

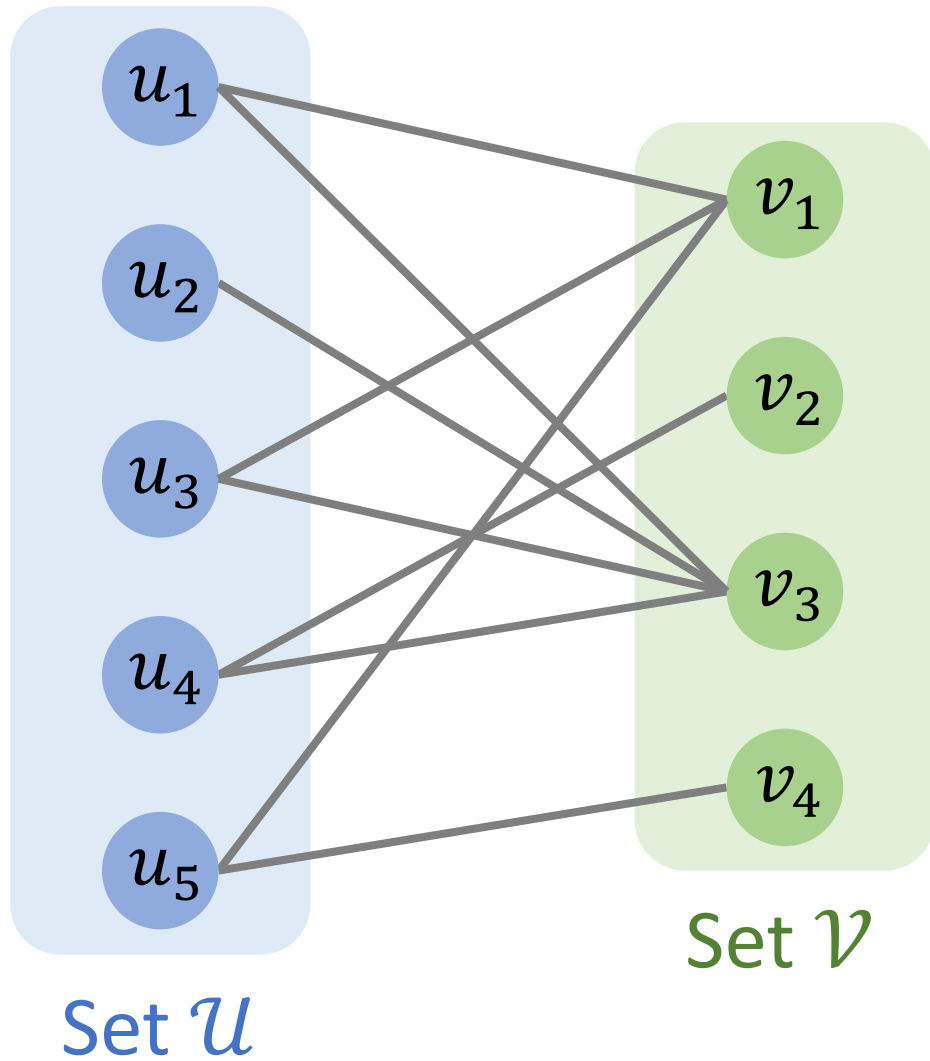
Maximum-Cardinality Bipartite Matching (MCBM)

Shusen Wang

<http://wangshusen.github.io/>

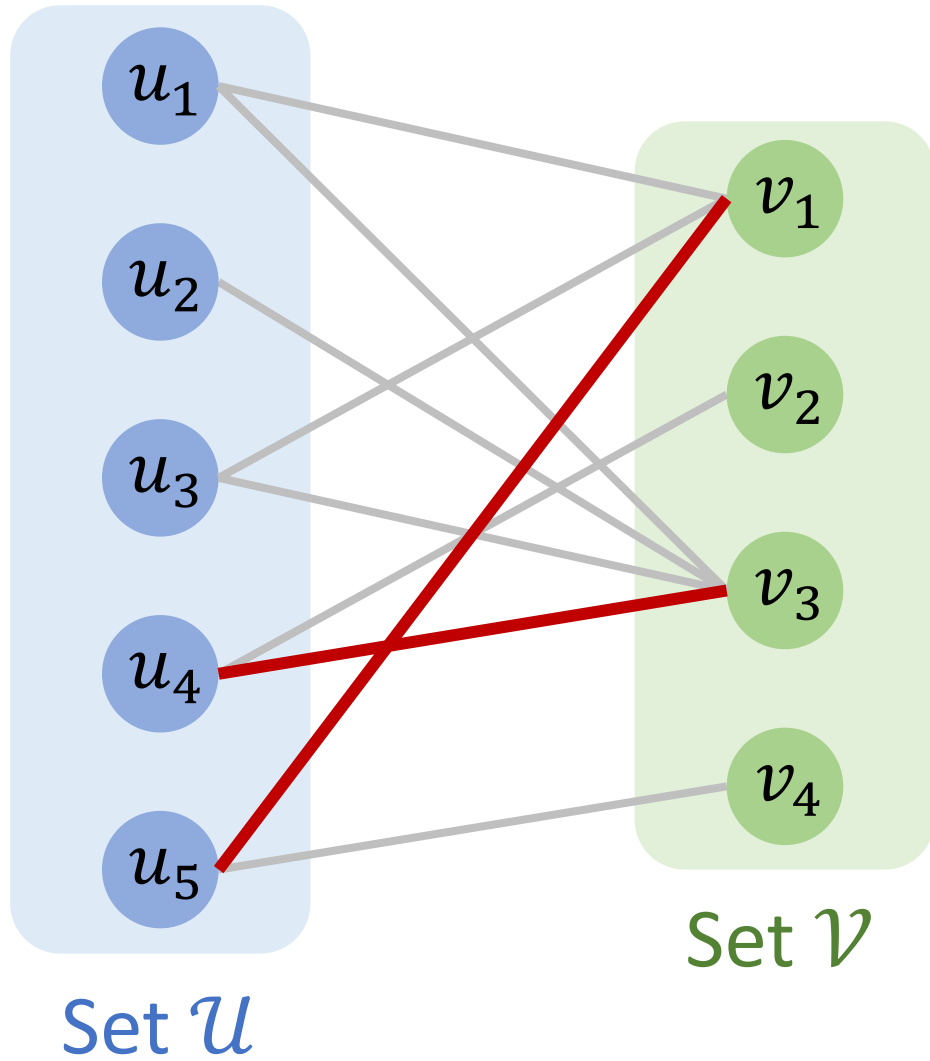
Definition of MCBM

Bipartite Matching



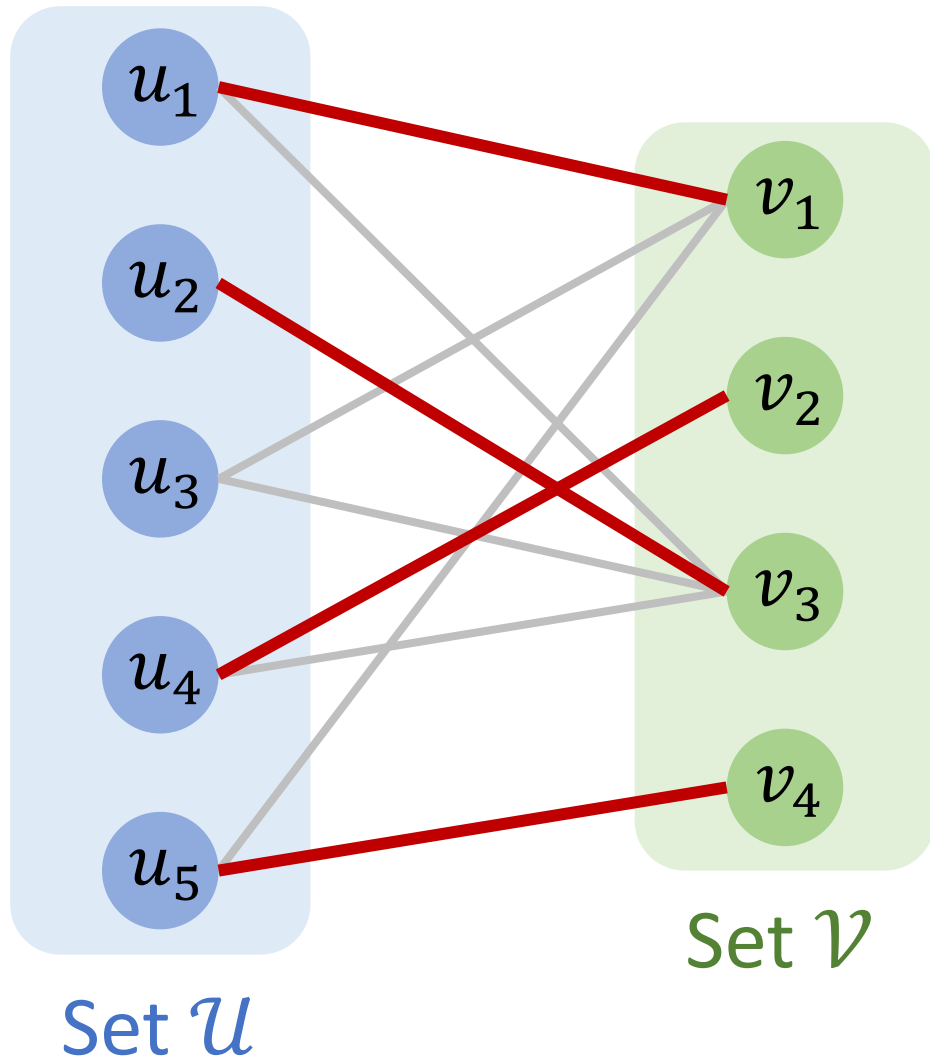
- Bipartite graph: $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$.
- **Matching** is a subset of edges without common vertices.
- Denote the **matching** by set $\mathcal{S} \subseteq \mathcal{E}$.

Bipartite Matching



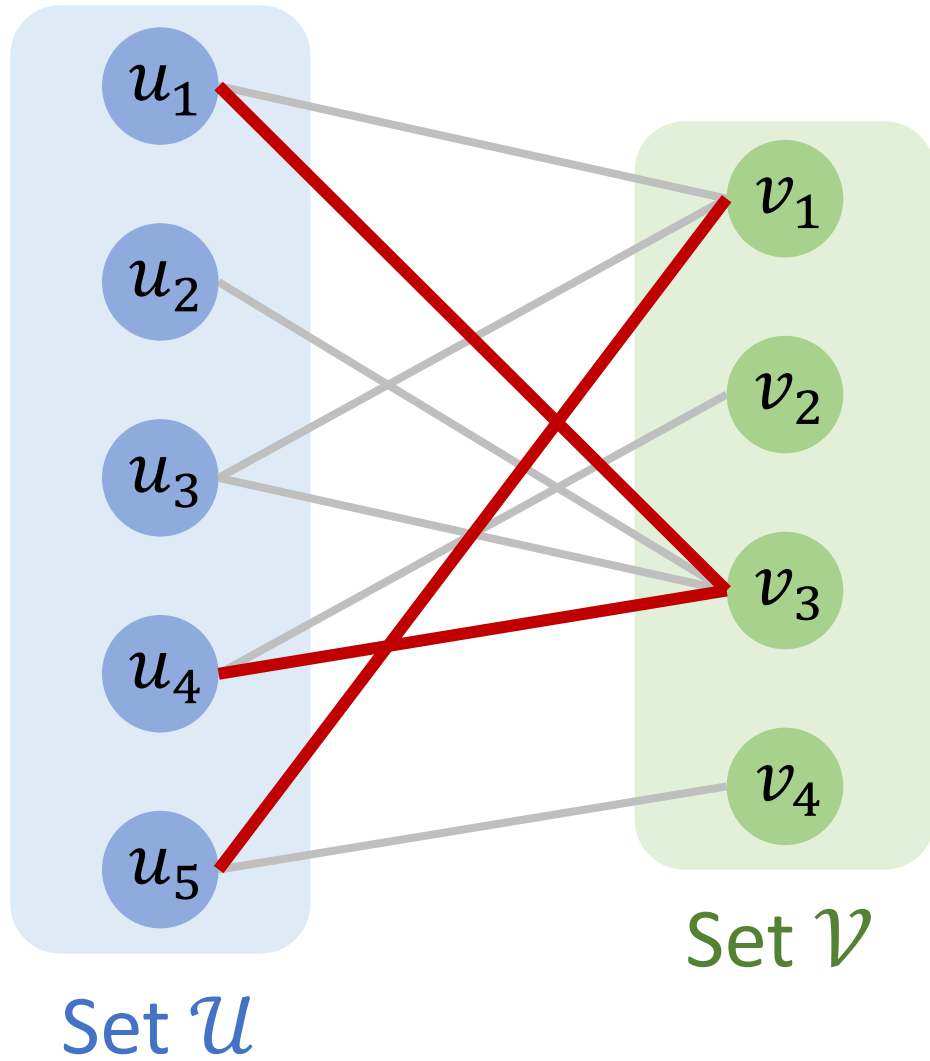
- $\mathcal{S} = \{e_{51}, e_{43}\}$ is a matching. (But not a maximum matching.)

Bipartite Matching



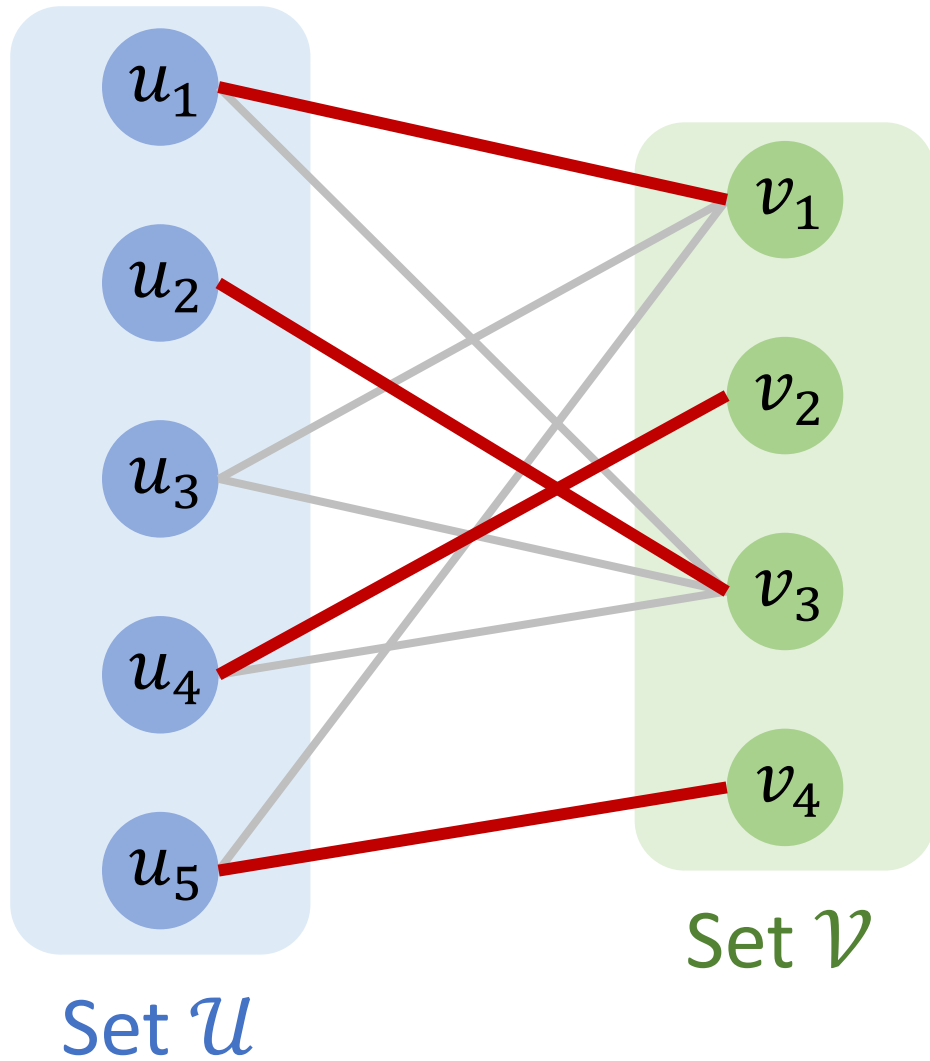
- $\mathcal{S} = \{e_{11}, e_{23}, e_{42}, e_{54}\}$ is a matching.

Bipartite Matching



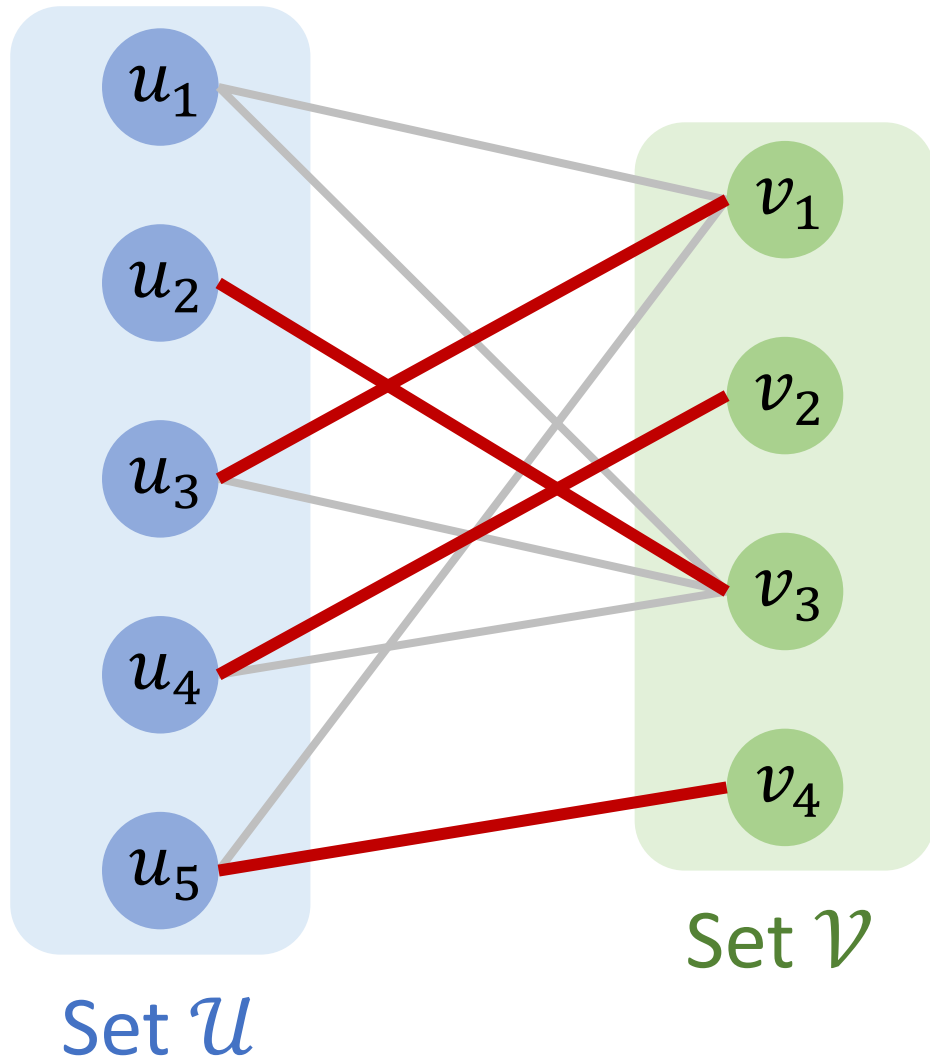
- $\{e_{13}, e_{51}, e_{43}\}$ is not a matching.
- Two edges have common vertex.

Maximum-Cardinality Bipartite Matching (MCBM)



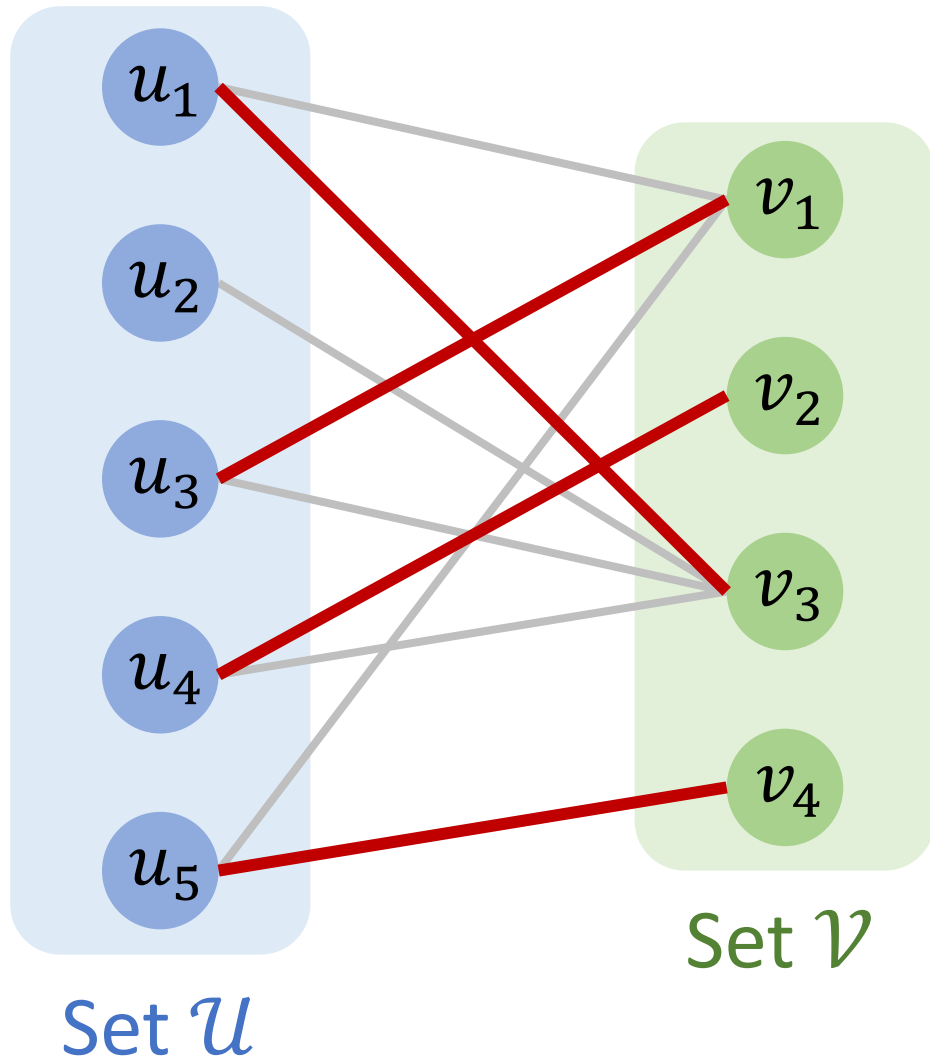
- **MCBM:** The matching \mathcal{S} that has the maximum cardinality $|\mathcal{S}|$.

Maximum-Cardinality Bipartite Matching (MCBM)



- **MCBM:** The matching \mathcal{S} that has the maximum cardinality $|\mathcal{S}|$.
- MCBM may not be unique.

Maximum-Cardinality Bipartite Matching (MCBM)

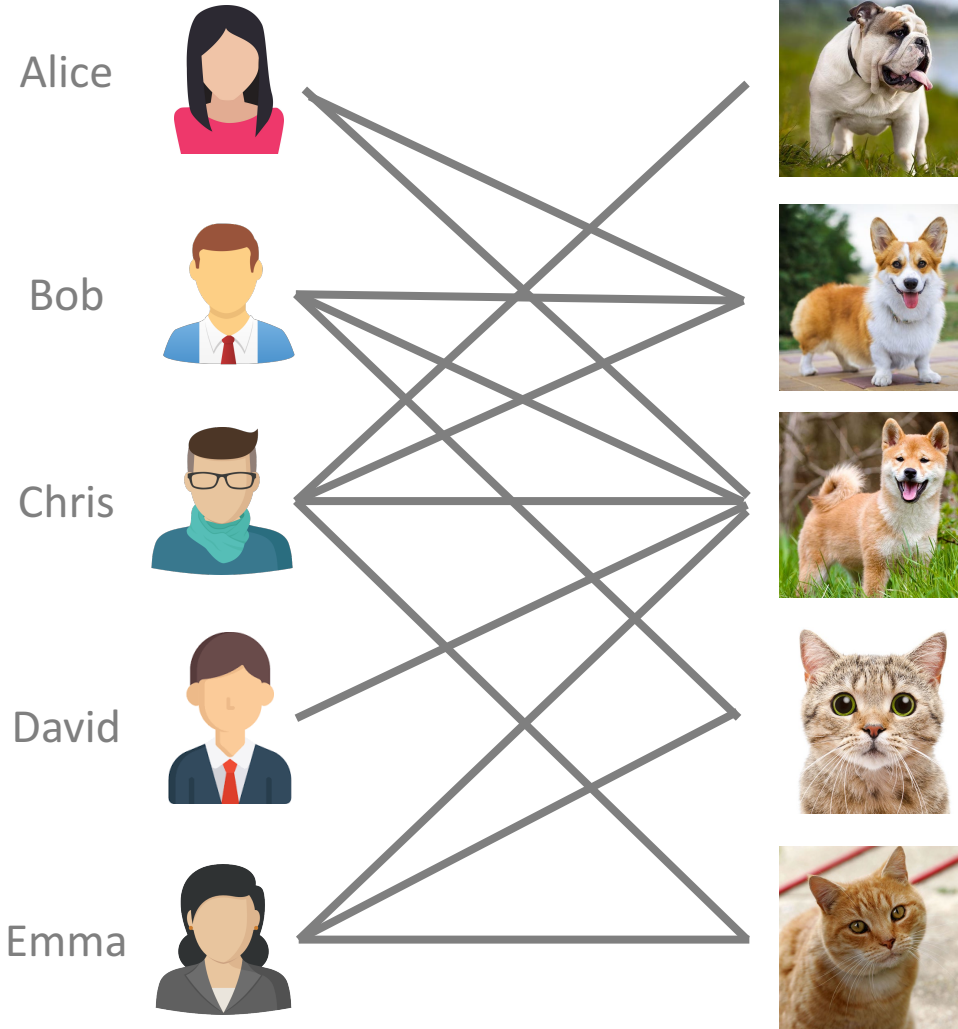


- **MCBM:** The matching \mathcal{S} that has the maximum cardinality $|\mathcal{S}|$.
- MCBM may not be unique.

Greedy Algorithm

People

Pets

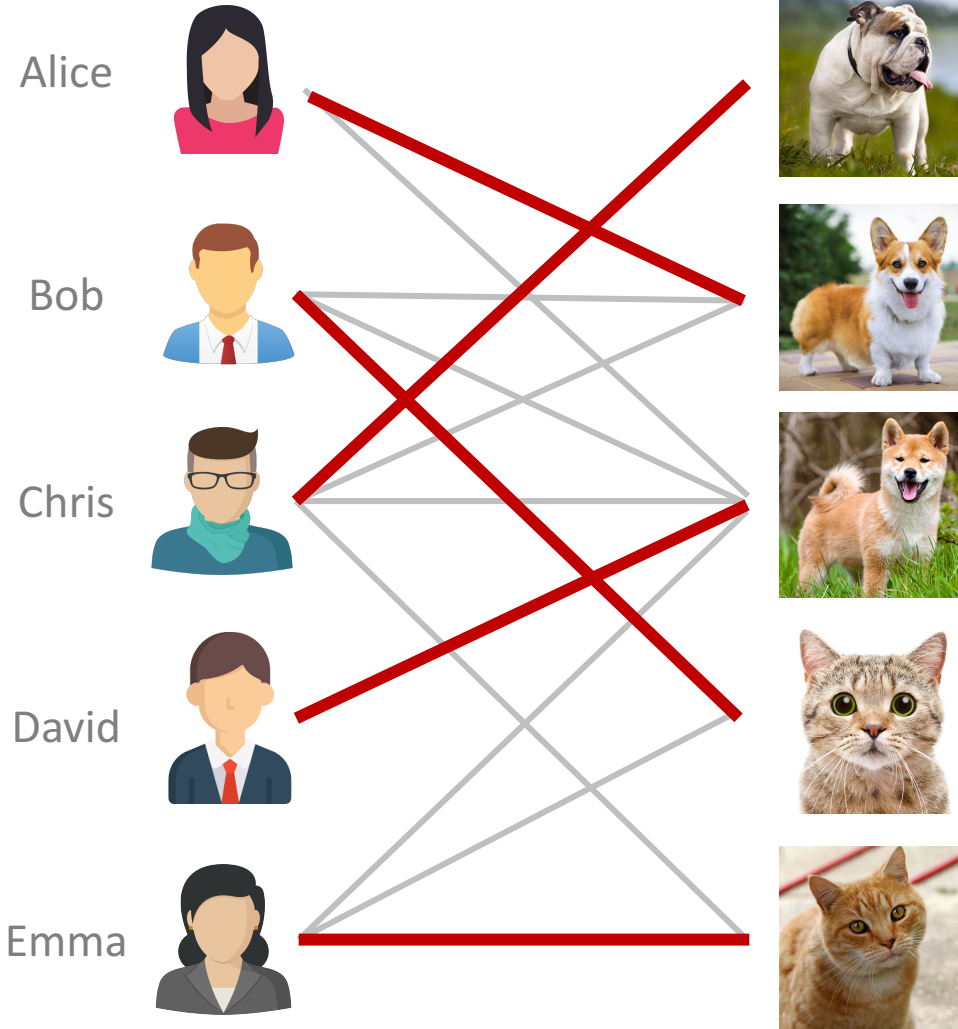


Pet adoption

- Bipartite graph: $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$.
- Set \mathcal{U} contains people.
- Set \mathcal{V} contains pets.
- Edges in \mathcal{E} are people's preference.
- **Goal:** Maximizing the cardinality of matching.

People

Pets

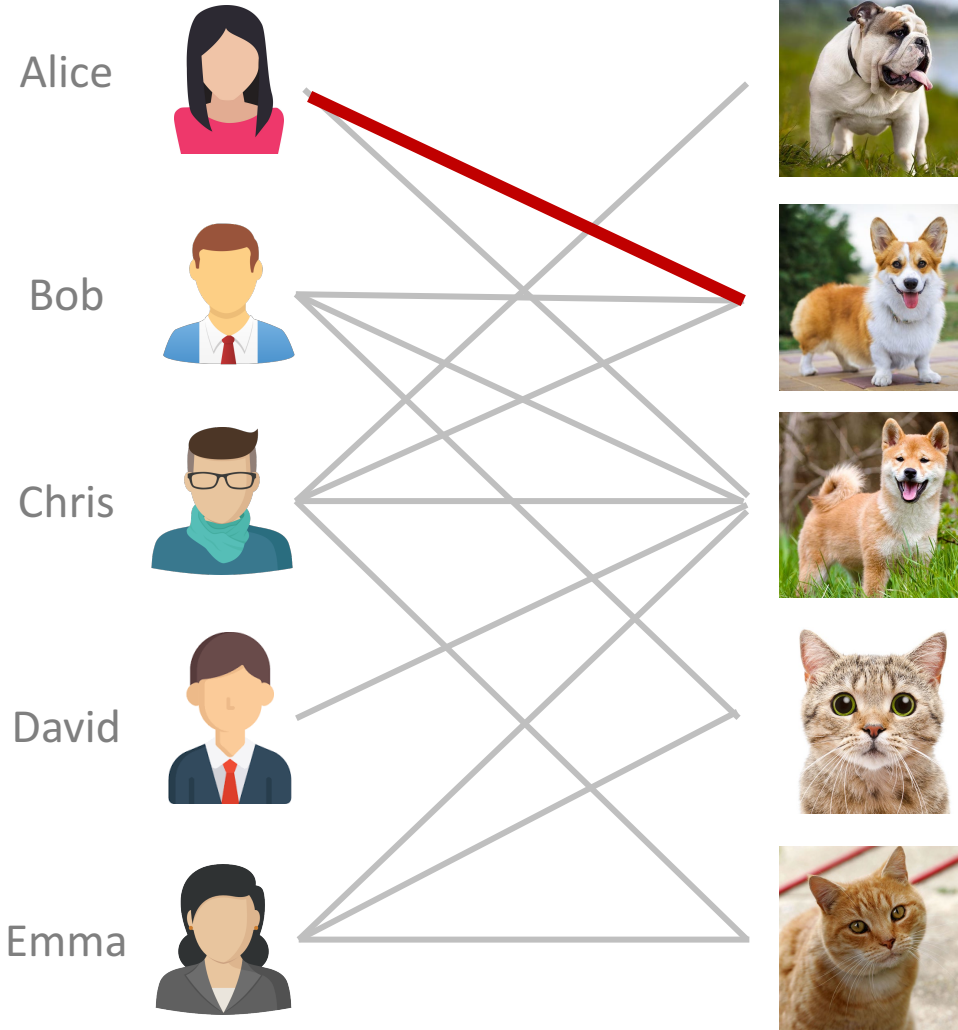


Pet adoption

- The cardinality of the maximum matching is 5.

People

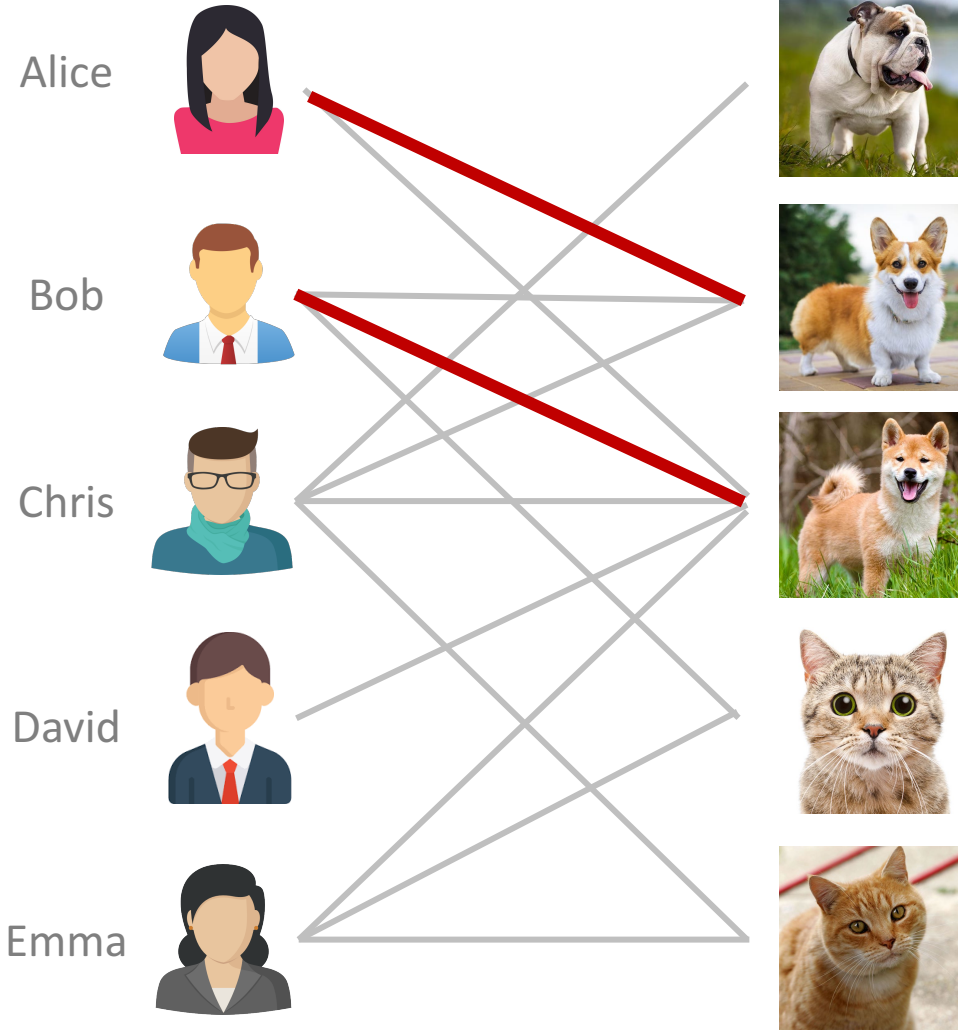
Pets



Greedy Algorithm

People

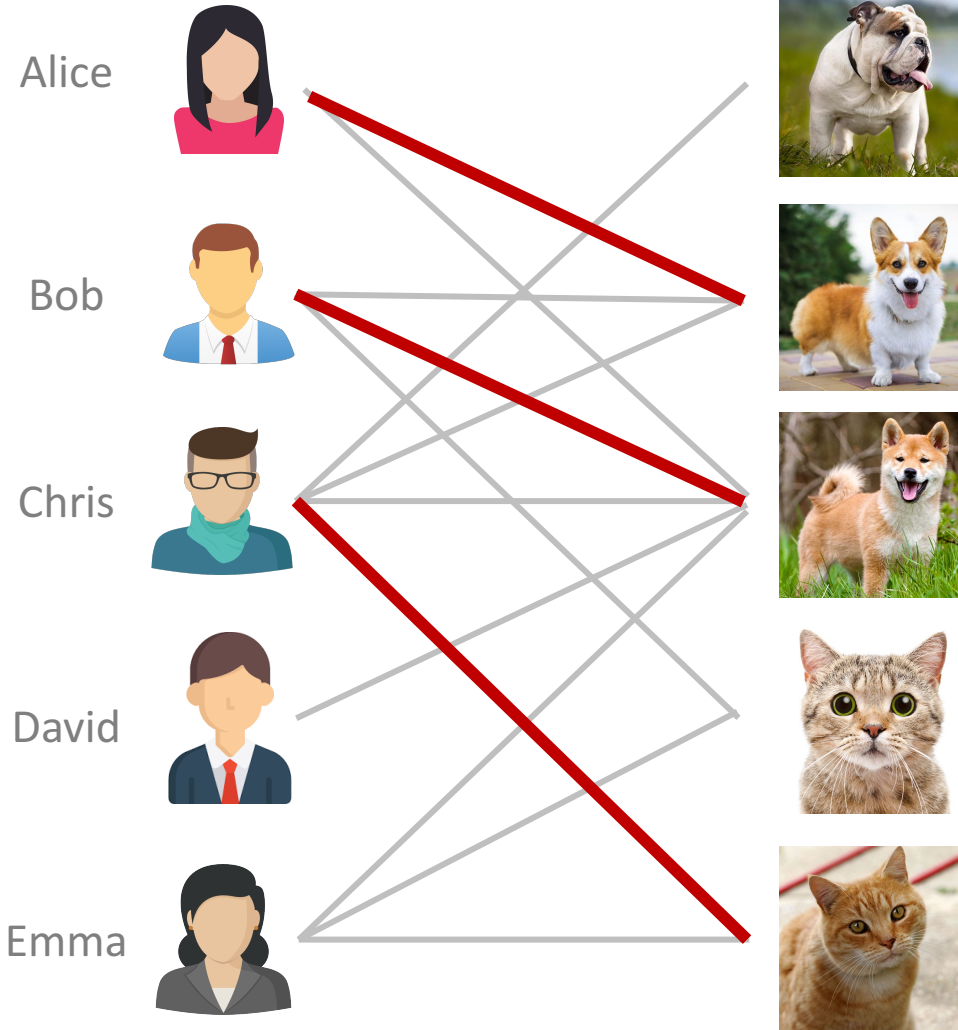
Pets



Greedy Algorithm

People

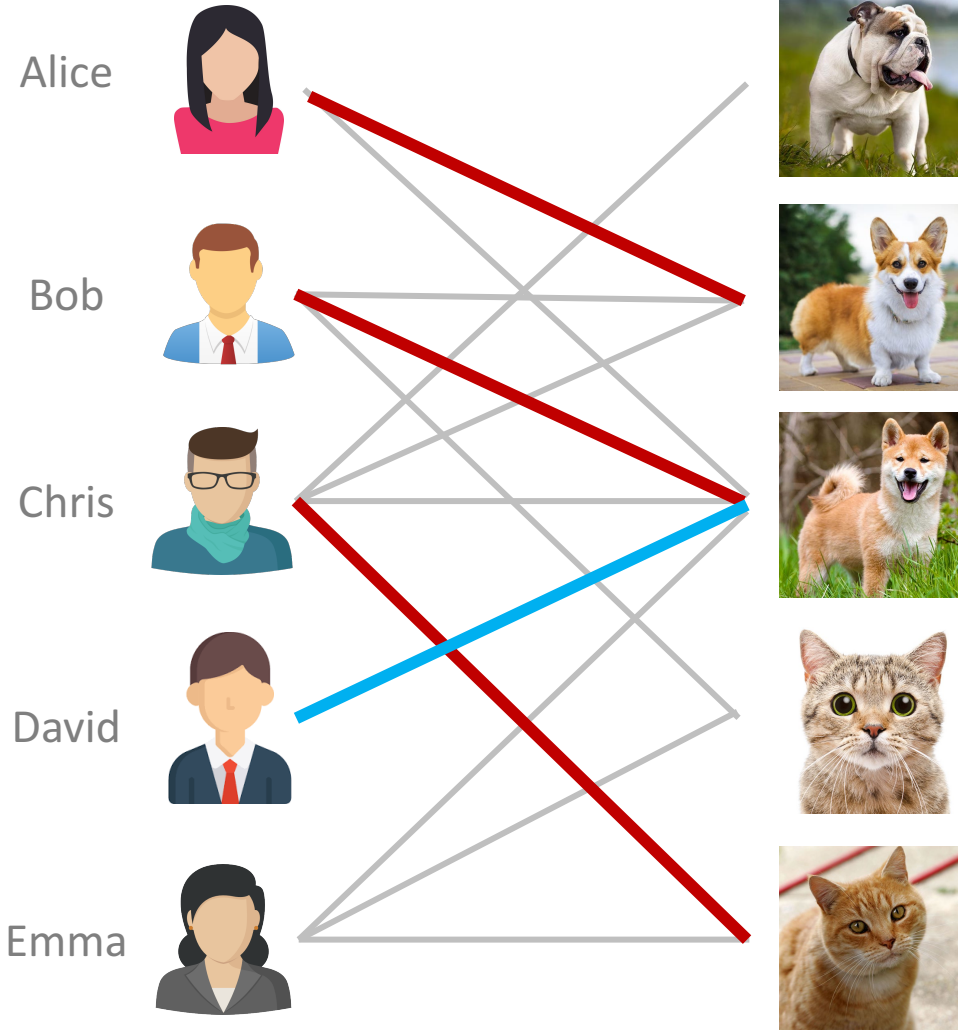
Pets



Greedy Algorithm

People

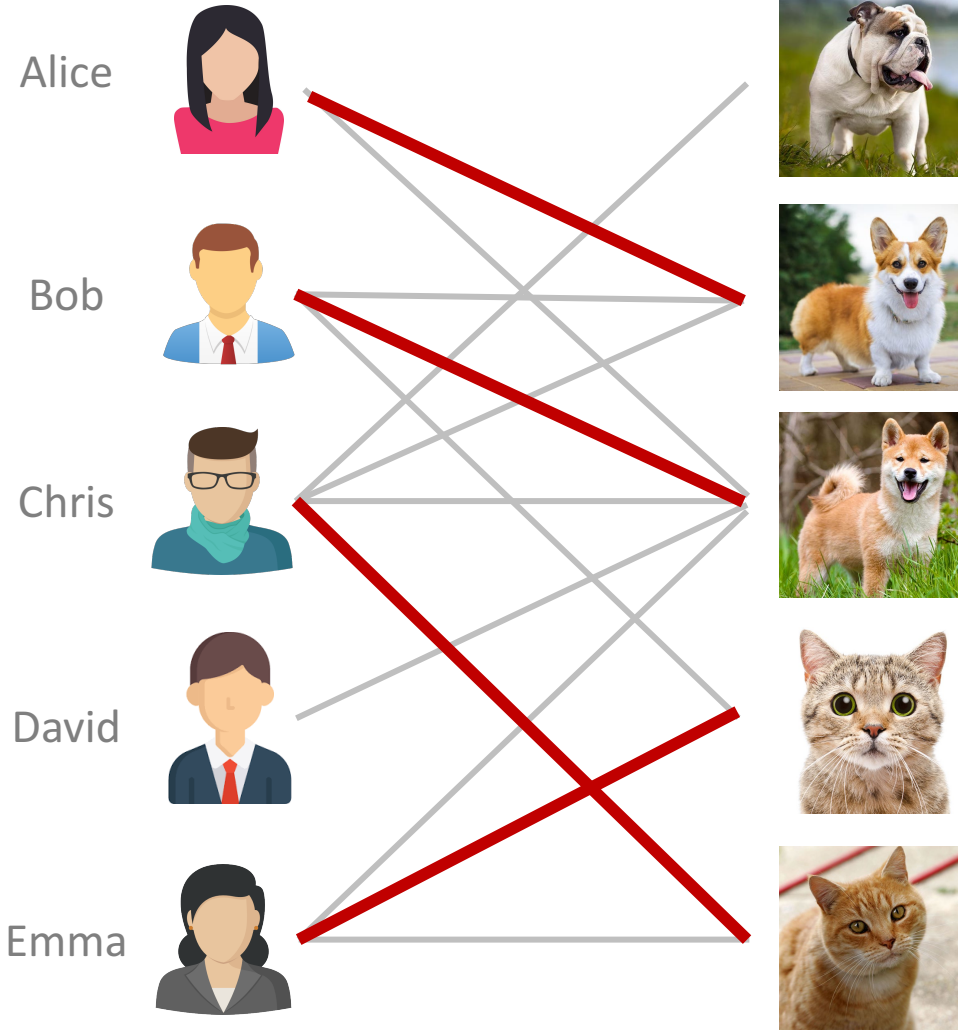
Pets



Greedy Algorithm

People

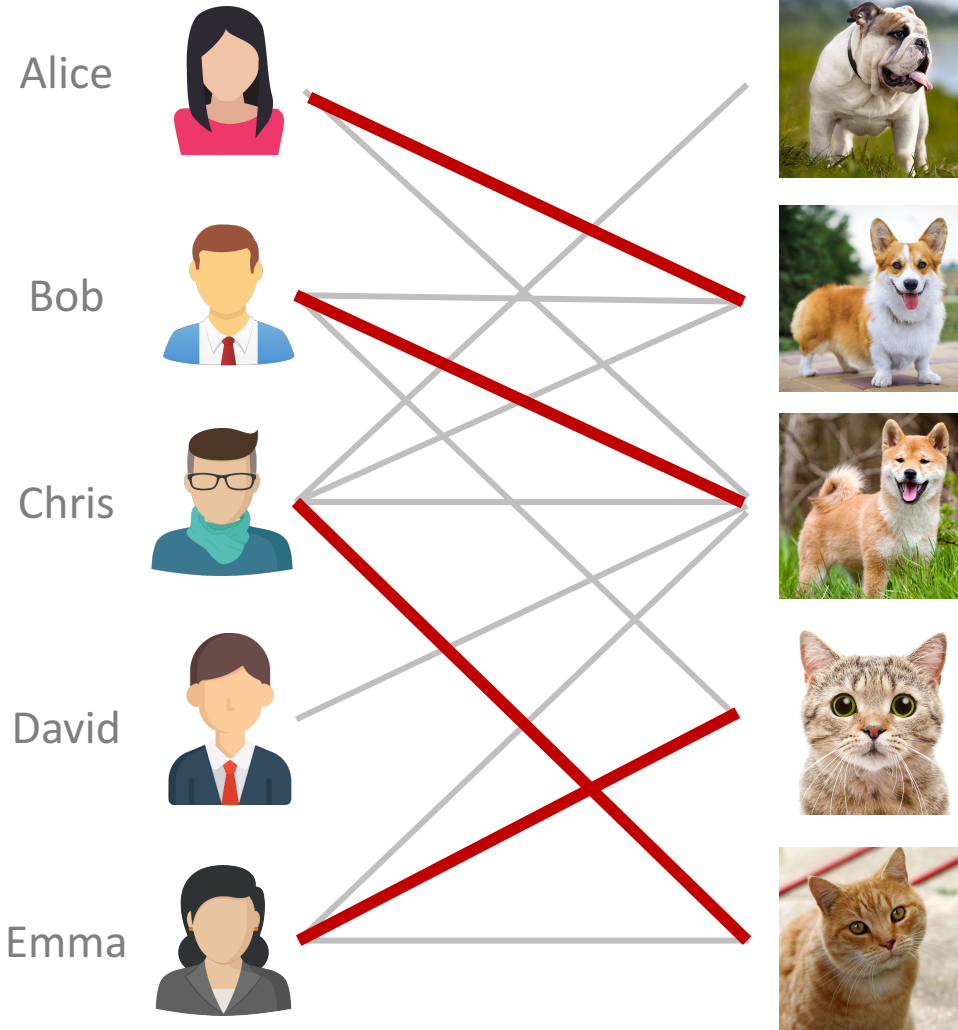
Pets



Greedy Algorithm

People

Pets



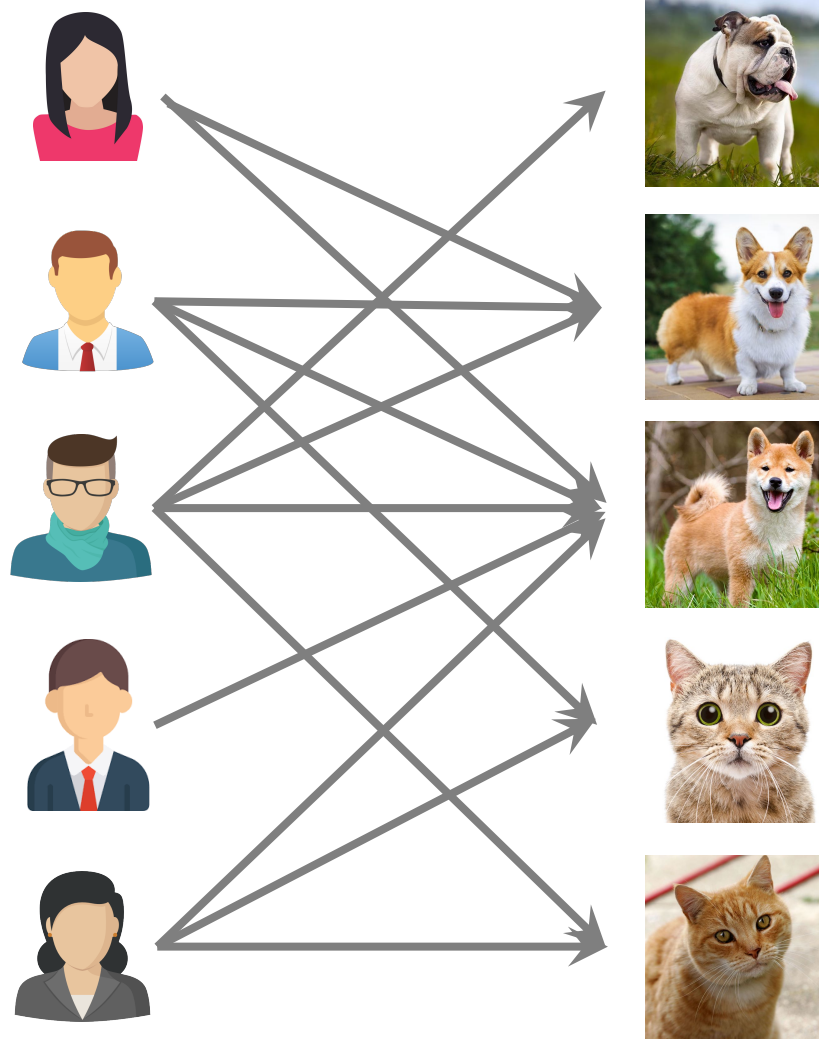
Greedy Algorithm

- The cardinality of matching is 4.
(Not the maximum.)

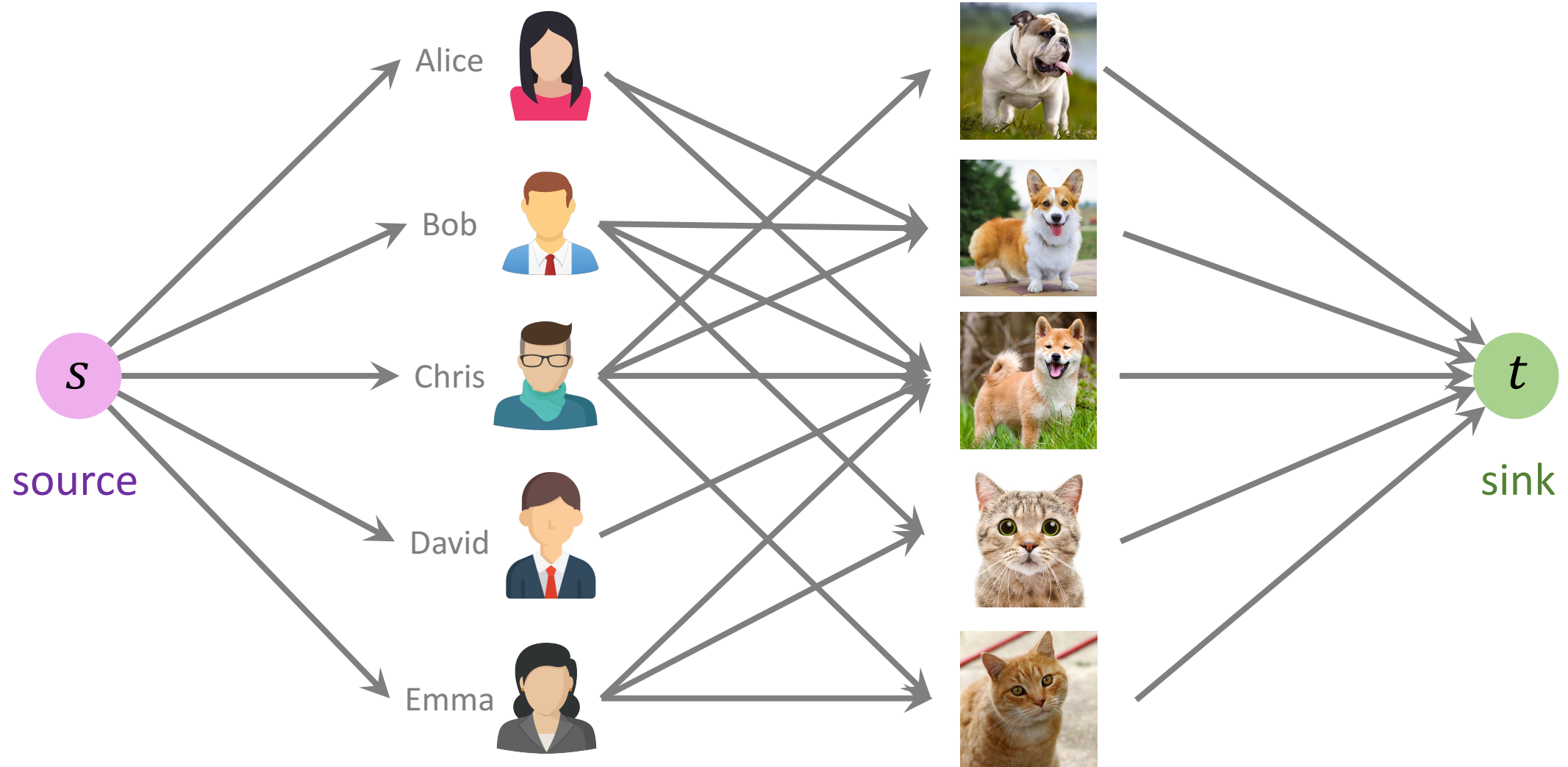
Greedy algorithm can fail!

Reducing Bipartite Matching to Network Flow

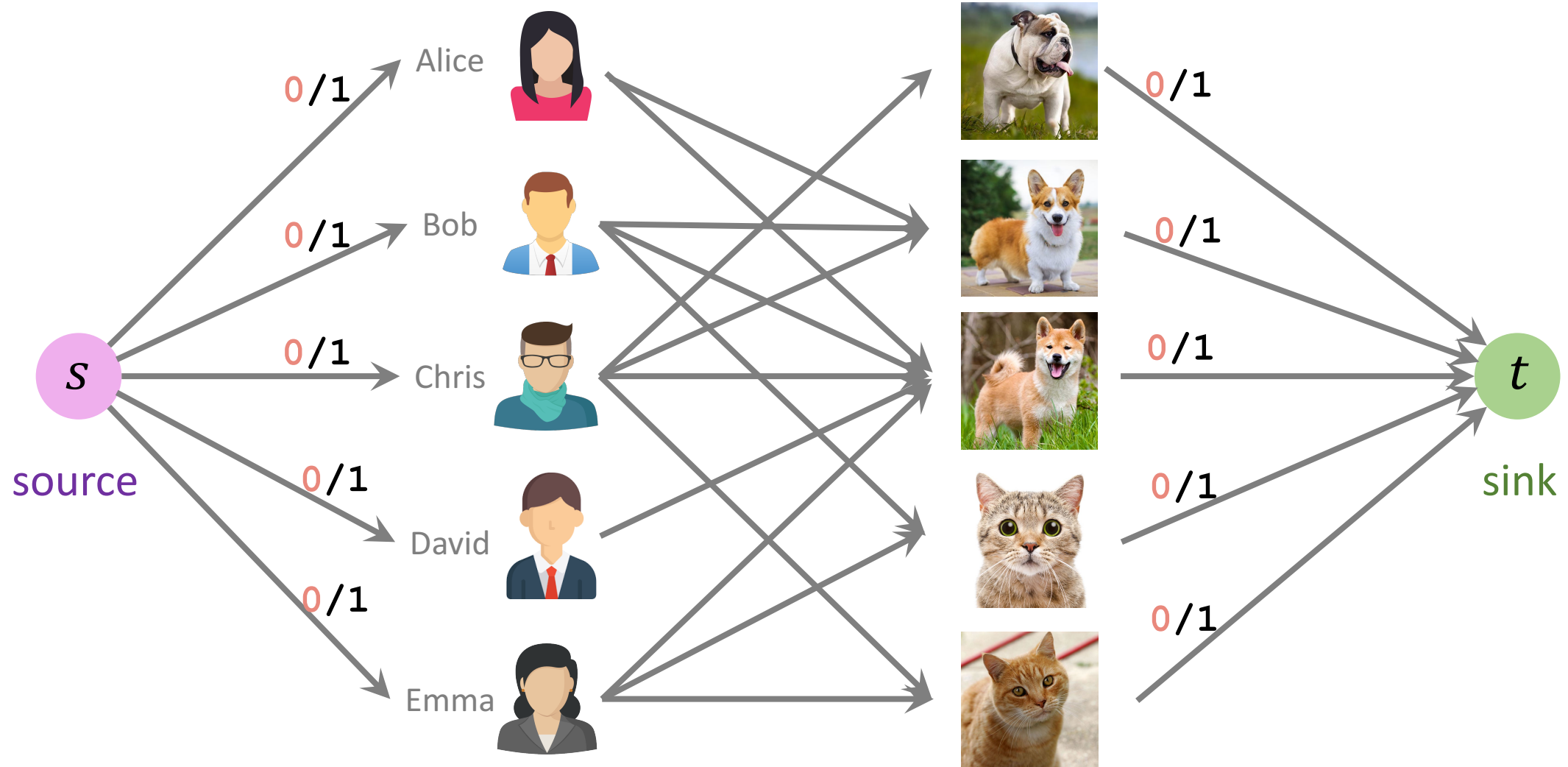
Make the edges directed



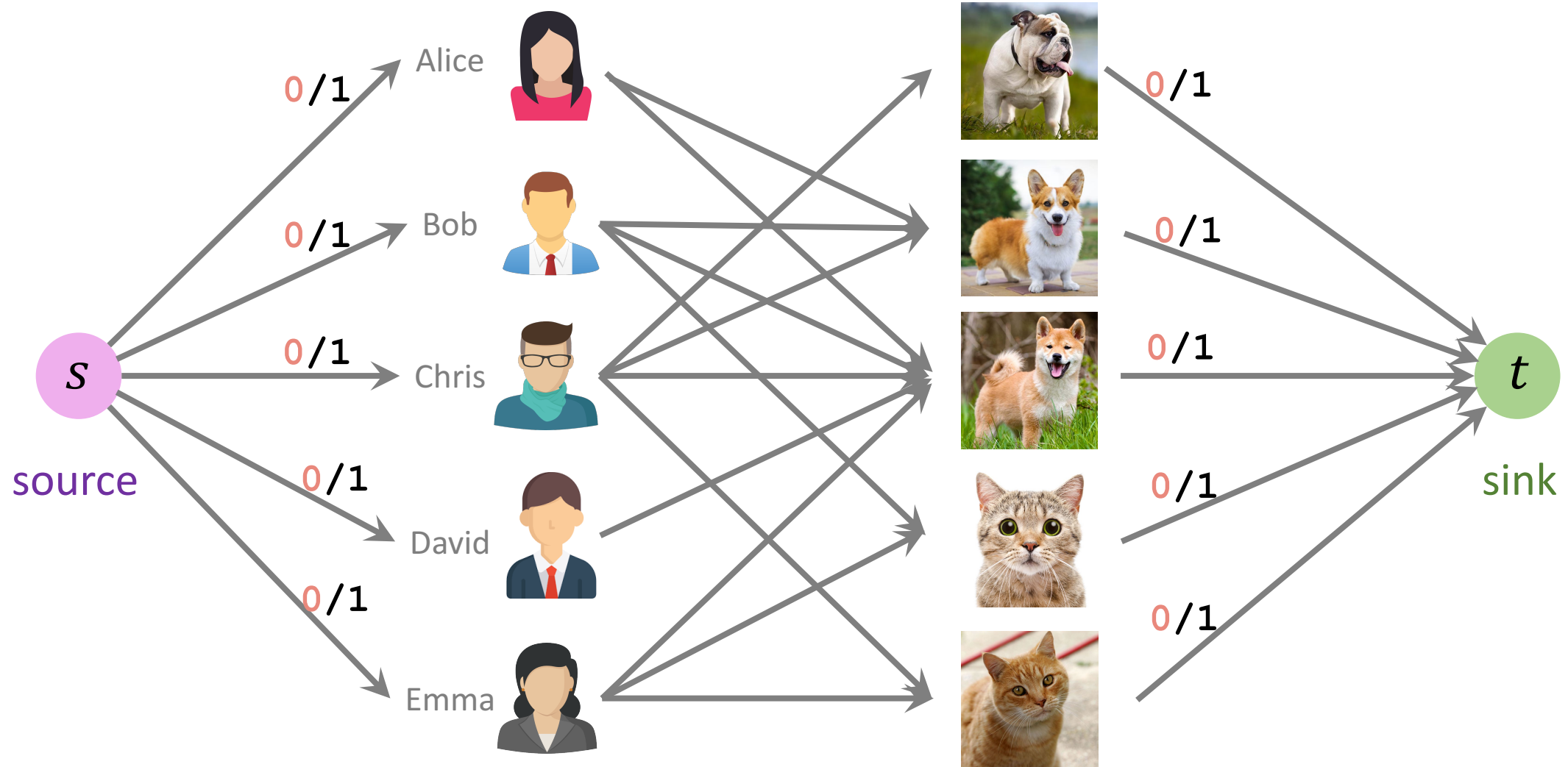
Add source and sink



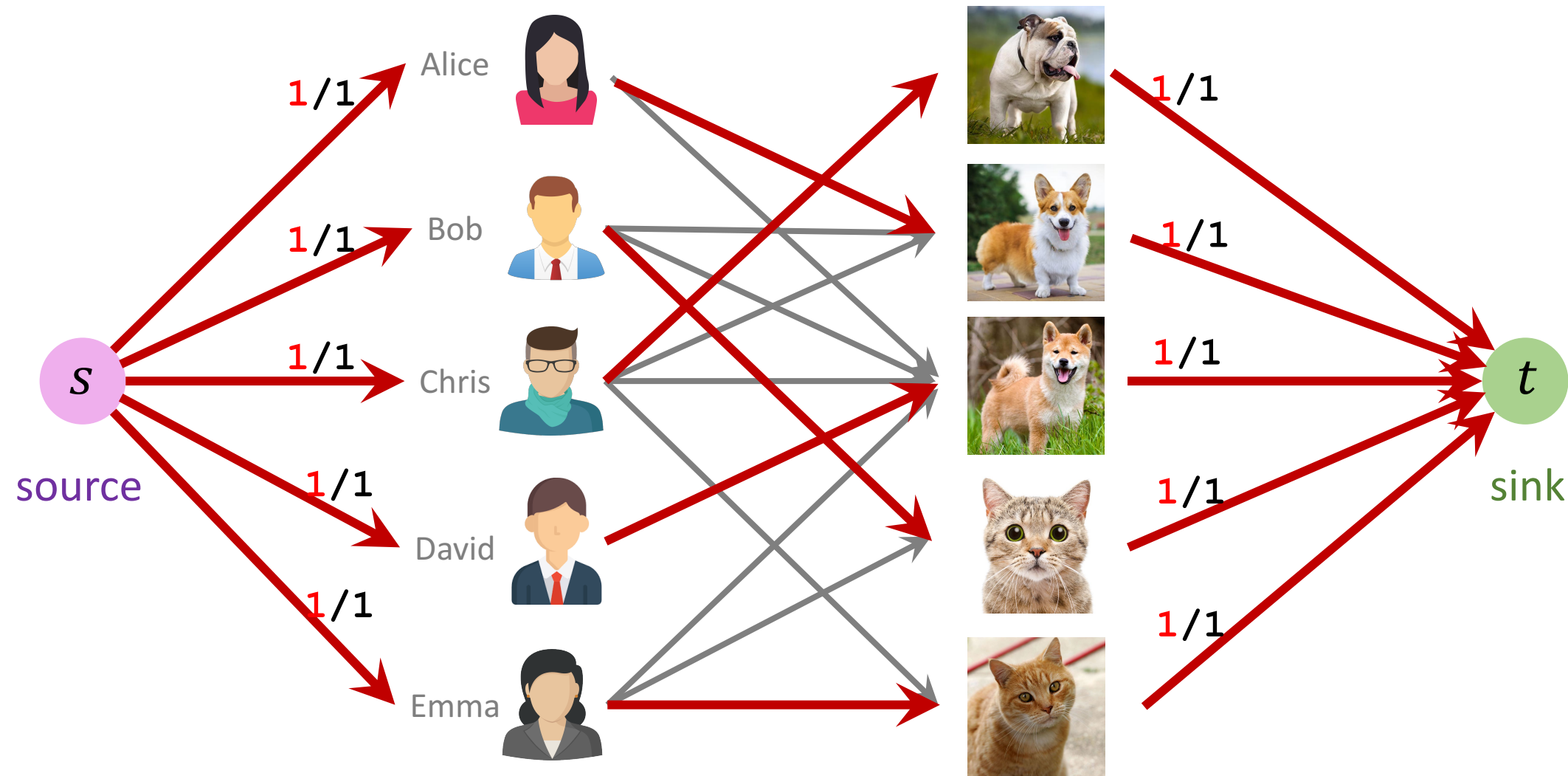
All the edge weights (capacities) are ones



Find the max-flow using any algorithm

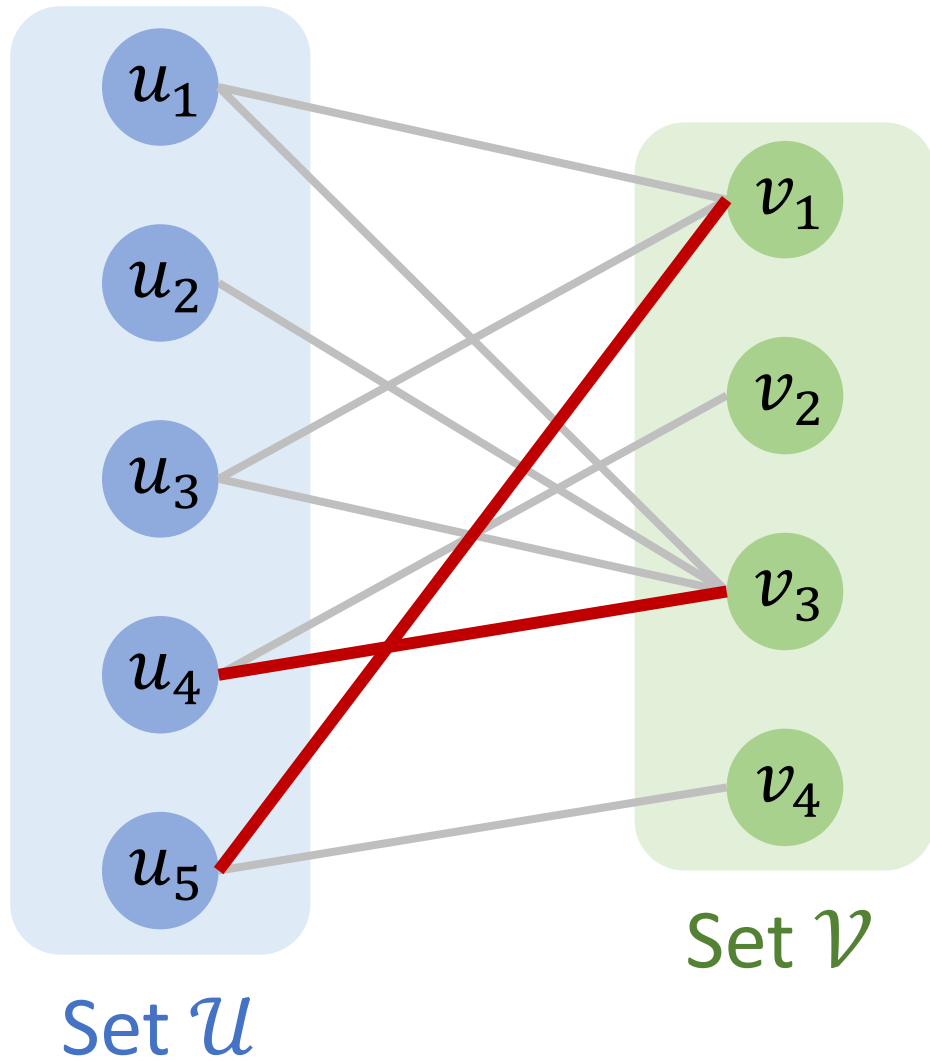


capacity of max-flow = cardinality of max-matching



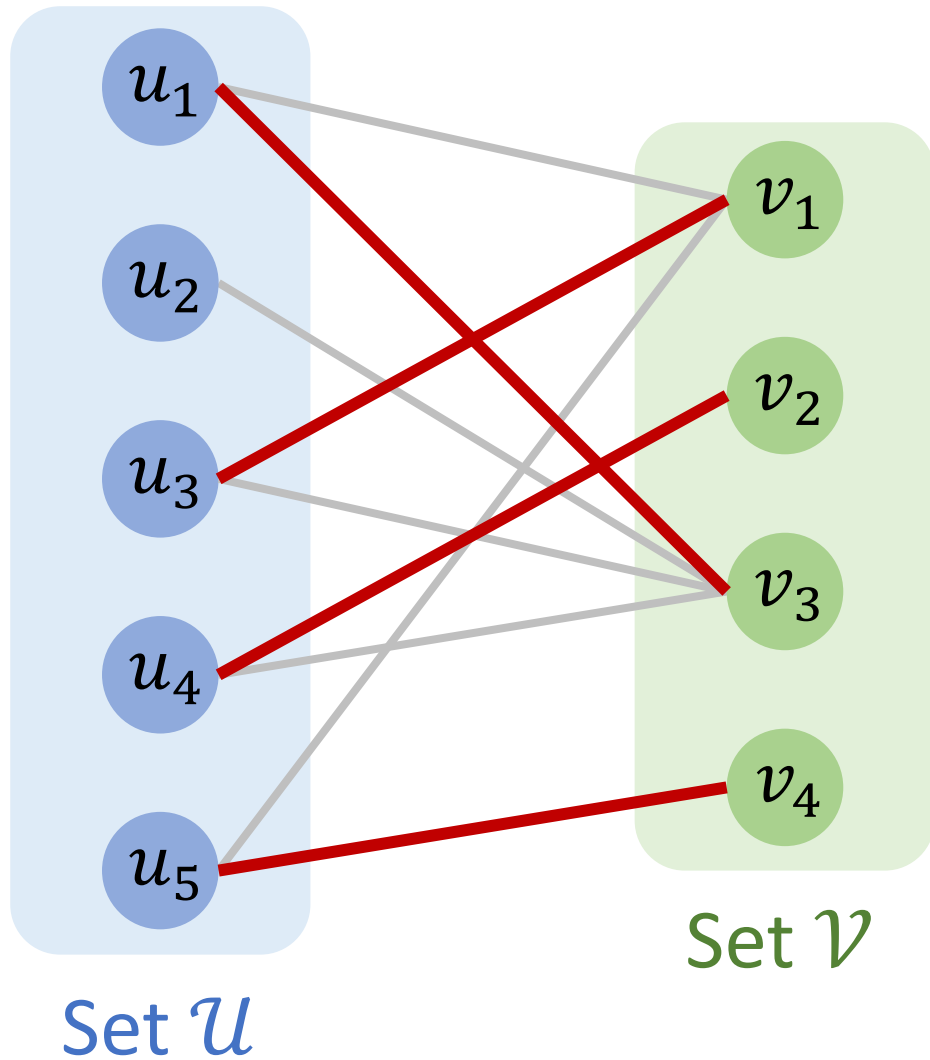
Summary

Bipartite Matching



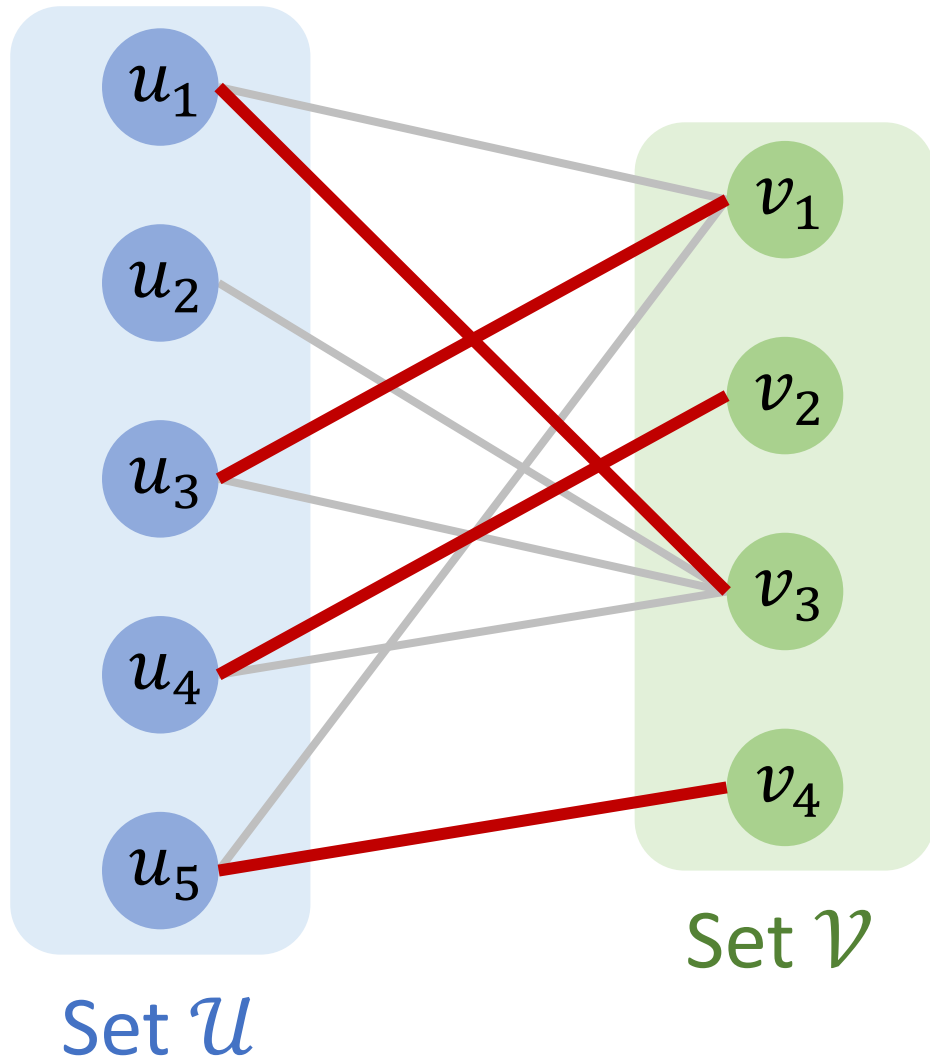
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Maximum Cardinality Bipartite Matching (MCBM)



- Given an **unweighted** bipartite graph.
- **MCBM**: Find matching \mathcal{S} that has the maximum cardinality $|\mathcal{S}|$.
- Bipartite matching in **unweighted** graphs is **easier** than in **weighted** bipartite graph.

Algorithms for finding MCBM



- Greedy algorithm can fail.
- Reduce MCBM to the max-flow problem and solve it using any max-flow algorithm.

Thank You!

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