Social networks

Agent-based modelling, Konstanz, 2024

Henri Kauhanen

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> **Update 25 June 2024**
>
> As some of you cleverly pointed out, there is actually a direct link from *Switzerland* to *Albert Einstein* on Wikipedia, rendering my six degrees of separation exercise somewhat vacuous…
> To better get the desired effect, try navigating between two pages which are less obviously related, such as *Easter Island* and *Albert Einstein*.
> Also: I've fixed a typo: "each node's side" → "each node's size".

Plan

- In our models so far, agent interactions have been either
  1. Completely random
  2. Random, but within a local spatial neighbourhood
- Today, we will take a step towards generalizing our models by assuming that agents are connected through a social network
- We will require *Graphs.jl* and *GraphPlot.jl*:

```
using Pkg
Pkg.add(["Graphs", "GraphPlot"])
```

Networks

- A **network** (also known as a **graph**) consists of
  1. A set of **nodes** (a.k.a. **vertices** (singular: *vertex*)) – for us, these are the agents

2. A set of **connections** (a.k.a. **links** or **edges**) between nodes – for us, these define the interaction pattern

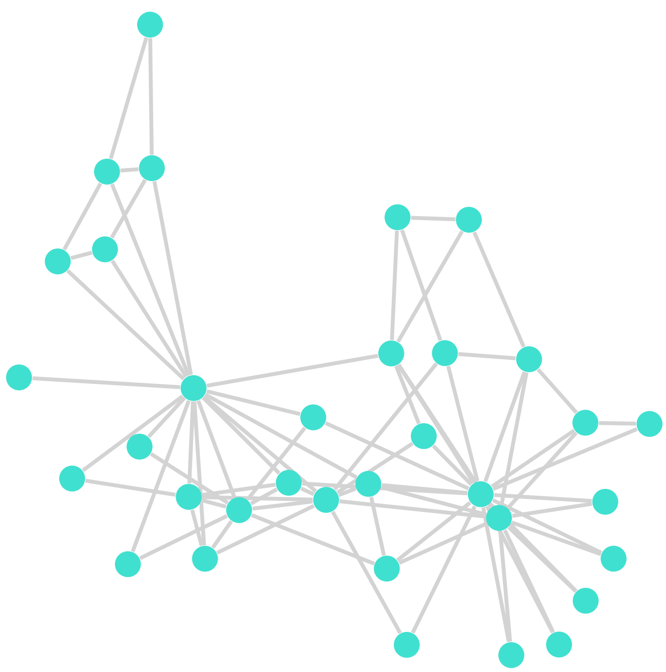
- Connections can be

  1. Unidirectional (A is connected to B, but B is not connected to A)
  2. Bidirectional (A is connected to B and B is also connected to A)
  3. Weighted or not

Drawing networks

- Nodes typically drawn as points / filled circles
- A unidirectional connection is drawn as an arrow
- A bidirectional connection is drawn as a line segment
- If connections are weighted, this can be represented e.g. by line width

Example (Zachary's karate club)



Networks in Julia

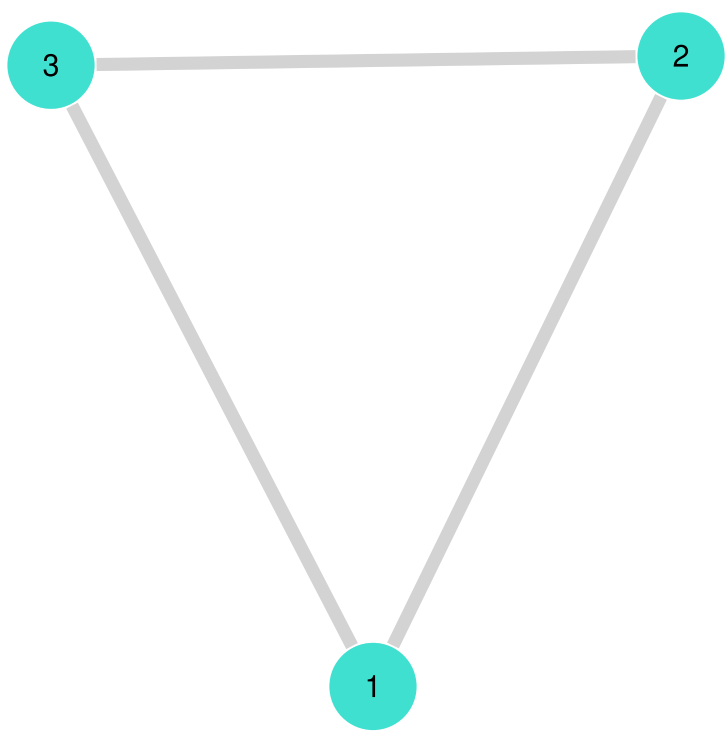
- In Julia, networks/graphs are handled by the Graphs.jl package
- Simple example: create an undirected graph of three nodes, connecting each node:

```
using Graphs, GraphPlot

G = Graph(3)

add_edge!(G, 1, 2)
add_edge!(G, 1, 3)
add_edge!(G, 2, 3)
```

Plotting networks

- Networks can be plotted with the `gplot` function from *GraphPlot.jl*:

```
gplot(G, nodelabel=1:3)
```

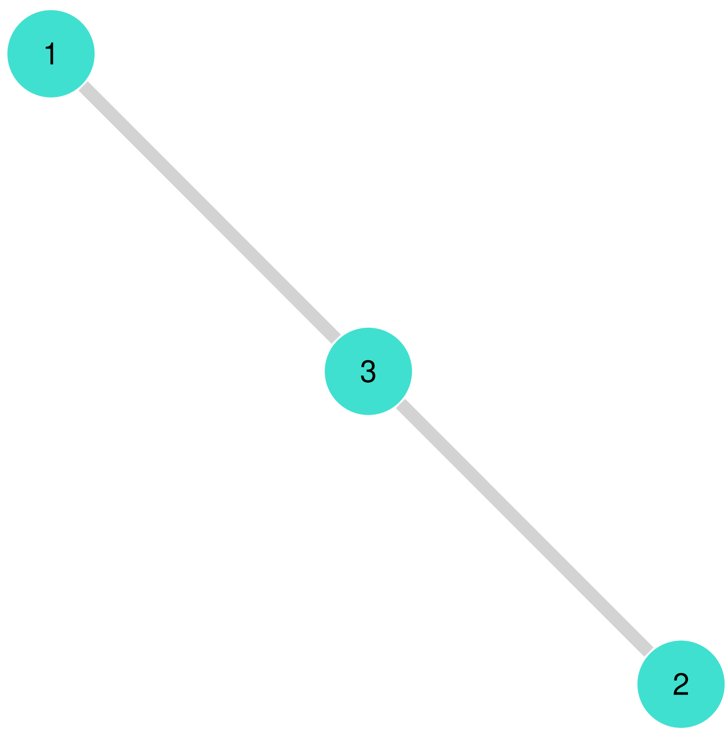


> **Important**
>
> The `gplot` function in fact returns something known as a "composition"; this may or may not be actually drawn as a picture, depending on your working environment. If you're in VSCode, a picture will be displayed, but if you're using the ordinary Julia REPL, you will not see a picture. In the latter case, you need to save the composition to an image file as follows:
>
> ```
> using Compose
>
> pic = gplot(G, nodelabel=1:3)
> draw(PNG("mypicture.png"), pic)
> ```

Removing connections

- `rem_edge!` can be used to remove existing connections:

```
rem_edge!(G, 1, 2)
gplot(G, nodelabel=1:3)
```

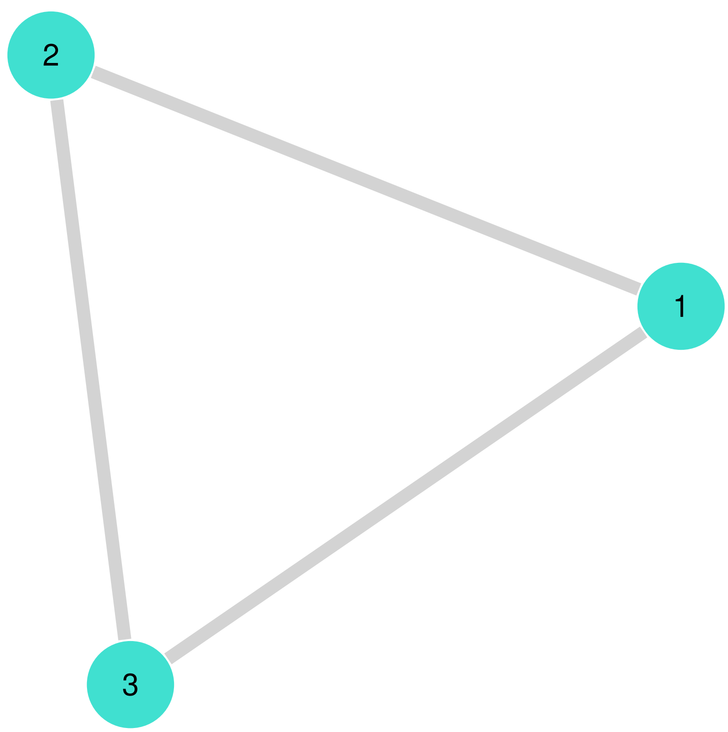


Constructing a network from an adjacency matrix

- Using lots of `add_edge!` calls quickly becomes tedious…
- Often, a better way of constructing a graph is by way of its **adjacency matrix**
- This is a matrix (two-dimensional array) of numbers such that:
  - if there is a 1 in the cell on the ith row, jth column, then nodes i and j are connected
  - if there is a 0 there, the nodes are not connected
- Example:

```
A = [0 1 1
     1 0 1
     1 1 0]

G2 = Graph(A)
```

```
gplot(G2, nodelabel=1:3)
```



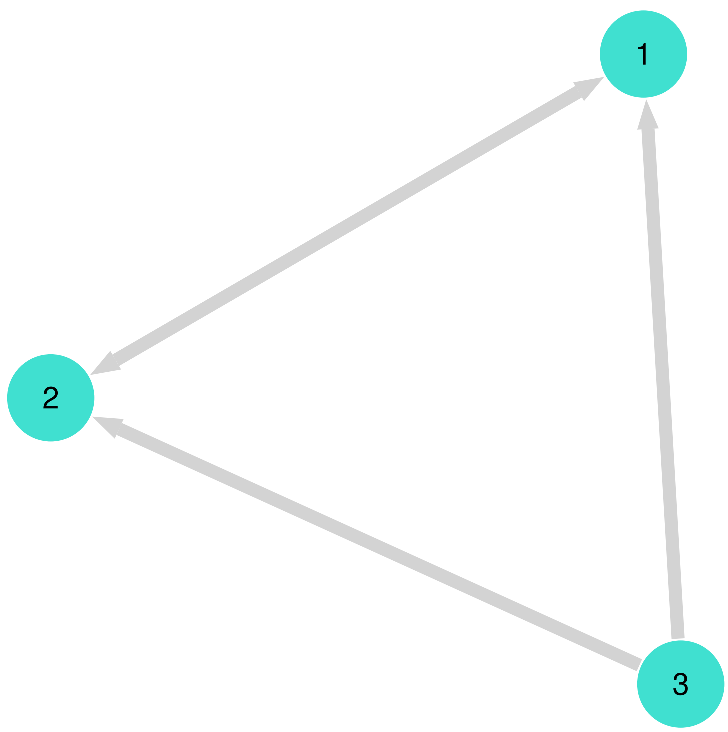
Directed graphs

- What if we need a network in which at least some connections are unidirectional?
- We need a **directed graph**, implemented by the `DiGraph` type

```
A = [0 1 0
     1 0 0
     1 1 0]

G3 = DiGraph(A)
```

```
{3, 4} directed simple Int64 graph
```

```
gplot(G3, nodelabel=1:3)
```



> ❗ Important
>
> Notice that, since `Graph` constructs an undirected graph, it expects a **symmetric** adjacency matrix as argument. If you try to pass an asymmetric adjacency matrix (such as

the one above) to `Graph`, you will get an error.
In other words, if your adjacency matrix is asymmetric, you are dealing with a directed graph, and you *must* use `DiGraph`.

Exercise

What kind of network do the following adjacency matrices represent? **Think about it first** (draw with your "mind's eye"), then implement the code and plot the graphs.

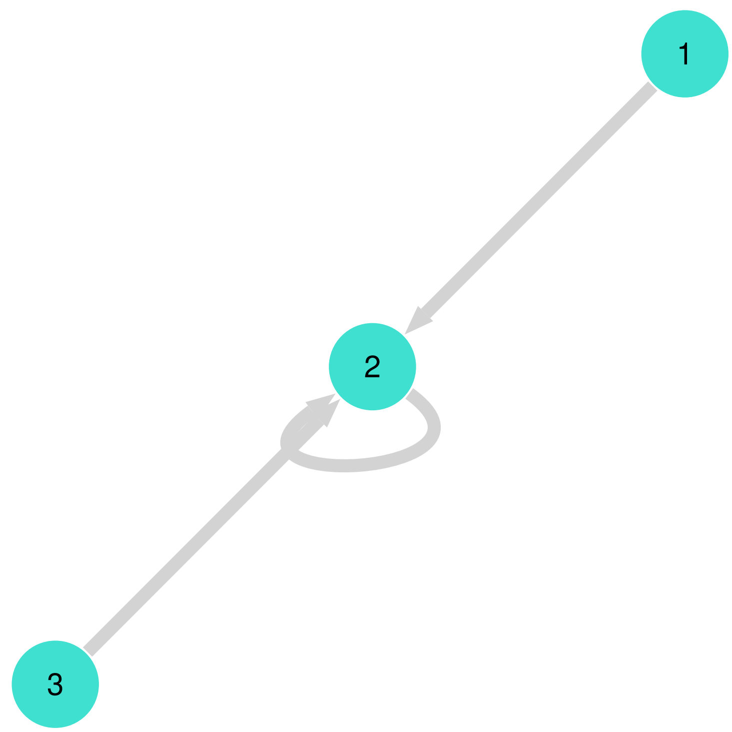
```
A = [0 1 0
     0 1 0
     0 1 0]
```

```
B = [0 0 0
     1 1 1
     0 0 0]
```

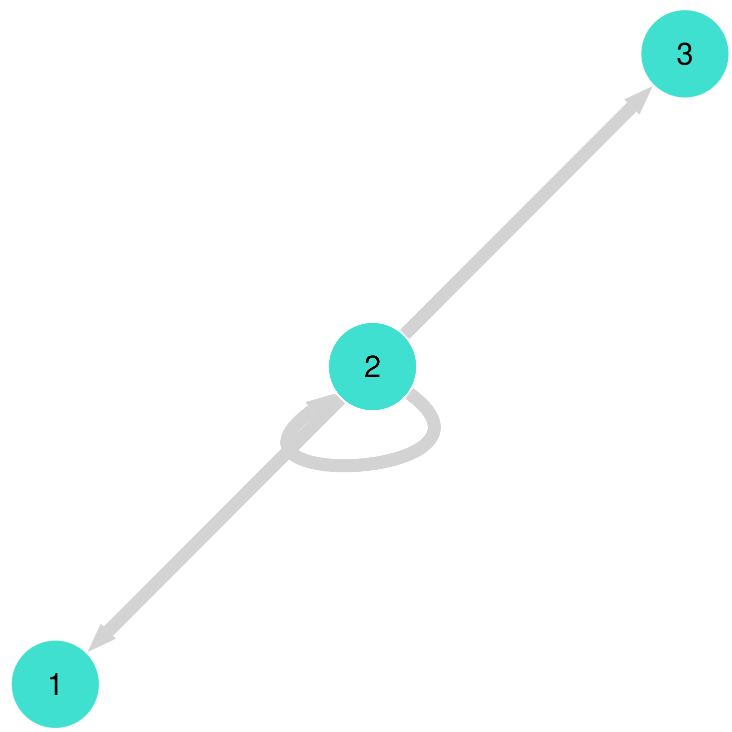
```
C = [1 0 0
     0 1 0
     0 0 1]
```

Solution

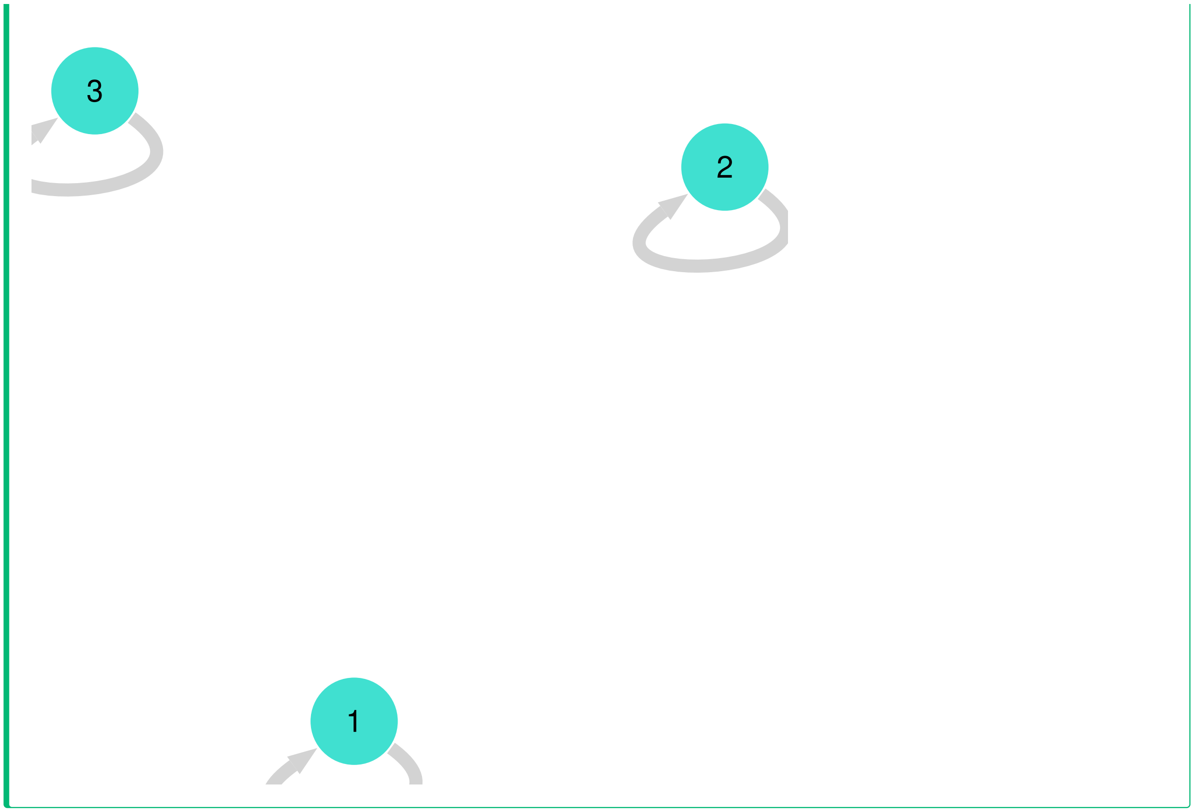
```
GA = DiGraph(A)
gplot(GA, nodelabel=1:3)
```




```
GB = DiGraph(B)
gplot(GB, nodelabel=1:3)
```




```
GC = DiGraph(C)
gplot(GC, nodelabel=1:3)
```



Graph generators

- For large networks, it would be tedious to construct them by hand…
- And in any case, we are rarely interested in the **precise** construction of a network
- What's more important are the **statistical characteristics** of the network
    - How many connections does a node have on average?
    - Are some nodes much more connected than others?
    - And so on.
- Large graphs with known statistical properties can be constructed using **generators**

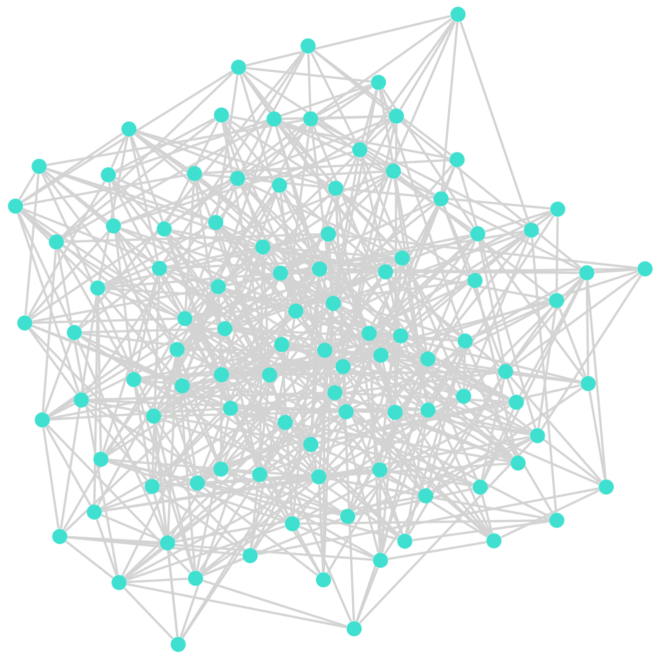
Erdős-Rényi graphs

- Take n nodes, initially unconnected. Cycle through each pair of nodes, connecting them with probability p.
    - In other words: At each node pair, you flip a biased coin that lands heads with prob. p and tails with prob. $1-p$. If you get heads, you connect the nodes; if tails, you don't.

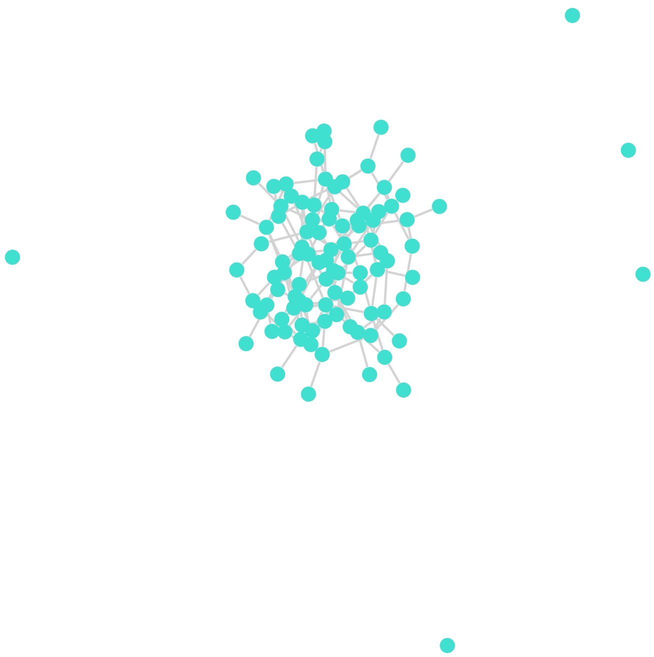
- This algorithm results in a so-called **Erdős-Rényi random graph**.
- In Graphs.jl, the `erdos_renyi` function can be used:

```
GER = erdos_renyi(100, 0.1)
GER2 = erdos_renyi(100, 0.03)
```

```
gplot(GER)
```



```
gplot(GER2)
```



Accessing graph properties

- Number of nodes (vertices):

```
nv(GER)
```

```
100
```

```
nv(GER2)
```

```
100
```

- Number of connections (edges):

```
ne(GER)
```

```
548
```

```julia
ne(GER2)
```

```
161
```

- Number of connections for each node (called the node's **degree**):

```julia
degree(GER)
```

```
100-element Vector{Int64}:
 11
 12
  9
 10
 18
 12
  9
 10
 10
  7
 11
 12
 12

  8
  7
 10
 14
  9
  6
 13
 13
  9
  9
  9
 12
```

```julia
degree(GER2)
```

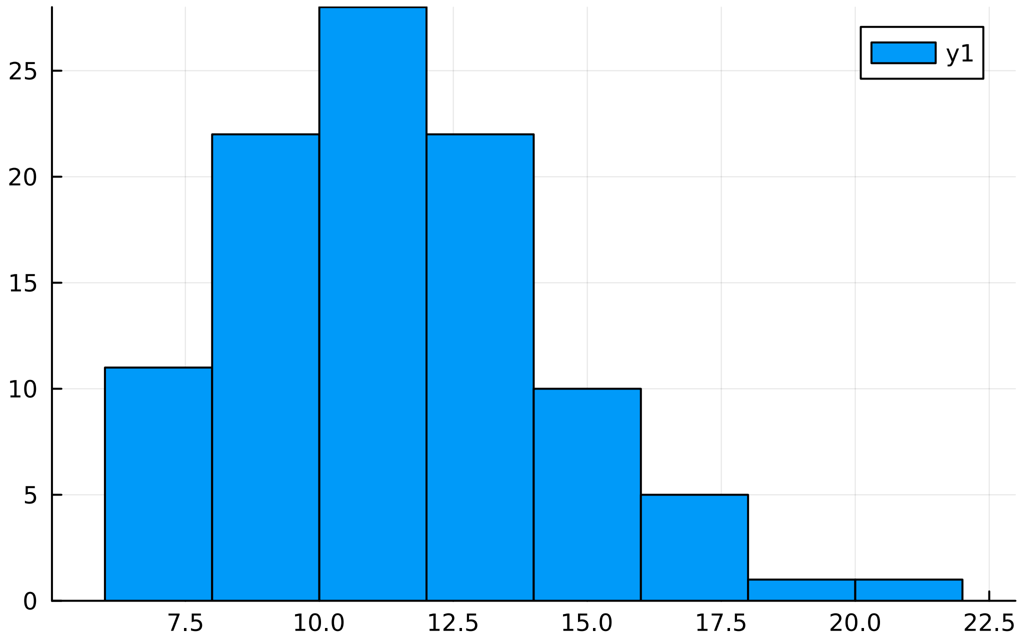
```
100-element Vector{Int64}:
 1
```

```
1
3
0
2
1
3
7
6
2
5
5
3

1
2
3
5
3
4
5
3
7
3
1
5
```

Degree distribution

- This makes it easy to plot a graph's **degree distribution**:

```
using Plots         # for the histogram() function
d = degree(GER)
histogram(d)
```



Exercise

Open Wikipedia on the page for *Switzerland*. Then, **using only links on the page**, try to navigate to the page for *Albert Einstein*.

How many links do you need to go through to reach the destination?

Answer

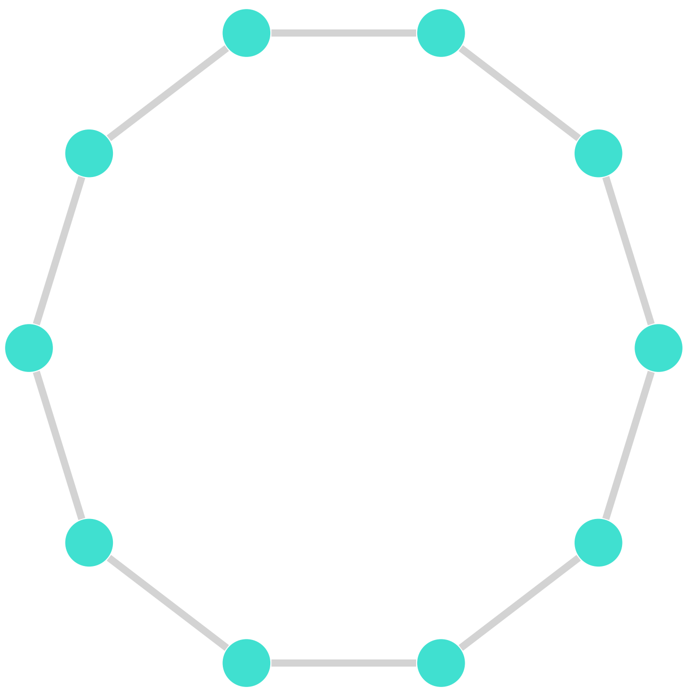
Here's one possible path:
Switzerland
→ *German*
→ *Germany*
→ *Education in Germany*
→ *Max Planck Institute for Plasma Physics*
→ *Physics*
→ *Albert Einstein*
That's 6 links.
But it's not the shortest path. You can also go, if you're in the know:
Switzerland
→ *University of Zurich*
→ *Albert Einstein*

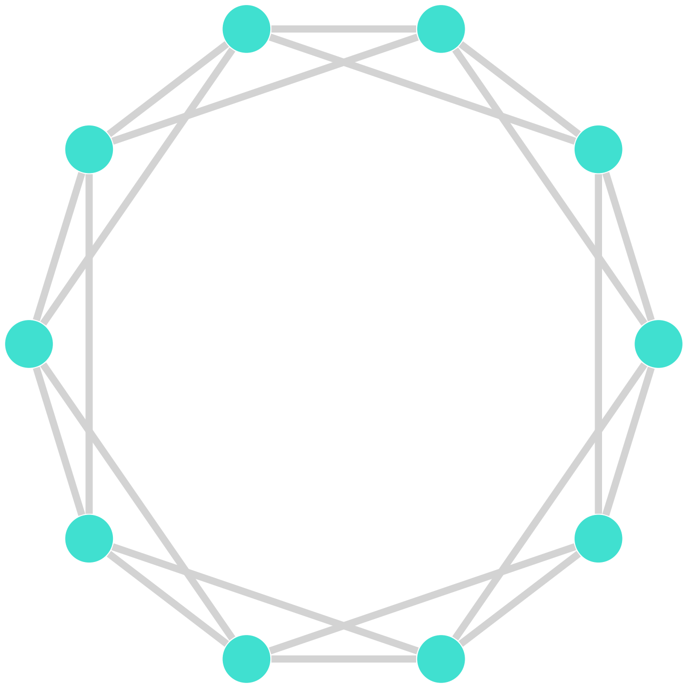
(2 links.)

"Six degrees of separation"

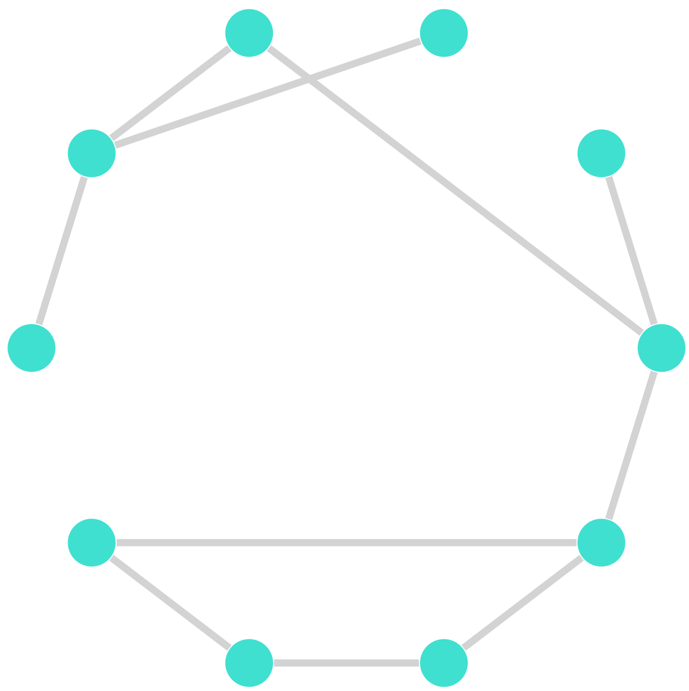
- Some networks have the "small-world property": there is a short path from every node to every other node
- The **Watts-Strogatz model** or **small world graph** is one way of modelling this
- To obtain such a graph, one does a random rewiring of a ring lattice

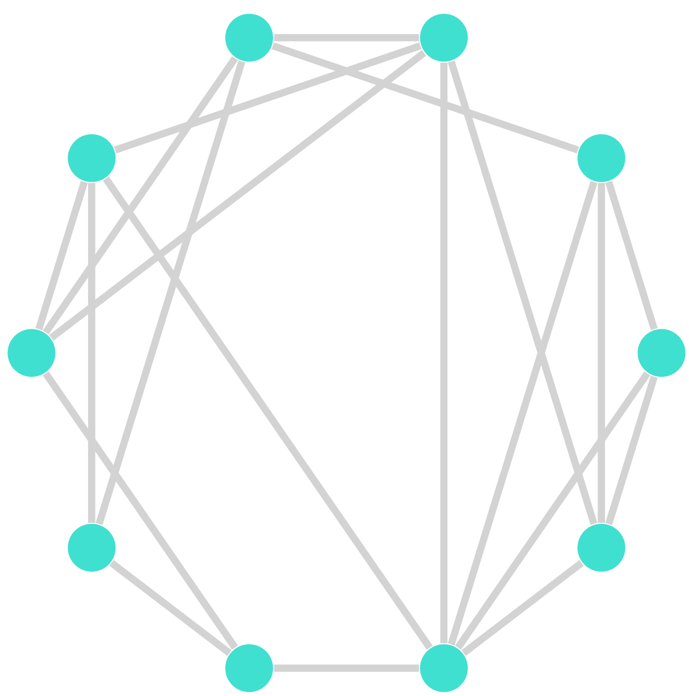
1. Start with a ring lattice (on the left, each node has $k = 2$ neighbours; on the right, each node has $k = 4$ neighbours):





2. Randomly rewire each edge with probability β to a randomly chosen destination (here, $\beta = 0.4$):

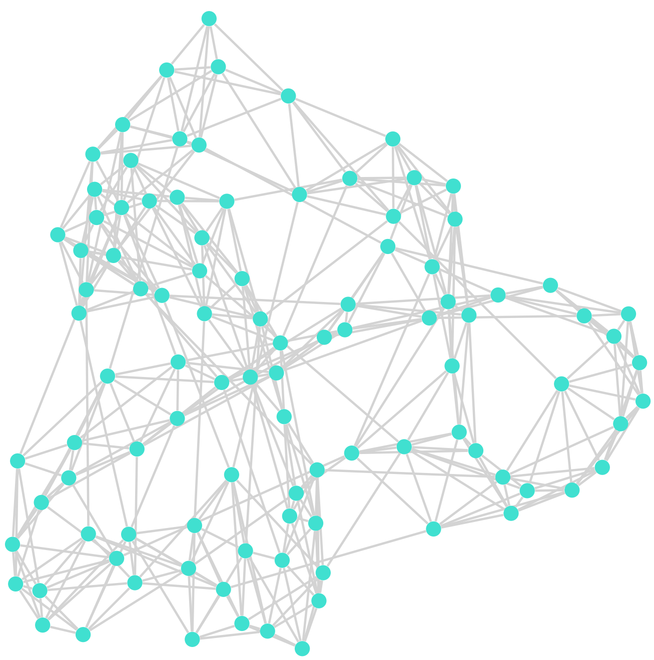




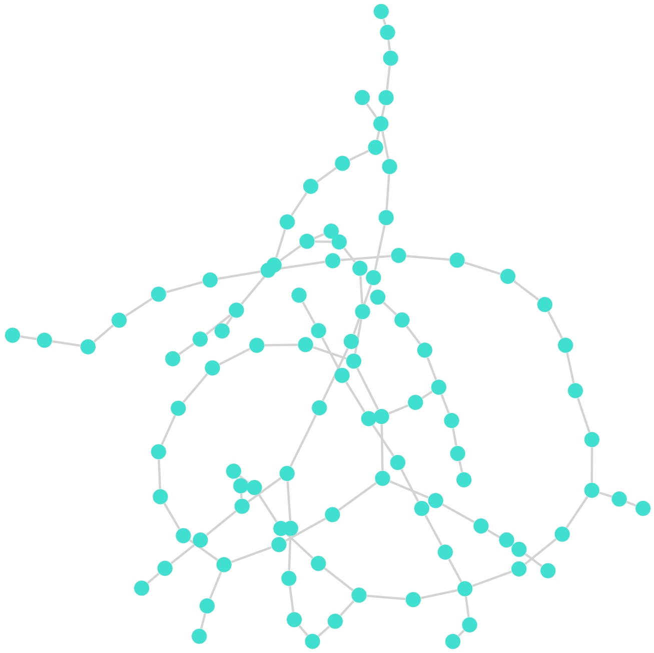
- In Graphs.jl, small-world networks can be created with `watts_strogatz(n, k, )`
  - `n`: number of nodes
  - `k`: initial degree of every node
  - : rewiring probability
- For example:

```
GWS1 = watts_strogatz(100, 8, 0.1)
GWS2 = watts_strogatz(100, 2, 0.1)
```

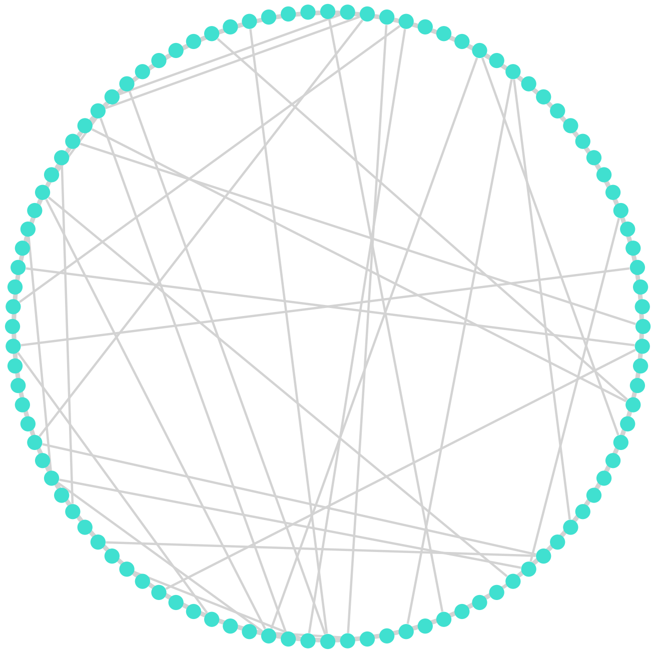
```
gplot(GWS1)
```



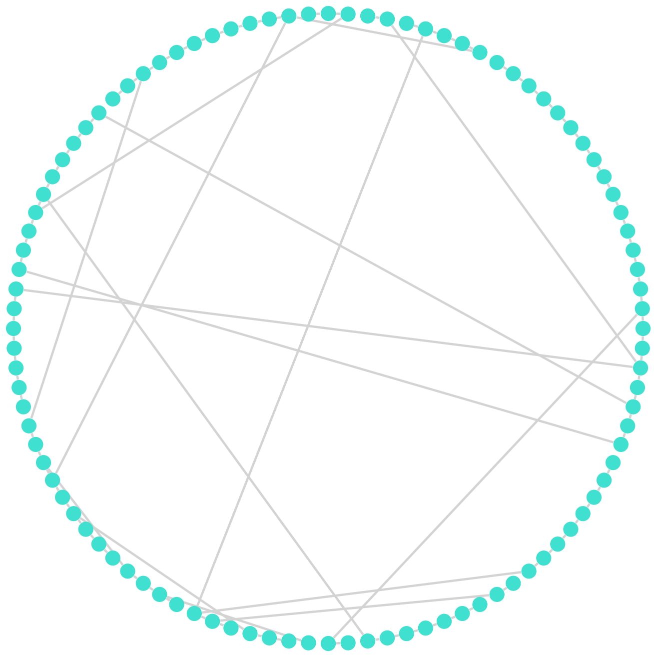
```
gplot(GWS2)
```



```
gplot(GWS1, layout=circular_layout)
```



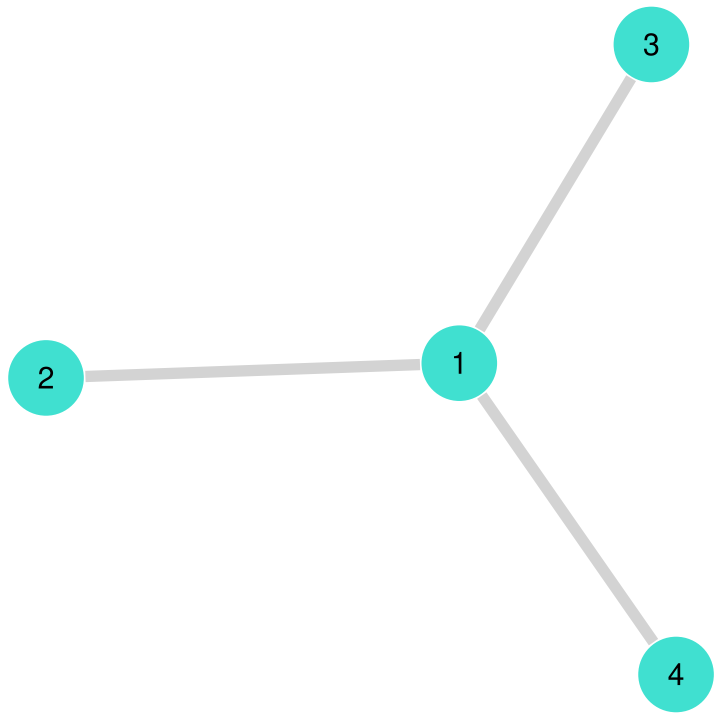
```
gplot(GWS2, layout=circular_layout)
```



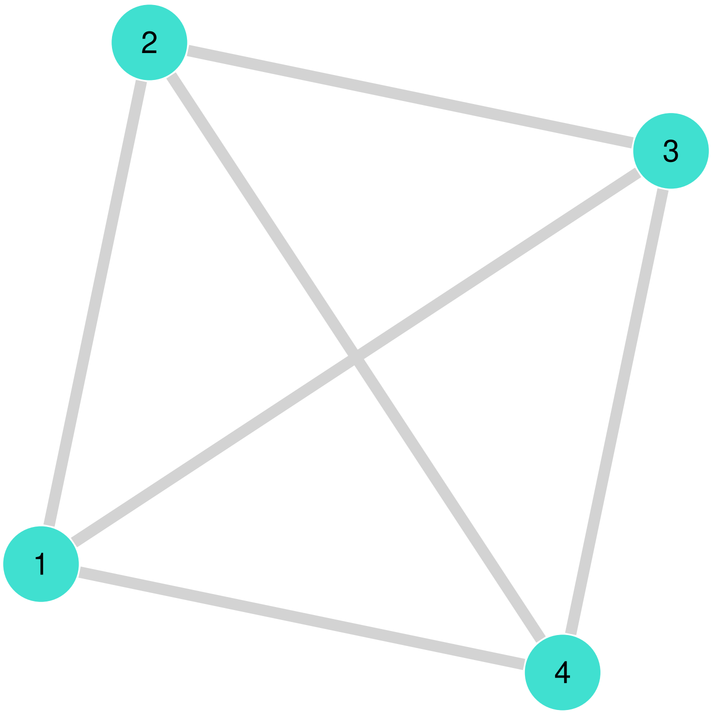
Clustering

- Short path lengths are not the only way in which small-world networks are interesting
- They also exhibit high clustering
- The **local clustering coefficient** of a node v is defined as the proportion of neighbours of v which are neighbours amongst themselves
- A highly clustered network contains **cliques**, subnetworks in which all nodes are connected to each other

Low clustering for node 1:



High clustering for node 1:



- In Graphs.jl, we can use `local_clustering_coefficient`
- E.g. to get the average local clustering coefficient:

```
using Statistics
G = erdos_renyi(100, 0.1)
mean([local_clustering_coefficient(G, v) for v in vertices(G)])
```

```
0.09162485991820357
```

```
using Statistics
G = watts_strogatz(100, 10, 0.1)
mean([local_clustering_coefficient(G, v) for v in vertices(G)])
```

```
0.5072842157842158
```

Exercise

1. Download and unzip the dolphin social network data from https://networkrepository.com/soc-dolphins.php

2. Import these data into Julia, construct a graph, and plot the network
3. Modify the plot so that each node's size is proportional to its degree
4. Plot a histogram of the degree distribution
5. Plot a histogram of the distribution of local clustering coefficient

You will need MatrixMarket.jl and GraphPlot.jl documentation

Going forward

- Next time, we will learn how to interface Graph.jl with Agents.jl, so that we can run ABM simulations on networks
- Homework:
  1. Read Smaldino (2023), chapter 9
  2. Complete the homework assignment

Smaldino, Paul E. 2023. *Modeling Social Behavior: Mathematical and Agent-Based Models of Social Dynamics and Cultural Evolution.* Princeton, NJ: Princeton University Press.