

# Variational learning

Solution | Agent-based modelling, Konstanz, 2024

Henri Kauhanen

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! Update 7 May 2024

Fixed bug in the `learn!` function. See the [lecture](#) for details.

First, I repeat the definitions of the custom types `VariationalLearner` and `LearningEnvironment` and the associated functions we defined in the lecture. (Without these definitions, none of what follows will work!)

```
using Plots          # for drawing plots
using StatsBase      # for the sample() function

mutable struct VariationalLearner
    p::Float64
    gamma::Float64
end

struct LearningEnvironment
    P1::Float64
    P12::Float64
    P2::Float64
end

function sample_string(x::LearningEnvironment)
    sample(["S1", "S12", "S2"], Weights([x.P1, x.P12, x.P2]))
end

function pick_grammar(x::VariationalLearner)
    sample(["G1", "G2"], Weights([x.p, 1 - x.p]))
end
```

```

function learn!(x::VariationalLearner, y::LearningEnvironment)
    s = sample_string(y)
    g = pick_grammar(x)

    if g == "G1" && s != "S2"
        x.p = x.p + x.gamma * (1 - x.p)
    elseif g == "G1" && s == "S2"
        x.p = x.p - x.gamma * x.p
    elseif g == "G2" && s != "S1"
        x.p = x.p - x.gamma * x.p
    elseif g == "G2" && s == "S1"
        x.p = x.p + x.gamma * (1 - x.p)
    end

    return x.p
end

```

learn! (generic function with 1 method)

## 1. Five learners

```

# create learning environment
london = LearningEnvironment(0.4, 0.5, 0.1)

# create 5 learners
gamma = 0.01
learner1 = VariationalLearner(0.5, gamma)
learner2 = VariationalLearner(0.5, gamma)
learner3 = VariationalLearner(0.5, gamma)
learner4 = VariationalLearner(0.5, gamma)
learner5 = VariationalLearner(0.5, gamma)

# simulate each learner for 1000 steps
trajectory1 = [learn!(learner1, london) for t in 1:1000]
trajectory2 = [learn!(learner2, london) for t in 1:1000]
trajectory3 = [learn!(learner3, london) for t in 1:1000]
trajectory4 = [learn!(learner4, london) for t in 1:1000]
trajectory5 = [learn!(learner5, london) for t in 1:1000]

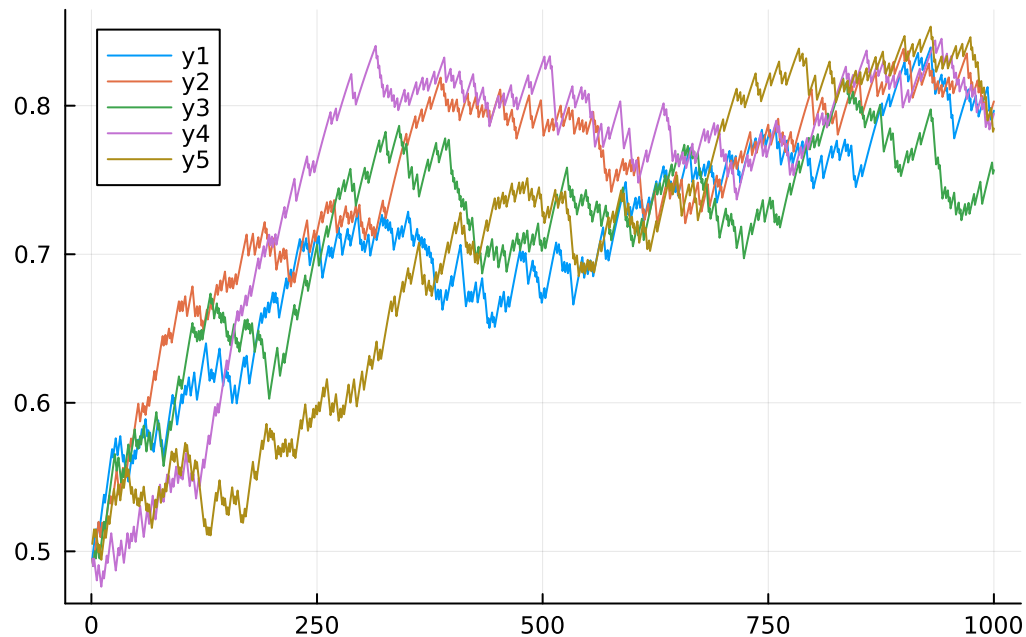
# plot

```

```

plot(1:1000, trajectory1)
plot!(1:1000, trajectory2)
plot!(1:1000, trajectory3)
plot!(1:1000, trajectory4)
plot!(1:1000, trajectory5)

```



## 2. Different learning rates

```

# create learning environment
london = LearningEnvironment(0.4, 0.5, 0.1)

# create 5 learners
gamma = 0.001
learner1 = VariationalLearner(0.5, gamma)
learner2 = VariationalLearner(0.5, gamma)
learner3 = VariationalLearner(0.5, gamma)
learner4 = VariationalLearner(0.5, gamma)
learner5 = VariationalLearner(0.5, gamma)

# simulate each learner for 1000 steps

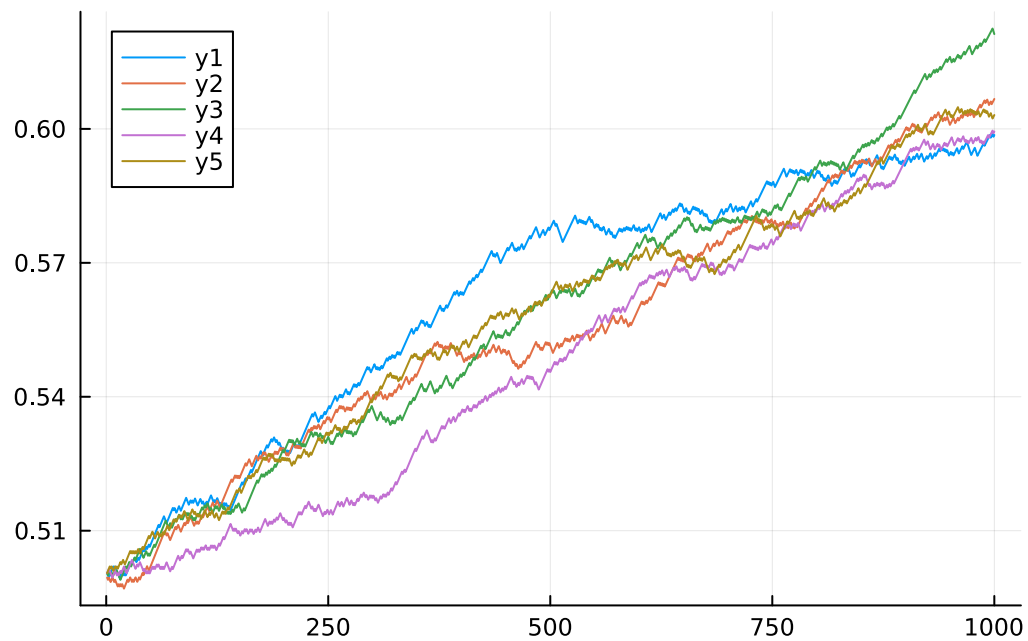
```

```

trajectory1 = [learn!(learner1, london) for t in 1:1000]
trajectory2 = [learn!(learner2, london) for t in 1:1000]
trajectory3 = [learn!(learner3, london) for t in 1:1000]
trajectory4 = [learn!(learner4, london) for t in 1:1000]
trajectory5 = [learn!(learner5, london) for t in 1:1000]

# plot
plot(1:1000, trajectory1)
plot!(1:1000, trajectory2)
plot!(1:1000, trajectory3)
plot!(1:1000, trajectory4)
plot!(1:1000, trajectory5)

```



```

# create learning environment
london = LearningEnvironment(0.4, 0.5, 0.1)

# create 5 learners
gamma = 1.0
learner1 = VariationalLearner(0.5, gamma)
learner2 = VariationalLearner(0.5, gamma)
learner3 = VariationalLearner(0.5, gamma)
learner4 = VariationalLearner(0.5, gamma)

```

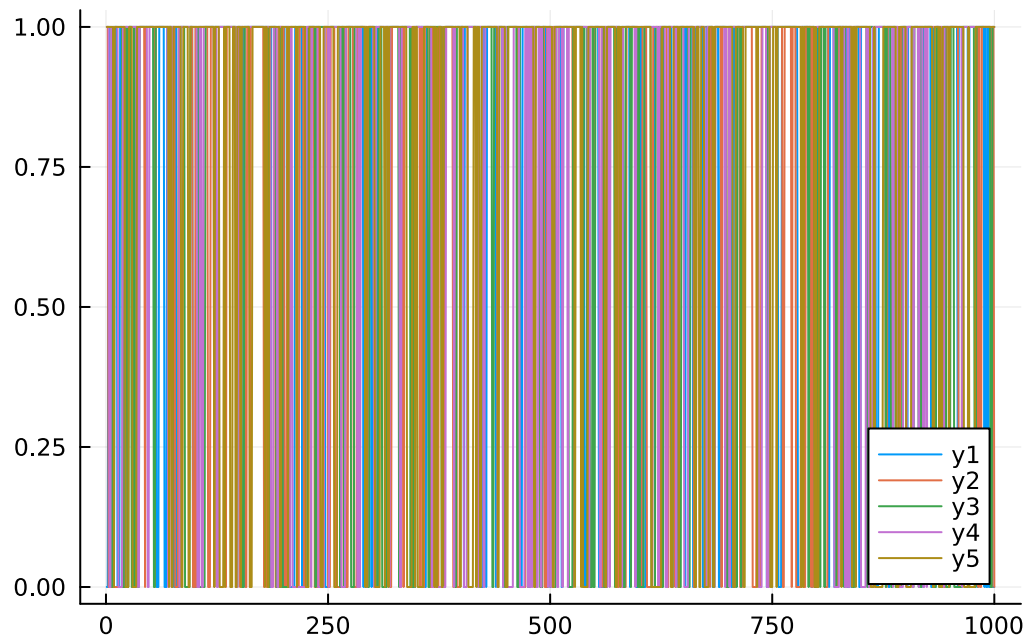
```

learner5 = VariationalLearner(0.5, gamma)

# simulate each learner for 1000 steps
trajectory1 = [learn!(learner1, london) for t in 1:1000]
trajectory2 = [learn!(learner2, london) for t in 1:1000]
trajectory3 = [learn!(learner3, london) for t in 1:1000]
trajectory4 = [learn!(learner4, london) for t in 1:1000]
trajectory5 = [learn!(learner5, london) for t in 1:1000]

# plot
plot(1:1000, trajectory1)
plot!(1:1000, trajectory2)
plot!(1:1000, trajectory3)
plot!(1:1000, trajectory4)
plot!(1:1000, trajectory5)

```



### 3. Varying the environment

Here I'm only doing this with the value of the learning rate  $\gamma = 0.01$ ; the principle will be exactly the same for any other value.

```

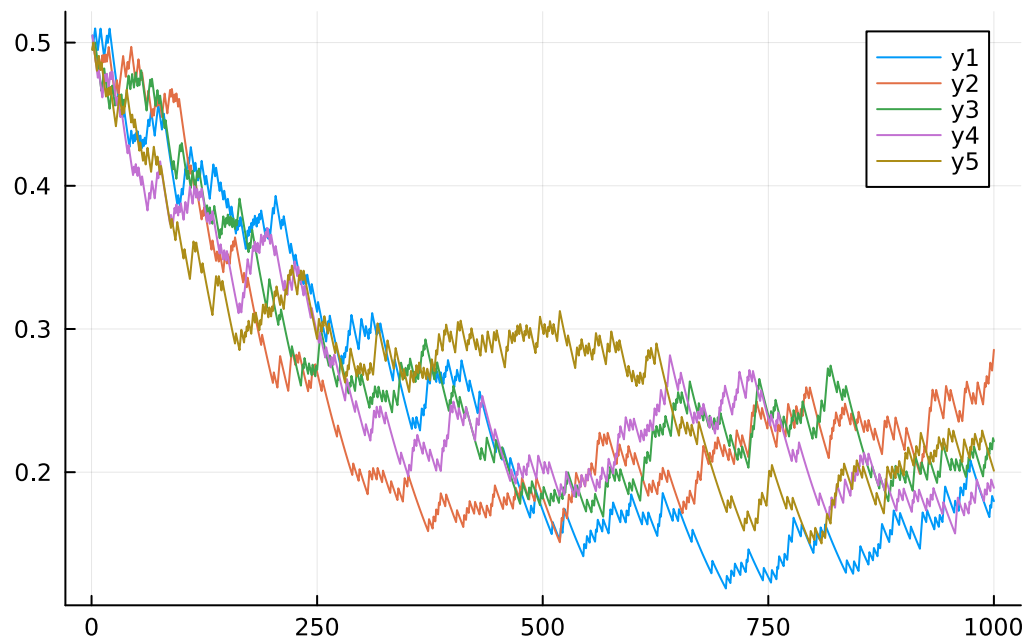
# create learning environment
prague = LearningEnvironment(0.1, 0.5, 0.4)

# create 5 learners
gamma = 0.01
learner1 = VariationalLearner(0.5, gamma)
learner2 = VariationalLearner(0.5, gamma)
learner3 = VariationalLearner(0.5, gamma)
learner4 = VariationalLearner(0.5, gamma)
learner5 = VariationalLearner(0.5, gamma)

# simulate each learner for 1000 steps
trajectory1 = [learn!(learner1, prague) for t in 1:1000]
trajectory2 = [learn!(learner2, prague) for t in 1:1000]
trajectory3 = [learn!(learner3, prague) for t in 1:1000]
trajectory4 = [learn!(learner4, prague) for t in 1:1000]
trajectory5 = [learn!(learner5, prague) for t in 1:1000]

# plot
plot(1:1000, trajectory1)
plot!(1:1000, trajectory2)
plot!(1:1000, trajectory3)
plot!(1:1000, trajectory4)
plot!(1:1000, trajectory5)

```



#### 4. What does this mean?

The effect of the learning rate parameter  $\gamma$  (**gamma**) is quite straightforward: the smaller the value of this parameter, the slower learning is. In the extreme case  $\gamma = 1$ , learning is in a sense as “fast” as it can possibly be: each learner switches between probability 1 of using  $G_1$  and probability 0 of using  $G_1$  at parsing failure.

The effect of the the probabilities P1, P12 and P2 of the learning environment is a bit less straightforward. In fact, a formula exists that allows one to predict the expected value of  $p$  (probability of using  $G_1$ ) a variational learner ends up with after a long period of learning, and that formula only depends on the environment’s probability parameters. We might look at this in detail later. For now, suffice it to say that the parameters have *some* such effect. More precisely, in the above simulations we see that

- When  $P1 = 0.4$  and  $P2 = 0.1$  (environment **london**), the learners end up with  $p \approx 0.8$ .
- When  $P1 = 0.1$  and  $P2 = 0.4$  (environment **prague**), the learners end up with  $p \approx 0.2$ .

#### 5. Prettifying the plots

```
# create learning environment
prague = LearningEnvironment(0.1, 0.5, 0.4)
```

```

# create 5 learners
gamma = 0.01
learner1 = VariationalLearner(0.5, gamma)
learner2 = VariationalLearner(0.5, gamma)
learner3 = VariationalLearner(0.5, gamma)
learner4 = VariationalLearner(0.5, gamma)
learner5 = VariationalLearner(0.5, gamma)

# simulate each learner for 1000 steps
trajectory1 = [learn!(learner1, prague) for t in 1:1000]
trajectory2 = [learn!(learner2, prague) for t in 1:1000]
trajectory3 = [learn!(learner3, prague) for t in 1:1000]
trajectory4 = [learn!(learner4, prague) for t in 1:1000]
trajectory5 = [learn!(learner5, prague) for t in 1:1000]

# plot
plot(1:1000, trajectory1, seriestype=:scatter)
plot!(1:1000, trajectory2, seriestype=:scatter)
plot!(1:1000, trajectory3, seriestype=:scatter)
plot!(1:1000, trajectory4, seriestype=:scatter)
plot!(1:1000, trajectory5, seriestype=:scatter)
xlabel!("learning iteration")
ylabel!("probability of G1")
title!("A variational learning trajectory")

```



A variational learning trajectory

