Harshil Kotamreddy
PHYS 2200-07

PHYS 2200 FINAL

1) a) $V = \frac{E}{B}$

$E = vB$

$E = (0.2 \cdot 3\times10^8)(12)$

$\boxed{E = 7.2\times10^8 \ \frac{N}{C}}$

b) $\vec{F} = q(\vec{v}\times\vec{B})$

$= qvB\sin\theta$

$= (-1.6\times10^{-19})(0.2\cdot 3\times10^8)(12)\sin(90°)$

$= -1.152\times10^{-10}$ N

Magnitude: 1.152×10^{-10} N
Direction: downwards

c) $R = \frac{mv}{|q|B} = \frac{(9.1\times10^{-31})(0.2\cdot 3\times10^8)}{(1.6\times10^{-19})(12)}$

$= \boxed{2.84\times10^{-5} \text{ m}}$

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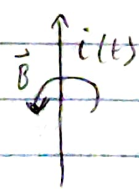
2) a) $B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0(120.3\cos(400t))}{2\pi(90\times10^{-2})} = 2.67\times10^{-5}\cos(400t)$

$B(t) = 2.67\times10^{-5}\cos(400t)$ T
Direction: out of the page



b) $\Phi = NBA\cos\theta$ $\quad N = 500 \quad A = \pi r^2 = \pi(10\times10^{-2})^2 = \pi\times10^{-2}$

$\Phi = (500)(2.67\times10^{-5}\cos(400t))(\pi\times10^{-2})\cos(0°)$

$\boxed{\Phi(t) = 4.19\times10^{-4}\cos(400t) \ T\cdot m^2}$

c) $\mathcal{E} = -\frac{d\Phi}{dt} = -(-(4.19\times10^{-4})(400)\sin(400t))$

$\boxed{\mathcal{E}(t) = 0.168\sin(400t) \ V}$

3) a) $C = \epsilon_0 \frac{A}{d}$

$= (8.85\times10^{-12}) \frac{(2.5)}{0.05\times10^{-3}}$

$= \boxed{4.425\times10^{-7}\ F}$

b) $Q = CV$

$12\times10^{-6} = (4.425\times10^{-7})V$

$V = \frac{12\times10^{-6}}{4.425\times10^{-7}}$

$\boxed{V = 27.12V}$

c) Yes, this is expected because the capacitor and inductor both charge and discharge

d) $w = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(100\times10^{-3})(4.425\times10^{-7})}} = 4.75\times10^{3}$

$f = \frac{w}{2\pi}$

$= \frac{4.75\times10^{3}}{2\pi}$

$\boxed{f = 7.57\times10^{2}\ Hz}$

e) $q = Q\cos(wt+\phi)$ $\phi = 0$ since capacitor is at full charge at $t=0$.

$q(t) = (12\times10^{-6})\cos(4.75\times10^{3}t)$

$q(20\times10^{-3}) = (12\times10^{-6})\cos((4.75\times10^{3})(20\times10^{-3}))$

$= 8.76\times10^{-6}\ C$

$q = CV$

$8.76\times10^{-6} = (4.425\times10^{-7})V$

$V = 18.34\ V$

$E = \frac{V}{d} = \frac{18.34}{0.05\times10^{-3}} = \boxed{3.67\times10^{5}\ \frac{N}{C}}$

f) $i = -wQ\sin(wt+\phi)$ $\qquad w = 4.75\times10^{3}$ rad/s $\qquad \phi = 0$

$i = -(4.75\times10^{3})(12\times10^{-6})\sin((4.75\times10^{3})t)$

$\frac{di}{dt} = -(4.75\times10^{3})^{2}(12\times10^{-6})\cos((4.75\times10^{3})t)$

$\mathcal{E} = -L\frac{di}{dt}$

$= -(100\times10^{-3})(-(4.75\times10^{3})^{2}(12\times10^{-6})\cos((4.75\times10^{3})t))$

$= 27.11\cos((4.75\times10^{3})t)$

$\mathcal{E} = -\frac{d\Phi}{dt}$

$\Phi(t) = -\int\mathcal{E}$

$= -\int 27.11\cos((4.75\times10^{3})t)$

$= \frac{-27.11}{(4.75\times10^{3})}\sin((4.75\times10^{3})t)$

$= -5.725\sin((4.75\times10^{3})t)$

$\Phi(20\times10^{-3}) = -5.725\sin((4.75\times10^{3})(20\times10^{-3}))$

$= \boxed{-4.16\times10^{-3}\ T\cdot m^{2}}$

4) a) $B = \mu_0 n I$ $\quad n = 900 \quad I = 0.5A$

$= (4\pi \times 10^{-7})(900)(0.5)$

$= \boxed{5.65 \times 10^{-4} T}$

b) $B = \frac{\mu_0 N I}{2\pi r}$ $\quad N = nL = (900)(1.2) = 1080$

$L = 2\pi r$

$= \frac{(4\pi \times 10^{-7})(1080)(0.5)}{1.2} = \boxed{5.65 \times 10^{-4} T}$

c) $\mathcal{E} = -\frac{d\Phi_B}{dt}$; $\Phi_B = B \cdot A$ $\quad B = B_0 \cos wt$

$= \pi r^2 B_0 \cos wt$

$\mathcal{E} = -\frac{d}{dt}(\pi r^2 B_0 \cos wt)$

$= w\pi r^2 B_0 \sin wt$

$I = \frac{\mathcal{E}}{R} = \frac{w\pi r^2 B_0 \sin wt}{R}$

when I is max, $\sin wt = 1$

$I = \frac{w\pi r^2 B_0}{R}$

$B_0 = \frac{I_{max} R}{\pi r^2 w} = \frac{(0.5)(2.4)}{\pi(0.1)^2(120\pi)} = \boxed{0.101 T}$

d) $P = I^2 R$

$= (0.5)^2 (2.4)$

$= \boxed{0.6 W}$

e) $\oint \vec{E} \cdot dl = \frac{d\Phi_{encl}}{dt}$

$E \cdot 2\pi r = \frac{d}{dt}(B \cdot A)$

$E = \frac{A}{2\pi r}\frac{dB}{dt}$ $\qquad \frac{dI}{dt} = \frac{\frac{1}{2}(0.5)}{5\times10^{-6}} = 5\times10^{4}$

$= \frac{\pi r^2}{2\pi r}\frac{d(\mu_0 n I)}{dt}$

$= \frac{1}{2} r \mu_0 n \frac{dI}{dt}$

$= \frac{1}{2}(0.05)(4\pi\times10^{-7})(900)(5\times10^{4})$

$= \boxed{1.4137\ V/m}$

Direction: Counter-clockwise

f) $\oint \vec{E} \cdot dl = \frac{d\Phi_{encl}}{dt}$

$E \cdot 2\pi R = \frac{d(BA)}{dt}$

$E \cdot 2\pi R = \pi r^2 \frac{dB}{dt}$ $\qquad \frac{dI}{dt} = \frac{\frac{1}{2}(0.5)}{5\times10^{-6}} = 5\times10^{4}$

$E = \frac{\cancel{\pi} r^2 \mu_0 n}{2\cancel{\pi} R}\frac{dI}{dt}$

$= \frac{r^2 \mu_0 n}{2R}\frac{dI}{dt}$

$= \frac{(0.1)^2(4\pi\times10^{-7})(900)}{2(0.25)}(5\times10^{4})$

$= \boxed{1.131\ V/m}$

Direction: Counter-clockwise

5) a) $\Phi_B = N \cdot B \cdot A = N \cdot B \cdot \pi r^2$

$= 680 \cdot \pi \cdot (0.12)^2 \, B_0 e^{-3t}$

$\boxed{\Phi_B(t) = 30.76 \, B_0 e^{-3t} \, T \cdot m^2}$

b) $\varepsilon = -\dfrac{d\Phi}{dt} = -(-3 \cdot 30.76 \, B_0 e^{-3t})$

$\boxed{\varepsilon(t) = 92.29 \, B_0 e^{-3t} \, V}$

c) $\varepsilon = I \cdot R = 0.6 \cdot 10 = 6$

$6 = 92.29 \, B_0 e^{-3t}$

$B_0 = \dfrac{6}{92.29 e^{-0.9}}$

$\boxed{B_0 = 0.16 \, T}$

d) $\int E \cdot dl = -\dfrac{d\Phi_B}{dt}$

$E \cdot 2\pi r = 92.29 \, B_0 e^{-3t}$

$E = \dfrac{92.29}{2\pi r} B_0 e^{-3t}$

$E = \dfrac{92.29}{2\pi(0.12)} B_0 e^{-3t}$

$\boxed{E(t) = 122.4 \, B_0 e^{-3t} \, V/m}$

6) a) $V_R = IR = (4.5)(90) = 405V$

$\boxed{V_R = 405V}$

$V_L = I X_L$ $\quad X_L = wL$ $\quad w = 2\pi f = 120\pi$

$= (4.5)(75.4)$ $\quad = (120\pi)(200\times10^{-3})$

$= 339.29V$ $\quad = 75.40$

$\boxed{V_L = 339.29V}$

$V_C = IX_C$ $\quad X_C = \frac{1}{wC} = \frac{1}{(120\pi)(20\times10^{-6})} = 1.326\times10^2$

$V_C = (4.5)(1.326\times10^2)$

$= 596.7V$

$\boxed{V_C = 596.7V}$

b) $\tan\phi = \frac{X_L - X_C}{R} = \frac{75.4 - 1.326\times10^2}{90}$

$\phi = \tan^{-1}\left(\frac{75.4 - 1.326\times10^2}{90}\right) = -0.57$ rads

Resistor: voltage and current are in phase, 0 radians
Capacitor: voltage lags behind current by 0.57 radians, -0.57 radians
Inductor: Voltage leads current by 0.57 radians, 0.57 radians

c) $V_R = V_R \cos wt$ $\quad V_p = IR = (4.5)(90) = 405$ $\quad w = 2\pi f = 120\pi$

$\boxed{V_R(t) = 405\cos(120\pi t)}$

$V_L = -IwL\sin(wt)$

$= -4.5(120\pi)(200\times10^{-3})\sin(120\pi t)$

$\boxed{V_L(t) = -339.29\sin(120\pi t)}$

$V_C = \frac{1}{wC}\sin(wt) = \frac{1}{(120\pi)(20\times10^{-6})}\sin(120\pi t)$

$\boxed{V_C(t) = 132.6\sin(120\pi t)}$

d) $V = ZI$

$$Z_R(t) = \frac{V_R(t)}{I} = \frac{405\cos(120\pi t)}{4.5}$$

$$\boxed{Z_R(t) = 90\cos(120\pi t)}$$

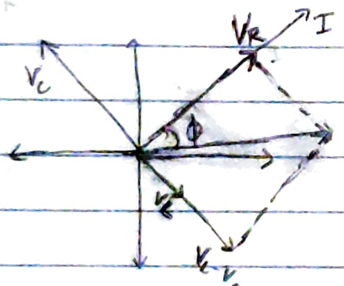
$$Z_L(t) = \frac{V_L(t)}{I} = \frac{-339.29}{4.5}\sin(120\pi t)$$

$$\boxed{Z_L(t) = -75.4\sin(120\pi t)}$$

$$Z_C(t) = \frac{V_C(t)}{I} = \frac{132.6}{4.5}\sin(120\pi t)$$

$$\boxed{Z_C(t) = 29.47\sin(120\pi t)}$$

e)



$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$= \tan^{-1}\left(\frac{339.29 - 596.7}{405}\right)$$

$$= \tan^{-1}(-6.36 \times 10^{-1})$$

$$\boxed{\phi = -0.57 \text{ rads}}$$

$$Z = \frac{V}{I} \qquad V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{405^2 + (339.2 - 596.7)^2}$$

$$Z = \frac{479.88}{4.5} \qquad = 479.88V$$

$$\boxed{Z = 106.64\,\Omega}$$

f) $V = IZ$

$V = (4.5)(106.64) = \boxed{479.88V}$

g) $V(t) = V\cos(\omega t) \qquad \omega = 2\pi f = 120\pi$

$$\boxed{V(t) = 479.88\cos(120\pi t)}$$

h) Energy is conserved if

$\overline{P}_s = \overline{P}_R + \overline{P}_L + \overline{P}_C$

$\overline{P}_s = \frac{1}{2} VI \cos\phi$ $\quad \overline{P}_L = 0$

$\overline{P}_R = \frac{1}{2} V_R I$ $\quad \overline{P}_C = 0$

Need to prove:

$\overline{P}_S = \overline{P}_R + 0 + 0$

$\frac{1}{2} VI \cos\phi = \frac{1}{2} V_R I$

$\cancel{\tfrac{1}{2}}\left(\frac{RI}{\cancel{\cos\phi}}\right)\cancel{I}\cancel{\cos\phi} = \cancel{\tfrac{1}{2}} V_R \cancel{I}$

$RI = V_R$

$V_R = V_R$ ✓

$V = ZI$

$V = \left(\sqrt{R^2 + (X_L - X_C)^2}\right) I$

$V = \left(\sqrt{R^2 + (R\tan\phi)^2}\right) I$

$V = RI\sqrt{1 + \tan^2\phi}$

$V = RI\sqrt{\sec^2\phi}$

② $V = \frac{RI}{\cos\phi}$

$\tan\phi = \frac{X_L - X_C}{R}$

$X_L - X_C = R\tan\phi$ ①

Thus, energy is conserved.

7) a) $I = \frac{P}{A}$ $\quad A =$ surface area of sphere $= 4\pi r^2$

$$I = \frac{P}{4\pi r^2} = \frac{180\,W}{4\pi(1.5)^2} = \boxed{6.37\,\frac{W}{m^2}}$$

b) $I = \frac{E_{max}^2}{2\mu_0 c}$

$$E_{max} = \sqrt{2\mu_0 c I}$$
$$= \sqrt{2(4\pi\times10^{-7})(3\cdot10^8)(6.37)}$$

$$\boxed{E_{max} = 69.3\,\frac{N}{C}}$$

$$B_{max} = \frac{E_{max}}{c} = \frac{69.3}{3\cdot10^8}$$

$$\boxed{B_{max} = 2.3\times10^{-7}\,T}$$

c) $\vec{E}(x,t) = \hat{j}\,E_{max}\cos(kx - \omega t)$

$\vec{B}(x,t) = \hat{k}\,B_{max}\cos(kx - \omega t)$

$\lambda = \frac{2\pi}{k}$ $\qquad \omega = ck = (3\cdot10^8)(1.12\times10^7) = 3.37\times10^{15}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{560\times10^{-9}} = 1.12\times10^7$

$$\boxed{\begin{aligned}\vec{E}(x,t) &= \hat{j}\,69.3\cos((1.12\times10^7)x - (3.37\times10^{15})t)\\ \vec{B}(x,t) &= \hat{k}\,(2.31\times10^{-7})\cos((1.12\times10^7)x - (3.37\times10^{15})t)\end{aligned}}$$

d) $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ $\qquad \vec{E}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & \vec{E}(x,t) & 0\\ 0 & 0 & \vec{B}(x,t)\end{vmatrix} = \vec{E}(x,t)\vec{B}(x,t)\,\hat{i}$

$$= \frac{1}{\mu_0}\vec{E}(x,t)\vec{B}(x,t)\,\hat{i}$$

$$= \frac{1}{(4\pi\times10^{-7})}(69.3)(2.31\times10^{-7})\cos^2((1.12\times10^7)x - (3.37\times10^{15})t)\,\hat{i}$$

$$\boxed{\vec{S} = 12.74\cos^2((1.12\times10^7)x - (3.37\times10^{15})t)\,\hat{i}}$$

8) a) $I = \frac{P}{A}$ $A = 2\pi r l$ (Area of a cylinder)

$l = 1$ bc $P = 80 \times 10^3$ per unit length

$$I = \frac{80 \times 10^3}{2\pi(0.7 \times 10^3)(1)}$$

$$\boxed{I = 18.19 \ W/m^2}$$

b) $I = \frac{E_{max}^2}{2\mu_0 c}$

$$E_{max} = \sqrt{2\mu_0 c I}$$

$$= \sqrt{2(4\pi \times 10^{-7})(3 \times 10^8)(18.19)}$$

$$\boxed{E_{max} = 117.11 \ N/C}$$

$$E_{max} = c B_{max}$$

$$B_{max} = \frac{E_{max}}{c} = \frac{117.11}{3 \times 10^8}$$

$$\boxed{B_{max} = 3.904 \times 10^{-7} \ T}$$

c) $\vec{E}(x,t) = \hat{j} E_{max} \cos(kx - wt)$ $W = 2\pi f$ $k = \frac{w}{c} = \frac{4.27 \times 10^6}{3 \times 10^8}$

$\vec{B}(x,t) = \hat{k} B_{max} \cos(kx - wt)$ $= 4.27 \times 10^6$ $= 1.42 \times 10^{-2}$

$$\boxed{\begin{aligned} \vec{E}(x,t) &= \hat{j} \ 117.11 \cos((1.42 \times 10^{-2})x - (4.27 \times 10^6)t) \\ \vec{B}(x,t) &= \hat{k} (3.904 \times 10^{-7}) \cos((1.42 \times 10^{-2})x - (4.27 \times 10^6)t) \end{aligned}}$$

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ $\vec{E} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \vec{E}(x,t) & 0 \\ 0 & 0 & \vec{B}(x,t) \end{vmatrix} = \vec{E}(x,t)\vec{B}(x,t)\hat{i}$

$$= \frac{1}{\mu_0} \vec{E}(x,t)\vec{B}(x,t)\hat{i}$$

$$= \frac{(117.11)(3.904 \times 10^{-7})}{(4\pi \times 10^{-7})} \cos^2((1.42 \times 10^{-2})x - (4.27 \times 10^6)t)\hat{i}$$

$$\boxed{\vec{S} = 36.38 \cos^2((1.42 \times 10^{-2})x - (4.27 \times 10^6)t)\hat{i}}$$

d) $\frac{S}{c^2} = \frac{E_{max} B_{max}}{\mu_0 c^2} = \frac{(117.11)(3.904 \times 10^{-7})}{(4\pi \times 10^{-7})(3 \times 10^8)^2}$

$= \boxed{4.04 \times 10^{-16}}$

e) $\frac{S}{c} = \frac{E_{max} B_{max}}{\mu_0 c} = \frac{(117.11)(3.904 \times 10^{-7})}{(4\pi \times 10^{-7})(3 \times 10^8)}$

$= \boxed{1.21 \times 10^{-7}}$

f) Absorbing surface:

$P = \frac{I}{c} = \frac{18.19}{(3 \times 10^8)} = \boxed{6.06 \times 10^{-8}}$

Reflecting surface:

$P = \frac{2I}{c} = \frac{2(18.19)}{3 \times 10^8} = \boxed{1.21 \times 10^{-7}}$