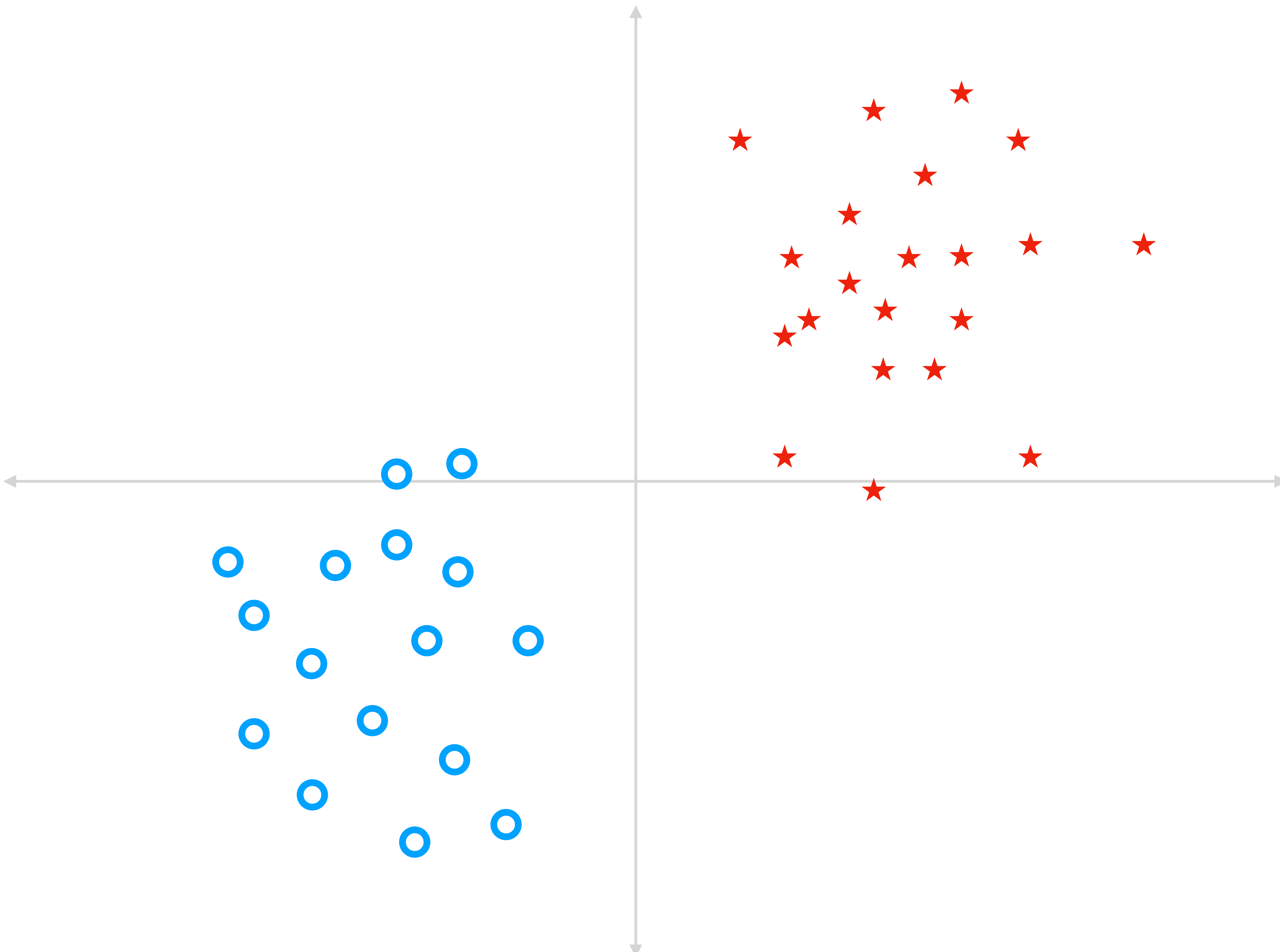
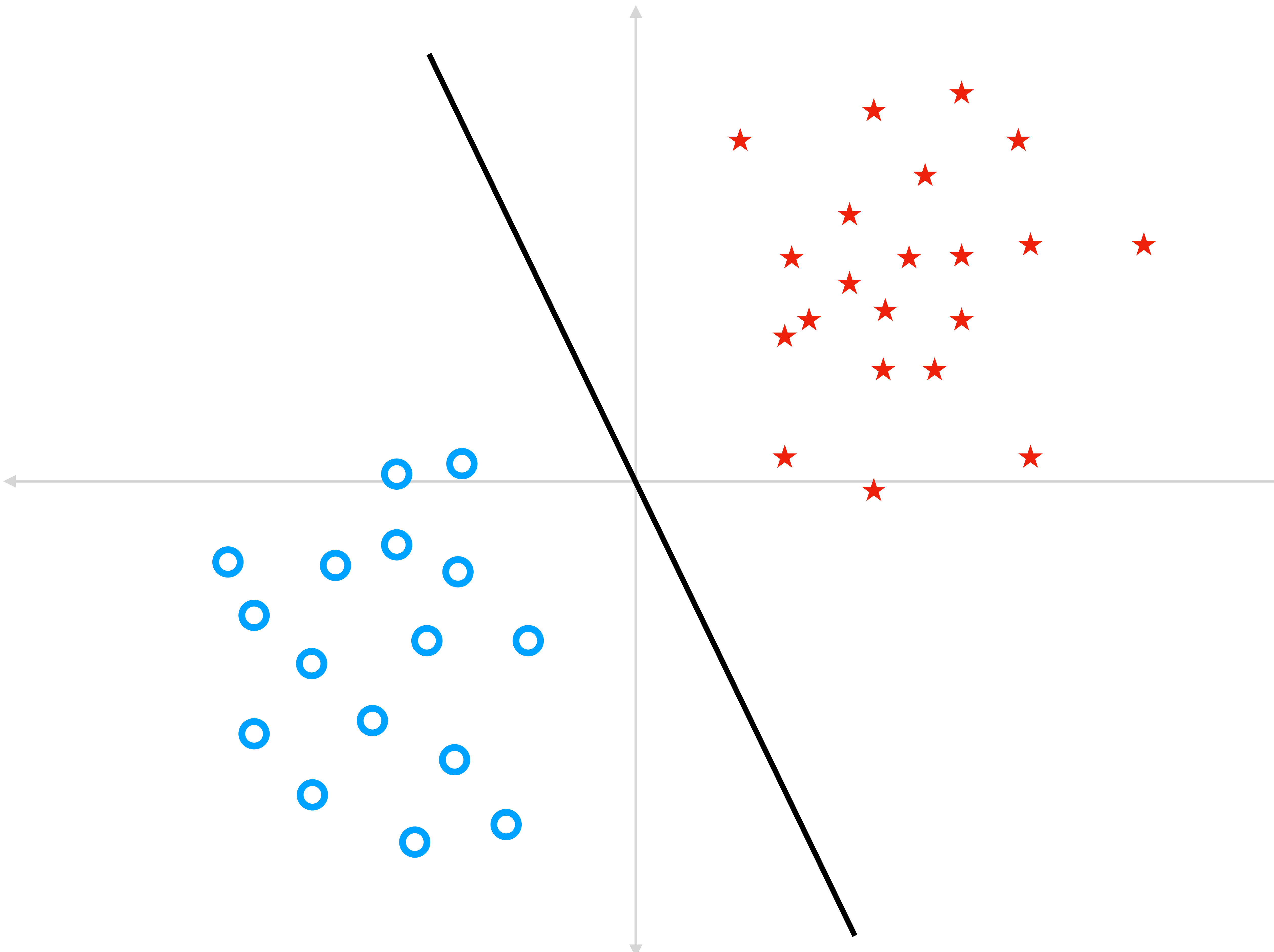
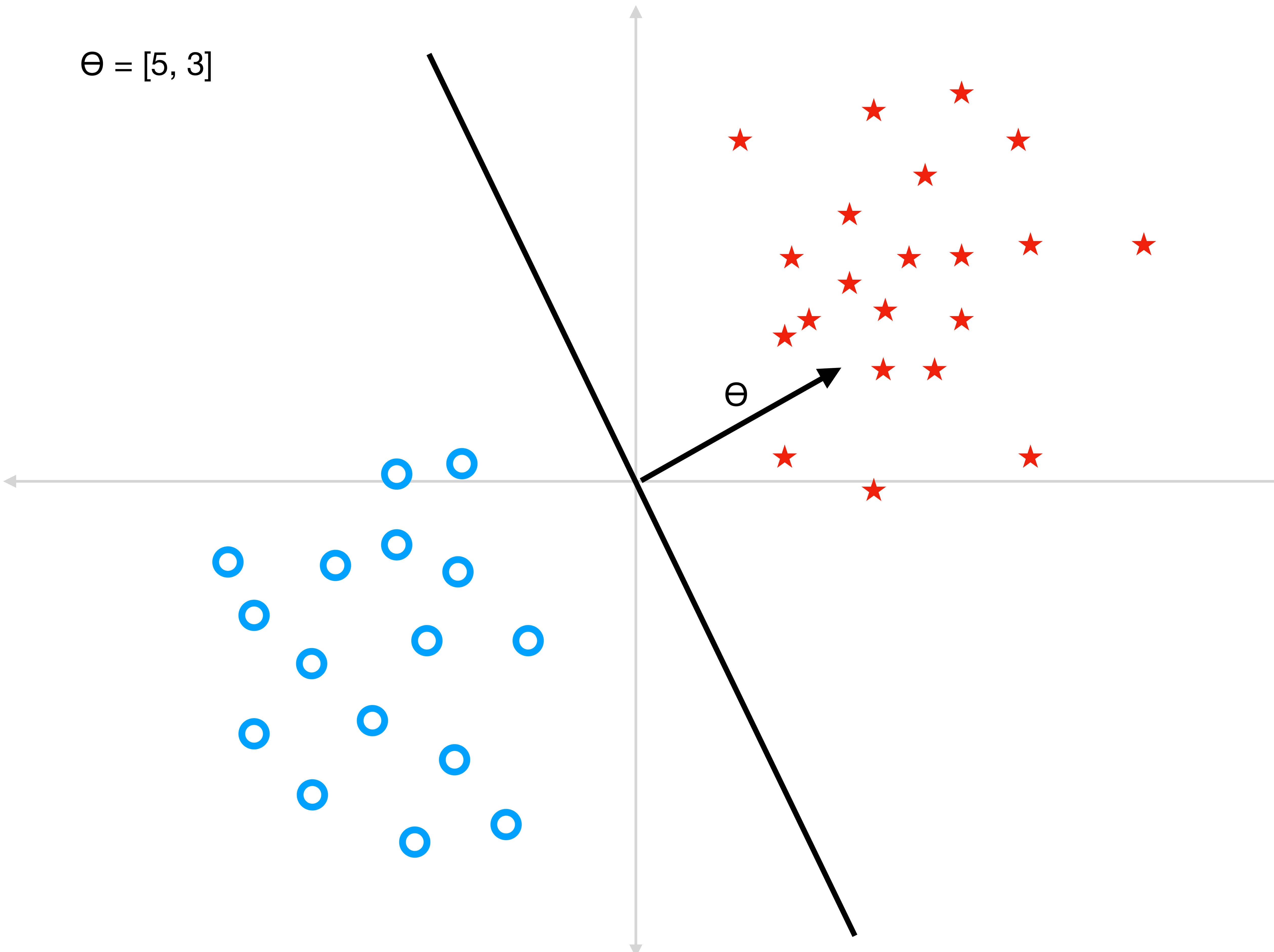




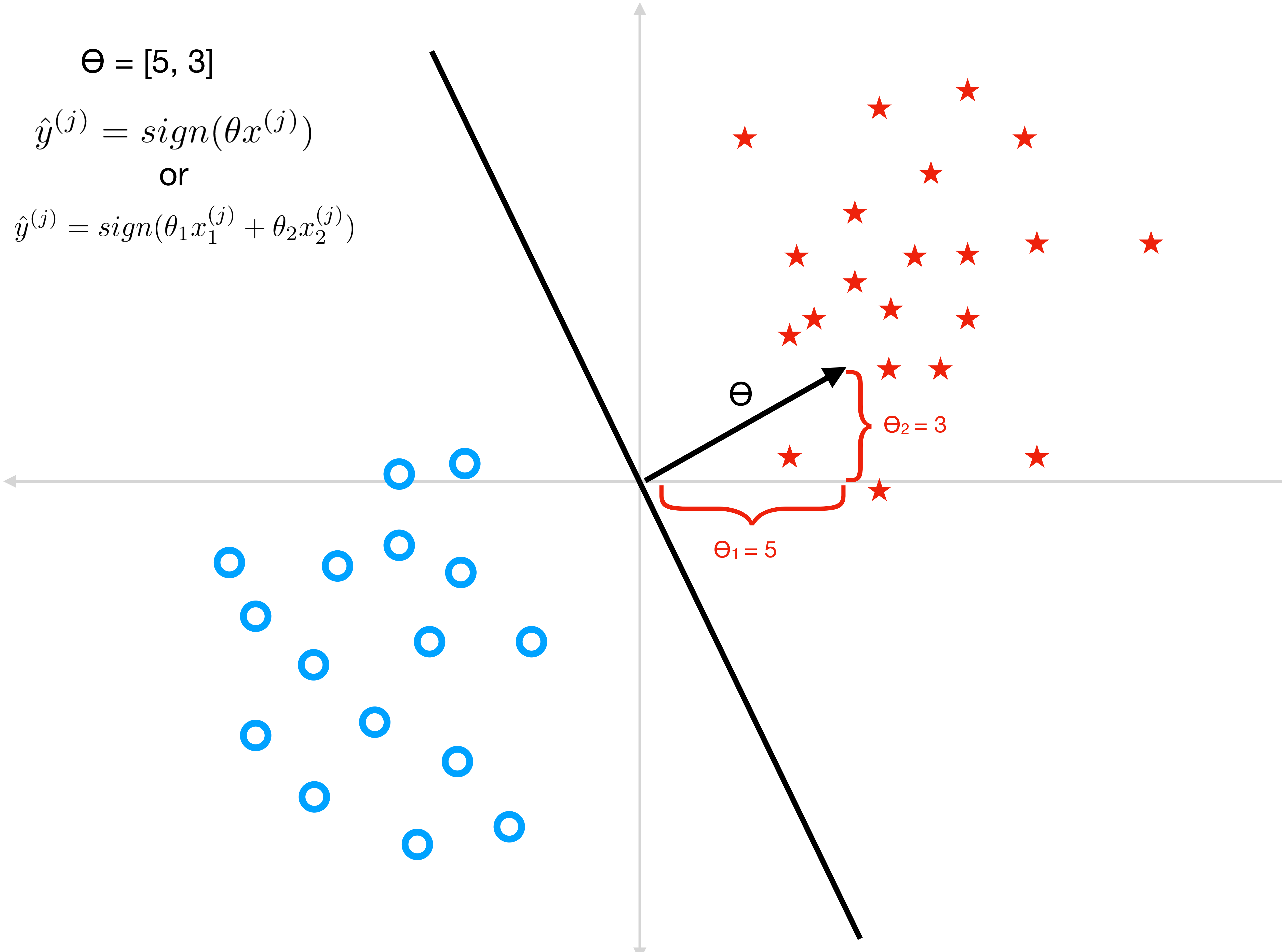

Θ = [5, 3]
Θ

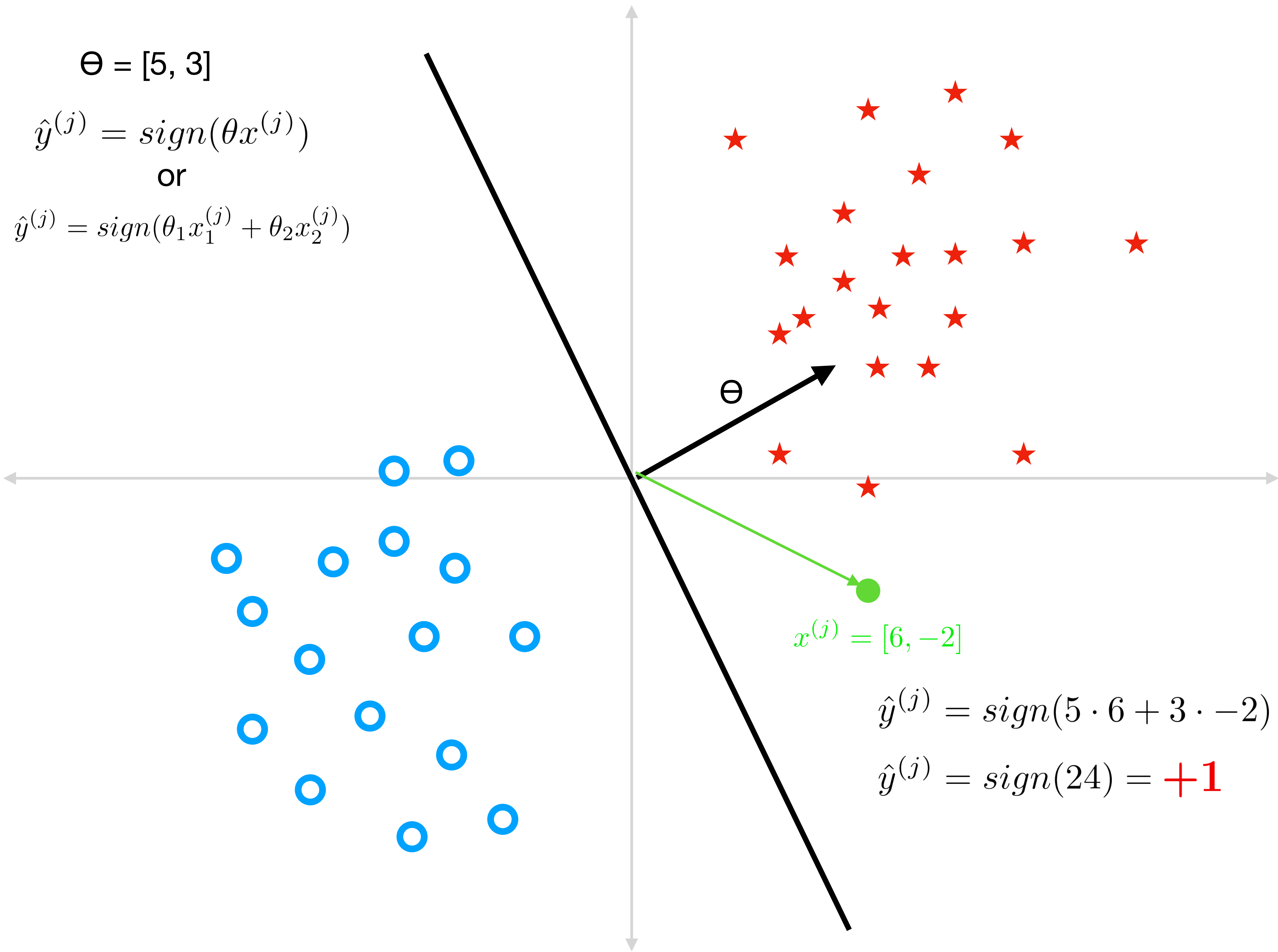
$\Theta = [5, 3]$

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

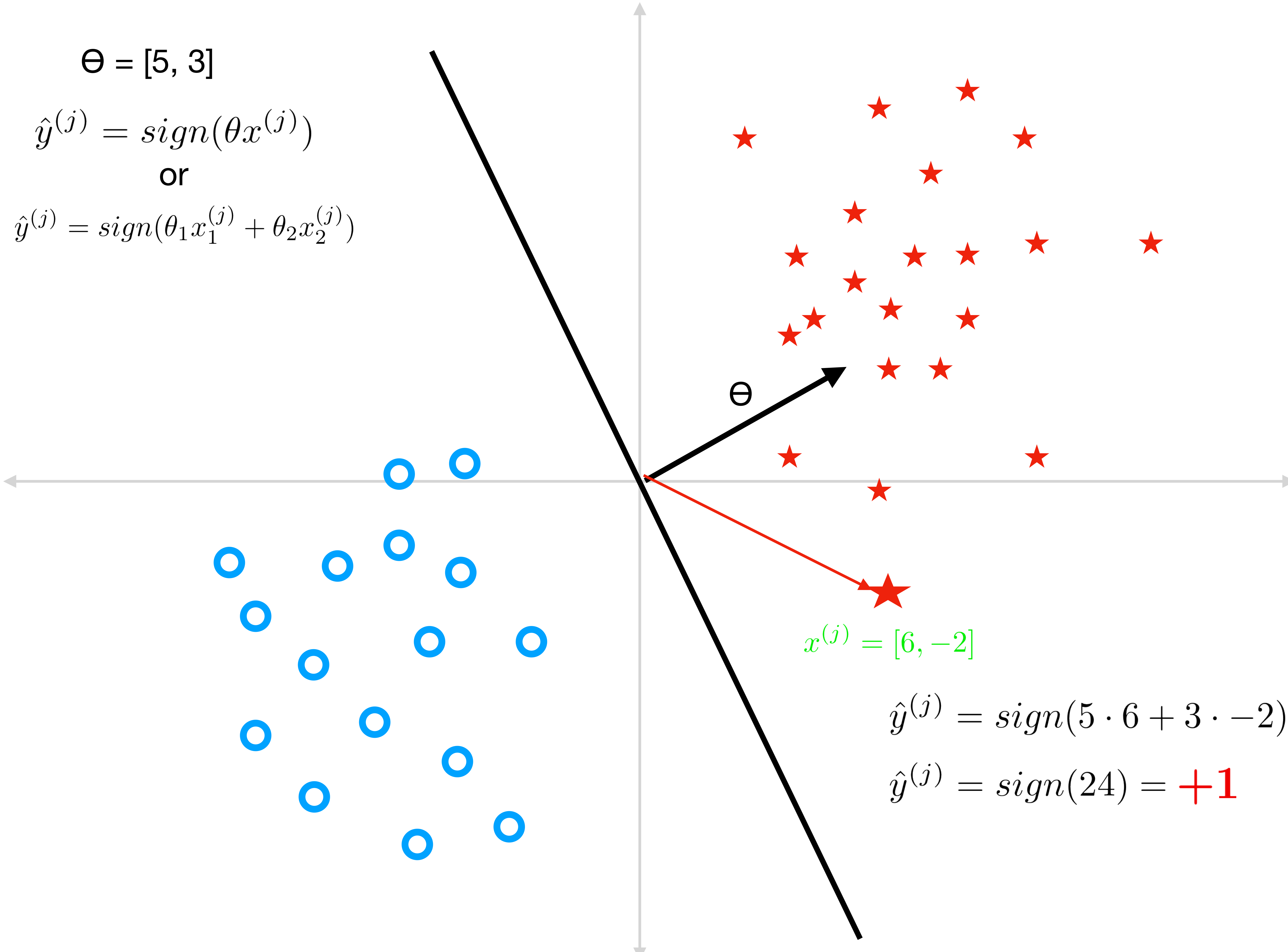
or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$





Θ = [5, 3]
$\hat{y}^{(j)} = sign(\theta x^{(j)})$
or
$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$
Θ
$x^{(j)} = [6, -2]$
$\hat{y}^{(j)} = sign(5 \cdot 6 + 3 \cdot -2)$
$\hat{y}^{(j)} = sign(24) = +1$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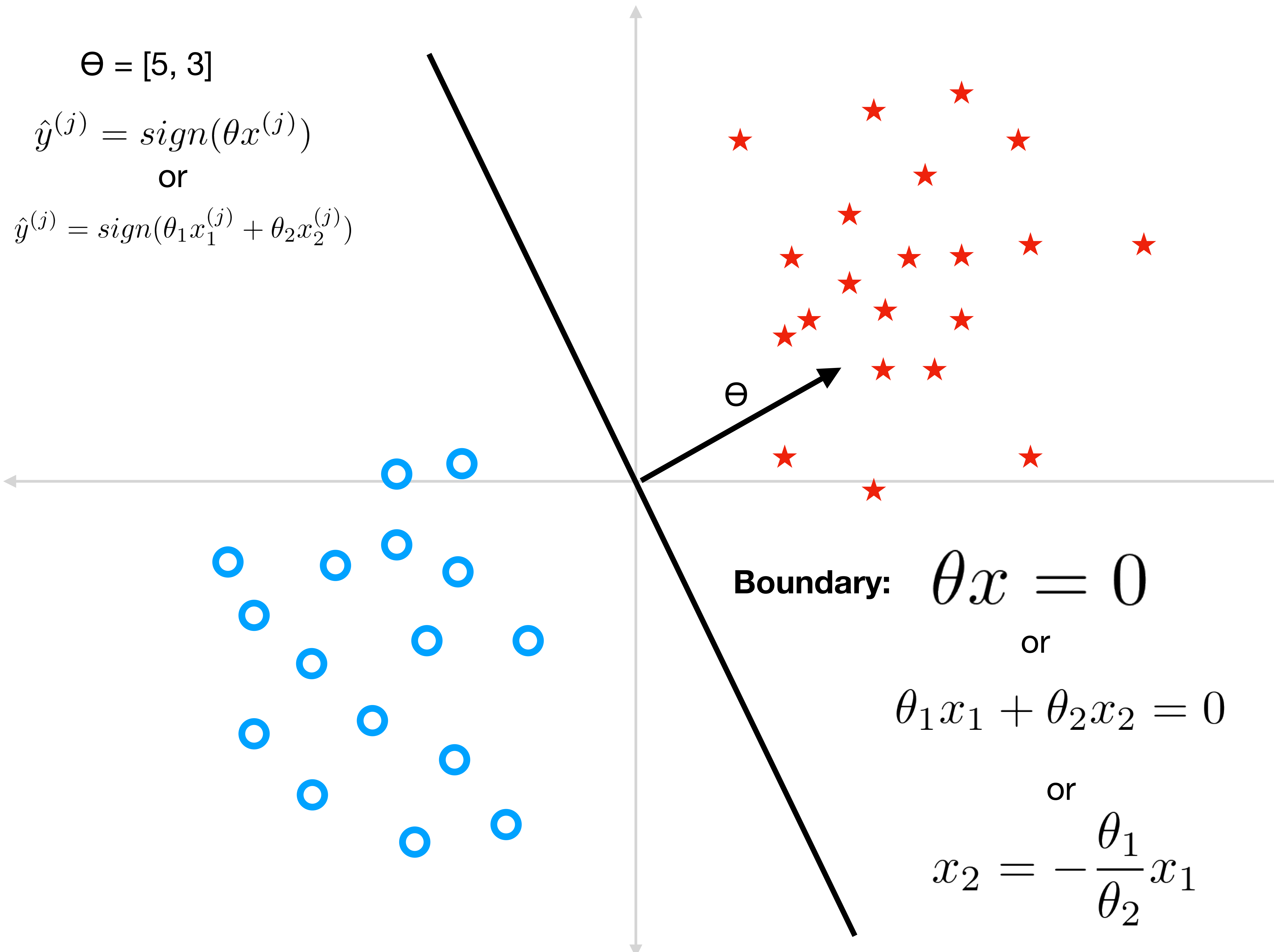


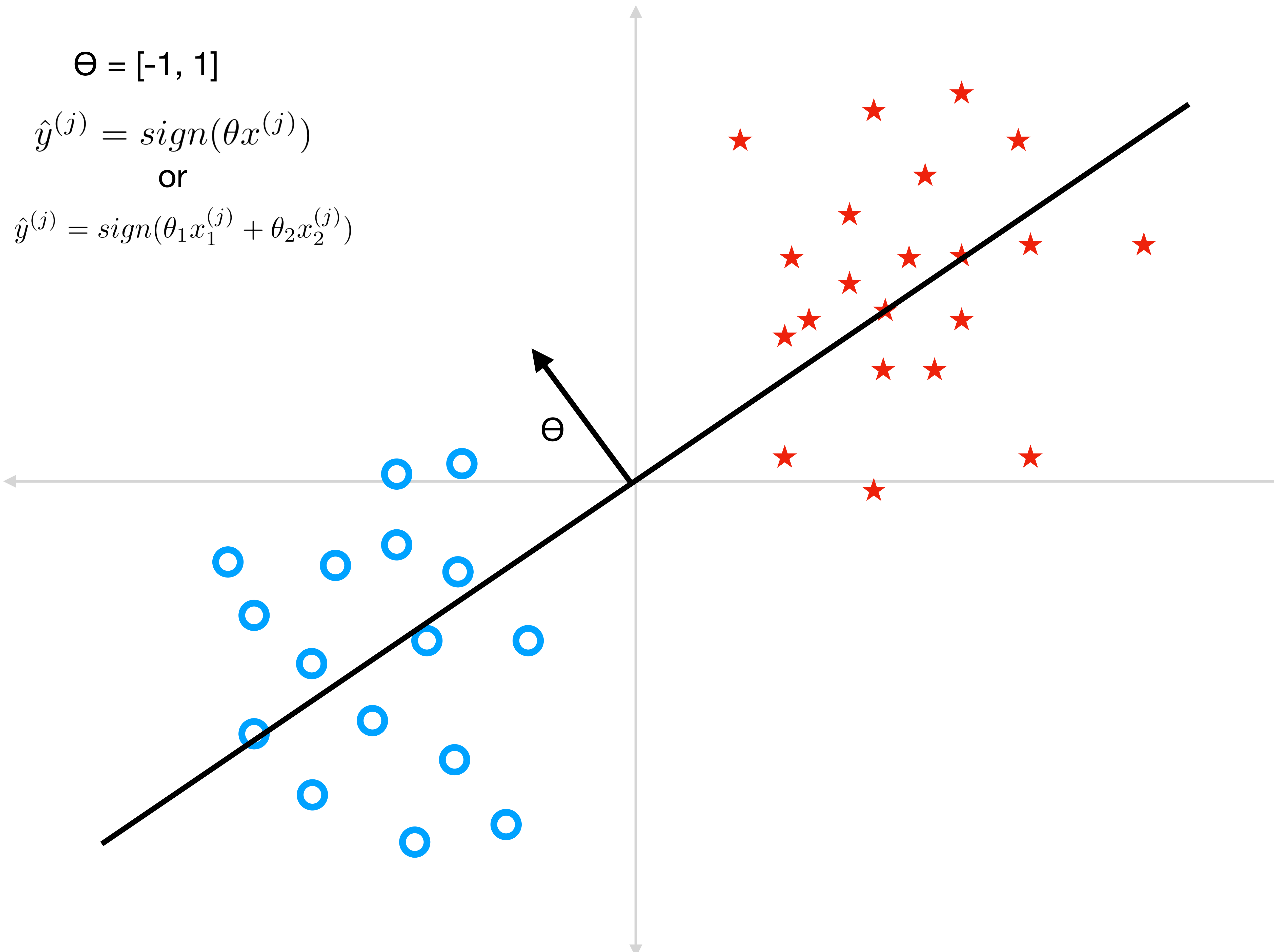
ϴ = [5, 3]
$\hat{y}^{(j)} = sign(\theta x^{(j)})$
or
$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$
ϴ
Boundary: $\theta x = 0$
or
$\theta_1 x_1 + \theta_2 x_2 = 0$
or
$x_2 = -\frac{\theta_1}{\theta_2} x_1$

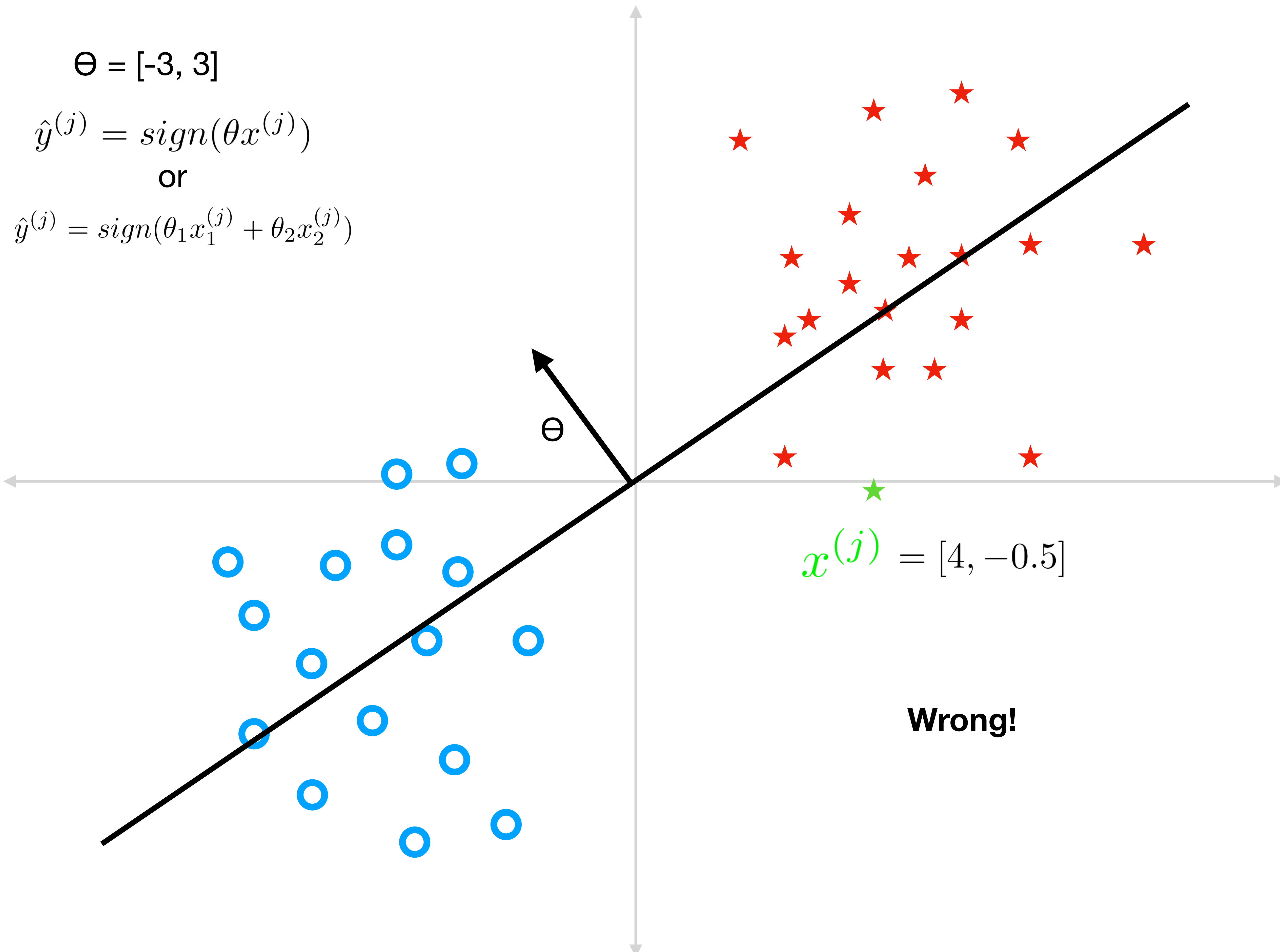
Θ = [-1, 1]

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



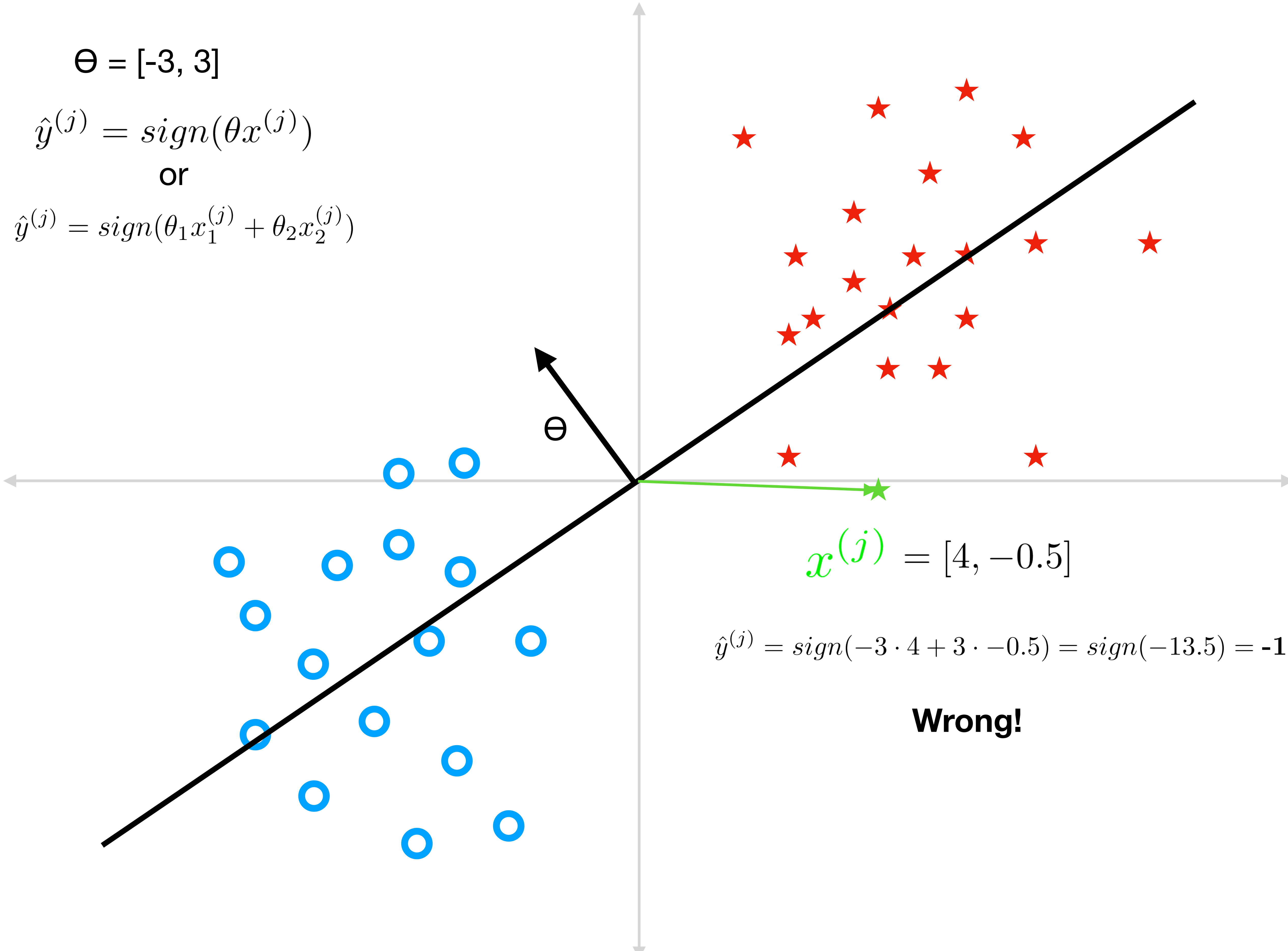

Θ = [-3, 3]
$\hat{y}^{(j)} = sign(\theta x^{(j)})$
or
$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$
Θ
$x^{(j)} = [4, -0.5]$
Wrong!

$\Theta = [-3, 3]$

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



$$x^{(j)} = [4, -0.5]$$

$$\hat{y}^{(j)} = sign(-3 \cdot 4 + 3 \cdot -0.5) = sign(-13.5) = \mathbf{-1}$$

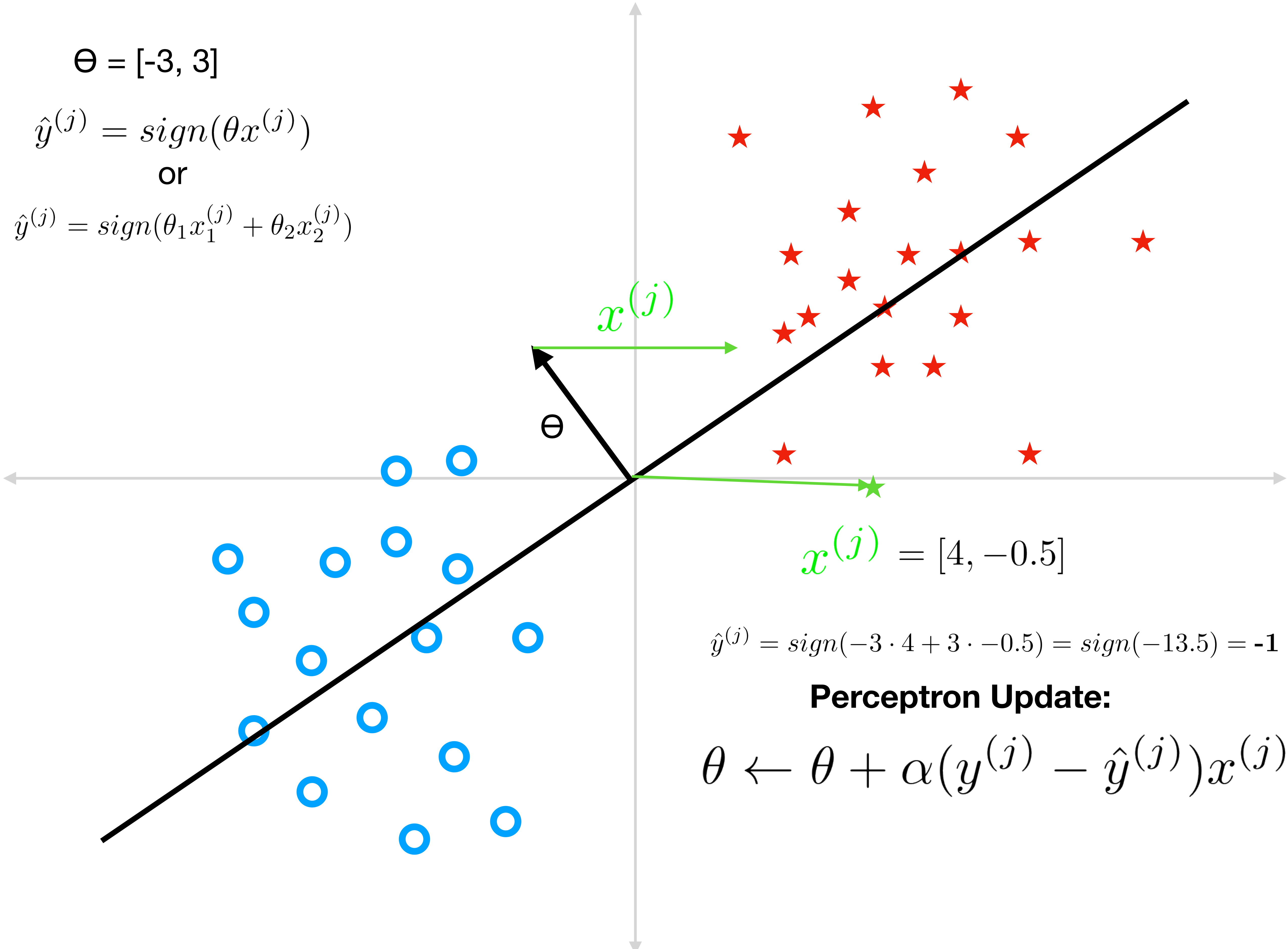
Wrong!

Θ = [-3, 3]

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



$$\hat{y}^{(j)} = sign(-3 \cdot 4 + 3 \cdot -0.5) = sign(-13.5) = \textbf{-1}$$

Perceptron Update:

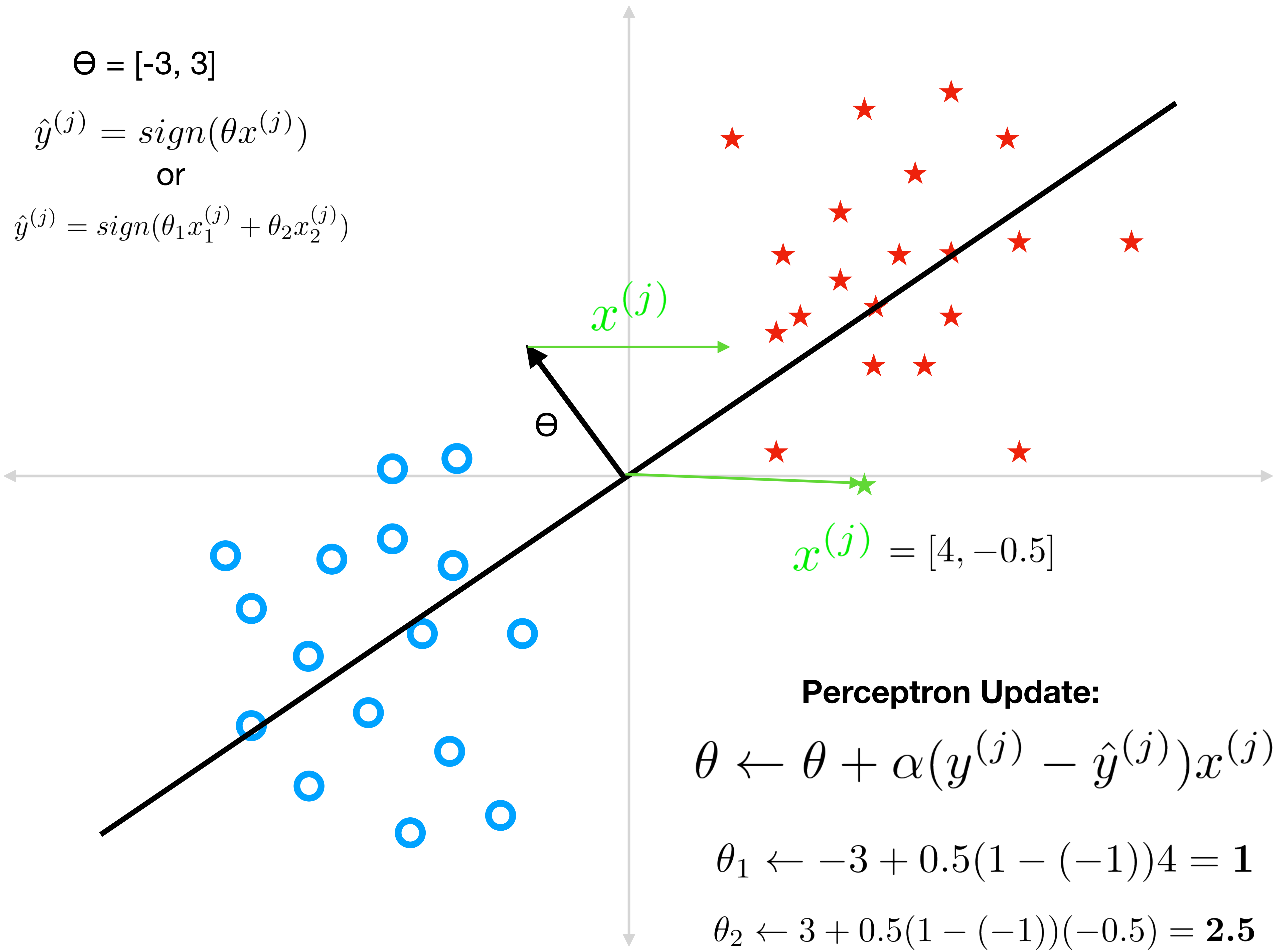
$$\theta \leftarrow \theta + \alpha(y^{(j)} - \hat{y}^{(j)})x^{(j)}$$

Θ = [-3, 3]

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



Perceptron Update:

$$\theta \leftarrow \theta + \alpha(y^{(j)} - \hat{y}^{(j)})x^{(j)}$$

$$\theta_1 \leftarrow -3 + 0.5(1 - (-1))4 = \mathbf{1}$$

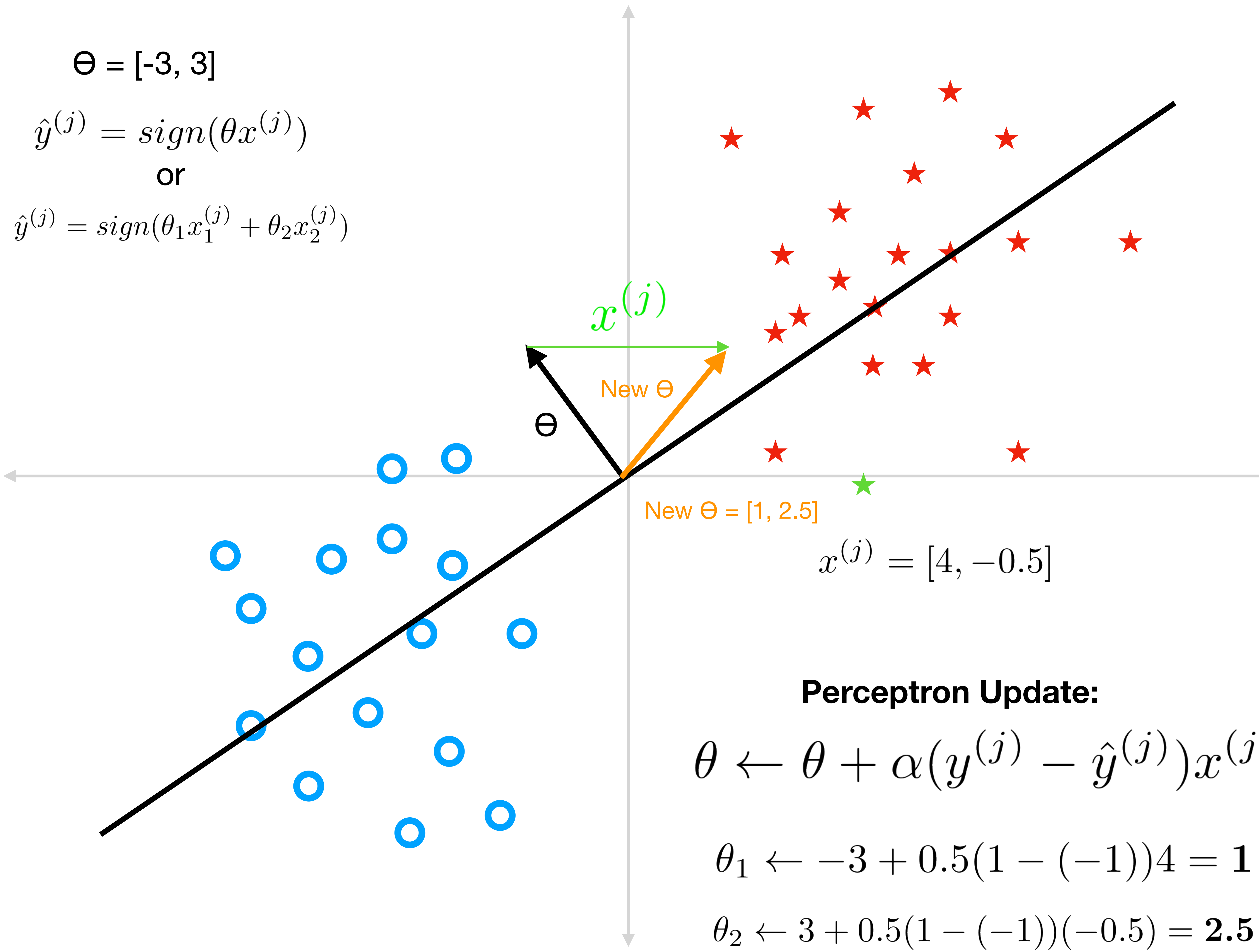
$$\theta_2 \leftarrow 3 + 0.5(1 - (-1))(-0.5) = \mathbf{2.5}$$

Θ = [-3, 3]

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



Perceptron Update:

$$\theta \leftarrow \theta + \alpha(y^{(j)} - \hat{y}^{(j)})x^{(j)}$$

$$\theta_1 \leftarrow -3 + 0.5(1 - (-1))4 = \mathbf{1}$$

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Θ = [-3, 3]

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

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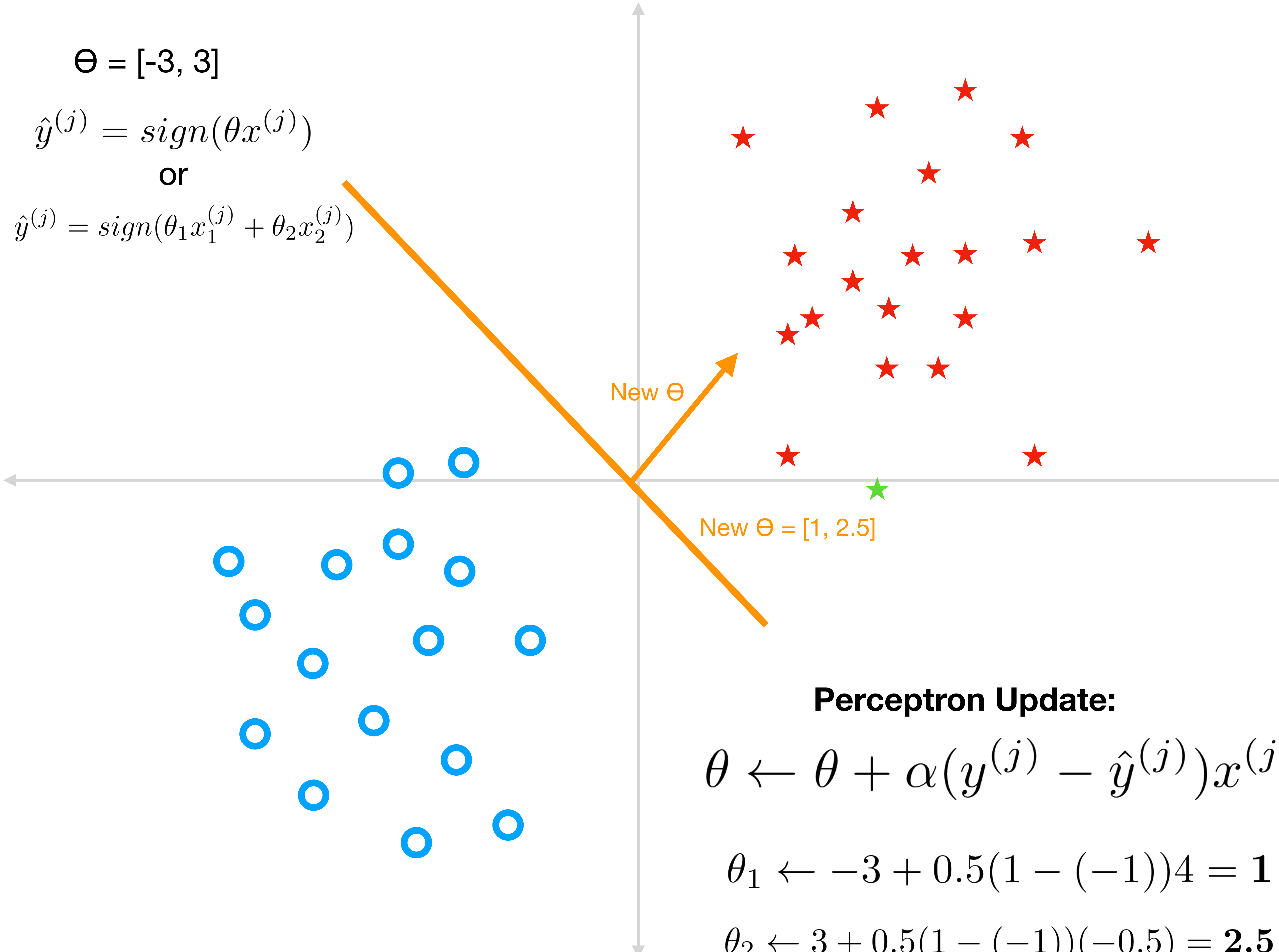


Perceptron Update:

$$\theta \leftarrow \theta + \alpha(y^{(j)} - \hat{y}^{(j)})x^{(j)}$$

$$\theta_1 \leftarrow -3 + 0.5(1 - (-1))4 = \mathbf{1}$$

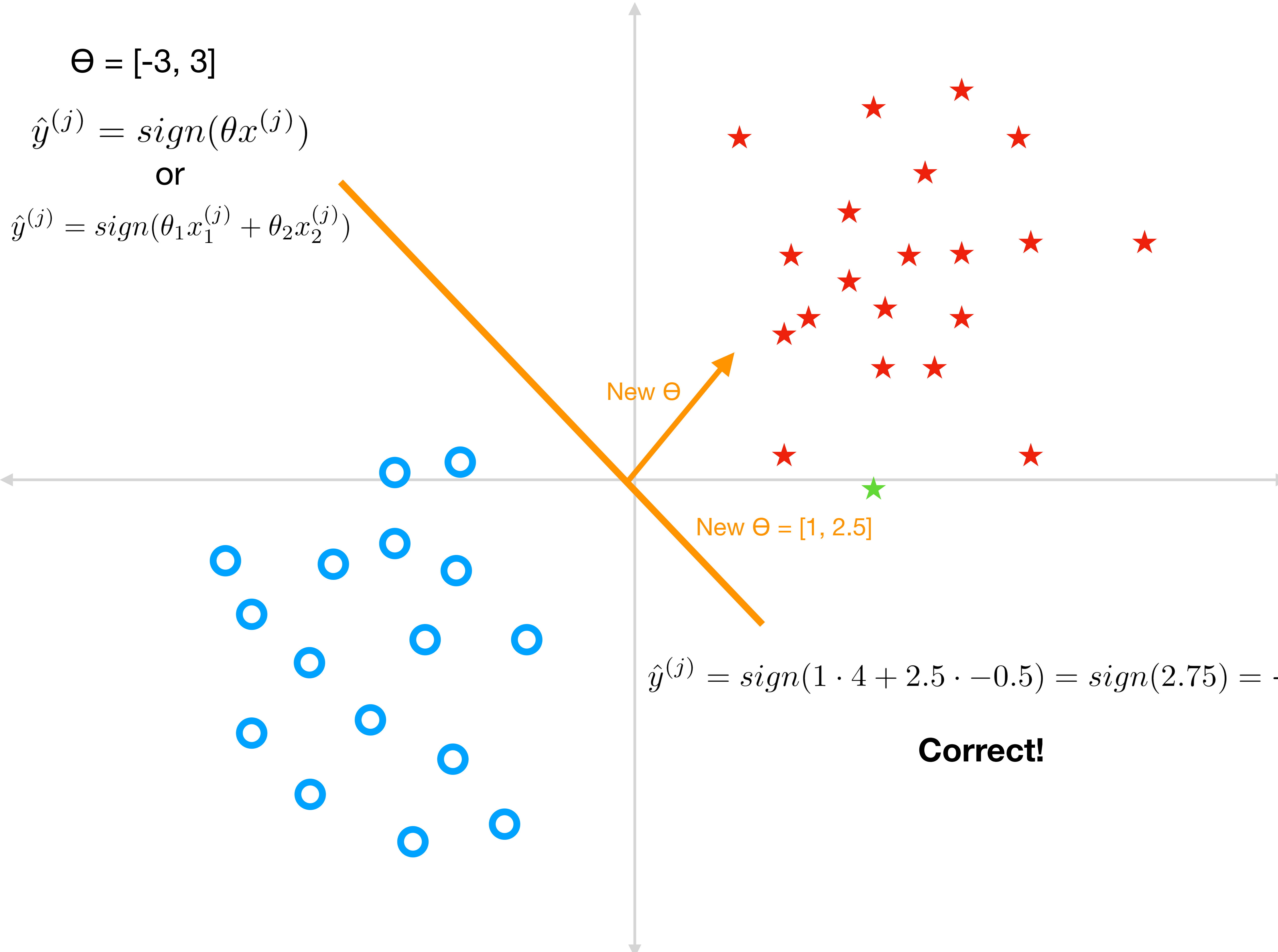
$$\theta_2 \leftarrow 3 + 0.5(1 - (-1))(-0.5) = \mathbf{2.5}$$

ϴ = [-3, 3]

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



$$\hat{y}^{(j)} = sign(1 \cdot 4 + 2.5 \cdot -0.5) = sign(2.75) = \mathbf{+1}$$

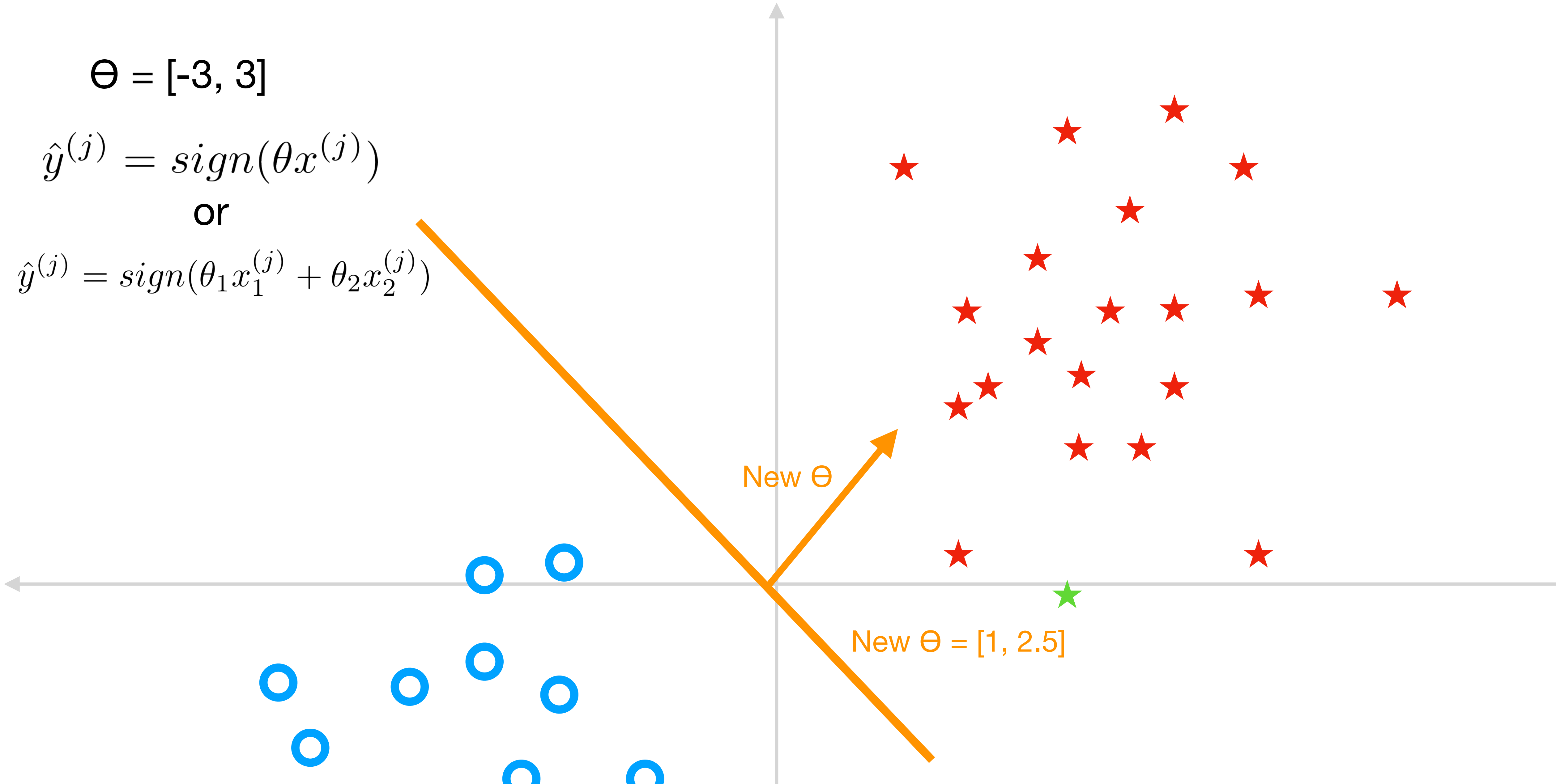
Correct!

Θ = [-3, 3]

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



$$\hat{y}^{(j)} = sign(1 \cdot 4 + 2.5 \cdot -0.5) = sign(2.75) = +1$$

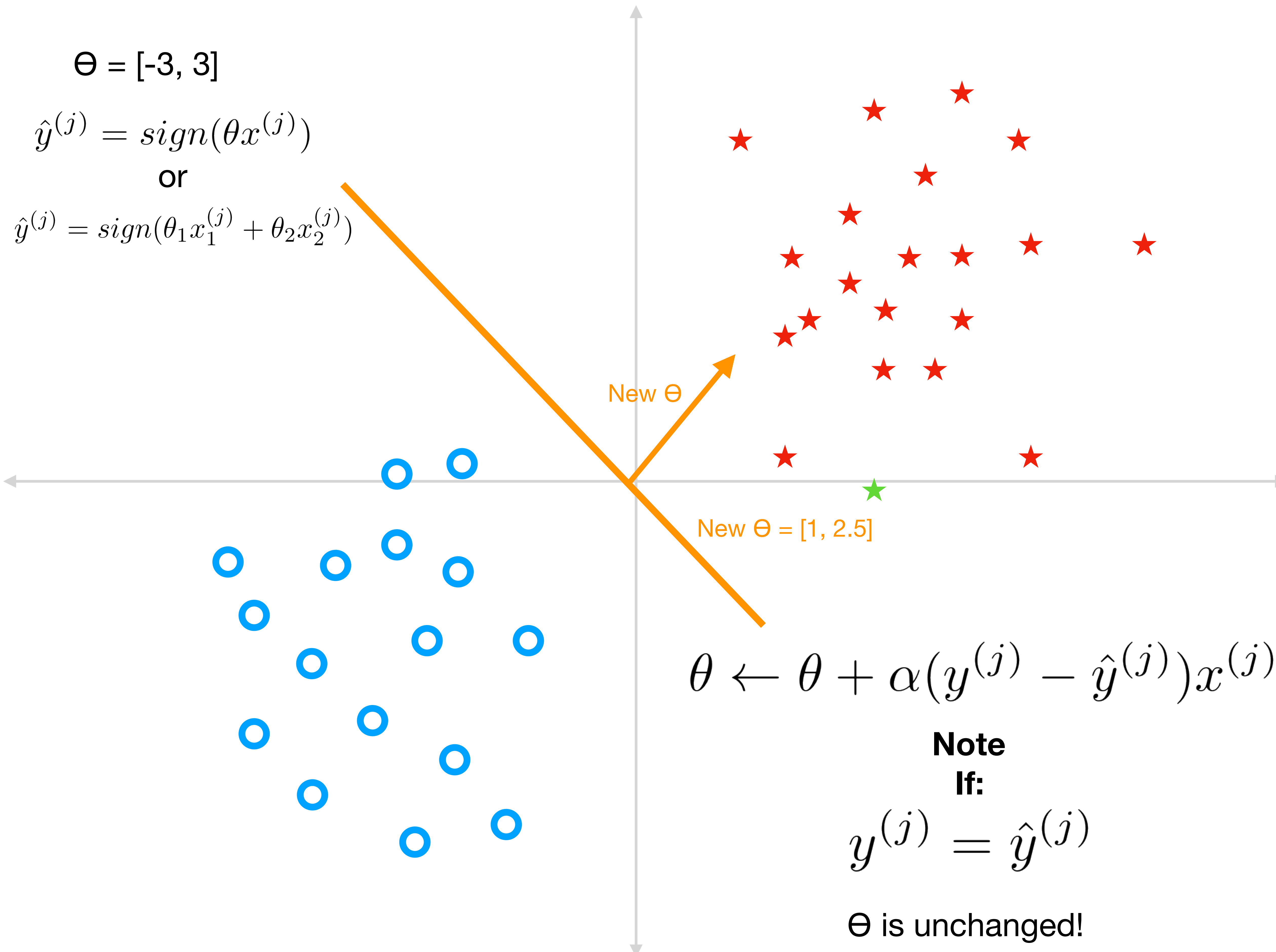
Correct!

Θ = [-3, 3]

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$

or

$$\hat{y}^{(j)} = sign(\theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



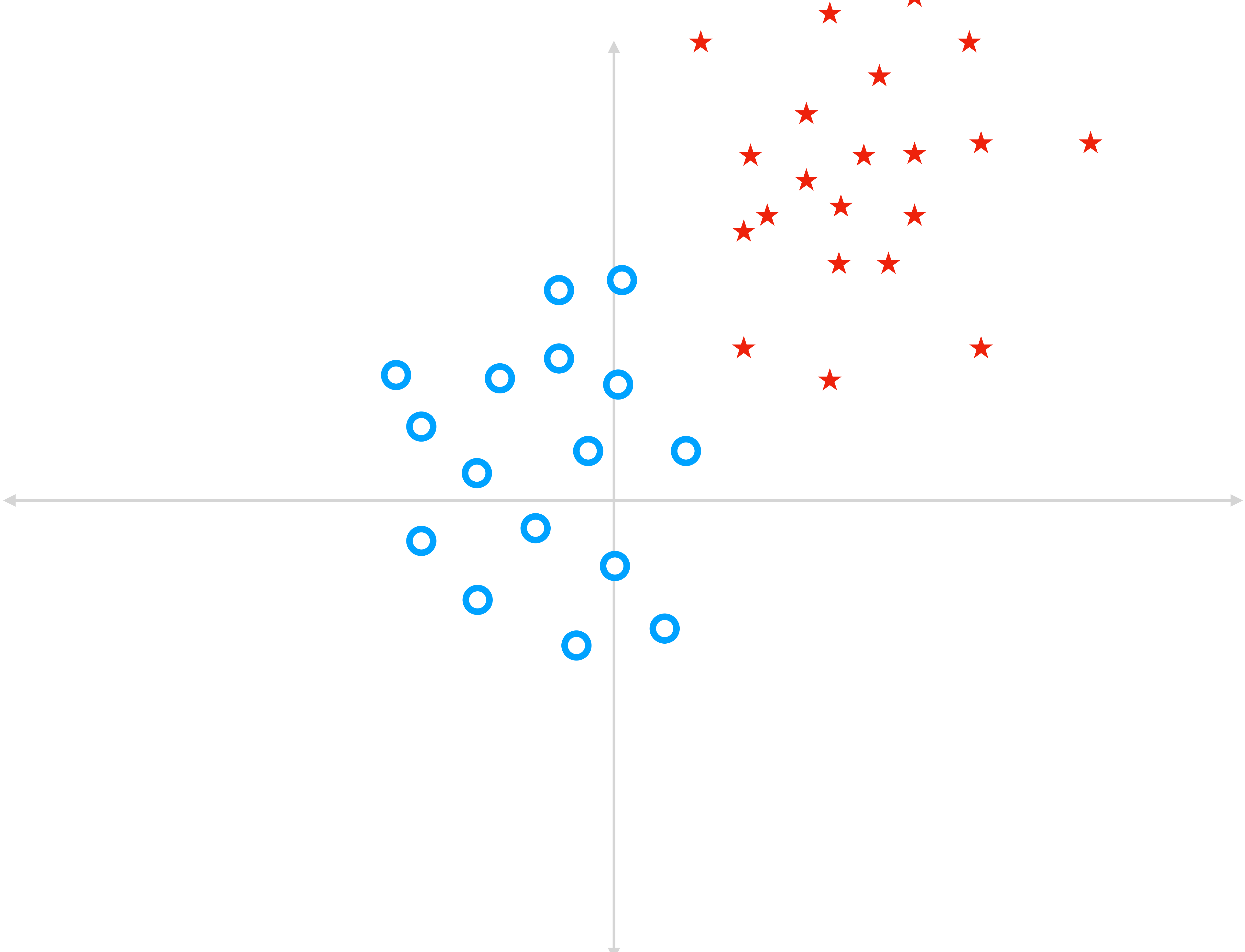
$$\theta \leftarrow \theta + \alpha(y^{(j)} - \hat{y}^{(j)})x^{(j)}$$

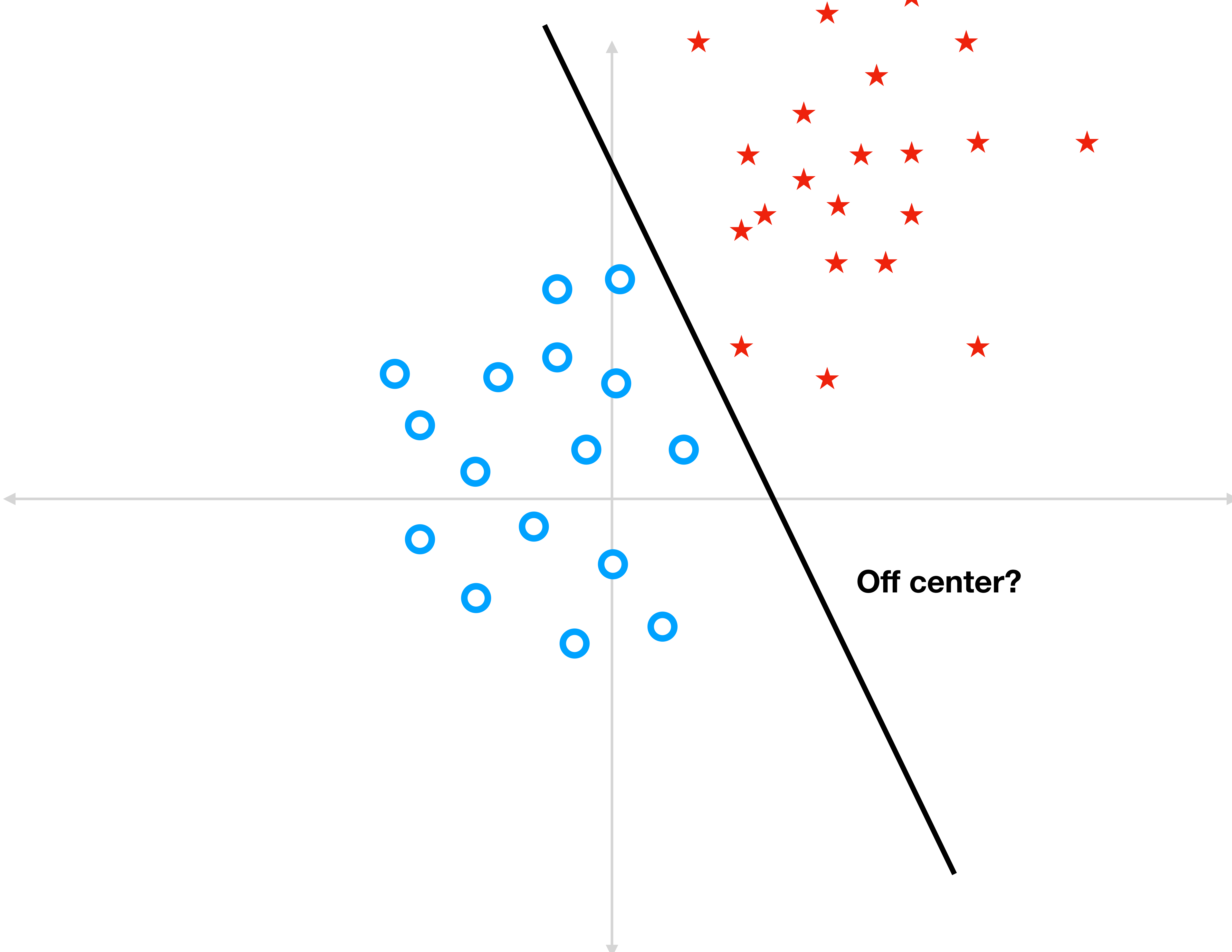
Note

If:

$$y^{(j)} = \hat{y}^{(j)}$$

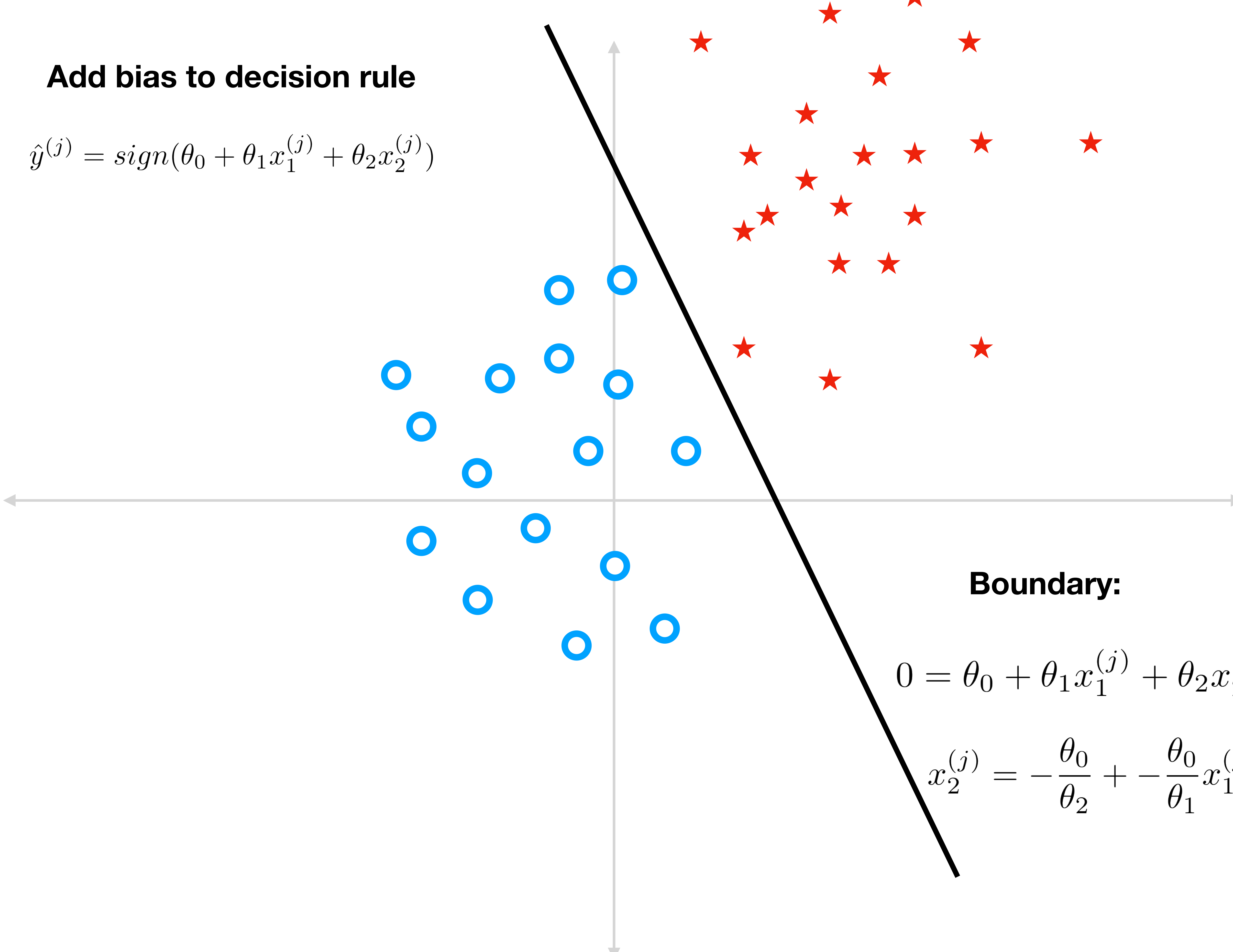
Θ is unchanged!




Off center?

Add bias to decision rule

$$\hat{y}^{(j)} = sign(\theta_0 + \theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



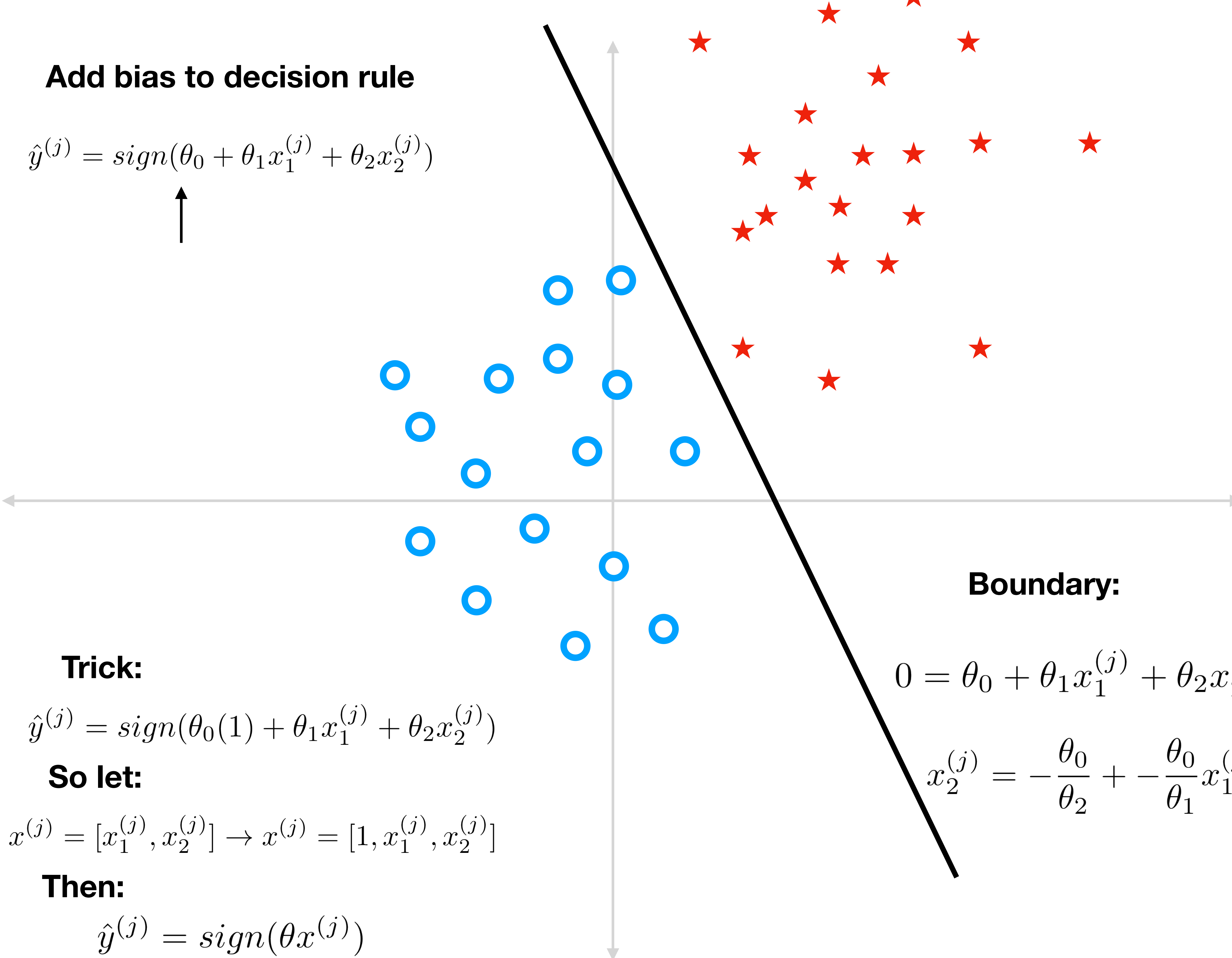
Boundary:

$$0 = \theta_0 + \theta_1 x_1^{(j)} + \theta_2 x_2^{(j)}$$

$$x_2^{(j)} = -\frac{\theta_0}{\theta_2} + -\frac{\theta_0}{\theta_1} x_1^{(j)}$$

Add bias to decision rule

$$\hat{y}^{(j)} = sign(\theta_0 + \theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$



Boundary:

$$0 = \theta_0 + \theta_1 x_1^{(j)} + \theta_2 x_2^{(j)}$$

$$x_2^{(j)} = -\frac{\theta_0}{\theta_2} + -\frac{\theta_0}{\theta_1} x_1^{(j)}$$

Trick:

$$\hat{y}^{(j)} = sign(\theta_0(1) + \theta_1 x_1^{(j)} + \theta_2 x_2^{(j)})$$

So let:

$$x^{(j)} = [x_1^{(j)}, x_2^{(j)}] \rightarrow x^{(j)} = [1, x_1^{(j)}, x_2^{(j)}]$$

Then:

$$\hat{y}^{(j)} = sign(\theta x^{(j)})$$