A model of language learning

Agent-based modelling, Konstanz, 2024

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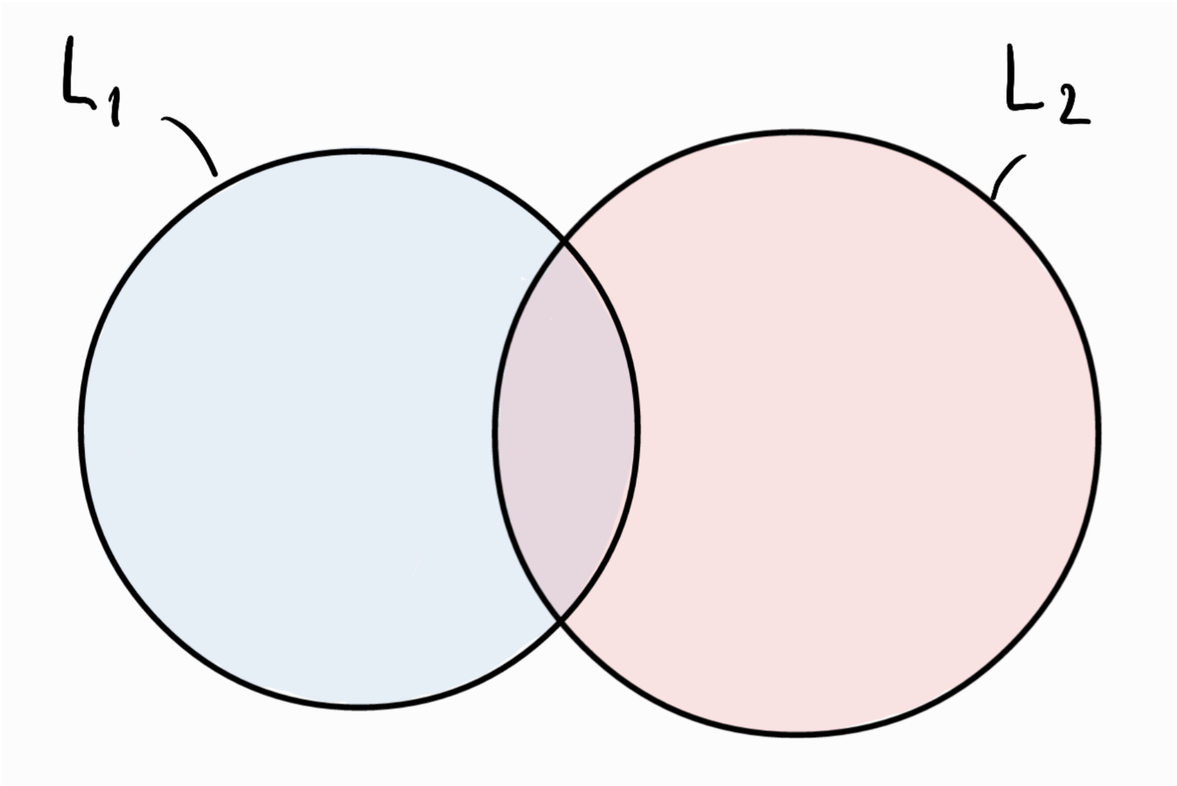
Plan

- Starting this week, we will put programming to good use
- We'll start with a simple model of language learning
  - Here, **learning = process of updating a linguistic representation**
  - Doesn't matter whether child or adult

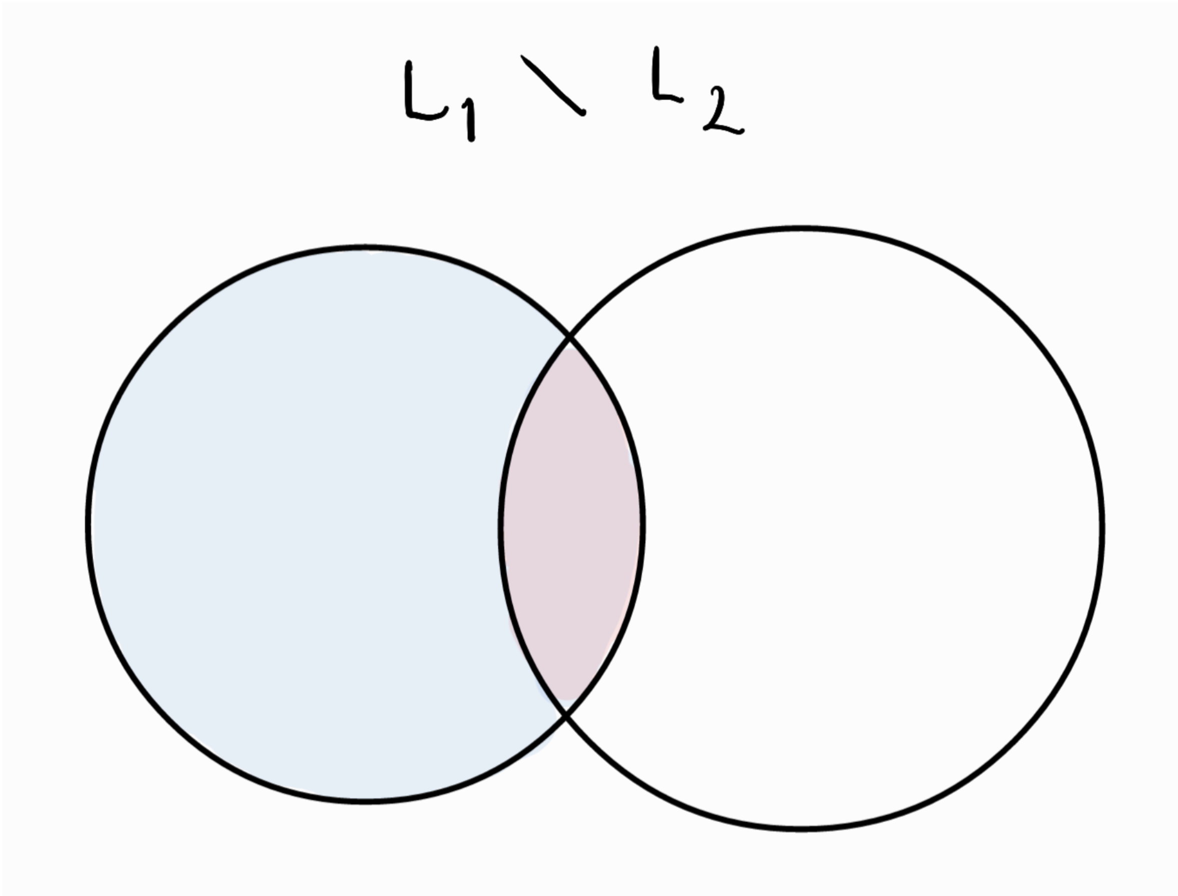
Grammar competition

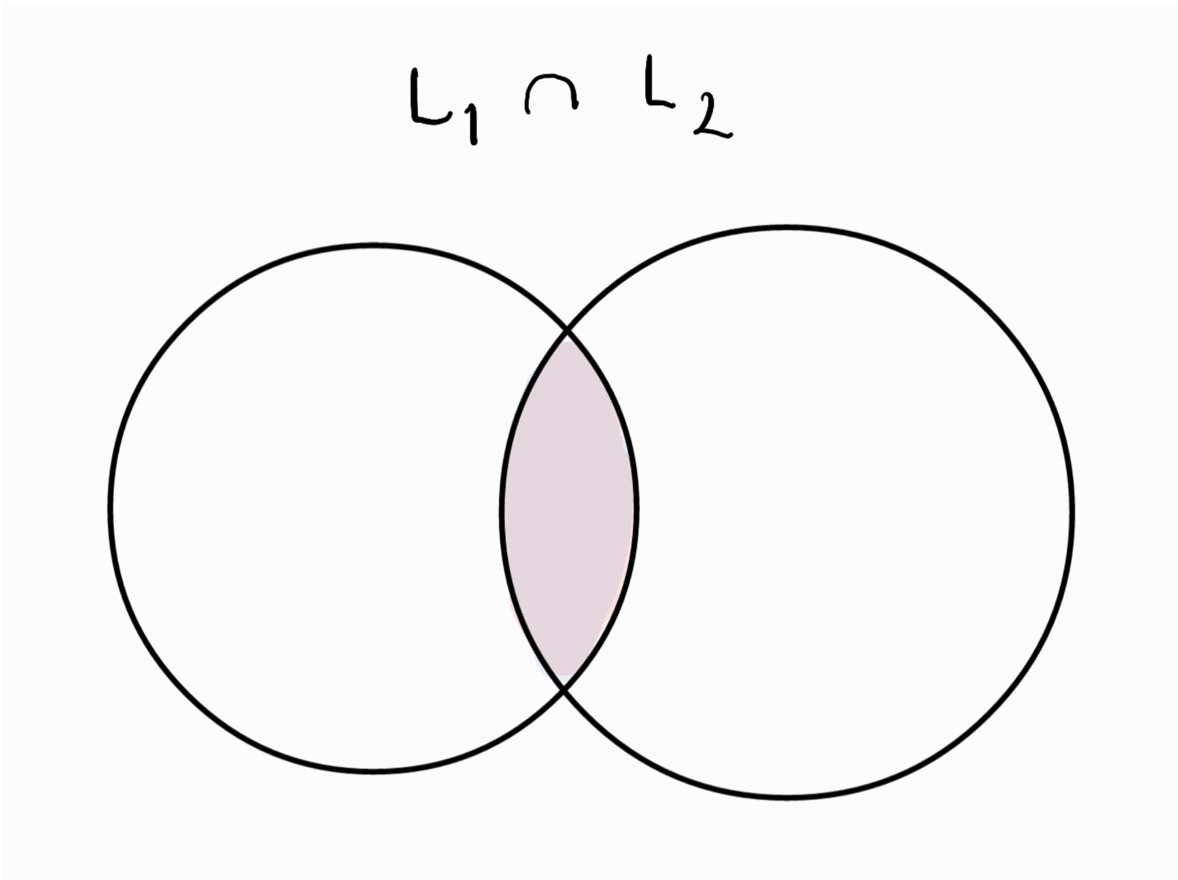
- Assume two grammars G_1 and G_2 that **generate** languages L_1 and L_2
  - language = set of strings (e.g. sentences)
- In general, L_1 and L_2 will be different but may overlap:

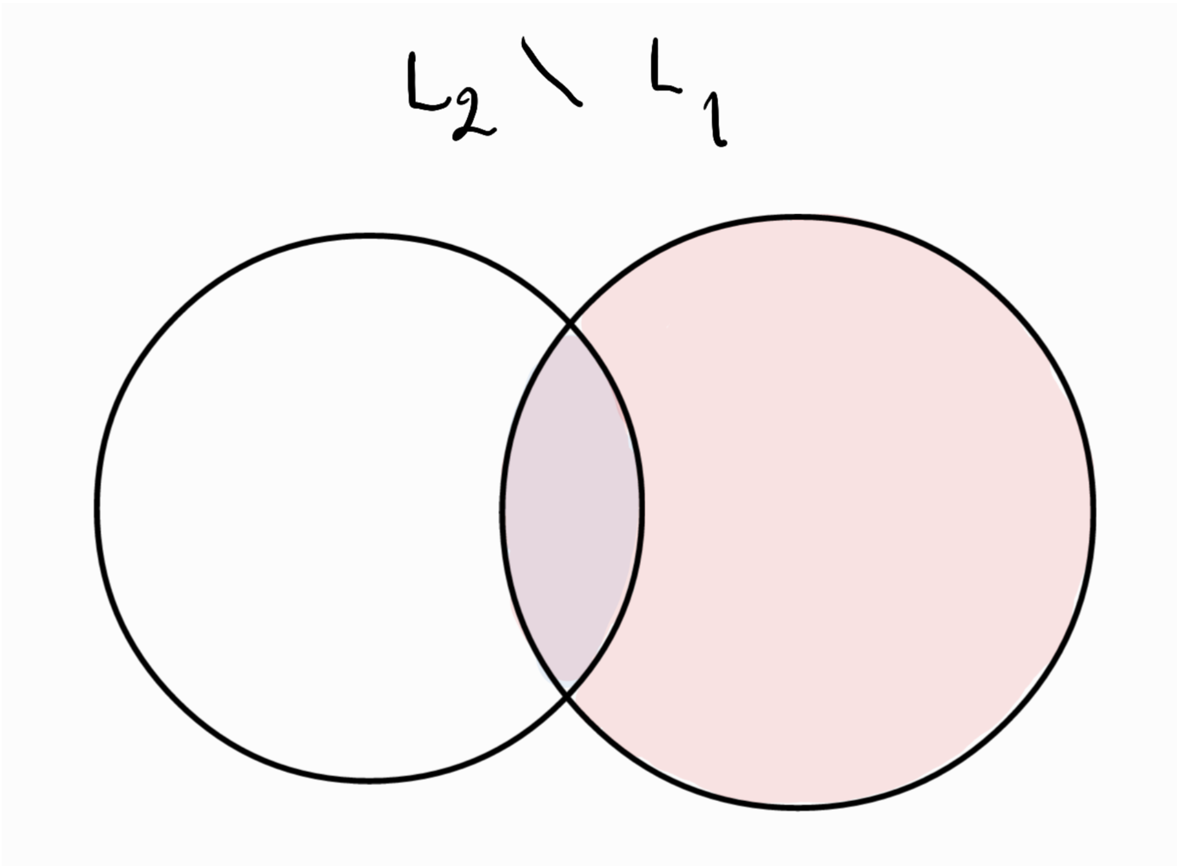


Grammar competition

- Three sets of interest: $L_1 \quad L_2$, $L_1 \cap L_2$ and $L_2 \quad L_1$

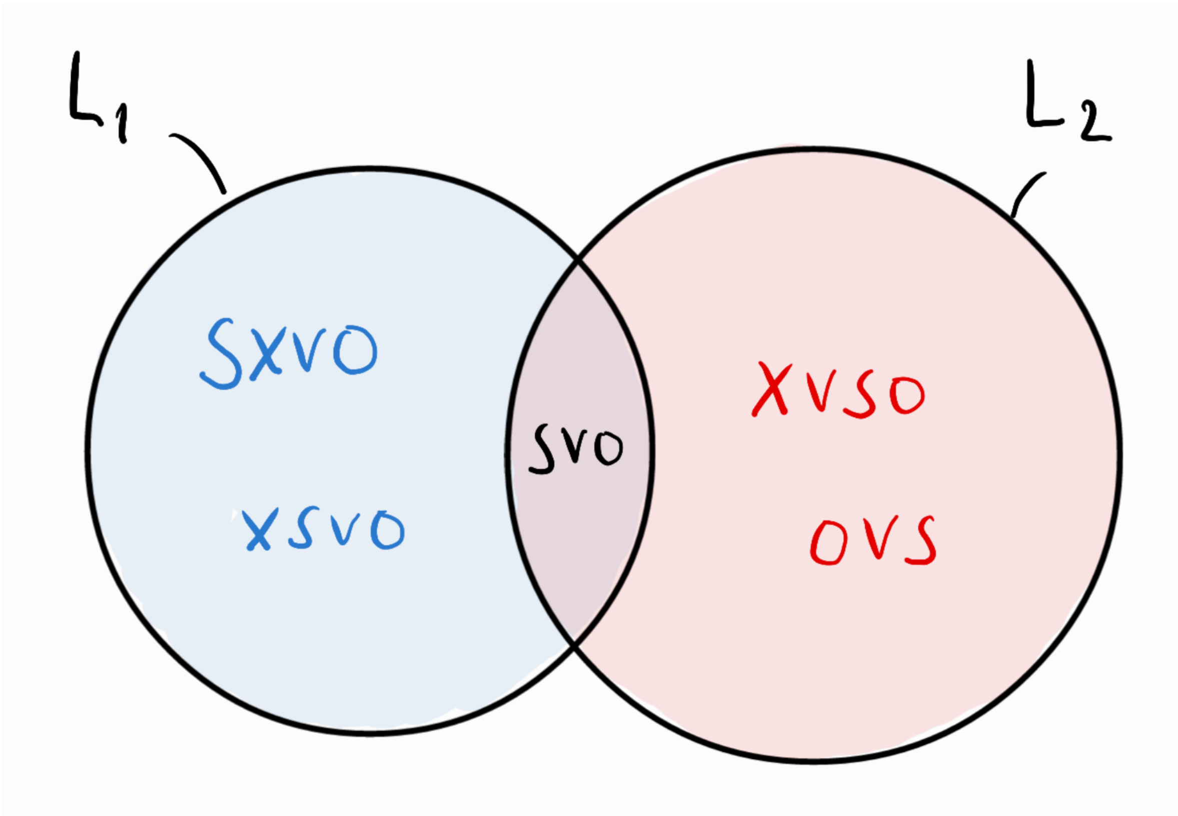

$L_1 \setminus L_2$


$L_1 \cap L_2$



Concrete example

- SVO (G_1) vs. V2 (G_2)



Grammar competition

- Suppose learner receives randomly chosen strings from L_1 and L_2
- Learner uses either G_1 or G_2 to parse incoming string
- Define p = probability of use of G_1
- **How should the learner update p in response to interactions with his/her environment?**

Variational learning

- Suppose learner receives string/sentence s
- Then update is:

Learner's grammar	String received	Update
G_1	$s \in L_1$	increase p
G_1	$s \in L_2 \quad L_1$	decrease p
G_2	$s \in L_2$	decrease p

Learner's grammar	String received	Update
G_2	$s \in L_1 \;\; L_2$	increase p

Exercise

How can we increase/decrease p in practice? What is the update formula?

> **Answer**
>
> One possibility (which we will stick to):
>
> - Increase: p becomes $p + \gamma(1 - p)$
> - Decrease: p becomes $p - \gamma p$
>
> The parameter $0 < \gamma < 1$ is a **learning rate**

Why this form of update formula?

- Need to make sure that always $0 \leq p \leq 1$ (it is a probability)
- Also notice:
  - When p is increased, what is added is $\gamma(1-p)$. Since $1-p$ is the probability of G_2, this means *transferring an amount of the probability mass of G_2 onto G_1*.
  - When p is decreased, what is removed is γp. Since p is the probability of G_1, this means *transferring an amount of the probability mass of G_1 onto G_2*.
  - Learning rate γ determines how much probability mass is transferred.

Plan

- To implement a variational learner computationally, we need:
  1. A representation of a learner who embodies a single probability, p, and a learning rate, γ
  2. A way to sample strings from $L_1 \;\; L_2$ and from $L_2 \;\; L_1$
  3. A function that updates the learner's p
- Let's attempt this now!

The struct

- The first point is very easy:

```
mutable struct VariationalLearner
  p::Float64
  gamma::Float64
end
```

Sampling strings

- For the second point, note we have three types of strings which occur with three corresponding probabilities
- Let's refer to the string types as `"S1"`, `"S12"` and `"S2"`, and to the probabilities as `P1`, `P12` and `P2`:

String type	Probability	Explanation
`"S1"`	`P1`	$s \in L_1 \quad L_2$
`"S12"`	`P12`	$s \in L_1 \cap L_2$
`"S2"`	`P2`	$s \in L_2 \quad L_1$

- In Julia, sampling from a finite number of options (here, three string types) with corresponding probabilities is handled by a function called `sample()` which lives in the `StatsBase` package
- First, install and load the package:

```
using Pkg
Pkg.add("StatsBase")
using StatsBase
```

Now to sample a string, you can do the following:

```
# the three probabilities (just some numbers I invented)
P1 = 0.4
P12 = 0.5
P2 = 0.1

# sample one string
sample(["S1", "S12", "S2"], Weights([P1, P12, P2]))
```

```
"S12"
```

Tidying up

- The above works but is a bit cumbersome – for example, every time you want to sample a string, you need to refer to the three probabilities
- Let's carry out a bit of software engineering to make this nicer to use
- First, we encapsulate the probabilities in a struct of their own:

```
struct LearningEnvironment
  P1::Float64
  P12::Float64
  P2::Float64
end
```

- We then define the following function:

```
function sample_string(x::LearningEnvironment)
  sample(["S1", "S12", "S2"], Weights([x.P1, x.P12, x.P2]))
end
```

```
sample_string (generic function with 1 method)
```

- Test the function:

```
paris = LearningEnvironment(0.4, 0.5, 0.1)
sample_string(paris)
```

```
"S12"
```

Implementing learning

- We now need to tackle point 3, the learning function which updates the learner's state
- This needs to do three things:
    1. Sample a string from the learning environment
    2. Pick a grammar to try and parse the string with
    3. Update p in response to whether parsing was successful or not

Exercise

How would you implement point 2, i.e. picking a grammar to try and parse the incoming string?

Answer

We can again use the `sample()` function from `StatsBase`, and define:

```
function pick_grammar(x::VariationalLearner)
  sample(["G1", "G2"], Weights([x.p, 1 - x.p]))
end
```

```
pick_grammar (generic function with 1 method)
```

Implementing learning

- Now it is easy to implement the first two points of the learning function:

```
function learn!(x::VariationalLearner, y::LearningEnvironment)
  s = sample_string(y)
  g = pick_grammar(x)
end
```

```
learn! (generic function with 1 method)
```

- How to implement the last point, i.e. updating p?

Aside: conditional statements

- Here, we will be helped by **conditionals**:

```
if COND1
  # this is executed if COND1 is true
elseif COND2
  # this is executed if COND1 is false but COND2 is true
else
  # this is executed otherwise
end
```

- Note: only the `if` block is necessary; `elseif` and `else` are optional, and there may be more than one `elseif` block

Aside: conditional statements

- Try this for different values of `number`:

```
number = 1

if number > 0
  println("Your number is positive!")
elseif number < 0
  println("Your number is negative!")
else
  println("Your number is zero!")
end
```

Comparison $\neq$ assignment

> **!** Important
>
> To compare equality of two values inside a condition, you **must** use a double equals sign, `==`. This is because the single equals sign, `=`, is already reserved for assigning values to variables.
>
> ```
> if 0 = 1    # throws an error!
>   println("The world is topsy-turvy")
> end
>
> if 0 == 1   # works as expected
>   println("The world is topsy-turvy")
> end
> ```

Exercise

- Use an `if ... elseif ... else ... end` block to finish off our `learn!` function
- Tip: logical "and" is `&&`, logical "or" is `||`
- Recall:

Learner's grammar	String received	Update
G_1	$s \in L_1$	increase p
G_1	$s \in L_2 \quad L_1$	decrease p
G_2	$s \in L_2$	decrease p

Learner's grammar	String received	Update
G_2	$s \in L_1 \quad L_2$	increase p

Answer

```
function learn!(x::VariationalLearner, y::LearningEnvironment)
  s = sample_string(y)
  g = pick_grammar(x)

  if g == "G1" && s == "S1"
    x.p = x.p + x.gamma * (1 - x.p)
  elseif g == "G1" && s == "S2"
    x.p = x.p - x.gamma * x.p
  elseif g == "G2" && s == "S2"
    x.p = x.p - x.gamma * x.p
  elseif g == "G2" && s == "S1"
    x.p = x.p + x.gamma * (1 - x.p)
  end

  return x.p
end
```

```
learn! (generic function with 1 method)
```

Testing our code

- Let's test our code!

```
bob = VariationalLearner(0.5, 0.01)
paris = LearningEnvironment(0.4, 0.5, 0.1)

learn!(bob, paris)
learn!(bob, paris)
learn!(bob, paris)
learn!(bob, paris)
learn!(bob, paris)
```

```
0.505
```

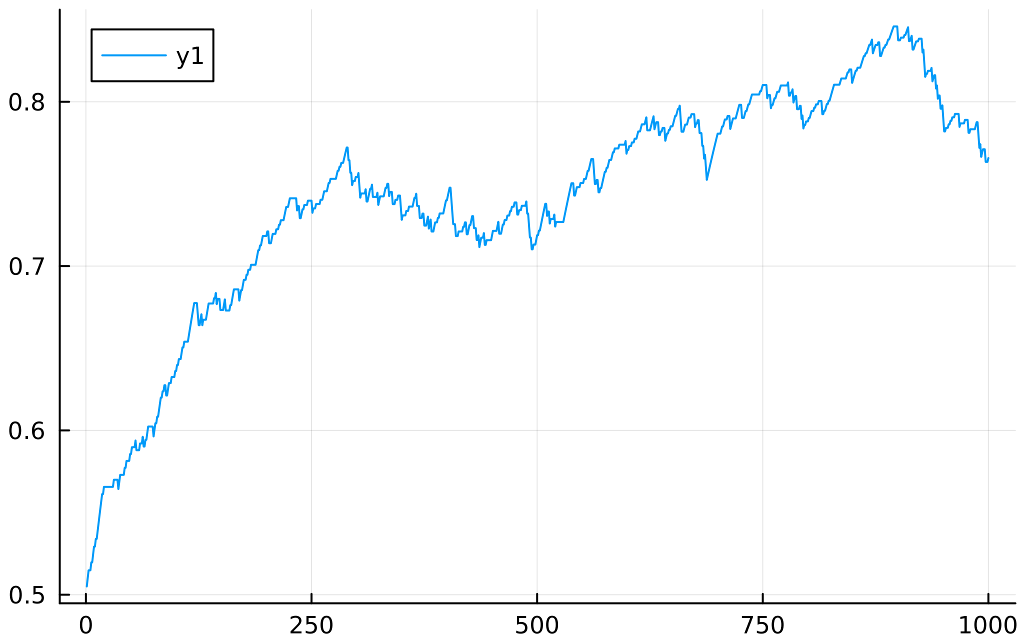
```julia
trajectory = [learn!(bob, paris) for t in 1:1000]
```

```
1000-element Vector{Float64}:
 0.505
 0.50995
 0.5148505
 0.5148505
 0.5148505
 0.519701995
 0.519701995
 0.52450497505
 0.5292599252995001
 0.5292599252995001
 0.5339673260465051
 0.5339673260465051
 0.5386276527860401

 0.779725655542553
 0.7719283989871275
 0.7742091149972562
 0.7664670238472836
 0.7688023536088108
 0.7711143300727227
 0.7711143300727227
 0.7711143300727227
 0.7634031867719955
 0.7634031867719955
 0.7634031867719955
 0.7657691549042756
```

Plotting the learning trajectory

```julia
using Plots
plot(1:1000, trajectory)
```



Bibliographical remarks

- For more about the notion of grammar competition, see Kroch (1989), Kroch (1994)
- Variational learner originally from Yang (2000), Yang (2002)
- Learning algorithm itself is old: Bush and Mosteller (1955)

Summary

- You've learned a few important concepts today:
  - Grammar competition and variational learning
  - How to sample objects according to a discrete probability distribution
  - How to use conditional statements
  - How to make a simple plot of a learning trajectory
- You get to practice these in the homework
- Next week, we'll take the model to a new level and consider what happens when several variational learners interact

References

Bush, Robert R., and Frederick Mosteller. 1955. *Stochastic Models for Learning*. New York, NY: Wiley.

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