

Stable Marriage Problem

Shusen Wang

<http://wangshusen.github.io/>

Bipartite Matching

Alex



Bob



Chris



Ada



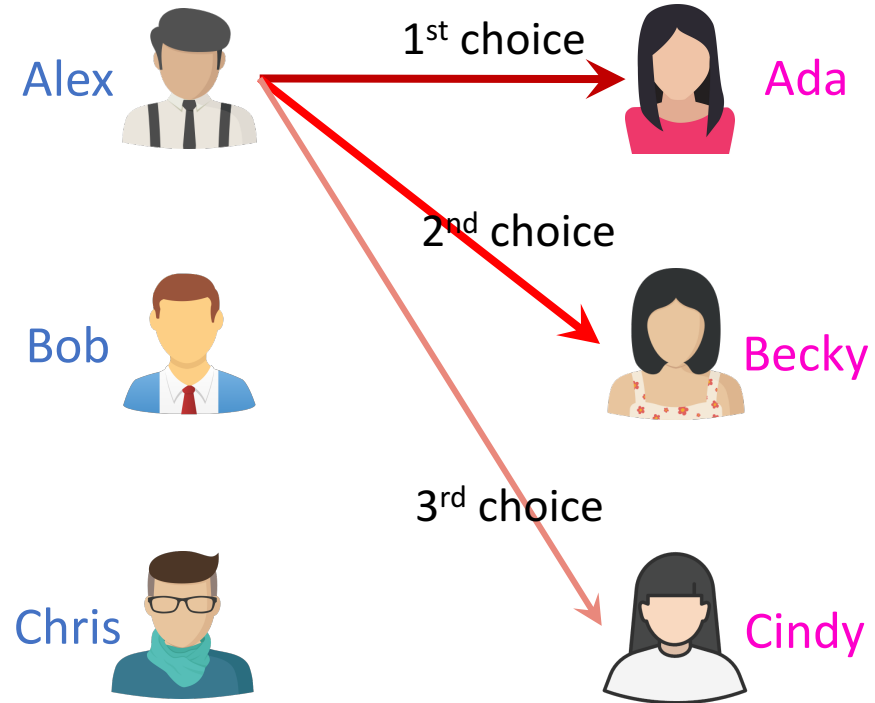
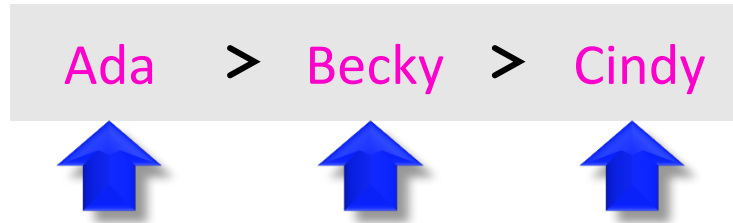
Becky



Cindy



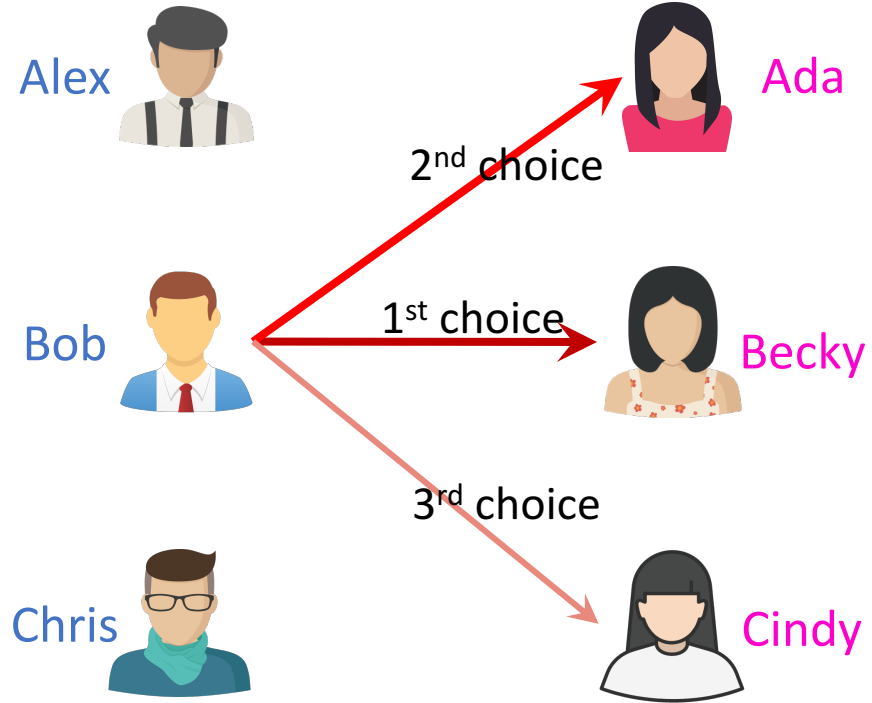
Bipartite Matching



Bipartite Matching

Ada > Becky > Cindy

Becky > Ada > Cindy

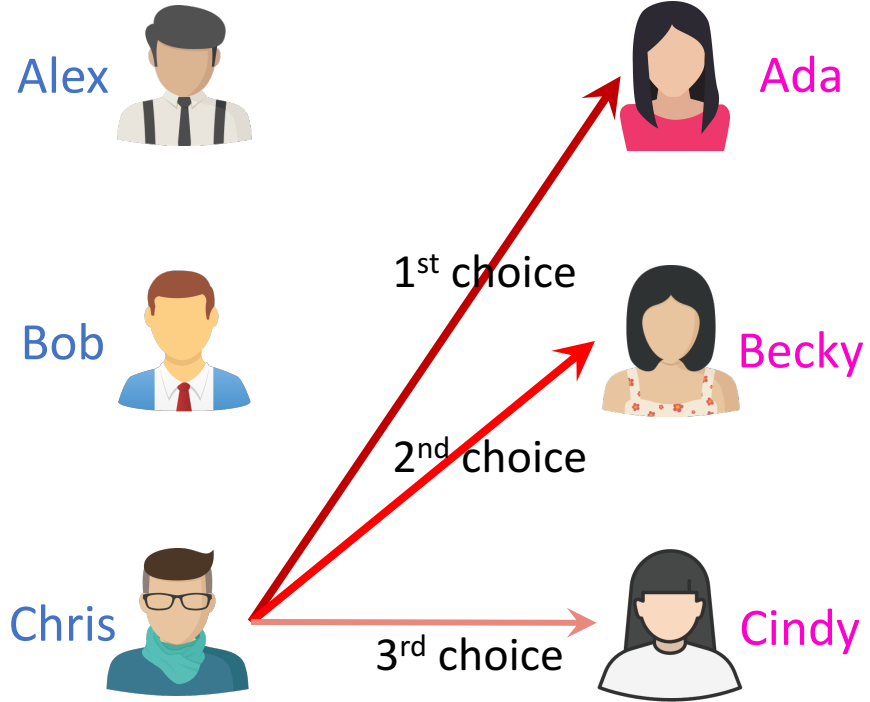


Bipartite Matching

Ada > Becky > Cindy

Becky > Ada > Cindy

Ada > Becky > Cindy

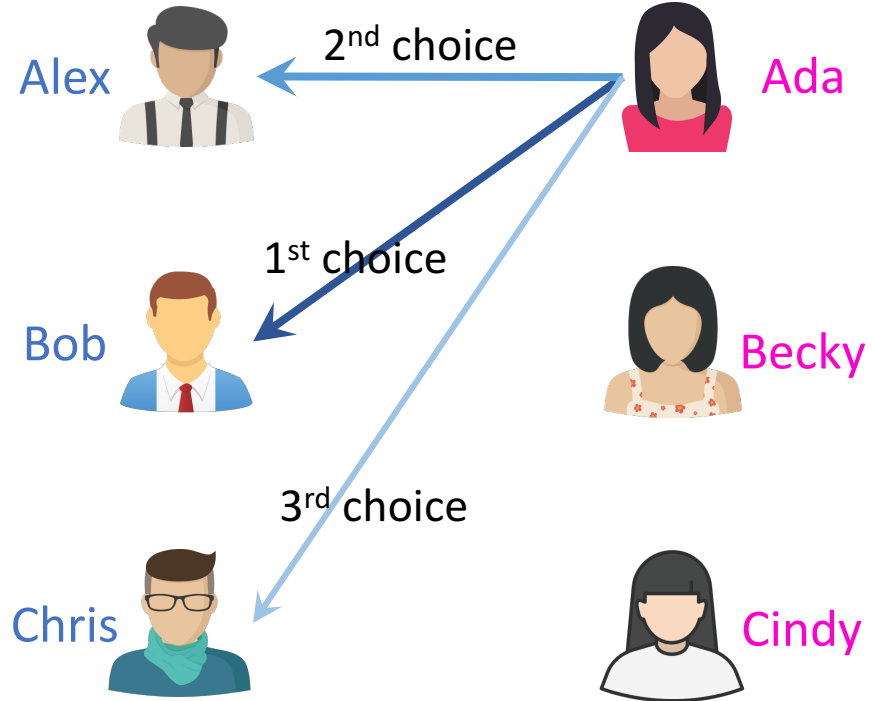


Bipartite Matching

Ada > Becky > Cindy

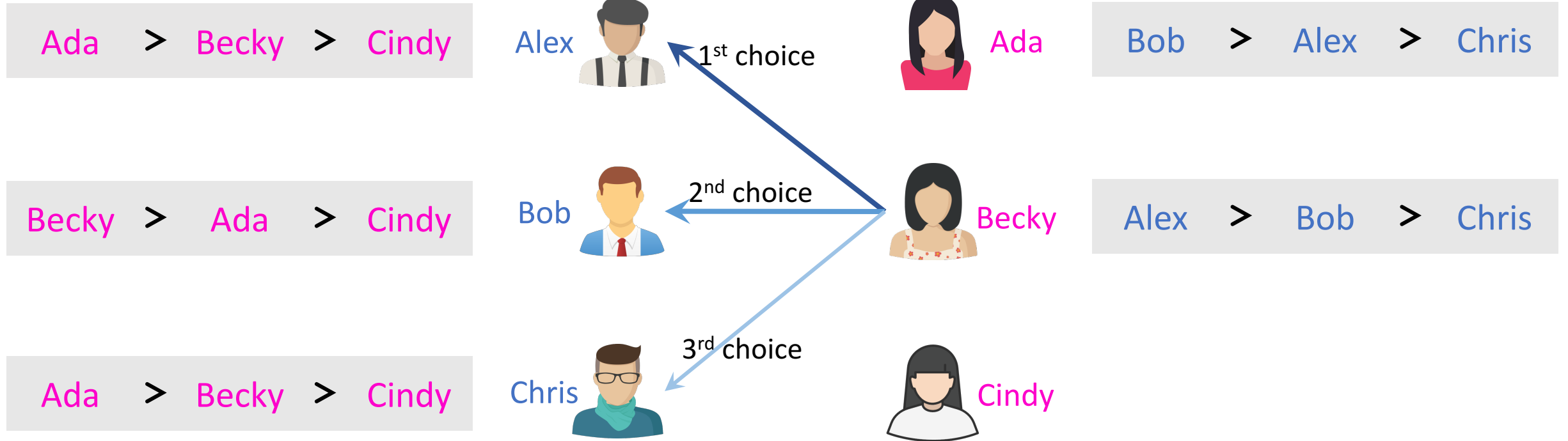
Becky > Ada > Cindy

Ada > Becky > Cindy



Bob > Alex > Chris

Bipartite Matching

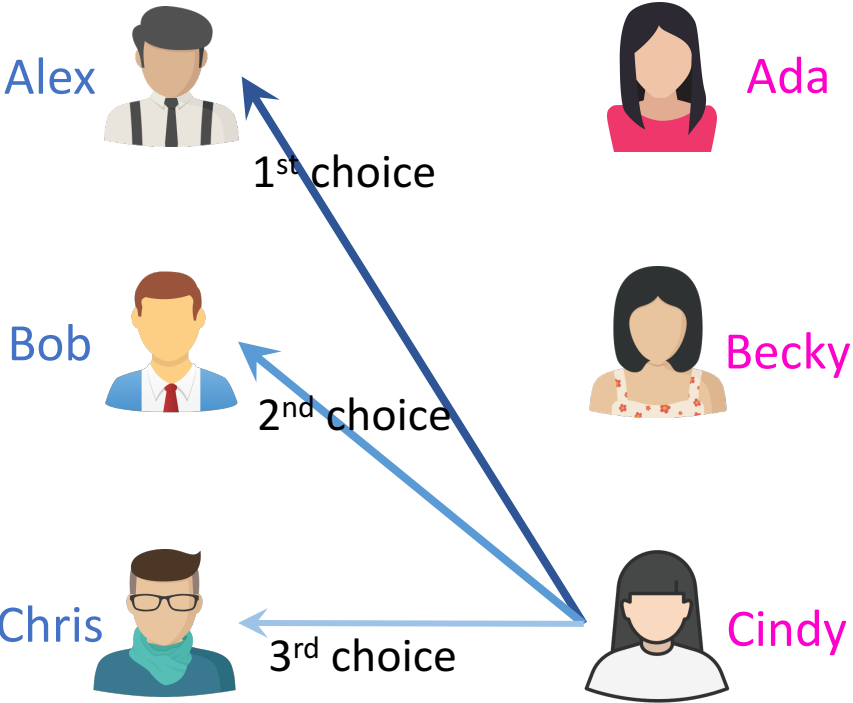


Bipartite Matching

Ada > Becky > Cindy

Becky > Ada > Cindy

Ada > Becky > Cindy



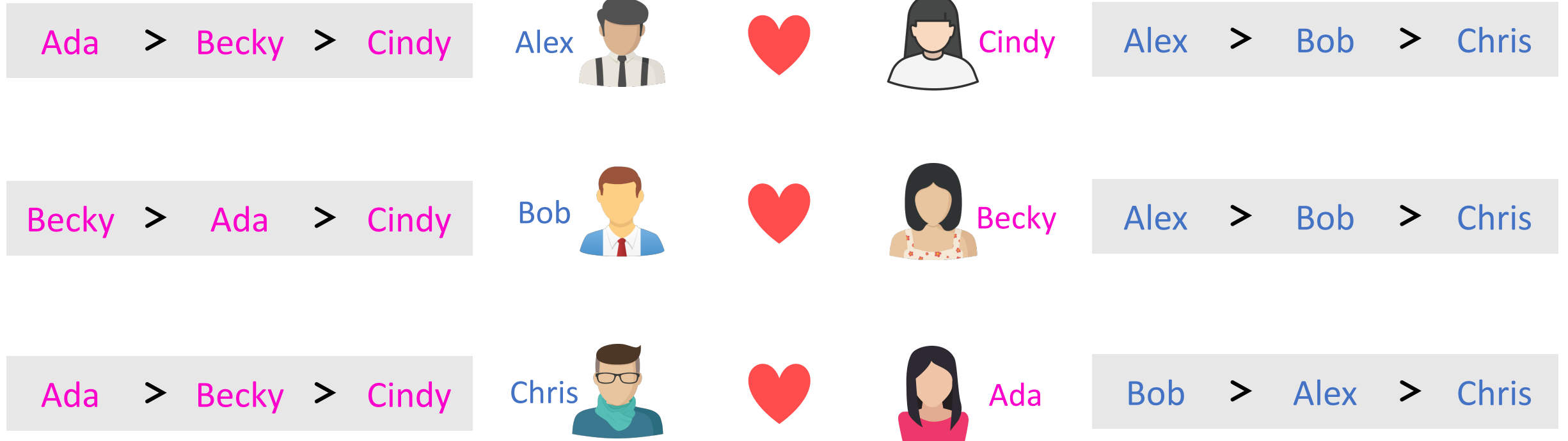
Bob > Alex > Chris

Alex > Bob > Chris

Alex > Bob > Chris

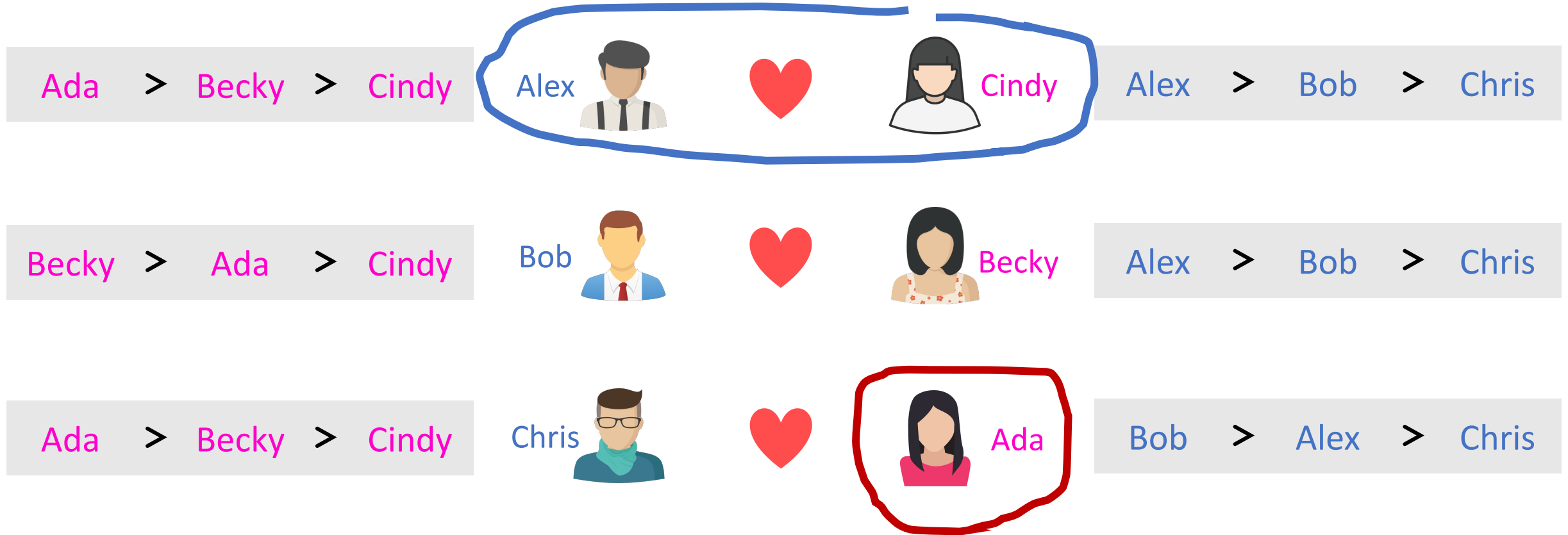
Stable Marriage

Is this a stable marriage?



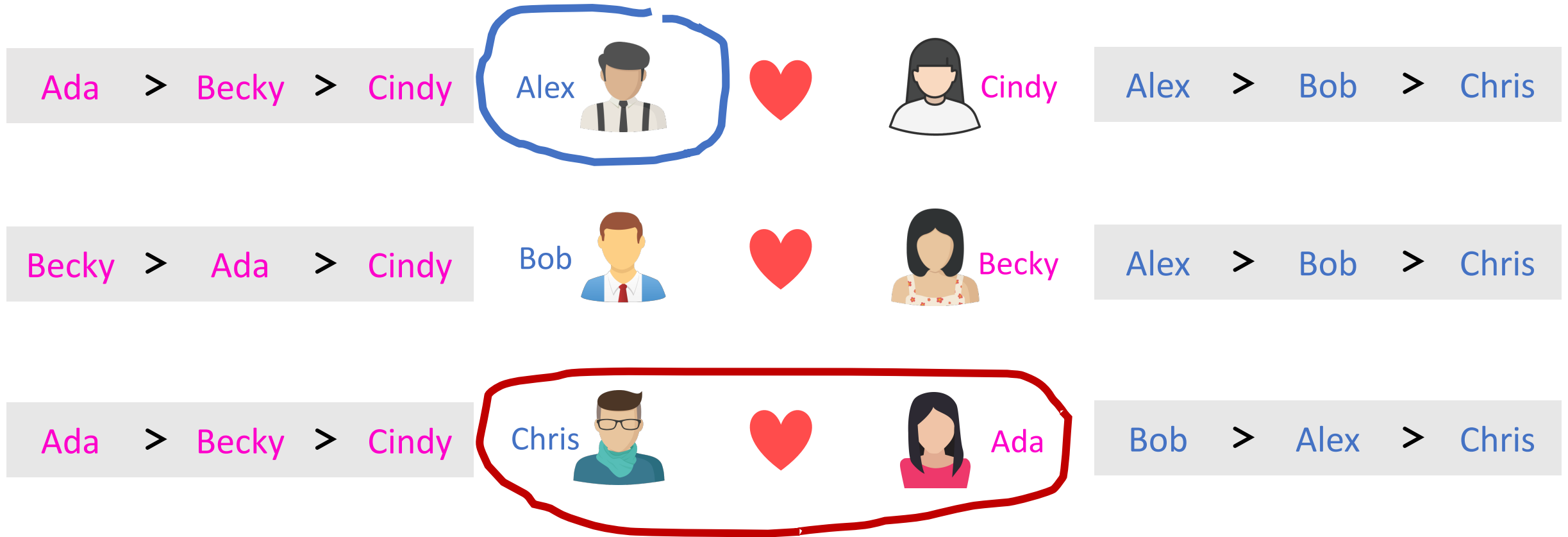
If a man and a woman (who are not spouses) prefer each other over their current spouses, then the marriage is **not stable**.

Is this a stable marriage?



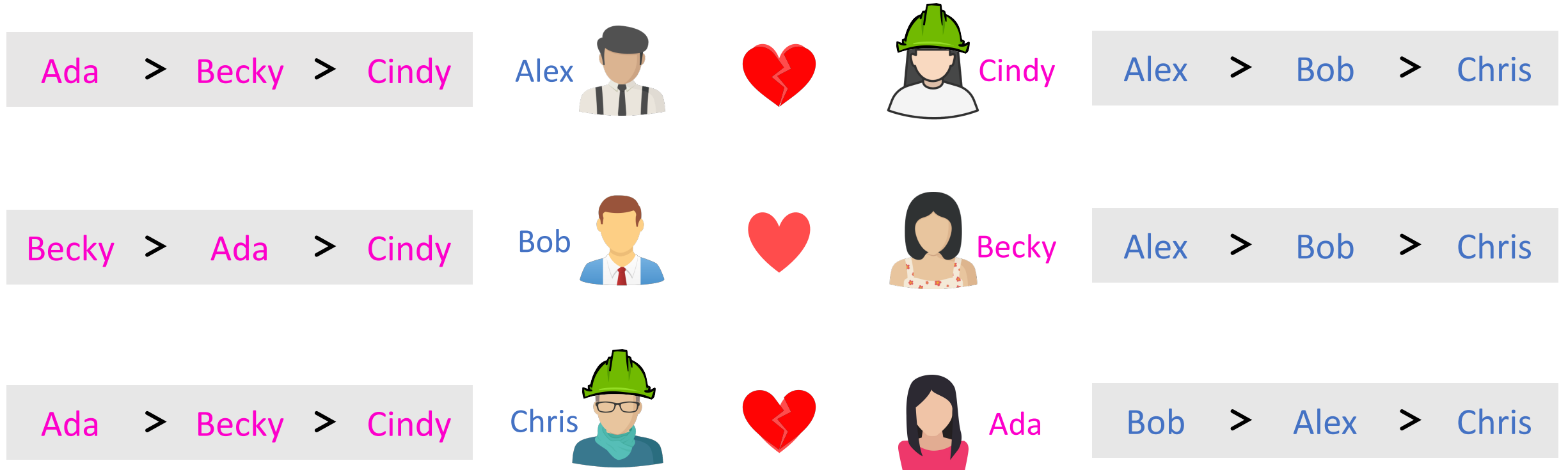
- Alex prefers Ada over his wife, Cindy.

Is this a stable marriage?



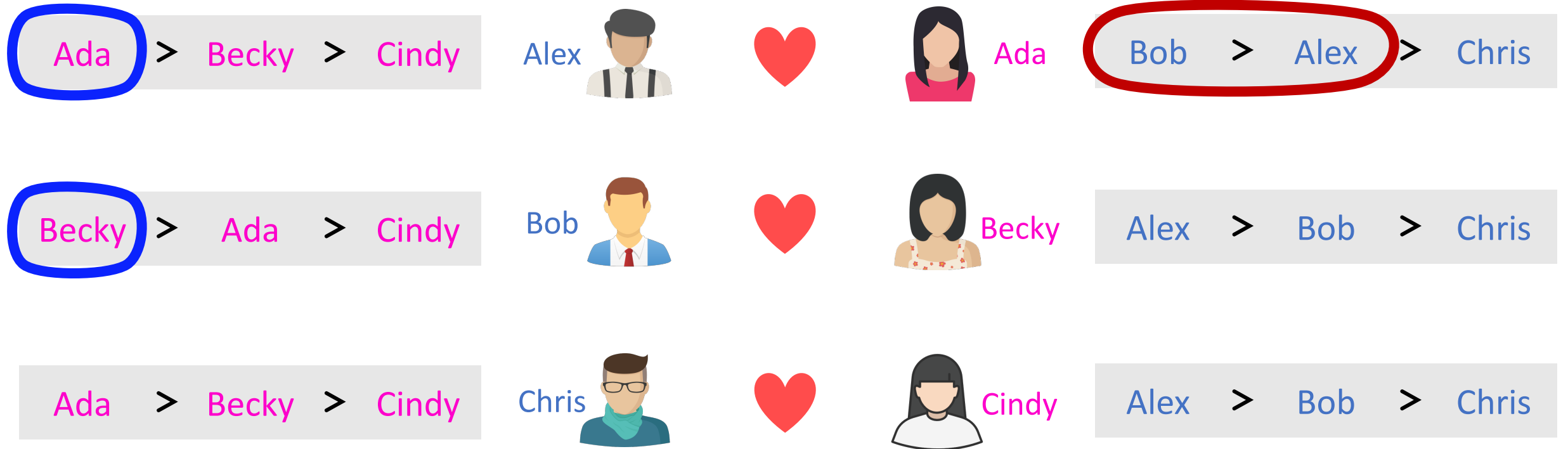
- Alex prefers Ada over his wife, Cindy.
- Ada prefers Alex over her husband, Chris.

This is **not** a stable marriage!

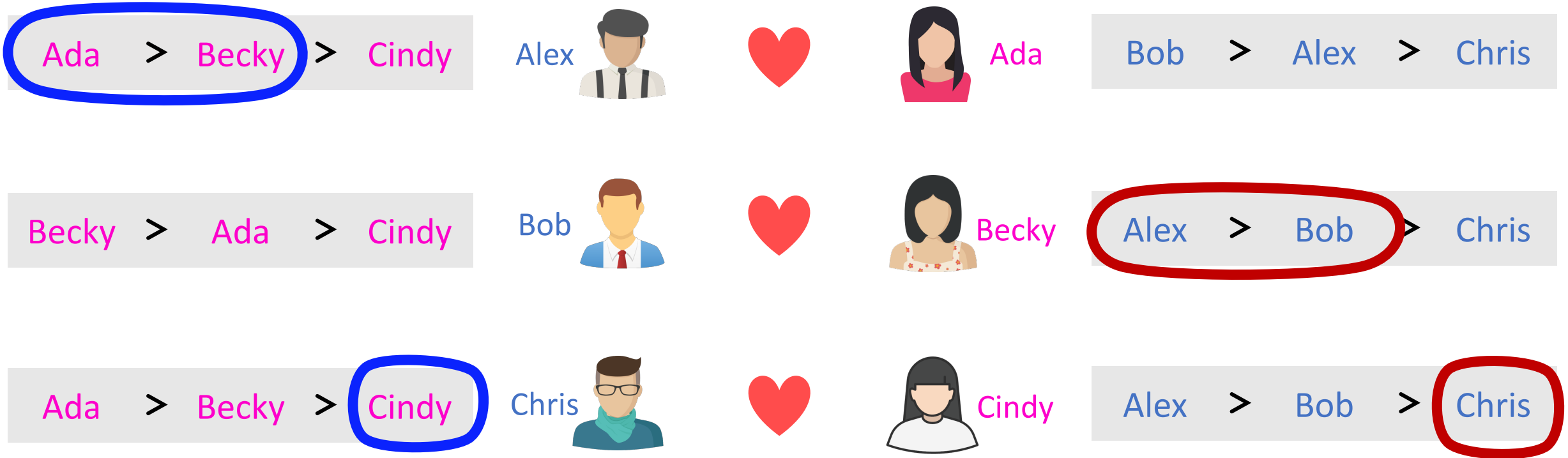


- Alex prefers Ada over his wife, Cindy.
- Ada prefers Alex over her husband, Chris.

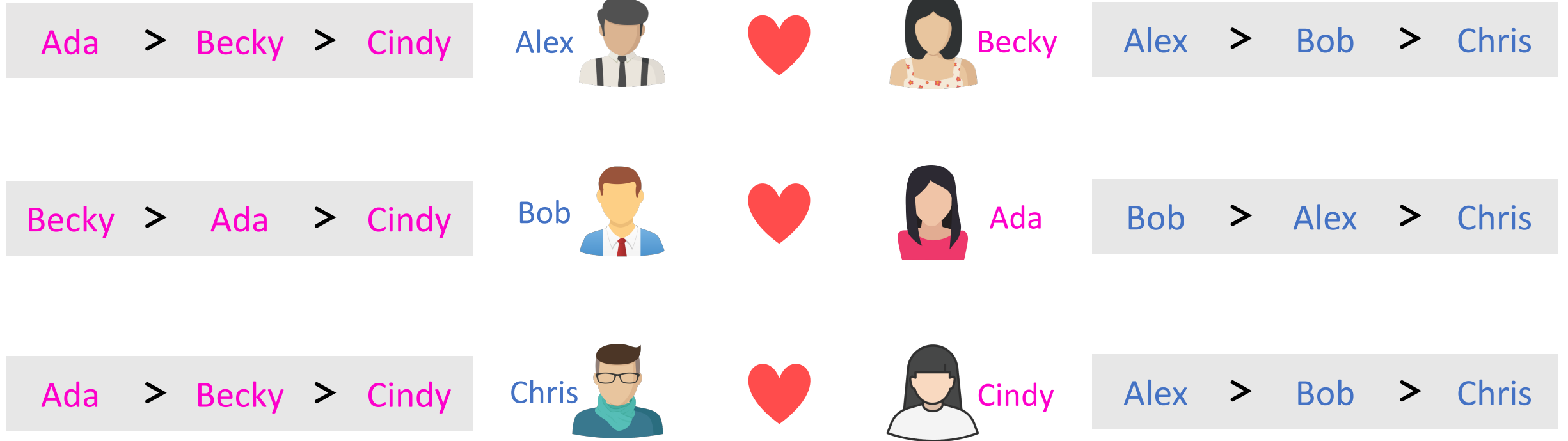
This is a stable marriage



This is a stable marriage



This is another stable marriage



Stable marriage is not unique.

Stable Marriage Problem

- Directed weighted bipartite graph: $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$.
 - \mathcal{U} : a set of men.
 - \mathcal{V} : a set of women.
 - The weights are the orders of preference.

Stable Marriage Problem

- Directed weighted bipartite graph: $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$.
 - \mathcal{U} : a set of men.
 - \mathcal{V} : a set of women.
 - The weights are the orders of preference.
- The numbers of men and women are the same: $|\mathcal{U}| = |\mathcal{V}| = n$.
- If a man and a woman both prefer each other over their current spouses, then the marriage is **not stable**.

Gale-Shapley Algorithm

- David Gale and Lloyd S. Shapley. [College Admissions and the Stability of Marriage](#). *American Mathematical Monthly*, 69 (1): 9–14, 1962.
- Time complexity: $O(n^2)$.
- To be introduced in the next lecture.

Thank You!

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