# Assignment_2

January 28, 2024

## 1 Problem 1: Linear Regression from Scratch (30 points)

[1]:
```
# import the necessary packages
import numpy as np
from matplotlib import pyplot as plt
np.random.seed(100)
```

Let's generate some data points first, by the equation $y = x - 3$.

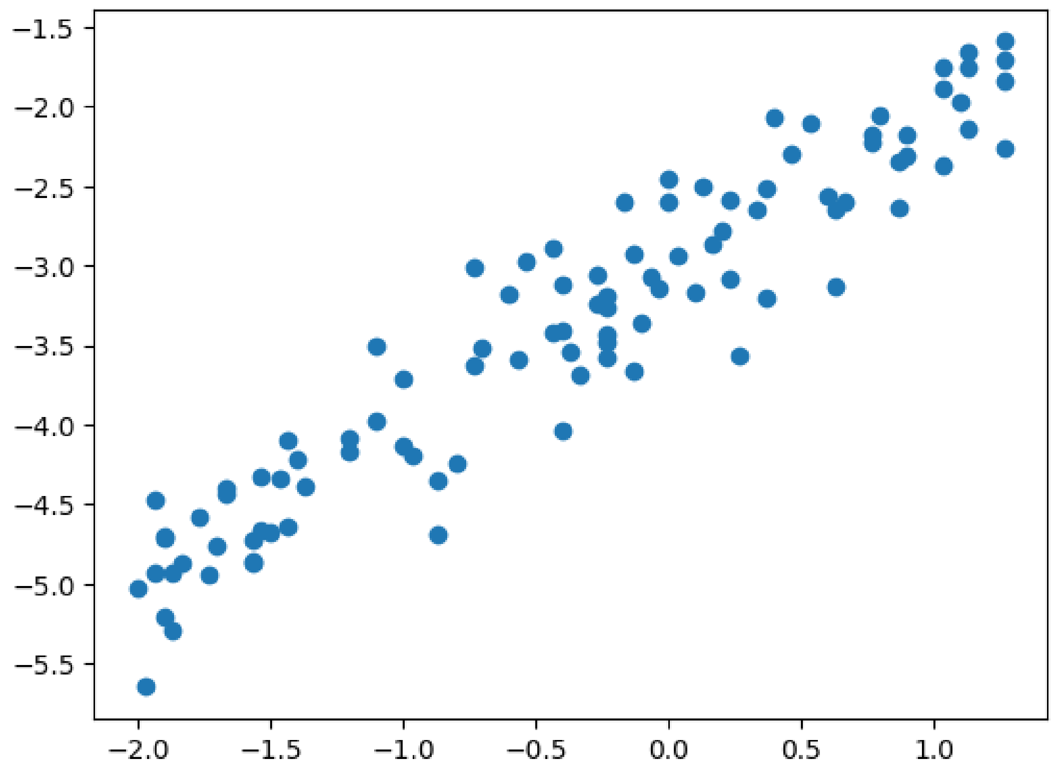
[2]:
```
x = np.random.randint(100, size=100)/30 - 2
X = x.reshape(-1, 1)

y = x + -3 + 0.3*np.random.randn(100)
```

Let's then visualize the data points we just created.

[3]:
```
plt.scatter(X, y)
```

[3]:
```
<matplotlib.collections.PathCollection at 0x7fb8930e2430>
```




## 1.1 1.1 Gradient of vanilla linear regression model (5 points)

In the lecture, we learn that the cost function of a linear regression model can be expressed as **Equation 1**:

$$J(\theta) = \frac{1}{2m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

The gredient of it can be written as **Equation 2**:

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)\left(x^{(i)}\right)$$

## 1.2 1.2 Gradient of vanilla regularized regression model (5 points)

After adding the L2 regularization term, the linear regression model can be expressed as **Equation 3**:

$$J(\theta) = \frac{1}{2m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j}^{n}(\theta_j)^2$$

The gredient of it can be written as **Equation 4**:

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{m}(\sum_{i}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)\left(x^{(i)}\right) + \lambda \sum_{i}^{n} \theta_j)$$

### 1.3 1.3 Implement the cost function of a regularized regression model (5 points)

Please implement the cost function of a regularized regression model according to the above equations.

### 1.4 1.4 Implement the gradient of the cost function of a regularized regression model (5 points)

Please implement the gradient of the cost function of a regularized regression model according to the above equations.

[4]:
```
def regularized_linear_regression(X, y, alpha=0.01, lambda_value=1, epochs=30):
    """
    :param x: feature matrix
    :param y: target vector
    :param alpha: learning rate (default:0.01)
    :param lambda_value: lambda (default:1)
    :param epochs: maximum number of iterations of the
           linear regression algorithm for a single run (default=30)
    :return: weights, list of the cost function changing overtime
    """

    m = np.shape(X)[0]  # total number of samples
    n = np.shape(X)[1]  # total number of features

    X = np.concatenate((np.ones((m, 1)), X), axis=1)
    W = np.random.randn(n + 1, )

    # stores the updates on the cost function (loss function)
    cost_history_list = []

    # iterate until the maximum number of epochs
    for current_iteration in np.arange(epochs):  # begin the process

        # compute the dot product between our feature 'X' and weight 'W'
        y_estimated = X.dot(W)

        # calculate the difference between the actual and predicted value
        error = y_estimated - y


########################################################################################
↪
```

```
######################################## Begin of Question 1.3␣
 ↪################################################
###############################################################################################
        ##### Please write down your code here:####

        # calculate the cost (MSE) (Equation 1)
        cost_without_regularization = (1/(2*m))*np.sum(np.power(error, 2))


        ##### Please write down your code here:####

        # regularization term
        reg_term = (lambda_value/(2*m))*np.sum(np.square(W))


        # calculate the cost (MSE) + regularization term (Equation 3)
        cost_with_regularization = cost_without_regularization + reg_term

###############################################################################################
 ↪
######################################## End of Question 1.3␣
 ↪##################################################
###############################################################################################
 ↪


###############################################################################################
 ↪
######################################## Begin of Question 1.4␣
 ↪################################################
###############################################################################################
 ↪

        ##### Please write down your code here:####

        # calculate the gradient of the cost function with regularization term␣
 ↪(Equation )
        gradient = (1/m)*(X.T.dot(error)+(lambda_value*W))


        # Now we have to update our weights
        W = W - alpha * gradient

###############################################################################################
 ↪
```

```
######################################### End of Question 1.4␣
↪#################################################
################################################################################################
↪



        # keep track the cost as it changes in each iteration
        cost_history_list.append(cost_with_regularization)

    # Let's print out the cost
    print(f"Cost with regularization: {cost_with_regularization}")
    print(f"Mean square error: {cost_without_regularization}")

    return W, cost_history_list
```

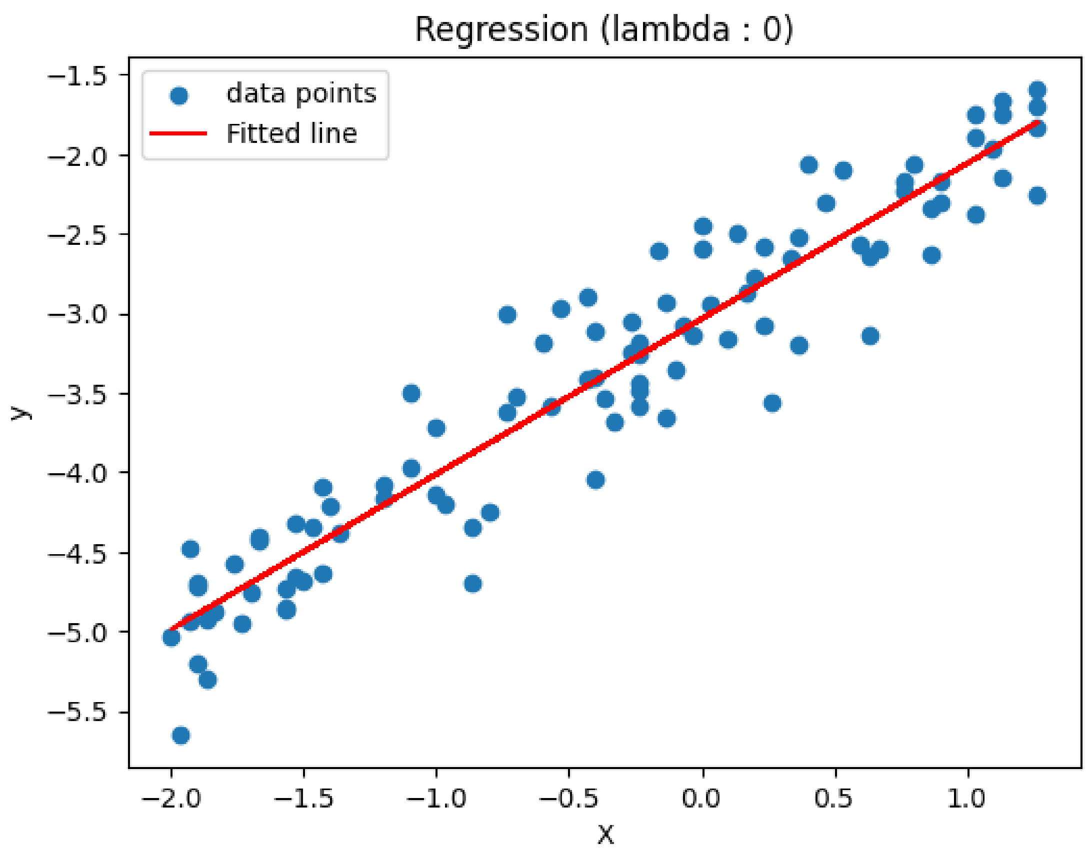
Run the following code to train your model.

Hint: If you have the correct code written above, the cost should be 0.5181222986588751 when $\lambda = 10$.

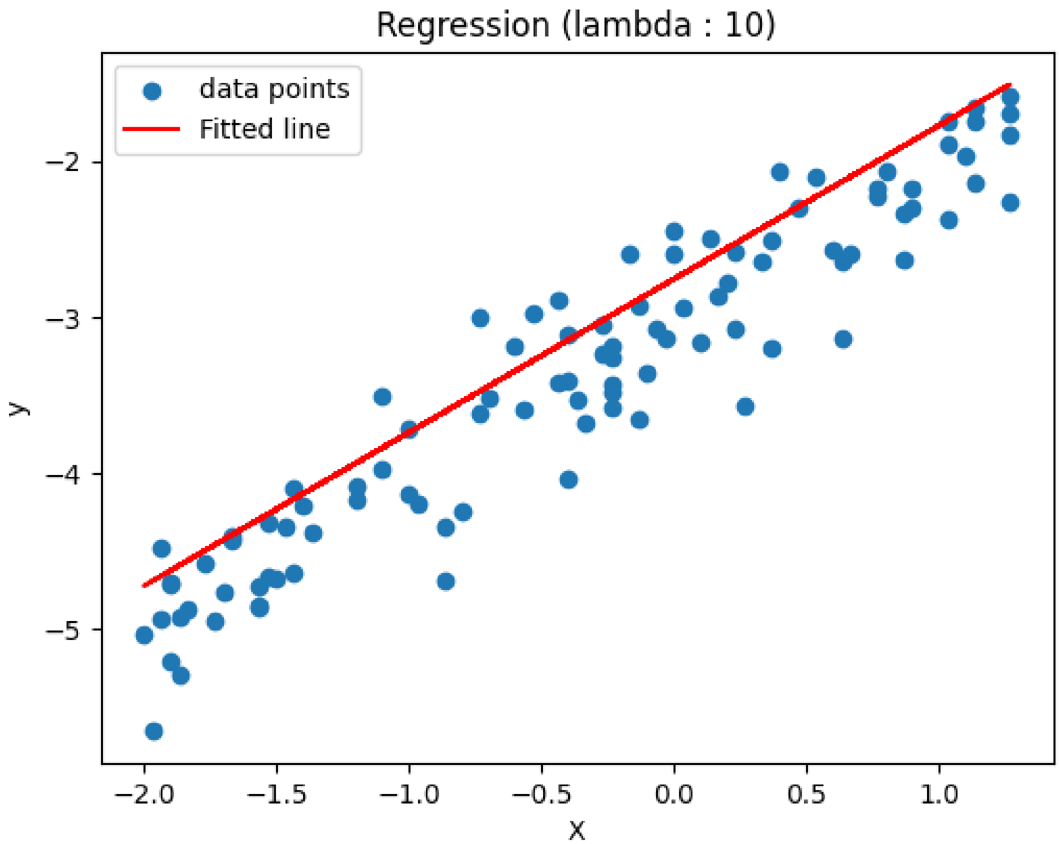
[5]:
```
lambda_list = [0, 10, 100, 1000, 10000]
for lambda_ in lambda_list:
    # calls regression function with different values of lambda
    weight, _ = regularized_linear_regression(X, y, alpha=0.01,
                                 lambda_value=lambda_, epochs=1000)

    fitted_line = np.dot(X, weight[1]) + weight[0]
    plt.scatter(X, y, label='data points')
    plt.plot(X, fitted_line, color='r', label='Fitted line')
    plt.xlabel("X")
    plt.ylabel("y")
    plt.title(f"Regression (lambda : {lambda_})")
    plt.legend()
    plt.show()
```

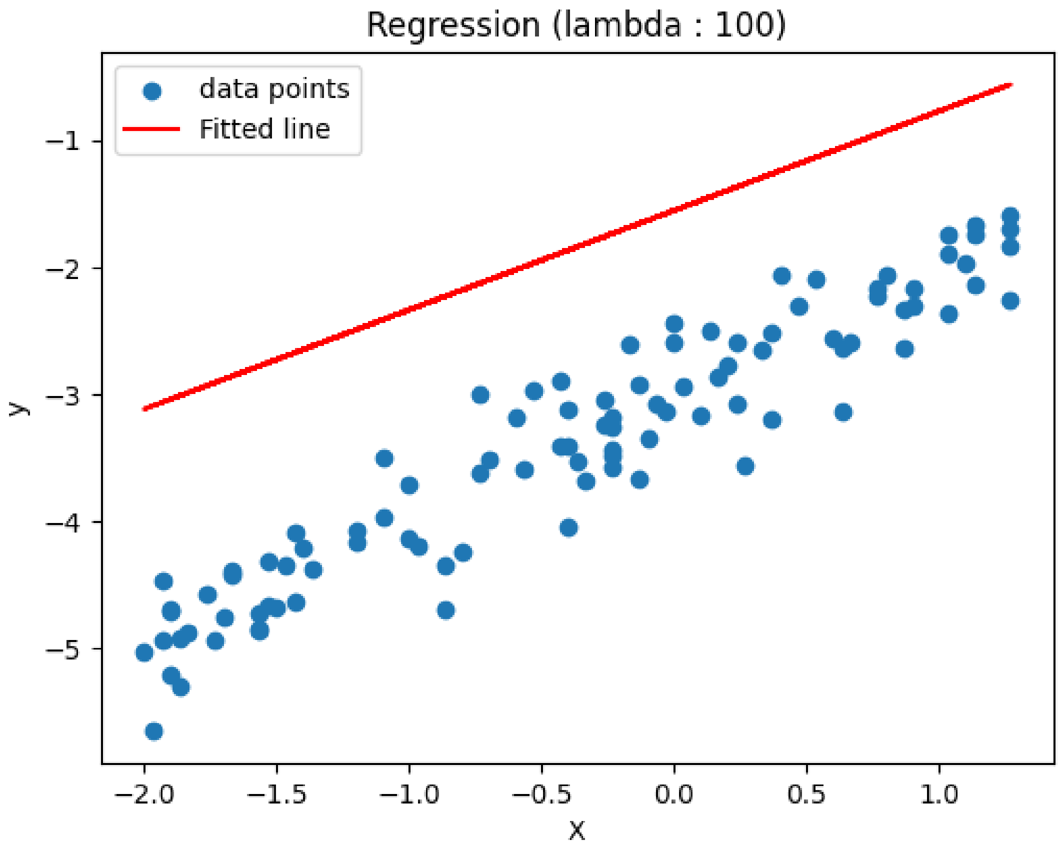
```
Cost with regularization: 0.05165888565058274
Mean square error: 0.05165888565058274
```



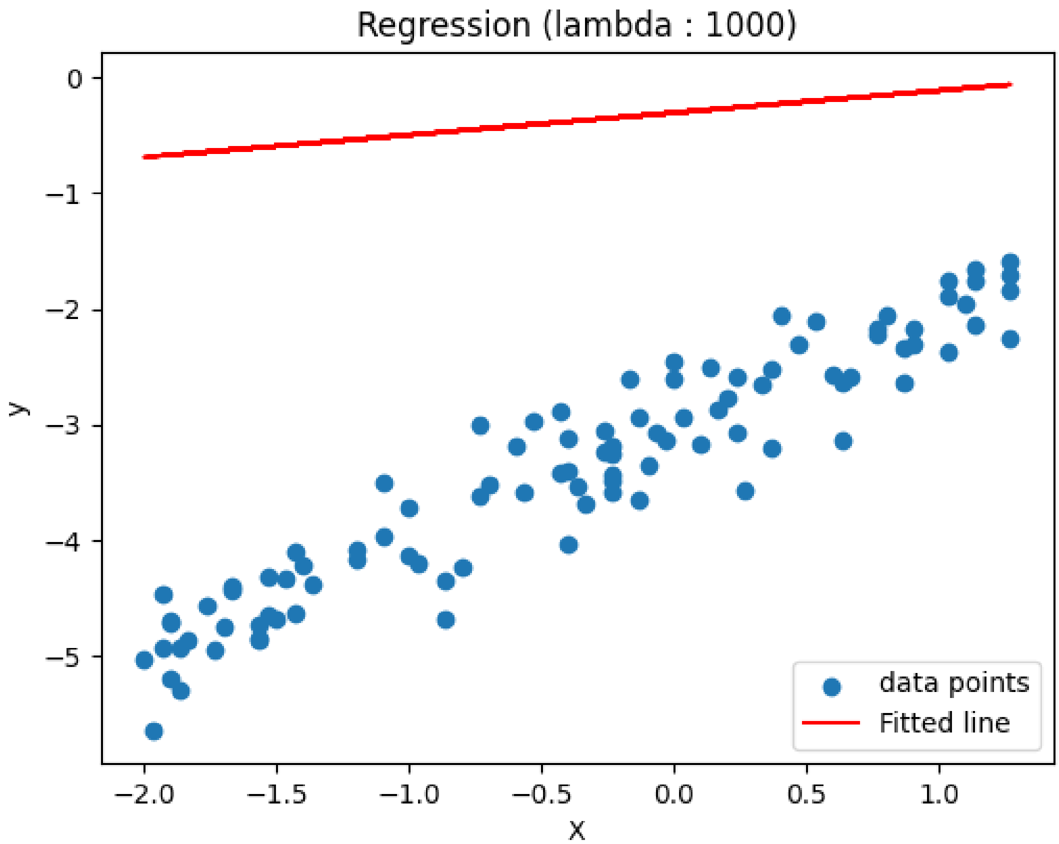

```
Cost with regularization: 0.5181225049184746
Mean square error: 0.08982014821513126
```



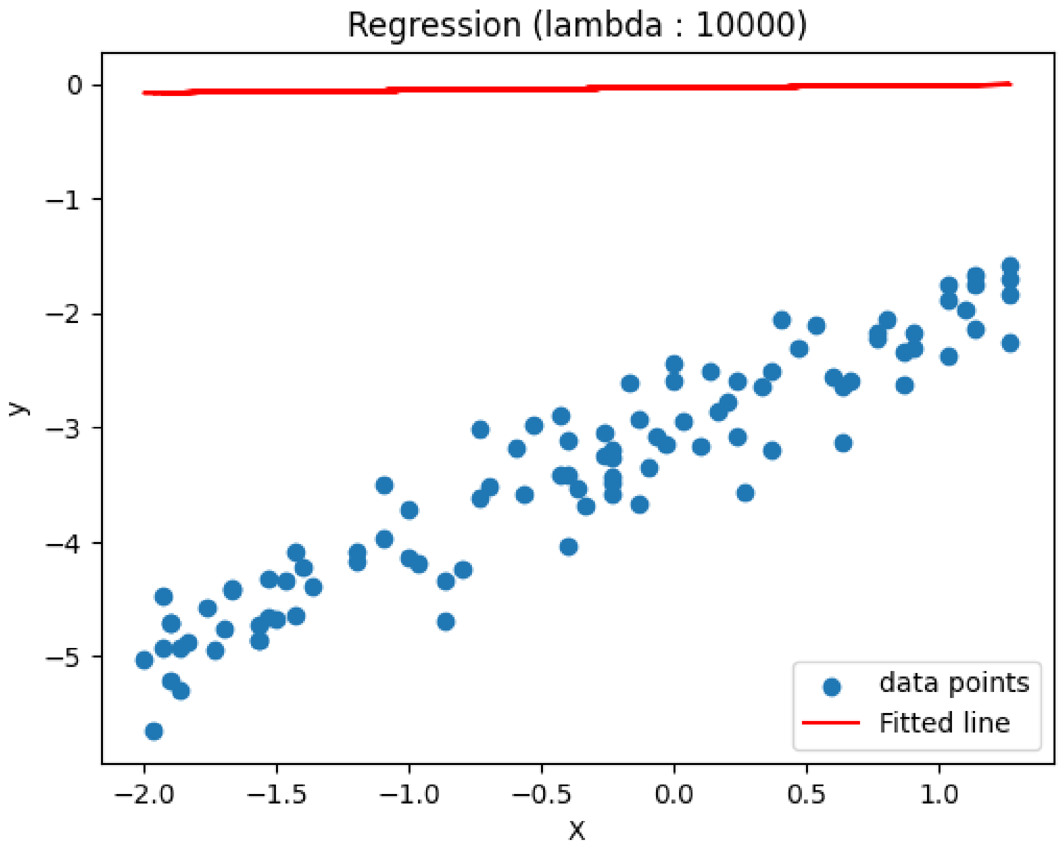

```
Cost with regularization: 2.793172488740026
Mean square error: 1.2785107029715972
```




```
Cost with regularization: 5.591464362606628
Mean square error: 4.946888025066496
```




```
Cost with regularization: 6.2426956269339735
Mean square error: 6.1614425833558135
```




### 1.5 1.5 Analyze your results (10 points)

According to the above figures, what's the best choice of $\lambda$?

Why the regressed line turns to be flat as we increase $\lambda$?

Your answer: 1. $\lambda = 0$
2. In regularized linear regression, increasing the value of the lambda tends to shrink the regression coefficient to zero. When the lambda value increases to a really large value, the magnitude of the regression coefficients should be very small, resulting in a flat regression line that is close to parallel to the x-axis.

## 2 Problem 2: Getting familiar with PyTorch (30 points)

[6]:
```
import mltools as ml
import torch

data = np.genfromtxt("data/curve80.txt")
X = data[:,0]
```

```
X = np.atleast_2d(X).T # code expects shape (M,N) so make sure it's␣
↪2-dimensional
Y = data[:,1] # doesn't matter for Y
Xtr,Xte,Ytr,Yte = ml.splitData(X,Y,0.75) # split data set 75/25
degree = 5
XtrP = ml.transforms.fpoly(Xtr, degree=degree, bias=False)
XtrP,params = ml.transforms.rescale(XtrP)

XtrP_tensor=torch.from_numpy(XtrP).float()
Ytr_tensor=torch.from_numpy(Ytr).reshape(-1,1).float()
XtrP_tensor=XtrP_tensor.view(-1,degree)
Ytr_tensor=Ytr_tensor.view(-1,1)
```

[7]:
```
linear_regressor = torch.nn.Linear(in_features=degree, out_features=1)
print(linear_regressor)
```

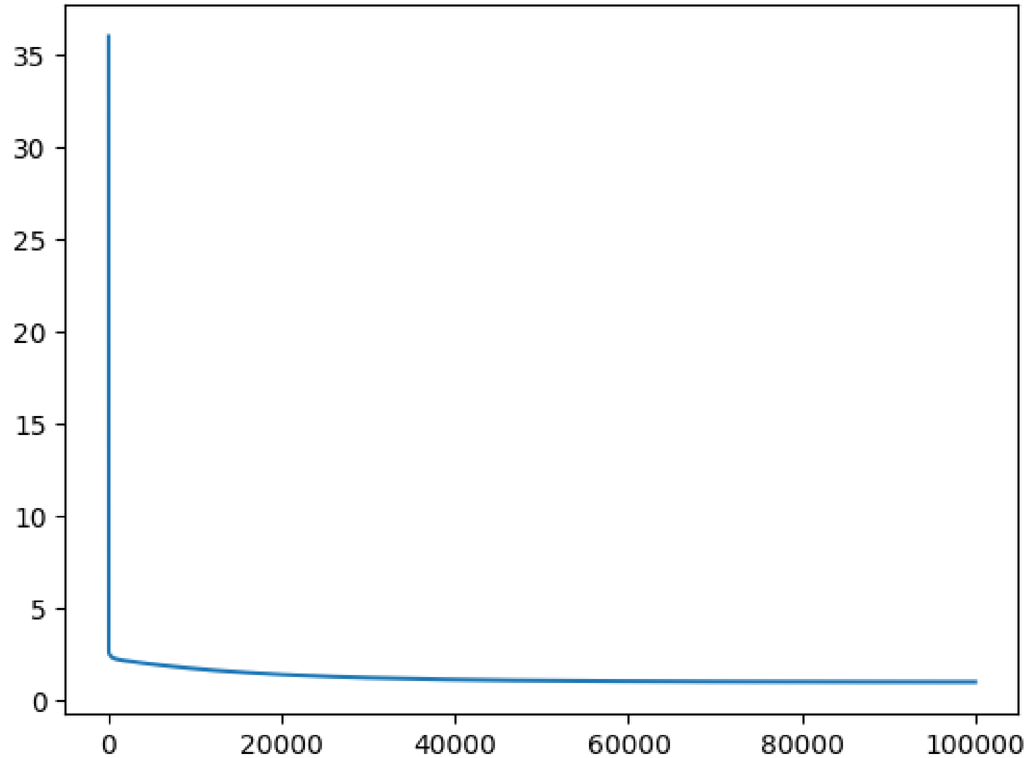
```
Linear(in_features=5, out_features=1, bias=True)
```

[8]:
```
criterion = torch.nn.MSELoss()
optimizer = torch.optim.SGD(linear_regressor.parameters(), lr=0.1)
epochs = 100000
```

[9]:
```
loss_record = []
for _ in range(epochs):
    optimizer.zero_grad()                        # set gradient to zero
    pred_y = linear_regressor(XtrP_tensor)
    loss = criterion(pred_y, Ytr_tensor)         # calculate loss function
    loss.backward()                              # backpropagate gradient
    loss_record.append(loss.item())
    optimizer.step()                         # update the parameters in the linear␣
↪regressor
```

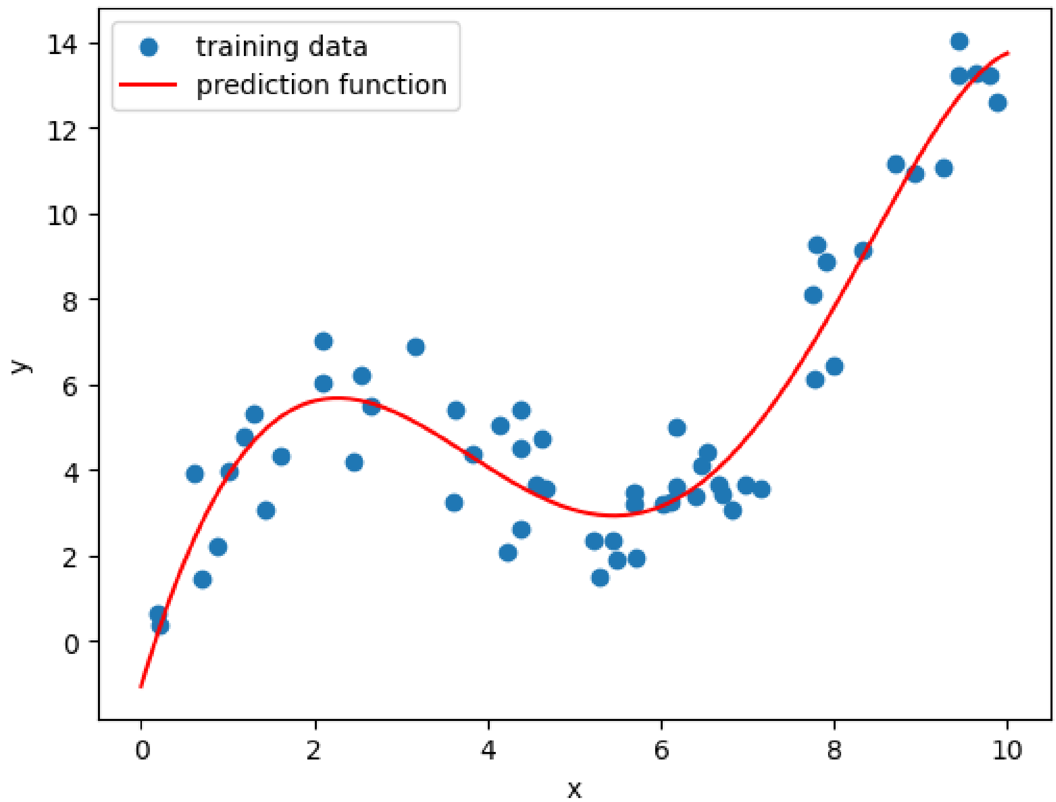
[10]:
```
plt.plot(range(epochs), (loss_record))
```

[10]: 
```
[<matplotlib.lines.Line2D at 0x7fb89b0c1ca0>]
```




[11]:
```
xs = np.linspace(0,10,200)
xs = xs[:,np.newaxis]
xsP, _ = ml.transforms.rescale(ml.transforms.
 ↪fpoly(xs,degree=degree,bias=False), params)
xsP_tensor = torch.from_numpy(xsP).float()
ys = linear_regressor(xsP_tensor)
plt.scatter(Xtr,Ytr,label="training data")
plt.plot(xs,ys.detach().numpy(),label="prediction function",color ='red')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
```

[11]:
```
<matplotlib.legend.Legend at 0x7fb89b46d9a0>
```




## 3 Statement of Collaboration

I do it by myself