NETWORK SCIENCE FINAL PROJECT

Catastrophic cascade of failures in interdependent networks

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The paper

Buldyrev, S. V., Parshani, R., Paul, G., Stanley, H. E., & Havlin, S. (2010). Catastrophic cascade of failures in interdependent networks. Nature, 464(7291), 1025-1028.

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LETTERS

Catastrophic cascade of failures in interdependent networks

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Complex networks have been studied intensively for a decade, but research still focuses on the limited case of a single, non-interacting network[1–14]. Modern systems are coupled together[15–19] and therefore should be modelled as interdependent networks. A fundamental property of interdependent networks is that failure of nodes in one network may lead to failure of dependent nodes in other networks. This may happen recursively and can lead to a cascade of failures. In fact, a failure of a very small fraction of nodes in one network may lead to the complete fragmentation of a system of several interdependent networks. A dramatic real-world example of a cascade of failures ('concurrent malfunction') is the electrical blackout that affected much of Italy on 28 September 2003: the shutdown of power stations directly led to the failure of nodes in the Internet communication network, which in turn caused further breakdown of power stations[20]. Here we develop a framework for understanding the robustness of interacting networks subject to such cascading failures. We present exact analytical solutions for the critical fraction of nodes that, on removal, will lead to a failure cascade and to a complete fragmentation of two interdependent networks. Surprisingly, a broader degree distribution increases the vulnerability of interdependent networks to random failure, which is opposite to how a single network behaves. Our findings highlight the need to consider interdependent network properties in designing robust networks.

Today's networks are becoming increasingly dependent on one another. Diverse infrastructures such as water supply, transportation, fuel and power stations are coupled together. We show that owing to this coupling, interdependent networks are extremely sensitive to random failure, such that a random removal of a small fraction of nodes from one network can produce an iterative cascade of failures in several interdependent networks. Electrical blackouts frequently result from a cascade of failures between interdependent networks, and the problem has been dramatically exemplified by the several large-scale blackouts that have occurred in recent years. In this Letter, we demonstrate a cascade of failures using real-world data from a power network and an Internet network (a supervisory control and data acquisition system) that were implicated in the blackout that affected much of Italy on 28 September 2003[20]. These two networks feature a bidirectional dependence such that power stations depend on communication nodes for control and communication nodes depend on power stations for their electricity supply.

Figure 1 shows the two networks and the connections between them, based on the real geographical locations. The figure exemplifies a situation in which an initial failure of only one power station may lead to an iterative cascade of failures that causes both networks to become fragmented. For an isolated single network, a significant number of the network nodes must be randomly removed before the network breaks down. However, when taking into account the dependencies between the networks, removal of only a small fraction of nodes can result in the complete fragmentation of the entire system.

To model interdependent networks, we consider for simplicity, and without loss of generality, two networks, A and B, with the same number of nodes, N. The functioning of node A_i ($i = 1, 2, \ldots, N$), in network A, depends on the ability of node B_i, in network B, to supply a critical resource, and vice versa. If node A_i stops functioning owing to attack or failure, node B_i stops functioning. Similarly, if node B_i stops functioning then node A_i stops functioning. We denote such a dependence by a bidirectional link, $A_i \leftrightarrow B_i$, that defines a one-to-one correspondence between nodes of network A and nodes of network B. Within network A, the nodes are randomly connected by A-links with degree distribution $P_A(k)$, where the degree, k, of each node is defined as the number of A-links connected to that node in network A. Analogously, within network B, the nodes are randomly connected by B-links with degree distribution $P_B(k)$ (Fig. 2).

We begin by randomly removing a fraction, $1 - p$, of the nodes of network A and removing all the A-links connected to these removed nodes (Fig. 2a). Owing to the dependence between the networks, all the nodes in network B that are connected to the removed A-nodes by $A \leftrightarrow B$ links must also be removed (Fig. 2b). Any B-links connected to the removed B-nodes are then also removed. As nodes and links are sequentially removed, each network begins to fragment into connected components, which we call clusters. The clusters in network A and the clusters in network B are different because each network is connected differently. A set of nodes, a, in network A and the corresponding set of nodes, b, in network B form a mutually connected set if (1) each pair of nodes in a is connected by a path that consists of nodes belonging to a and links of network A, and (2) each pair of nodes in b is connected by a path that consists of nodes belonging to b and links of network B. We call a mutually connected set a mutually connected cluster if it cannot be enlarged by adding other nodes and still satisfy the conditions above. Only mutually connected clusters are potentially functional.

To identify these mutually connected clusters, we first define the a_1-clusters as the clusters of network A remaining after a fraction $1 - p$ of the A-nodes are removed as the result of an attack or malfunction (Fig. 2b). This state of the networks is the first stage in the cascade of failures. Next we define the b_1-sets as the sets of B-nodes that are connected to a_1-clusters by $A \leftrightarrow B$ links. According to the definition of mutually connected clusters, all the B-links connecting different b_1-sets must be removed. Because the two networks are connected differently, each b_1-set may split into several clusters, which we define as b_2-clusters (Fig. 2c). The b_1-sets that do not split, and hence coincide with a_1-clusters, are mutually connected. This state of the networks is the second stage in the cascade of failures. In the third stage, we determine all the a_3-clusters (Fig. 2d), in a similar way, and in the fourth stage we determine all the b_4-clusters. We

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INTRODUCTION AND MODEL

Cascade of failure

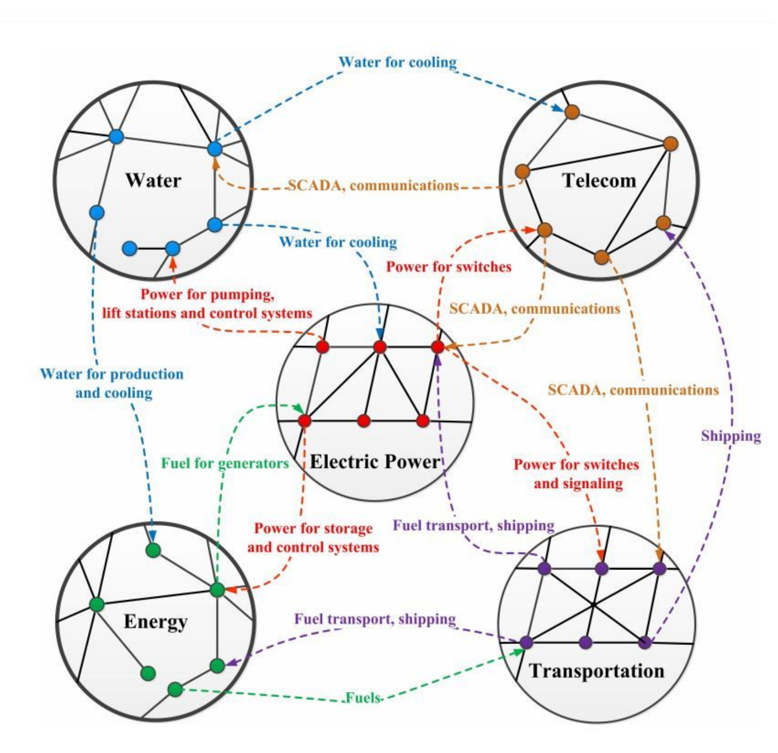
01

NETWORKS INTERDEPENDENCE (1/3)

- Past studies focused on the study of individual networks
- Modern networks are more and more interdependent
- Interdependent networks are more prone to **failures** → **Cascade of Failures**
- **Robustness:** How complex networks reacts to node failures

NETWORKS INTERDEPENDENCE (2/3)

- Example of interdependent networks:



[1]

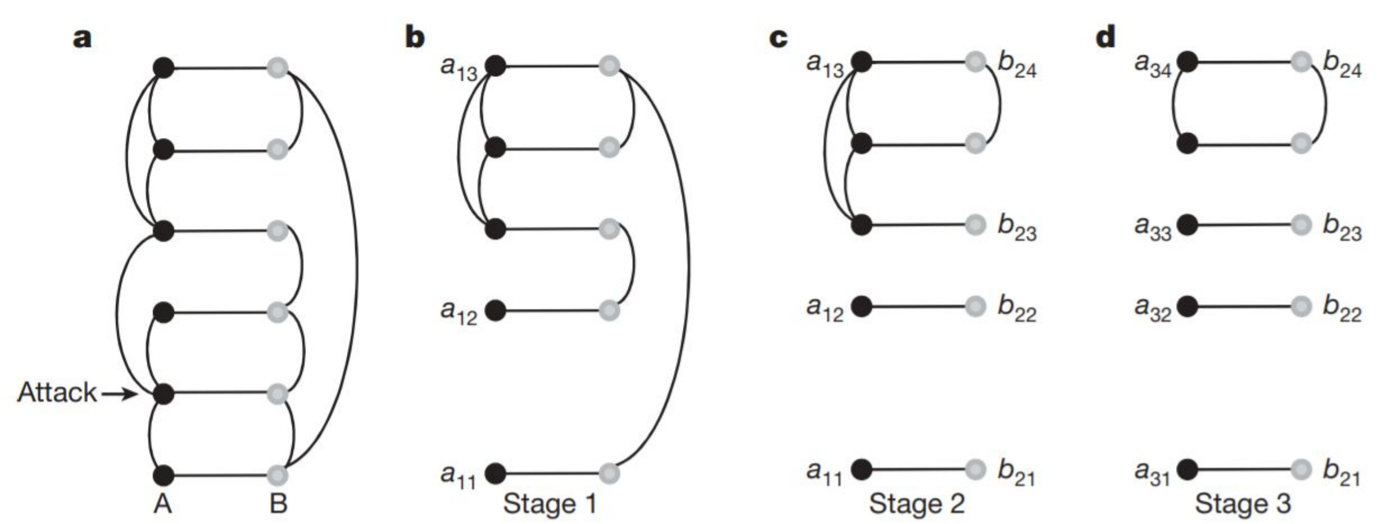
NETWORKS INTERDEPENDENCE (3/3)

- Ordinary interruptions → many infrastructure interdependencies and interactions can be safely ignored

- EMP attack scenario → expected to affect the different infrastructures simultaneously through multiple electronic component disruptions and failures over a wide geographical area

- Understanding cross-cutting interdependencies and interactions is critical to assess the **recoverability** of the full system

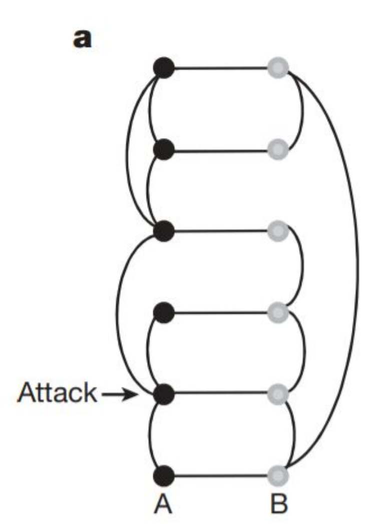
CASCADE OF FAILURES MODEL (1/5)



[2]

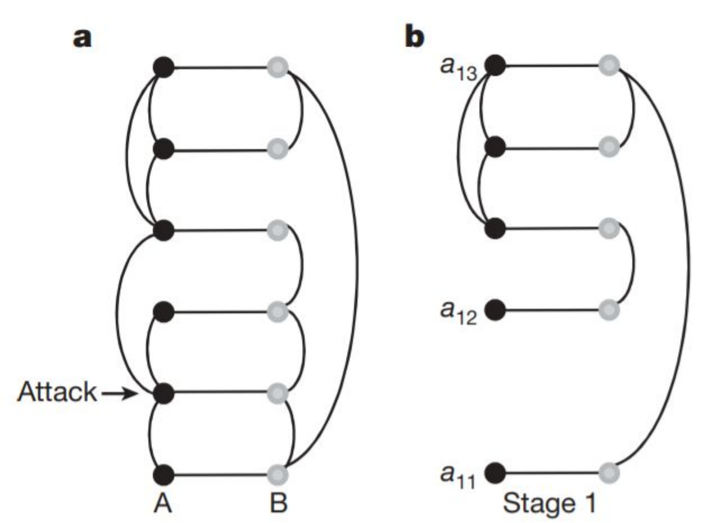
CASCADE OF FAILURES MODEL (2/5)

- **Initial condition**: Union network (A+B). One node in A is attacked.



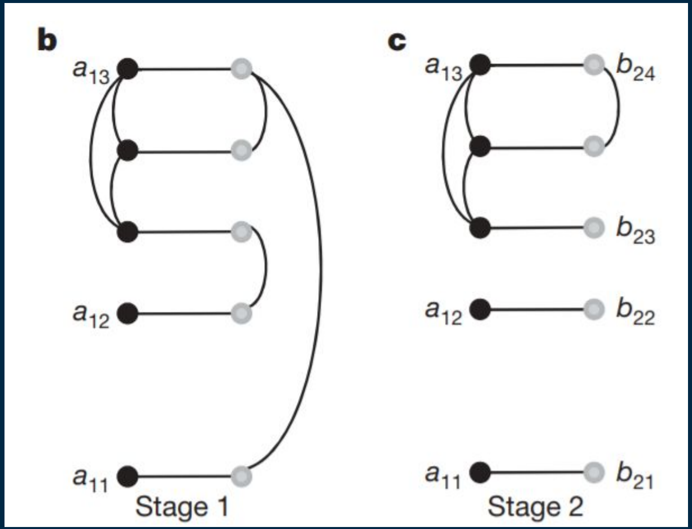
CASCADE OF FAILURES MODEL (3/5)

- **<u>Stage 1</u>**: A failure in A leads to failures in B.



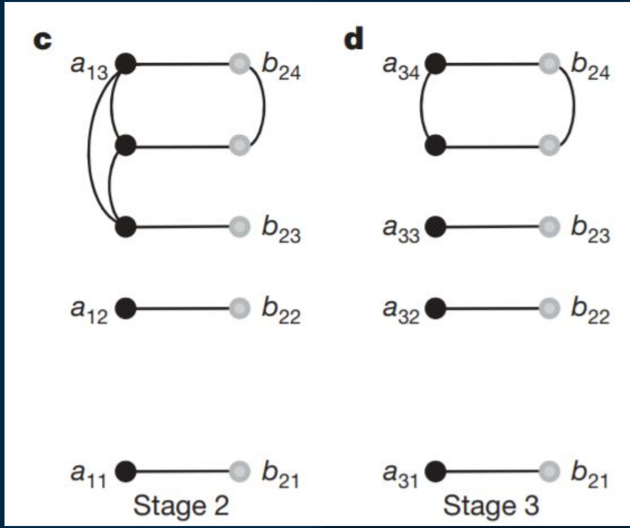
CASCADE OF FAILURES MODEL (4/5)

- **Stage 2**: Some edges in B are removed because they do not depend on the same nodes on A anymore.



CASCADE OF FAILURES MODEL (5/5)

- **Stage 3**: Repeat stage 2 for edges in A.



PROJECT GOAL

- Replicate Buldyrev et al. (2010) study: find networks robustness
  - Erdős–Rényi Networks
  - Scale-free Networks
  - Real world Dataset: Paris Multilayer Transportation Network

METHODS

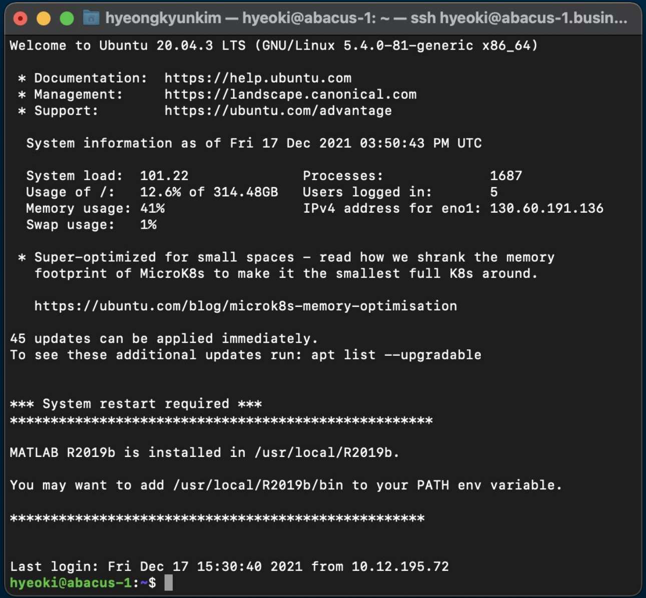
Resources and Procedure

02

RESOURCES (1/2)

Python

Jupyter Notebook

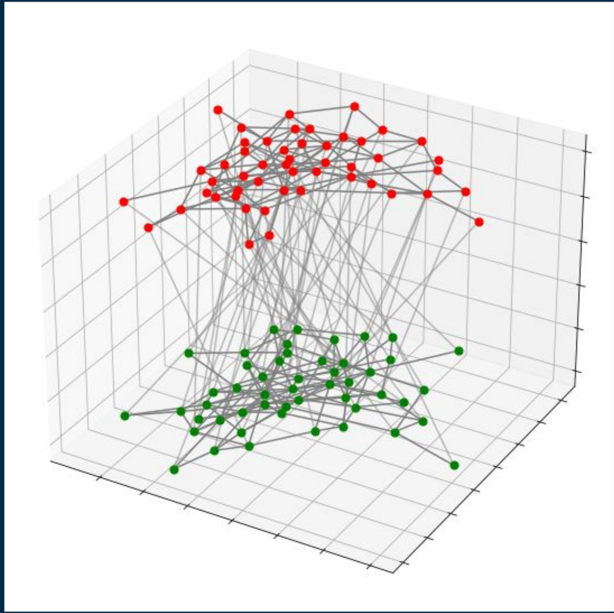


RANDOM NETWORKS 1

- **Erdos - Renyi Model (ER)**
  - N = 500 / 1000 / 2000
  - Average Degree <k> = 4
  - Using the method of 'Networkx' Erdos-Renyi Random Graph
  - Union and Connect every node in two ER network(A,B) set as followed;
    - Network A : One node

      ↕

      Network B : One randomly selected node



Sample N = 50 / <k> = 4

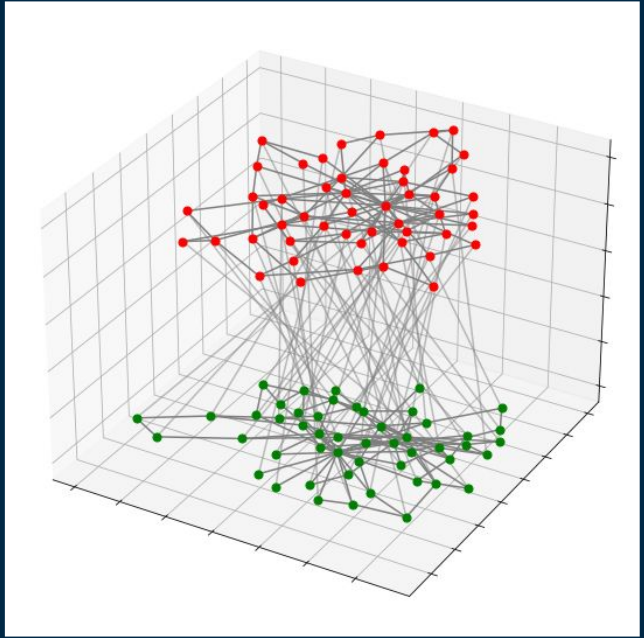
RANDOM NETWORKS 2

❑ Scale-Free Model (SF)

- N = 1000
- λ = 2.8 / 2.9 / 3.0
- Average Degree <k> = 4
- Using the library of 'power law' to create power law distribution
- Using the method of 'Networkx' configuration model to create Network
- Union and Connect every node in two ER network(A,B) set as followed;
  - Network A : One node

    ↕

    Network B : One randomly selected node



Sample N = 50 / λ = 3 / <k> = 4

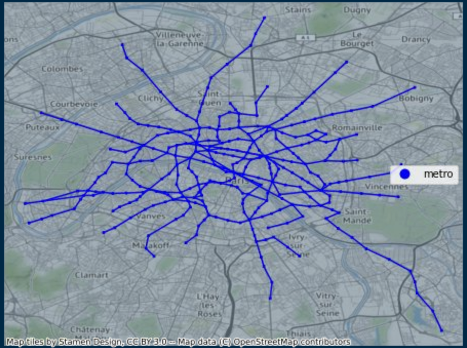
REAL WORLD NETWORKS

- Paris Multilayer Transport Network (Metro and Train Networks)
  - Github LINK
  - Nodes: transportation stops
  - Edges: connections between the stops

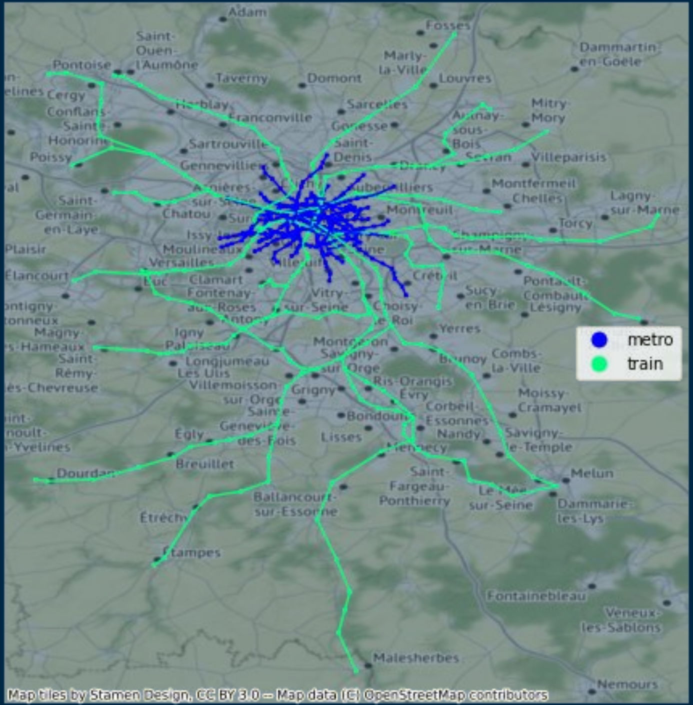
	NODES	EDGES
METRO	303	356
TRAIN	241	244
Crosslayer	M: 56 / T: 28	64

REAL WORLD NETWORKS

Metro

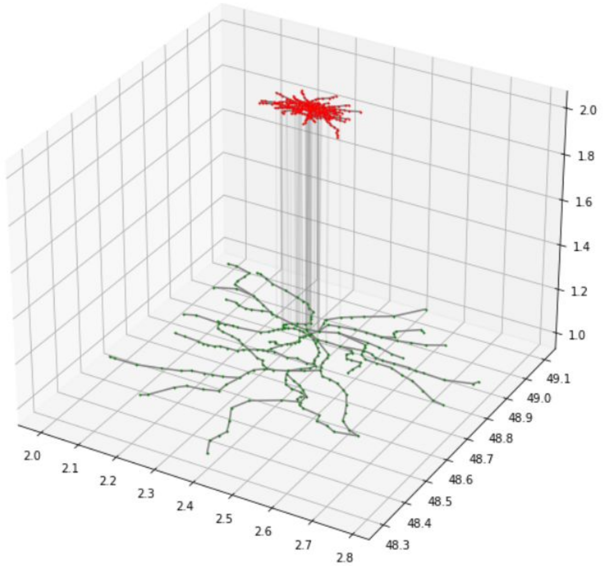


Train

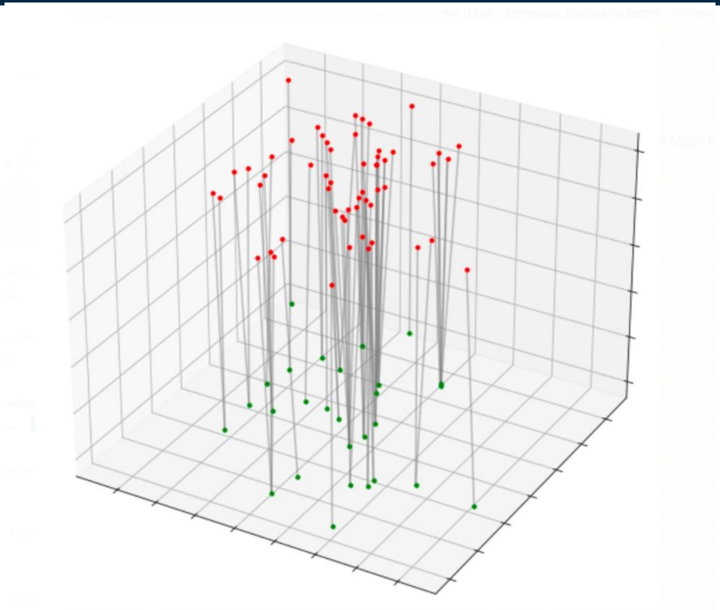


Interdependent Network Geographic
(Metro - Train)

REAL WORLD NETWORKS

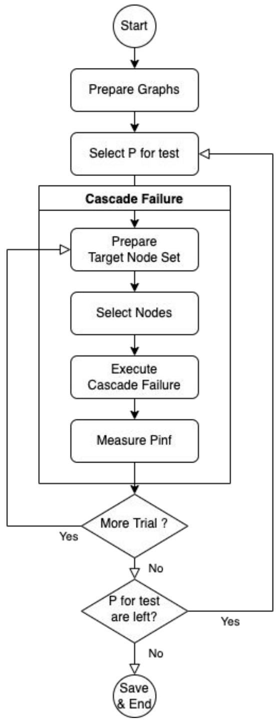


Interdependent Network Layered
(Metro - Train)



Interdependent Network Crosslayer Edges
(Metro - Train)

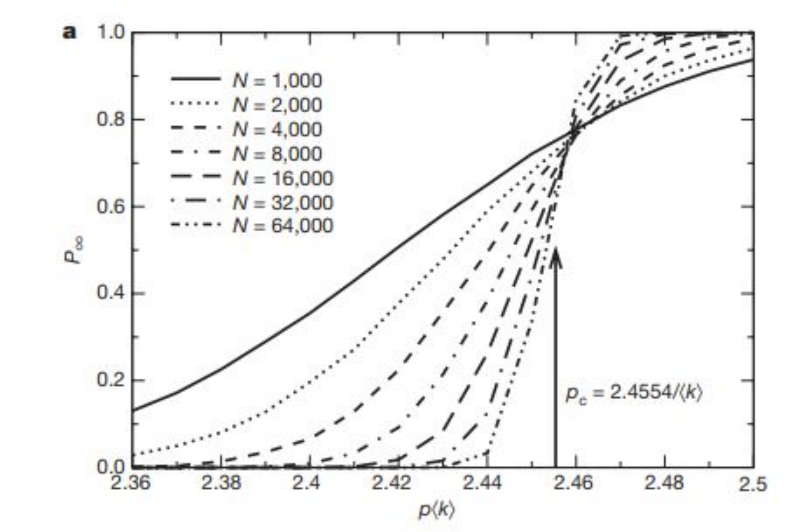
Procedure of Experiment



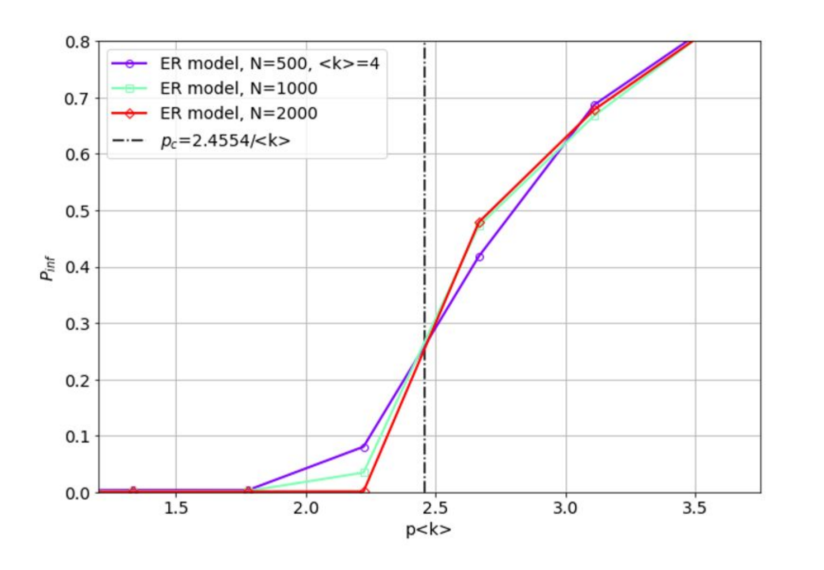
RESULTS AND DISCUSSION

03

ER-Networks

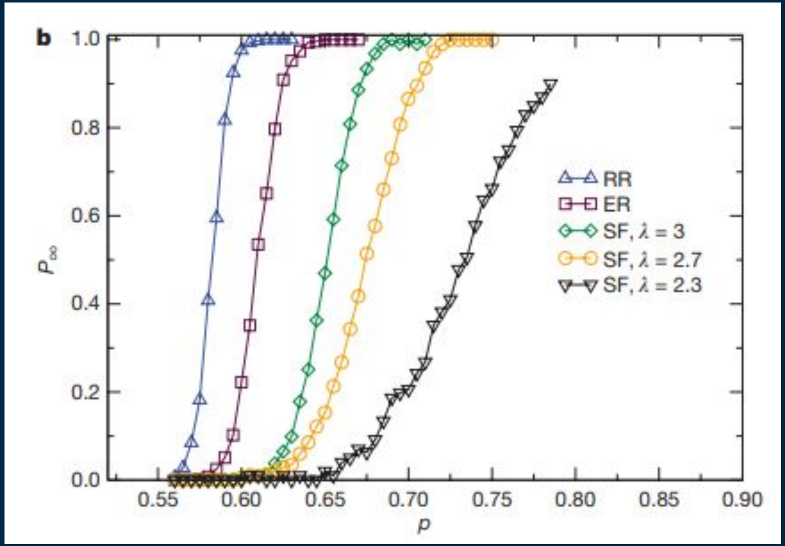


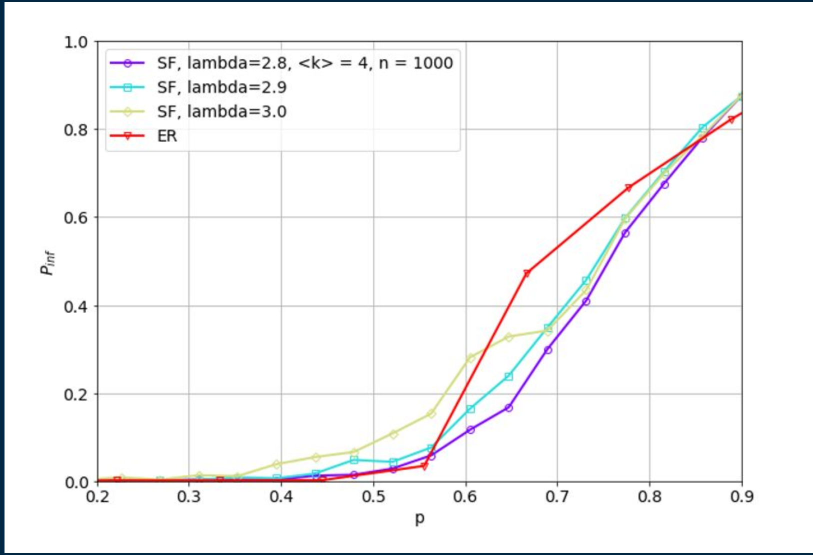
a
N = 1,000
N = 2,000
N = 4,000
N = 8,000
N = 16,000
N = 32,000
N = 64,000
P_∞
$p_c = 2.4554/\langle k \rangle$
$p\langle k \rangle$
1.0
0.8
0.6
0.4
0.2
0.0
2.36
2.38
2.4
2.42
2.44
2.46
2.48
2.5



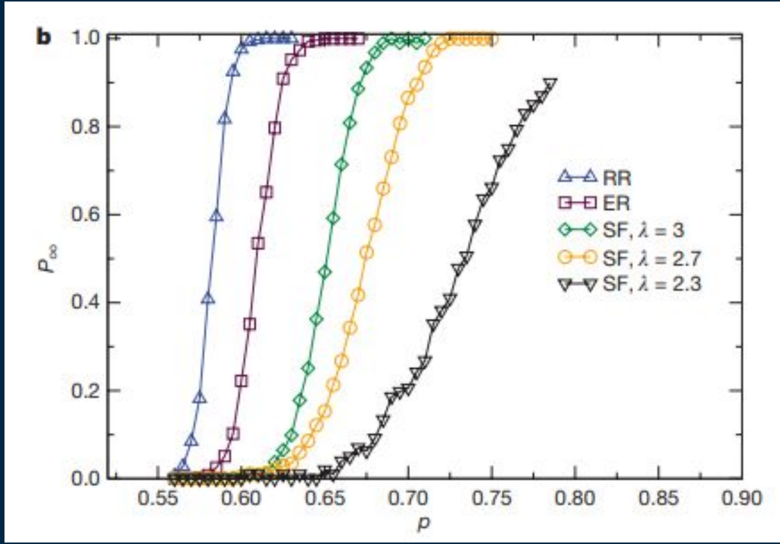
ER model, N=500, <k>=4
ER model, N=1000
ER model, N=2000
p_c=2.4554/<k>
P_{inf}
p<k>
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1.5
2.0
2.5
3.0
3.5

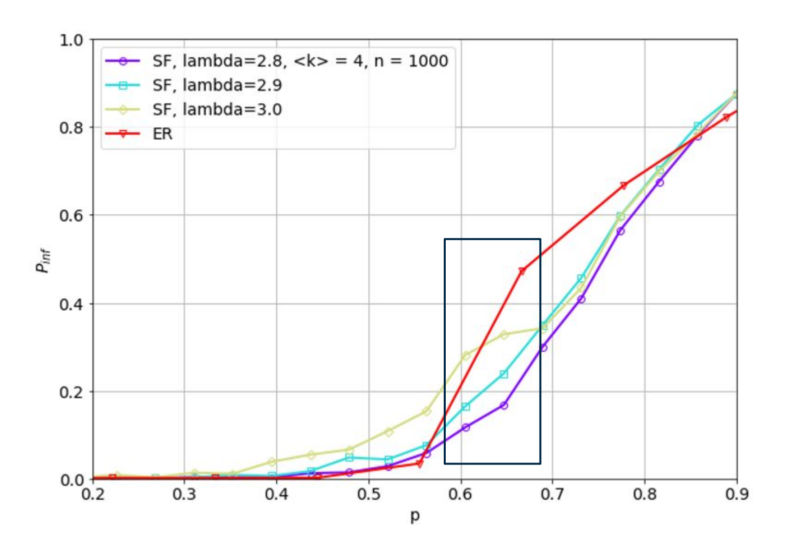
Scale-free Networks



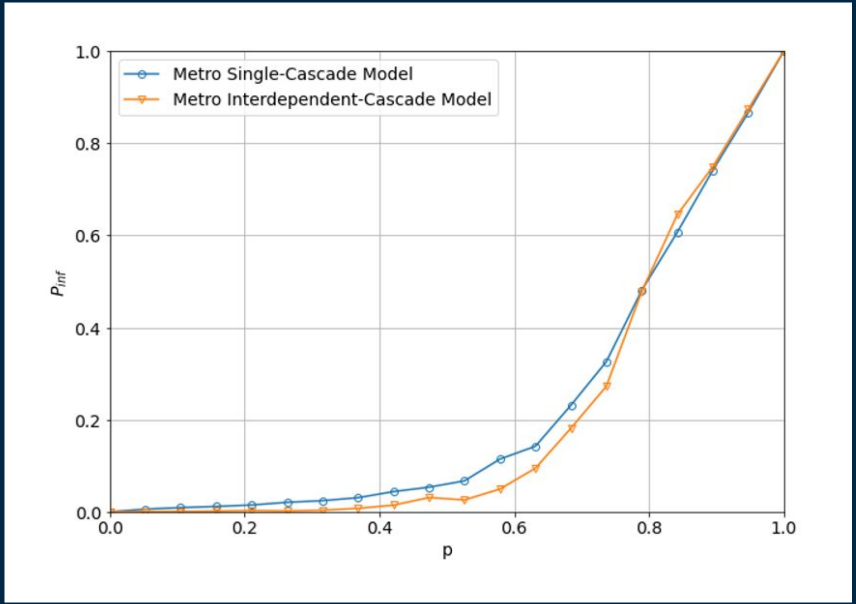


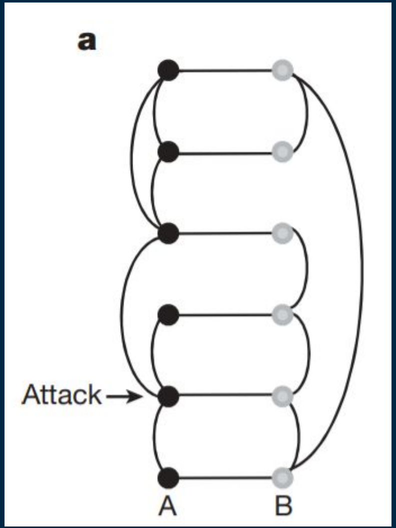
Scale-free Networks



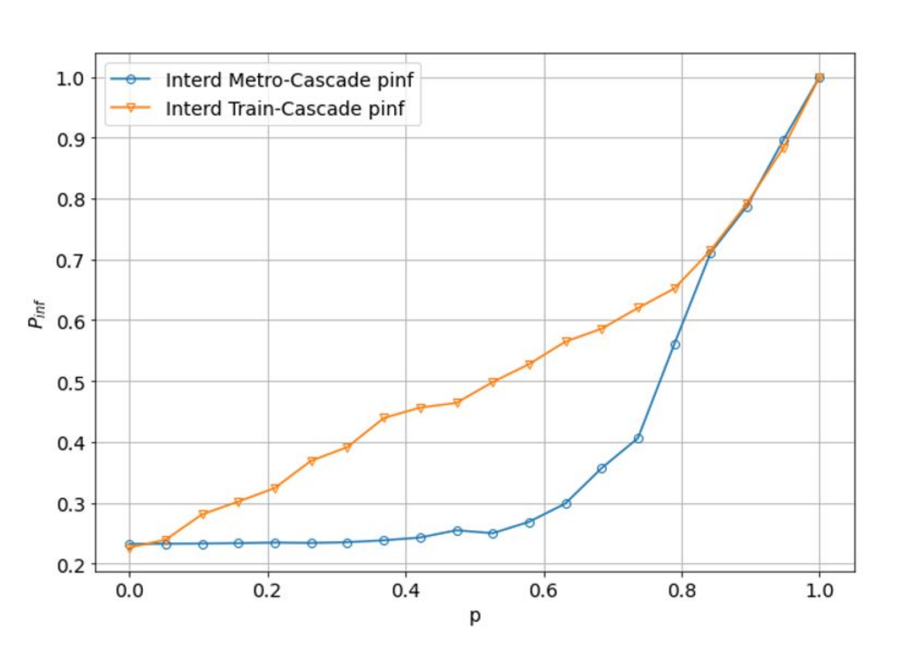


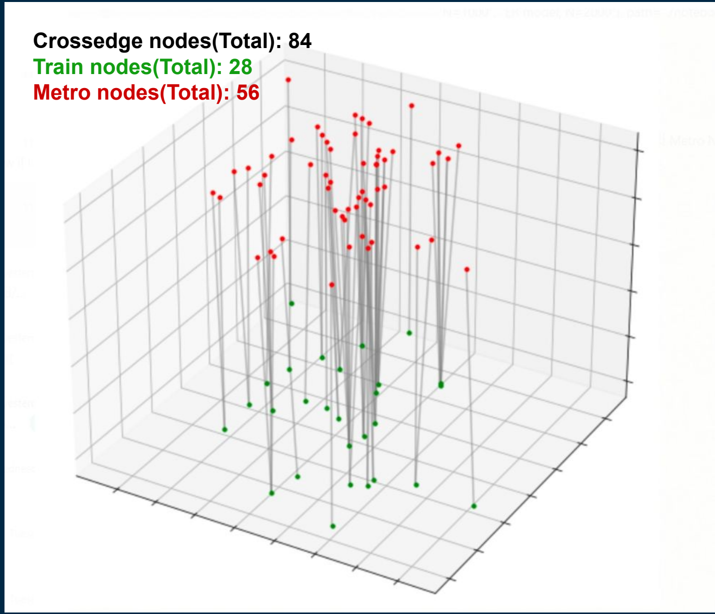
Paris Transportation Networks





Paris Transportation Networks





Reflection & future work

- Understanding iterative process of a cascade of failure in paper
  What we tried
  - first approach :
  all foreign neighbours in both nodes needed to exist on the same cluster component.
  - second approach:
  require at least one foreign neighbours of both sets needed to be on the same cluster component of the other network

- Using real world dataset with larger number of nodes and interconnection which also follows power-law distribution
  What we used:
  Transportation network - small number of nodes and interconnection, $\langle k \rangle = 2$

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IMAGES SOURCES

- [1]:https://en.wikipedia.org/wiki/Cascading_failure#/media/File:Interdependent_relationship_among_different_infrastructures.tif
- [2]:https://www.nature.com/articles/nature08932/figures/2
- [3]: https://www.pngall.com/python-programming-language-png
- [4]: https://de.wikipedia.org/wiki/Project_Jupyter
- [5]:https://github.com/networkx
- [6]:https://geopandas.org/en/stable/about/logo.html
- [7]:https://matplotlib.org/3.1.1/api/_as_gen/matplotlib.pyplot.title.html