

Supplier Search and Market Concentration *

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Abstract

This paper studies how changes in input market frictions reshape the distribution of firm sizes and aggregate productivity in general equilibrium. I develop a quantitative model of supplier search in which firms incur fixed costs to locate and bargain with input suppliers. The framework provides a microfoundation for how input trade affect firm performance and resource allocation. Empirical evidence from firm-level import data motivates the model through four key patterns: (i) the dispersion in the number of imported varieties across firms has widened, (ii) the sales distribution among importers has become more unequal, (iii) larger firms pay lower input prices, and (iv) firms in municipalities with better digital infrastructure expand their supplier networks faster, with stronger effects for larger firms. In general equilibrium, reductions in input search frictions reallocate resources toward more productive importers, raising real GDP by 9 percent but also increasing market concentration by 6.6 percent. A counterfactual analysis shows that a 10 percent tariff on imported inputs would largely offset these gains while reducing market concentration.

Keywords: Search costs; Input markets; Firm-to-firm trade; Market concentration; Misallocation; Tariffs

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1 Introduction

Market concentration has risen steadily since the 1990s across many advanced economies, raising concerns about weaker competition and its implications for productivity and allocative efficiency. Over the same period, input markets have undergone profound changes. Improvements in digital communication, logistics, and capital accumulation in emerging economies such as China have made it easier and cheaper for firms to source intermediate goods from abroad. These developments represent broad improvements in input market efficiency: search frictions have fallen, and more efficient intermediate suppliers are now accessible to a larger set of firms. This paper argues that these changes are central to understanding the twin patterns of rising aggregate productivity and rising market concentration.

When input markets become more efficient, firms gain access to a broader and higher quality set of suppliers, lowering marginal costs and raising productivity, as shown by Amiti and Konings (2007); Goldberg et al. (2010); Antràs et al. (2022); Halpern et al. (2015). These gains, however, are uneven because exposure to international supply chains varies widely across firms. Some firms maintain large and diversified supplier networks, while others rely on few or exclusively domestic suppliers. Consequently, improvements in input markets, whether through lower search frictions, rising supplier productivity, or better connectivity, can widen the firm size distribution and increase concentration even as aggregate efficiency improves. This paper asks: How do changes in input market efficiency reshape productivity and market concentration?

It is known that changes in trade patterns can reshape the firm size distribution in profound ways. For example, Amiti and Heise (2025) show that intensified import competition raises market concentration among local firms by displacing less productive domestic producers. This perspective, however, focuses on trade in final goods, while the role of intermediate inputs in shaping firm dynamics has received much less attention. Importing final goods competes directly with domestic producers and reduces demand for their output, a demand shock. By contrast, importing inputs lowers marginal costs and improves effective quality or variety, creating a productivity shock. This distinction shifts the focus from how trade reallocates demand across firms (Eaton et al., 2011) to how heterogeneity in access to international suppliers influences production efficiency of firms. Understanding this margin is important because intermediate inputs account for roughly 56% of global trade (Miroudot et al., 2009) and dominate the EU’s import basket. Imported inputs also play a large quantitative role, making input markets essential for explaining recent trends in firm concentration.

To motivate and discipline the model, I use linked Swedish administrative data and document four patterns in manufacturing import behavior. First, there is wide dispersion in the number of imported varieties across firms, with the upper tail now sourcing many more inputs than twenty years ago. Second, the sales distribution among importing firms has become markedly more unequal over the same period. Third, more productive firms pay lower input prices, as unit values decline systematically with firm size and imported quantities. Fourth, exploiting the staggered rollout of high-speed internet as a proxy for improved input market connectivity, I find that firms in municipalities with greater fiber coverage expand their supplier networks about 7 percent faster, and more productive firms expand a further 19 percent faster.

To interpret these patterns, I develop a quantitative general equilibrium model of input trade with frictions in buyer-supplier relationship formation and contracting. Domestic firms are heterogeneous in productivity and choose how many suppliers to engage with and how much to source from each. The framework is designed to analyze a broad set of input market forces, including changes in supplier productivity, search frictions, and trade policy, that jointly determine input prices, network structure, and aggregate outcomes. I compare equilibria under different parameterizations of these forces and map simulated moments to the administrative data, which allows me to infer the magnitude of key input market frictions and assess their contribution to the observed evolution of firm outcomes.

The model highlights asymmetric gains from lower search costs. Incumbent importers expand their supplier networks, and firms near the importing margin enter international supply chains that were previously out of reach, whereas firms that never search are largely unaffected directly. In general equilibrium, however, non-importers become relatively less productive than importers, leading to declining sales and profits. Quantitatively, a 33 percent reduction in search costs that matches the empirical patterns increases market concentration by about 6.6 percent and raises real GDP by roughly 9 percent.

I also study a policy counterfactual in which the government imposes a uniform tariff on imported intermediates. A 10 percent tariff is sufficient to offset both the output gains and the increase in market concentration associated with the decline in search costs over the past two decades, indicating that input trade policy can materially influence both aggregate performance and industry structure.

This paper contributes to the literature in three ways.

First, I uncover the role of changes in input markets as a determinant of market concentration. In doing so, I complement existing studies that emphasize other drivers of concentration, such as rising entry barriers (Covarrubias et al., 2020), intensified import competition (Amiti and Heise, 2025), falling market spanning costs due to advances in information technology (Aghion et al., 2023), and the rise of intangible capital (Crouzet and Eberly, 2019; Weiss, 2020; Bajgar et al., 2025).

Second, this paper connects to the growing literature on firm-to-firm input trade. Empirically, I document rising dispersion in the number of imported varieties and substantial input price heterogeneity using Swedish import data, consistent with Atalay (2014). Prior studies attribute such patterns to buyer market power (Morlacco, 2020; Rubens, 2023), input quality differences (Kugler and Verhoogen, 2012), and match-specific frictions (Burstein et al., 2024). I extend this evidence by documenting new correlations between price gaps and firm characteristics and by emphasizing additional empirical regularities. On the theoretical side, I build a model of buyer–supplier relationships that combines bargaining (Alviarez et al., 2023), search (Eaton et al., 2022a), and matching (Eaton et al., 2022b) in a unified framework. The model allows for endogenous relationship formation with bargaining over both quantities and prices and parallels the random job search model of McCall (1970). It contributes to the labor search and matching literature by incorporating explicit search costs, two-sided heterogeneity, and multidimensional bargaining. Finally, the paper relates to research showing that importing intermediate goods raises productivity by improving access to cheaper or higher-quality inputs (Amiti and Konings, 2007; Goldberg et al., 2010; Halpern et al., 2015; Gopinath and Neiman, 2014). While these studies focus on policy-driven or cross-sectional variation, I show that improvements in input market efficiency have both productivity and distributional consequences.

Third, I show that importing intermediate goods can create distortions, contrasting with the findings of Edmond et al. (2015), who show the opposite effect for imports of consumer goods. This places my work within the misallocation literature pioneered by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), and further developed by Dhingra and Morrow (2019). I study how search costs in supply chains affect allocative efficiency for downstream firms. My theoretical framework, which models search and bargaining between buyers and suppliers, generates endogenous price gaps and allows me to assess their impact on aggregate output. This introduces a novel mechanism, whereby the wedge depends directly on firms’ productivity. This mechanism provides an alternative explanation why some countries gain more from trade liberalization, because places with institution can outsource more inputs, as explained in Boehm and Oberfield (2020).

The remainder of the paper is organized as follows. Section 2 describes the data and Section 3 documents the four facts. Section 4 develops the model, while Section 5 and 6 illustrate the main mechanisms by introducing simplified models. Section 7 maps model predictions to the data and quantifies the impact of falling search costs on productivity and concentration. Section 8 presents the tariff counterfactual. Section 9 concludes.

2 Data

I assemble a comprehensive firm level panel from high quality Swedish registry data that spans 1998 to 2021 by combining three sources. The first source contains detailed import transactions. The second source provides firm financial statements with standard size and productivity measures. The third source reports municipality level diffusion of fast internet. I merge these sources at the firm year level and attach municipality characteristics by firm location and year. In this study, I focus on manufacturing firms.

2.1 Import Data

The import dataset is organized at the firm–product–country–year level. For each observation, I observe both the customs value and the physical quantity, which allows me to construct unit prices as value divided by quantity. I define a variety as an eight-digit product code by country, which corresponds to a supplier variety in the model. The panel covers manufacturing firms from 1998 to 2021.

The data come from the import component of the *Utrikeshandel med varor* (Foreign Trade in Goods) dataset compiled by Statistics Sweden (SCB). These firm-level import records report, for each year, the country of shipment, the HS8 product code, the import value, and the physical quantity (usually weight). For some products, the dataset also includes an additional variable, “Other Quantities” (for example, number of pieces for pencils or m^2 for curtains). A list of such products is available in the section *Övrigt om varukoder* on the SCB website.¹ I use these records to compute unit values, which I interpret as prices. For each firm f , variety v , and year t , I define the unit price as:

$$P_{f,v,t} = \frac{\text{Deflated Value}_{f,v,t}}{\text{Quantity}_{f,v,t}},$$

where the import value is deflated using the aggregate CPI to adjust for inflation.

¹<https://www.scb.se/...>

To remove variation across products and countries, I compute the within-variety-year relative (residualized) price:

$$p_{f,v,t} = \log\left(\frac{P_{f,v,t}}{\bar{P}_{v,t}}\right),$$

where the benchmark price $\bar{P}_{v,t}$ is the value-weighted average price for variety v in year t :

$$\bar{P}_{v,t} = \frac{\sum_f \text{Deflated Value}_{f,v,t}}{\sum_f \text{Quantity}_{f,v,t}}.$$

I also construct firm-level indices for input price and quantity. The input price index measures the average log deviation of firm-specific prices from variety-level averages, weighted by import value:

$$\text{InputPrice}_{f,t} = \frac{\sum_v \text{Value}_{f,v,t} \log\left(\frac{P_{f,v,t}}{\bar{P}_{v,t}}\right)}{\sum_v \text{Value}_{f,v,t}}.$$

Analogously, the input quantity index is defined as:

$$\text{InputQuantity}_{f,t} = \frac{\sum_v \text{Value}_{f,v,t} \log\left(\frac{\text{Quantity}_{f,v,t}}{\text{Quantity}_{v,t}}\right)}{\sum_v \text{Value}_{f,v,t}}.$$

For example, suppose a firm imports 10,000 SEK worth of German cars at a 20% higher price than the average German car buyer and 3,000 SEK worth of Chinese headphones at a 30% lower price. The firm-level price index is then:

$$\text{FirmPriceIndex} = \frac{(1.2 \times 10,000) + (0.7 \times 3,000)}{10,000 + 3,000}.$$

The dataset includes all imports originating from outside the European Union (including Switzerland and EEA countries) and intra-EU imports for firms whose total annual imports exceed a reporting threshold. This threshold has gradually increased over time, from about SEK 1.5 million in the early 2000s to roughly SEK 9 million in recent years (see SCB (2018) for details).²

For data cleaning, I exclude observations with total import value below 100 SEK (approximately 9 USD) and those where the unit price is more or less than 15 times the

²Intra-EU trade is subject to this reporting threshold. When the United Kingdom left the EU in 2021, imports below the threshold from the UK that had previously been excluded began to appear in the data. This results in a mechanical increase in the number of small import records in 2021. To address this issue, I exclude post-2020 UK observations from trend plots.

average variety price, as these likely reflect non-arm’s-length transactions, typographical errors, or placeholder values.

A limitation of the data is that if a firm imports the same variety from multiple suppliers within a year, the dataset reports only one aggregated record.

2.2 Balance sheet data and other supporting datasets

To characterize importing firms, I link the import data to firms’ balance sheets using a common firm identifier available in both datasets. I use the industry codes (SNI) reported in the balance sheet data to restrict the sample to manufacturing firms. This linkage also allows me to compare the characteristics of importing firms with those of the broader firm population, as documented in [Data Appendix](#).

From the balance sheet data, I obtain firm-level measures of size, input expenditures, and productivity. I focus primarily on sales and employment, and I construct a measure of total factor productivity (TFP). I first compute revenue-based productivity (TFPR) under a Cobb–Douglas production function:

$$\text{TFPR}_{f,t} = \frac{p_{f,t} y_{f,t}}{\left(w_{f,t} \ell_{f,t}\right)^{\alpha_\ell} K_{f,t}^{\alpha_k} \left(\sum_u T_{f,t,u}\right)^{1-\alpha_\ell-\alpha_k}}.$$

Here, $p_{f,t} y_{f,t}$ denotes firm f ’s nominal output (price times quantity) in year t , $w_{f,t} \ell_{f,t}$ is the wage bill, $K_{f,t}$ is the capital stock, and $T_{f,t,u}$ are intermediate input expenditures (indexed by input type u). All variables are observed in firms’ financial statements.

I set the rental rate of capital to $r = 0.15$, following standard values in the literature. The elasticities α_ℓ and α_k are obtained using the Cobb–Douglas cost-share property: under cost minimization and competitive input markets, each elasticity equals the input’s expenditure share. I assume these elasticities are constant within an industry and equal to the industry’s average expenditure shares. Provided the firm uses strictly positive labor, intermediate inputs, and capital, the TFPR measure is well defined.

I then estimate physical productivity (TFPQ) as:

$$\text{TFPQ}_{f,t} = \frac{\text{TFPR}_{f,t}}{p_{f,t}}.$$

Here, $p_{f,t}$ denotes the firm-level output price. I obtain $p_{f,t}$ from the producer price index (PPI) or from an export database. A limitation is that PPI coverage is concentrated among large firms, which are often exporters, so this sample is biased toward larger firms. In the Appendix, I construct model-based output prices that adjust for product quality and allow for a broader set of firms to be included. The conclusions are robust to using these model-based prices.

2.3 Internet data

As a proxy for search costs, I use variation in internet infrastructure that facilitates communication between firms and suppliers. To measure digital infrastructure, I use municipality-level internet connection data from the Swedish Post and Telecom Authority (PTS).³

The dataset provides annual information on internet coverage by technology across municipalities, including the total number of firms and the number of firms with access to each type of connection. From these data, I construct the share of firms in each municipality with access to fiber broadband, which I interpret as a proxy for lower search costs. The variable is available annually from 2010 to 2023.

Each establishment in the administrative data is identified by a municipality code, which allows me to merge the internet coverage data to firms' geographic locations. This linkage enables me to study how differences in local digital infrastructure affect firms' sourcing behavior and the number of suppliers they connect with.

3 Stylized facts

In this section, I present the main empirical patterns that motivate the model and analysis. I use a set of empirical moments and regularities to illustrate how reductions in search costs can influence firms' behavior in input markets. In particular, I study how firms of different sizes and productivity levels respond to an environment with lower search costs. I also document long-run trends in the distribution of firm size and market concentration in input markets.

³<https://statistik.pts.se/telekom-och-bredband/mobiltackning-och-bredband/dokument/>

3.1 Trends in input markets

Search costs have declined substantially over the past three decades due to rapid improvements in transport and communication technologies. This trend is also reflected in Sweden’s internet infrastructure. In 2010, only about 27% of commercial establishments were connected to the internet through fiber, whereas by 2021 this share had increased to 79%. Given the mechanisms discussed in the previous sections, such reductions in search costs are expected to induce important distributional changes within input markets. In this subsection, I show how key variables in the input market have evolved and discuss how these patterns may translate into changes in market concentration in the final goods market.

3.1.1 Dispersion within the import market

I define a variety as a unique country–product pair. To illustrate changes in sourcing patterns, I compare the number of varieties imported across firms at different points of the distribution. For comparability over time, I normalize the number of imported varieties in each percentile by its value in 1998.

I find that, except for the top percentiles, most firms have experienced little change in the number of imported varieties. In contrast, firms in the upper part of the distribution (particularly at the 90th and 99th percentiles) have expanded their variety of imports by up to 20%. Because the number of imported varieties reflects firms’ search activities, this pattern suggests that lower search costs have had differential effects across the firm size distribution.

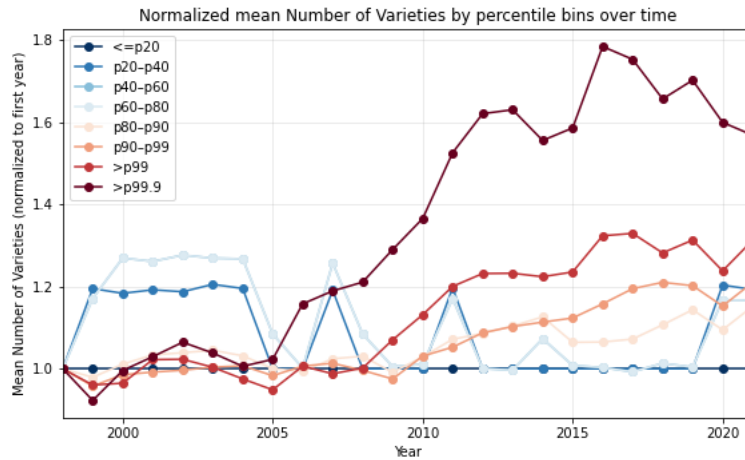


Figure 1: Changes in the number of imported varieties across firm percentiles

A large part of this pattern is driven by the entry of smaller firms into importing. When I restrict the sample to a balanced panel of firms that import every year in the data, all firms show an increase in the number of imported varieties relative to 1998.

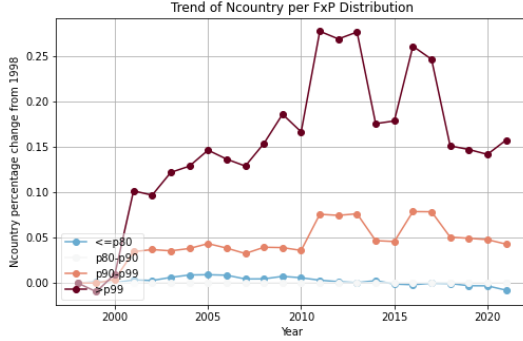


Figure 2: Number of source countries per firm–product pair

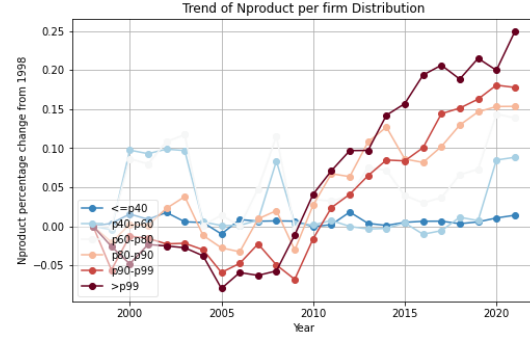


Figure 3: Number of products sourced per firm

The same pattern emerges when examining the number of countries from which each firm sources a given eight-digit product (left panel) or the number of distinct eight-digit products sourced by each firm in a given year (right panel). In both cases, firms at the top of the distribution expand their sourcing scope more rapidly than others.

3.1.2 Importing firm size dispersion

In this subsection, I examine changes in firm size, measured by annual sales, across percentiles from 1998 to 2021. Specifically, I analyze how firm sizes at different points of the distribution have evolved relative to their 1998 levels, normalizing each percentile by its 1998 value.

The results show a substantial reduction in firm size at the median and lower percentiles. For instance, the 20th percentile firm in 2021 is approximately 50% smaller than the 20th percentile firm in 1998. This large decline likely reflects the effect of lower search costs, which previously acted as a barrier to entry into importing. As search costs fell, less productive firms that had faced prohibitively high entry barriers could begin to import. This pattern indicates that the cost of entering international supply chains has declined considerably.

In contrast, firm sizes at the upper end of the distribution have grown substantially, by up to 75% at the 99th percentile and 125% at the 99.9th percentile. This expansion likely reflects reduced search costs that enable larger incumbent firms to expand their supplier base, as discussed in the previous subsection. Among firms that were already importers in 1998, sales increased across the entire distribution, suggesting that incumbents also

benefited from the reduction in search frictions. The corresponding graph is shown in the [Data Appendix](#).

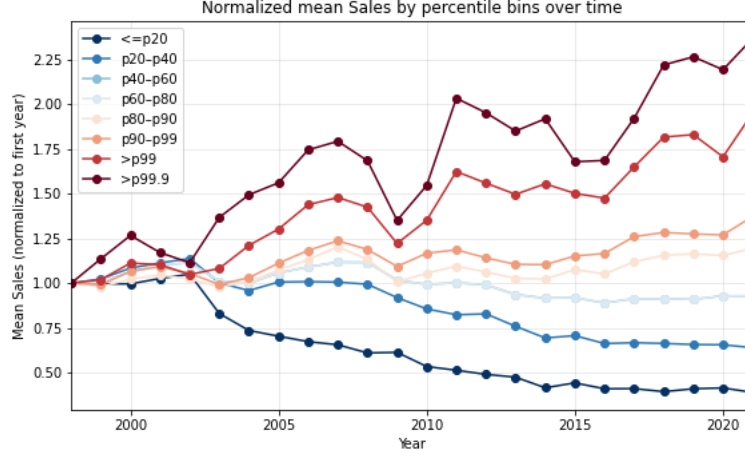


Figure 4: Changes in importing firm size across percentiles, 1998–2021

Overall, these findings point to increasing dispersion in firm size within the importing sector. A plausible explanation is that falling search costs simultaneously lower entry barriers for small firms and reduce expansion costs for large firms, thereby affecting both the extensive and intensive margins of the firm-size distribution.

3.2 Static Moments

In this section, I describe how can change in search cost in the input market affect firm size distribution. In the first part of the this section, I use internet connection as a measure for search cost to see how it affect firms behaviour for acquiring inputs. In the second part, I look at how prices are different for different buyers and explains why.

3.2.1 Input Prices and Firm Size

In this section, I primarily examine what determines input prices, as we know that it is another important channel that input markets can affect productivity and allocative efficiency.

I examine whether larger firms pay lower input prices by estimating

$$\log \text{Input Price}_{f,t} = \beta_0 + \beta_1 \log \left(\frac{\text{Size}_{f,t}}{\text{Size}_{i,t}} \right) + \epsilon_{f,t}, \quad (1)$$

where $\log \text{Input Price}_{f,t}$ is the firm-year value-weighted average of import unit values, and $\text{Size}_{f,t}$ is measured by either (i) the number of employees or (ii) the value-weighted average of import quantities. Both size measures are demeaned by their industry-year average $\overline{\text{Size}}_{i,t}$ so that β_1 is an industry-year-relative elasticity. I have also include estimates using other size measures, such as sales or TFPQ, in the [Data Appendix](#).

Table 1: Regression Results: Input Prices and Firm Size

	(1) Employment	(2) Import Quantity
log No. of Workers	-0.1629*** (0.0017)	—
Input Quantity	—	-0.2774*** (0.0014)
Observations	172,602	172,602
R^2	0.0534	0.2837

Notes: Dependent variable is log value-weighted import unit value at the firm-year level. Size measures are demeaned by industry \times year means. Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

I find a strong negative relationship: a 10% larger firm pays about 1.6% lower input prices when size is proxied by employment and about 2.8% lower when proxied by import quantities. This is consistent with bargaining power or market power on the buyer’s side. When buyer firms have better outside options, they can leverage it and obtain a better deal.

It is also important to note that the slope is similar every year. That indicates while there is some oligonistic marketing power, the power haven’t been changing much over time.

3.2.2 Connectivity, search costs, and imported-variety expansion

Evidence from the import market shows growing dispersion in firms’ input-variety choices alongside rising concentration. Many forces could generate these patterns, but I focus on *buyer search costs* as a parsimonious mechanism linking the micro margin of variety acquisition to aggregate market structure. When search frictions fall, firms can locate and bargain with additional suppliers more easily. This expands variety networks, especially for larger firms, and through both intensive and extensive margins, reshapes the firm-size distribution and contributes to higher concentration. To provide suggestive evidence consistent with this channel, I relate the growth in a firm’s imported varieties to local improvements in digital connectivity.

From the internet data, I use the percentage of firm that have access to internet through fiber as a proxy for (inverse) search cost. I choose the this technology as the proxy instead of xDSL/cable because fiber is the fastest and most realiable internet technology, which is optimal for video calls. Therefore, having fiber connection can significantly reduce communication costs with suppliers. The fiber rollout in Sweden can be seen in the [Data Appendix](#).

Let $\% \Delta \text{Varieties}_{f,t}$ denote the relative change in the number of imported input varieties for firm f in year t . I estimate

$$\begin{aligned} \% \Delta \text{Varieties}_{f,t} = & \beta_1 \text{FiberCoverage}_{m,t} + \beta_2 \left[\mathbf{1}\{\text{Sales}_{f,t} > \overline{\text{Sales}_{i,t}}\} \times \text{FiberCoverage}_{m,t} \right] \\ & + \text{FE}_t + \text{FE}_i + \text{FE}_m + \epsilon_{f,t}, \end{aligned} \quad (2)$$

where $\text{FiberCoverage}_{m,t}$ is the fraction of firms with fiber access in municipality m and year t , $\mathbf{1}\{\cdot\}$ indicates “big” firms with sales above the contemporaneous industry mean $\overline{\text{Sales}_{i,t}}$, and FE_t , FE_i , and FE_m are year, industry, and municipality fixed effects. Standard errors are clustered by municipality.

Table 2: Connectivity and the speed of imported-variety expansion

	(1)	(2)
Fiber coverage	0.1485*** (0.0160)	0.1064*** (0.0184)
Fiber \times 1(Big)	—	0.1817*** (0.0325)
Observations	112,022	112,022
R^2	0.0013	0.0020
Industry FE	Yes	Yes
Time FE	Yes	Yes
Municipality FE	Yes	Yes

Notes: Dependent variable is the relative change in a firm’s number of imported input varieties, $\% \Delta \text{Varieties}_{f,t}$. “Big” denotes firms with sales above the industry mean in year t . Standard errors clustered by municipality in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

The estimates in Table 2 indicate that variety growth is faster where fiber coverage is higher, and the interaction term implies a substantially larger response for bigger firms. While the R^2 values are small, which is consistent with firm-level heterogeneity and municipality-level measurement, these patterns align with the model channel in

which lower search costs disproportionately expand supplier networks at the top of the distribution.

3.3 Summary

I begin by documenting four empirical facts about Swedish manufacturers' importing behavior and then build a model designed to match these facts and reveal the underlying mechanism. First, the distribution of imported input varieties has fanned out over time, with the upper tail importing many more varieties than two decades ago, and the sales distribution has stretched in tandem. Second, larger firms pay lower unit prices for otherwise comparable imported inputs. Third, firms located in municipalities that gained high-speed internet earlier expanded their supplier networks faster—especially the already large and productive firms. This fact is consistent with a story of falling search costs in input markets.

The first fact motivates a tight link in the model between a firm's supplier network and its scale. I allow firms to search for and add foreign input varieties, and I map variety breadth into effective productivity and sales. When search frictions fall, firms with better fundamentals are more likely to find and retain valuable suppliers, pushing out the top tail of both the variety and sales distributions. This mechanism is motivated by the observed widening in the number of imported varieties and in sales dispersion.

The second fact motivates bargaining in buyer–supplier relationships. In the model, input prices are negotiated. A firm's outside option improves as its supplier set deepens, so larger buyers extract better terms. This delivers an endogenous unit-cost schedule that declines with buyer size, matching the empirical gradient in input prices. I discipline the bargaining parameters with external estimates.

The third fact is consistent with heterogeneous responses to changes in input markets. I use the rollout of fiber connectivity as a proxy for lower search frictions, with market level connectivity reflecting variation in search costs. In markets with higher connectivity, firms with larger supplier networks tend to expand their variety of inputs more than smaller or non-importing firms. This pattern, showing stronger variety growth in better connected markets and more pronounced effects among large firms, helps discipline the search component of the model and shapes its aggregate implications for prices and market concentration.

On top of the 4 main empirical facts, I document also two other empirical regularities for model validation. First, firms that receive a lower weighted-average input price tends

to have higher profit share. Also, unsurprisingly, more productive firms import more varieties

On top of my own findings, I also adopt 2 additional features in my model from recent trade literature. The first one concerns the time dependency of the supplier network. Martin et al. (2023) suggests that an median buyer-supplier relationship last around an year. Therefore, as my model period is an year, having a repeated static model is not unreasonable and simplifies the numerical exercises by a lot. The second is related to inventory. Alessandria et al. (2010) point out that average company import internationally every 150 days, which indicate that inventory also should not matter for most firms in a yearly model.

4 Theory

Based on my empirical findings, I build a model that describe buyer-supplier relationship. The model builds on a standard framework with monopolistic competition in the final goods market. Downstream firms, interpreted as buyers, produce final goods using a composite of intermediate inputs. The novel contribution lies in introducing heterogeneity and frictions into the input procurement process. Specifically, intermediate inputs differ in variety, and downstream firms must search for upstream suppliers that produce differentiated inputs. Upon matching, buyers and suppliers engage in bilateral bargaining over both price and quantity. This structure allows the model to capture how search frictions and contractual bargaining jointly shape input variety, firm-to-firm link formation, and ultimately production outcomes.

The economy consists of upstream suppliers and downstream producers. Each upstream supplier u uses labor l to produce an intermediate input x_u according to a linear production technology:

$$x_u = z_u l,$$

where z_u denotes the supplier's productivity. The cost of production is $\frac{w_{\text{foreign}}}{z_u}$, where w_{foreign} is the foreign wage. Without loss of generality, I henceforth assume w_{foreign} to be 1, so all the efficiency measure of the supplier is captured by z_u .⁴ Productivity z_u are observable to the buyer once match is formed.

Downstream firms combine a continuum of differentiated intermediate inputs with labor to produce the final good. The production function of a representative downstream firm

⁴It will be same if I just named another efficiency variable $e_u = \frac{w_f}{z_u}$, but as I don't distinguish between different foreign countries that doesn't matter.

is given by

$$y = zX^\alpha l^{1-\alpha}, \quad X = \left(\sum_u x_u^\rho \right)^{\frac{1}{\rho}},$$

where y is output, z is the downstream firm's productivity, and l is the amount of labor used in final production. The term X represents the composite of all inputs sourced from upstream suppliers. The parameter $\alpha \in (0, 1)$ captures the expenditure share on intermediate inputs, while $\rho \in (0, 1]$ determines the elasticity of substitution across input varieties. A higher ρ implies that inputs are more easily substitutable, whereas a lower ρ corresponds to greater complementarity among input varieties.

The model timing is as followings: all downstream firms begin the period without any inputs. Each firm first decides whether to pay a search cost to look for suppliers or to stop searching altogether. A firm that chooses to search is randomly matched with a potential supplier. Upon meeting, the two parties bargain over a binding contract that specifies both the quantity of inputs to be delivered and the corresponding payment. After reaching an agreement, the downstream firm decides whether to terminate its search or to incur the search cost again to potentially find additional suppliers.

When all firms have completed their search and chosen to stop, production of the final good takes place simultaneously across all firms. The entire sequence of search, matching, bargaining, and production occurs instantaneously within the model period. To characterize equilibrium behavior, I solve the model by backward induction, starting from the production stage.

Now, consider a downstream firm with productivity z that has already completed its search process and is matched with N upstream suppliers indexed by $u = 1, \dots, N$. Let $\mathbf{x} = (x_1, x_2, \dots, x_N)$ denote the vector of input quantities purchased from these suppliers. The composite input used in final production is given by

$$X(\mathbf{x}) = \left(\sum_{u=1}^N x_u^\rho \right)^{\frac{1}{\rho}}.$$

Given its existing input bundle \mathbf{x} , the downstream firm chooses labor l to maximize profits. The value of operating without further search, denoted $V^{NS}(z, \mathbf{x})$, is

$$V^{NS}(z, \mathbf{x}) = \max_{l \geq 0} \left[z X(\mathbf{x})^\alpha l^{1-\alpha} - w l \right],$$

subject to product-market demand

$$y = C \left(\frac{p}{P} \right)^{-\epsilon},$$

where w is the wage rate, C is aggregate consumption, p is the firm's price, P is the aggregate price index, and $\epsilon > 1$ is the elasticity of substitution across differentiated final goods. Notice that this is the familiar monopolistic competition demand curve. This stage captures the production and pricing decisions of a firm that has concluded its supplier search and operates given its current network of input relationships.

There is a closed form solution that can express V^{NS} in terms of only state (z, X) and parameters. See details of derivations in the [Theory Appendix](#).

Before making a decision to stop searching, a downstream firm with productivity z and an existing input portfolio \mathbf{x} decides whether to continue searching for additional suppliers or to stop and proceed directly to production. The firm's value function is

$$V(z, \mathbf{x}) = \max \left\{ V^S(z, \mathbf{x}), V^{NS}(z, \mathbf{x}) \right\},$$

where $V^S(z, \mathbf{x})$ is the value of continuing to search for new suppliers, and $V^{NS}(z, \mathbf{x})$ is the value of stopping, as defined in previous step.

If the firm chooses to search, it incurs a search cost κ measured in labor units and faces random matching with potential upstream suppliers. The expected value of searching is

$$V^S(z, \mathbf{x}) = \int V^m(z, z_u, \mathbf{x}) dF(z_u) - w\kappa,$$

where z_u denotes the productivity of a prospective supplier drawn from distribution $F(z_u)$, and w is the wage rate. The term $V^m(z, z_u, \mathbf{x})$ represents the expected value upon matching with a supplier of productivity z_u , prior to bargaining over the terms of trade. After paying the search cost, the firm proceeds to Stage 1, where it negotiates a binding contract specifying the input quantity x_{new} and the corresponding transfer payment T .

When a downstream firm with productivity z and existing input bundle \mathbf{x} meets a new upstream supplier with productivity z_u , the two parties negotiate a binding contract over the quantity of the new input x_u and the transfer payment T . The outcome of this bilateral negotiation maximizes a Nash product of buyer and seller surpluses:

$$\max_{T, x_u \geq 0} \left(V(z, \mathbf{x}_{\text{new}}) - T - V(z, \mathbf{x}) \right)^\theta \left(T - \frac{w_{\text{foreign}}}{z_u} x_u \right)^{1-\theta},$$

where $\theta \in (0, 1)$ denotes the bargaining weight of the buyer, $V(z, \mathbf{x}_{\text{new}})$ is the downstream firm's value after adding the new input, and $V(z, \mathbf{x})$ is its pre-match value. The term T represents the payment from the buyer to the supplier, and $\frac{w_{\text{foreign}}}{z_u} x_u$ is the

supplier's production cost, which depends on the foreign wage w_{foreign} and the supplier's productivity z_u .

The first term in the Nash product captures the buyer's surplus from obtaining the new input, while the second term captures the seller's surplus from the transaction. The negotiated outcome determines both the optimal quantity x_u traded and the transfer T . The value of a successful match to the downstream firm is then given by

$$V^m(z, z_u, \mathbf{x}) = V(z, \mathbf{x}_{\text{new}}) - T,$$

which represents the firm's continuation value after accounting for the payment to the new supplier.

4.1 Recursive CES aggregator and state compression

Now, I have always use (z, \mathbf{x}) as state variables, where \mathbf{x} is a vector that is growing. If we assume each upstream suppliers is one variety, the length of \mathbf{x} can go up to 1,000 for bigger firms. This large state space poses significant difficulties in solving the model. In this subsection, I will provide a solution to this computation problem.

Consider a downstream firm with productivity z that has already formed relationships with N upstream suppliers providing input quantities $\mathbf{x} = (x_1, \dots, x_N)$. Inputs are aggregated through a CES composite

$$X = \left(\sum_{u=1}^N x_u^\rho \right)^{1/\rho}, \quad \rho \in (-\infty, 1],$$

so that final output depends on X and labor as specified in the environment. When the firm successfully bargains with a new supplier for quantity $x_{\text{new}} \geq 0$, the input vector expands to $\mathbf{x}_{\text{new}} = (x_1, \dots, x_N, x_{\text{new}})$ and the composite updates exactly as

$$X_{\text{new}} = \left(\sum_{u=1}^N x_u^\rho + x_{\text{new}}^\rho \right)^{1/\rho} = \left(X^\rho + x_{\text{new}}^\rho \right)^{1/\rho}.$$

This identity implies that the history of individual matches can be summarized by the scalar state X . The continuation value can therefore be written as

$$V(z, \mathbf{x}) \equiv V(z, X),$$

and future negotiations with prospective suppliers only require the pair (z, X) together

with the draw of the supplier's productivity. In particular, if a meeting yields productivity z_u , the contracting problem selects (x_{new}, T) to solve the Nash program given in the bargaining subsection, and the next period state becomes $X_{\text{new}} = (X^\rho + x_{\text{new}}^\rho)^{1/\rho}$.

Notation. x_u denotes the quantity bought from supplier u . X is the CES composite of all purchased inputs. The parameter ρ governs substitution across input varieties, with elasticity $\sigma = 1/(1 - \rho)$ under the standard CES parameterization. The symbol x_{new} is the quantity negotiated with a newly met supplier, and X_{new} is the updated composite after adding this input.

Methodological contribution. The recursion

$$X_{\text{new}} = (X^\rho + x_{\text{new}}^\rho)^{1/\rho}$$

allows the dynamic buyer problem with an expanding set of differentiated input relationships to be solved on a low dimensional state that consists of (z, X) rather than the full vector \mathbf{x} . This reduction is exact under CES aggregation and does not rely on symmetry or distributional approximations. It delivers substantial computational gains for value function iteration and policy iteration because one replaces an ever growing vector of inputs with a single sufficient statistic for procurement history. To our knowledge, prior work uses CES aggregation mainly to obtain tractable demand systems, nested aggregator structures, or sufficient statistics, but not to implement recursive state compression for search and bargaining over input relationships. This subsection formalizes that compression and uses it to build a tractable quantitative model of firm to firm matching with differentiated inputs.

5 Simplified model

I use a stripped down environment to highlight how a change in the search cost κ alters input choices and output. I remove supplier heterogeneity by setting $z_u = \bar{z}_u = 1$ for all potential suppliers. I set the bargaining weight to $\theta = 1$, which gives the buyer full bargaining power. Under these assumptions the negotiated input price equals the seller's unit production cost, $1/\bar{z}_u$ in units of the buyer's numeraire, wage.

5.1 Objective and reduced contracting problem

With no seller surplus, the one match contracting problem is

$$\max_{x \geq 0} \left[V(z, X_{\text{new}}) - V(z, X) - \frac{1}{z_u} x \right],$$

where X is the current CES composite of inputs and $X_{\text{new}} = (X^\rho + x^\rho)^{1/\rho}$ after purchasing quantity x from the newly met supplier. To study comparative statics in a transparent way, I consider a collapsed version that chooses the full procurement path directly. The collapsed problem is equivalent as the original sequential setup because there are no randomness and no strategic interaction as buyer have full power. I impose symmetry across relationships, so $x_i = x$ for all $i \in \{0, \dots, N\}$. The problem becomes

$$\max_{N \in \mathbb{N}_+, x \geq 0} \left\{ K_1 \left[z \left(\sum_{i=0}^N x_i^\rho \right)^{\frac{\alpha}{\rho}} \right]^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} - \frac{1}{z_u} Nx - Nw\kappa \right\},$$

where K_1 collects constants and demand shifters, $\alpha \in (0, 1)$, and $\epsilon > 1$ is the demand elasticity in the final goods market. Symmetry implies $\sum_{i=0}^N x_i^\rho = Nx^\rho$.

5.2 Optimal number of searches

The optimal number of supplier searches satisfies

$$N^* = \left[\frac{K_1 z^{\epsilon-1} \bar{z}_u^{\alpha(\epsilon-1)}}{w \kappa} \right]^\phi, \quad \phi = \frac{\rho}{\rho - \alpha(\epsilon - 1)(1 - \rho)}.$$

I use the following expression for K_1 in the computations:

$$K_1 = \epsilon^{-\epsilon} \left(\frac{w}{(\epsilon - 1)(1 - \alpha)} \right)^{(1-\alpha)(1-\epsilon)} \left(\frac{C}{P^{-\epsilon}} \right) (\alpha(\epsilon - 1))^{\alpha(\epsilon-1)} \frac{\alpha(1 - \rho)}{\rho}.$$

The comparative static with respect to search costs is

$$\frac{\partial \ln N^*}{\partial \ln \kappa} = -\phi.$$

A proportional decline in κ raises N^* by the same proportion across all z when $\phi > 0$, which requires $\rho - \alpha(\epsilon - 1)(1 - \rho) > 0$. Higher substitutability across inputs (higher ρ) increases the gain from additional relationships and raises the elasticity of N^* with respect to κ .

5.3 Discrete choice and dispersion

Because N is discrete, firms adjust the number of relationships in steps. Large firms, for which the marginal value of an extra link varies smoothly with z , display near continuous adjustments. Small firms face lumpy changes as they cross thresholds at which an additional relationship becomes profitable.

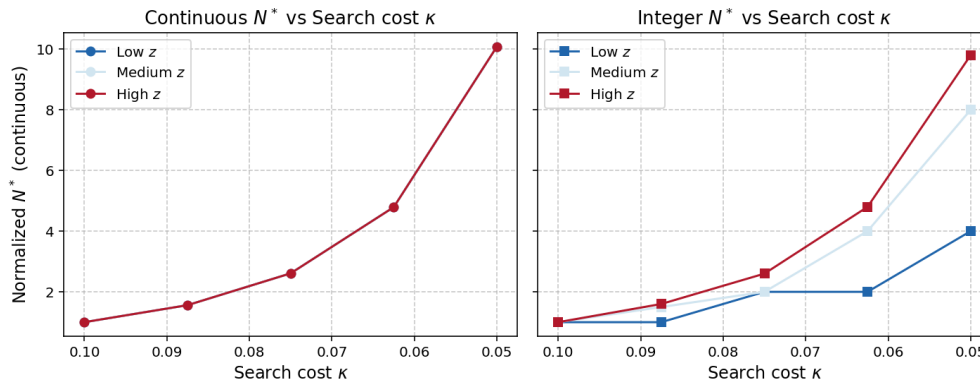


Figure 5: Discrete supplier choice generates lumpy adjustment for small firms and smoother adjustment for large firms.

5.4 Extensive margin and entry

A lower search cost also shifts the extensive margin. Some small firms that previously chose zero imported relationships now find it profitable to enter. The figure fixes the position for clarity.

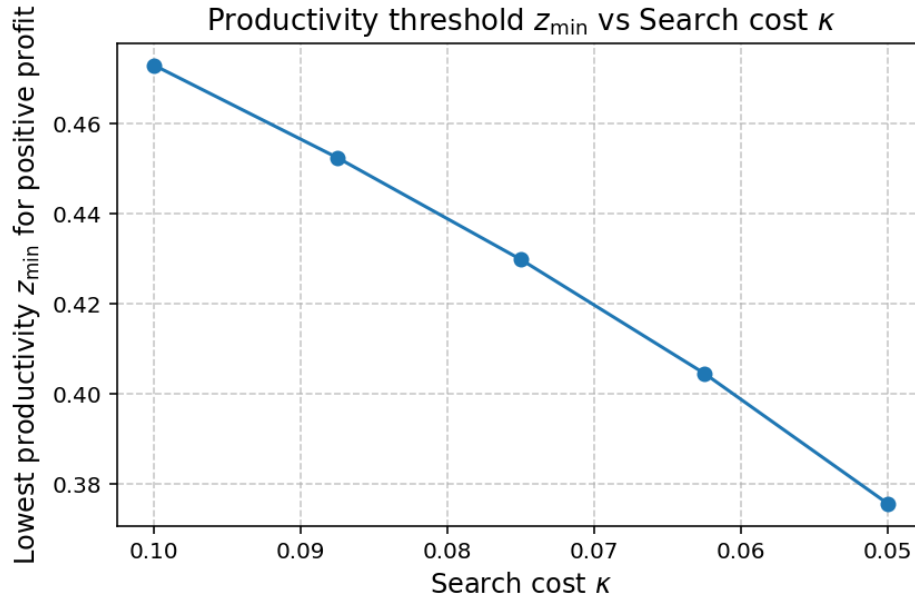


Figure 6: A decline in κ increases entry at the extensive margin.

5.5 Summary

A reduction in the search cost increases the optimal number of supplier relationships on the intensive margin through the elasticity ϕ . The discreteness of N generates heterogeneous adjustment, with smooth changes for large firms and lumpy jumps for small firms. The same reduction in search costs also activates the extensive margin by inducing entry among small firms. These forces together reshape the distribution of firms across input relationships and output levels.

6 Price dispersion with identical suppliers

I now relax the assumption that the buyer has full bargaining power. Sellers receive weight $1 - \theta$ in the bargaining process, and the input quantity is chosen efficiently to maximize the joint surplus. To isolate the role of bargaining, I continue to assume that all potential suppliers have identical productivity \bar{z}_u .

The efficient bargaining problem is

$$\max_{x \geq 0} \left(V(z, X_{\text{new}}) - V(z, X) - \frac{1}{\bar{z}_u} x \right),$$

where $V(z, X_{\text{new}})$ is the buyer's continuation value after forming a new supplier relationship, and $V(z, X)$ is the buyer's outside option prior to the new match. The first order condition defines the efficient input $x^*(z, \bar{z}_u)$, which increases with buyer productivity,

$$\frac{dx^*(z, \bar{z}_u)}{dz} > 0.$$

When there is no randomness in the productivity of the next potential supplier, the buyer's outside option is the value of meeting another supplier with the same productivity \bar{z}_u , net of the search cost $w\kappa$:

$$V(z, X) = V(z, X_{\text{new}}) - w\kappa.$$

Because suppliers are identical, the same efficient input level $x^*(z, \bar{z}_u)$ applies to any potential match.

Under Nash bargaining with weights $(\theta, 1 - \theta)$ for the buyer and the seller respectively, the total transfer from the buyer to the supplier is

$$T = (1 - \theta)[V(z, X_{\text{new}}) - V(z, X)] + \theta \frac{1}{\bar{z}_u} x^*.$$

Substituting the outside option expression yields

$$T = (1 - \theta)\left\{V(z, X_{\text{new}}) - [V(z, X_{\text{new}}) - w\kappa]\right\} + \theta \frac{1}{\bar{z}_u} x^*,$$

which simplifies to

$$T = (1 - \theta)w\kappa + \theta \frac{1}{\bar{z}_u} x^*.$$

The payment therefore consists of a fixed component $(1 - \theta)w\kappa$ that shares the buyer's surplus and a variable component $\theta \frac{1}{\bar{z}_u} x^*$ reflecting the supplier's production cost.

Dividing through by x^* gives the per unit input price,

$$p_u = \frac{T}{x^*} = (1 - \theta) \frac{w\kappa}{x^*} + \theta \frac{1}{\bar{z}_u}.$$

Because x^* increases with productivity z , the term $\frac{w\kappa}{x^*}$ decreases with z , and more productive buyers pay a lower input price p_u . The degree of price heterogeneity across buyers depends on the relative magnitudes of $\frac{1}{\bar{z}_u}$ and κ : when either \bar{z}_u or κ increases, dispersion in input prices also rises.

See Appendix 9 for the full derivation of the sequential version of this model.

The example above illustrates one mechanism through which the model generates differential effects across firms of different sizes. However, this is not the only source of heterogeneity. Owing to the love of variety structure of the input aggregator X , firms with higher productivity z employ a larger number of input varieties. Consequently, even in the absence of explicit unit price dispersion $\left(\frac{\sum^T}{\sum^x}\right)$ as analyzed in this section, their effective unit price, measured as total spending per composite input $\frac{\sum^T}{X}$, is lower. The CES aggregation therefore reinforces the cost advantage of more productive firms.

7 Quantitative Model and Results

7.1 Full Equilibrium Model

In the full model, I add further features. First of all, there is outside options for production for firms. It is unrealistic to assume firms cannot produce without imports. The outside option is to assume there is a domestic supplier D that use only labor to produce. I normalize $z_D = 1$ and assume that market is perfectly competitive such that domestic inputs M is provided at price $p_M = w$.

Therefore, after firms search (or not search), the production function becomes:

$$y = z \max\{X, M\}^\alpha l^{1-\alpha}$$

7.2 General Equilibrium

In equilibrium, aggregate labor demand equals the fixed labor supply \bar{L} ,

$$\int (l_f + n_f \kappa) df = \bar{L},$$

where both production labor l_f and search effort $n_f \kappa$ depend on the demand shifter CP^ϵ . The general equilibrium solution is computed using a nested fixed point algorithm. In the outer loop, I update the aggregate demand term CP^ϵ until the labor market clears. In the inner loop, I solve firms' dynamic problems by value function iteration, simulate the distribution of firms over productivity and input stocks (z, X) , compute implied labor demand, and feed the result back to the outer loop to update CP^ϵ .

The household's budget constraint is given by

$$\max_{c_i} C = \left(\int (y_i Q_i^\alpha)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad \text{s.t.} \quad PC \leq wL_t + \pi + \text{export},$$

where the export term reflects that, since the economy imports some intermediate inputs, an equivalent value of domestic goods must be exported to balance trade. I assume these exports are sold at the domestic price P .

The goods market clearing condition

$$\int \mathbb{E}[y|z] dz = C$$

is automatically satisfied and therefore does not impose an additional equilibrium restriction. In the CES aggregator, aggregate demand C is defined as the integral of firm-level demands, and each firm's output y exactly equals the quantity demanded in equilibrium. Because total expenditure across varieties equals total revenue by construction of the CES price index, aggregate supply equals aggregate demand identically once firms produce their equilibrium outputs.

7.3 Numerical Solution and Simulation

The full model have no closed form solutions, so I resolve to numerical method. I first calculate the policies of firms given states z and X . $X = (\sum_u^{N_i} x_{i,u}^\rho)^{\frac{1}{\rho}}$ is sufficient as a state variable to represent the history of contracted inputs $\{x_{i,1}, x_{i,2}, \dots x_{i,u}\}$. Observe that if I add a new x_{new} to list, so that the new aggregate input is:

$$\begin{aligned} X_{new} &= \left(\sum_u^{N_i} x_{i,u}^\rho + x_{new}^\rho \right)^{\frac{1}{\rho}} \\ &= (X^\rho + x_{new}^\rho)^{\frac{1}{\rho}} \end{aligned}$$

I solve the model by a standard VFI procedure. In the PE model, I first make an initial guess on the search value function V^S and calculate not-search V^{NS} based on the closed form solution above. Based on V^S and V^{NS} , I solve the firm search policy and get a guess V . Assuming this guess of V is true, I solve the policy functions for payment T and quality-adjusted quantity xq by grid searching. Then, I calculate the matching value V^m by plugging in the optimal T and new state $X^N Q^N$ and update the expected search value V^S and V . The process stops when the value functions converge.

To speed up the VFI, I prove that the bargaining problem is strictly concave in T (See [Theory Appendix](#)). So I can use a first order condition of T given fixed x :

$$\begin{aligned}(1 - \theta)\text{Buyer Surplus} &= \theta\text{Supplier Surplus} \\ \theta\left(\frac{T - K(x)}{V(x) - T}\right)^{1-\theta} &= (1 - \theta)\frac{V(x) - T}{T - K(x)} \\ T &= (1 - \theta)V(x) + \theta K(x)\end{aligned}$$

where $K(x) = \frac{x}{z_u}$ and $V(x) = V(z, X_{new}) - V(z, X)$. And plug this into the original bargaining problem, I get:

$$\max_x \theta^\theta (1 - \theta)^{1-\theta} (V(z, X_{new}) - V(z, X) - \frac{x}{z_u})$$

In this way, I reduce the 2D optimization to 1D and I can ignore the constants $\theta^\theta(1 - \theta)^{1-\theta} > 0$. V is not concave in x because of the discrete search cost. Therefore, I have to find the optimization with grid-search.

I simulate the full model with various different parameters to understand 1) how does search cost change allocative efficiency and concentration 2) how does economy in different condition subject to this friction.

I first initiate a set of downstream firms with productivity z_d according to the distribution $F(z)$. Given a set of parameters and the policies, I simulate forward. The downstream firms choose to search according to the search policy and its state. The downstream firms will get a random draw of supplier z_u and gain intermediate good and pay according to the policy functions that I solved previously. I then obtained a panel of simulated inputs x , payments to suppliers T and search cost κ .

7.4 Calibration

In this subsection, I describe the strategy of calibrating the model. I take some of the parameters from external sources and other calibrate data moments and model moments. I use the following parameters:

Table 3 lists the parameters I take as given in the quantitative exercise. Buyer bargaining power is set to $\theta = 0.83$, taken from Alvarez et al. (2023). They develop a theory of market power that link buyer' market share, supplier market share and prices. Then,

Table 3: Externally set parameters

Variable	Definition	Value(s)	Source
θ	Buyer bargaining power	0.83	Alviarez et al. (2023)
ϵ	Final market elasticity	4	Broda and Weinstein (2006)
ρ	Input elasticity	4	Broda and Weinstein (2006)
α	Cobb–Douglas intermediate share	0.7	Intermediate share (manufacturing)
μ_{z_u}	Mean of supplier z_u	-.2	Assumed equals to μ_z
σ_{z_u}	Std. of supplier z_u	0.45	Assumed equals to σ_z

Table 4: Calibrated parameters and matched moments

Variable	Definition	Value(s)	Moment	Data	Model
μ_z	Mean of buyer z	-0.2	Top 1% vs. median sales ratio	414 \rightarrow 948	606 \rightarrow 874
σ_z	Std. of buyer z	0.45	Top 1% sales share	0.44 \rightarrow 0.49	0.479 \rightarrow 0.494
κ_t	Search cost	0.3 \rightarrow 0.2	Mean number of varieties	19 \rightarrow 23	23 \rightarrow 30

they use firm-to-firm trade data to estimate the bargaining power of U.S. firms. On the demand side, I adopt standard benchmark elasticities from Broda and Weinstein (2006): the final-goods market elasticity $\epsilon = 4$ and an input elasticity of substitution across varieties equal to 4. This is also consistent with the Swedish-specific elasticity from Boppart et al. (2023), where they find a median elasticity of 4.39 within industry. The Cobb–Douglas share on intermediates is $\alpha = 0.7$, chosen to match the intermediate-input expenditure share in manufacturing. For supplier productivity, I normalize the mean to the downstream scale, setting $\mu_{z_u} = -.2$, and choose a dispersion $\sigma_{z_u} = 0.45$ to mirror the downstream distribution. None of these parameters are targeted in the calibration moments below; they serve as conventional benchmarks and normalizations.

Table 4 reports the calibration linking model primitives to sales and sourcing moments. To calibrate the moments, I begin with buyer productivity z , which I assume is lognormally distributed. This leaves two parameters to pin down, so I target two sales-based moments because physical productivity is difficult to measure accurately with available data. Specifically, within industries with at least five firms (to avoid mechanically concentrated sectors), I compute (i) the top-1% sales share and (ii) the ratio of sales at the 99th percentile to the median; I then take the median estimate across industries. Anchoring the location parameter at $\mu_z = -0.2$ aligns the model’s top-1% vs. median sales ratio with the data, rising from 606 \rightarrow 874 in the model versus 414 \rightarrow 948 in the data. While setting the dispersion to $\sigma_z = 0.45$ matches the increase in the top-1%

sales share ($0.479 \rightarrow 0.494$ in the model vs. $0.44 \rightarrow 0.49$ in the data). As a cross-check, the top-1% vs. mean ratio in the data also increases from 87 to 100. Given the lack of reliable quality- and price-adjusted productivity measures, using the sales distribution avoids additional structure (e.g., TFPR or labor-productivity corrections) and ties the two lognormal parameters to the level and the tail of the distribution, respectively. Finally, I discipline the time path of search frictions by lowering the per-period search cost from $\kappa_t : 0.3 \rightarrow 0.2$, which reproduces the observed growth in the mean number of imported varieties (data: $19 \rightarrow 23$; model: $23 \rightarrow 30$).⁵

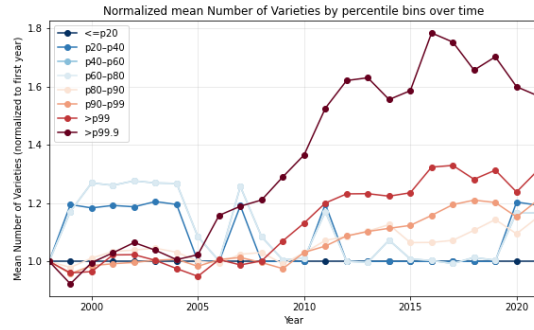
7.5 Firm Size Dispersions

To gauge the aggregate role of search frictions, I lower the buyer search cost parameter by one-third in the calibrated model and recompute the stationary equilibrium. Declines in search costs make it easier for firms to locate and bargain with additional suppliers. The resulting expansion in supplier networks is uneven across firms: more productive firms add varieties faster, while some smaller firms enter. The distributional consequence is a widening of firm size dispersion and, in the aggregate, higher market concentration.

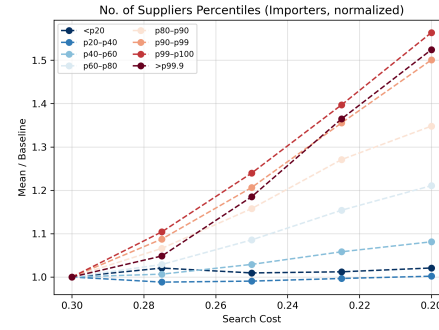
Figure 7 documents the evolution of the number of imported input varieties at the firm level, my proxy for supplier links, in the data and in the model. Both series display a pronounced right-tail expansion. Figure 8 shows the dispersion of firm sales across the distribution; the model reproduces the broad rise in dispersion observed in the data over time. These distributional changes underpin the aggregate increase in concentration reported in Table 5. Additional cuts (balanced panel, industry-by-year fixed effects, arm’s-length and non-EU subsamples) are reported in the [Data Appendix](#).

In the data, firm size distribution is increasing, that assembles the first part of the graph. On one hand, smaller firms get to enter the market, but the firms that expand the fastest are the most productive firms. The reason is the discrete increase of suppliers and the concavity of profit function. When search cost is high, let’s assume the most productive firm have 20 suppliers while the median firm has 1. While adding one extra supplier is not a lot for a firm with 20 suppliers, a increase from 1 to 2 is a big jump. That’s why at the start big firms expand the quickest. However, when the search cost decrease enough that less productive firms also start to expand, they expand quicker because the big firms have less incentive to further grow because of the concavity of the profit function.

⁵In the simulations I truncate the sales distribution at the 1st and 99th percentiles.

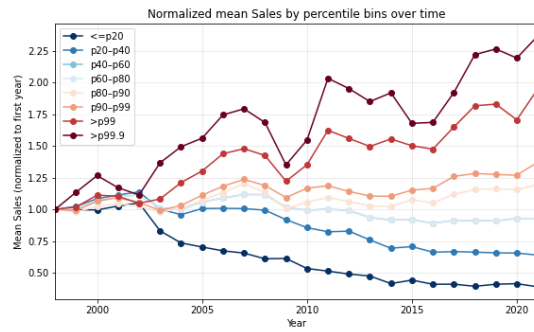


(a) Data

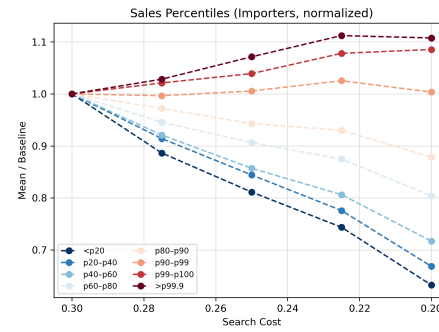


(b) Model

Figure 7: Number of imported input varieties: data vs. model



(a) Data



(b) Model

Figure 8: Firm-size dispersion (sales): data vs. model

7.6 Price level, GDP and Concentration

Given my parameterization, this experiment implies a **10%** decline in the aggregate price level and a **9%** increase in real GDP. On the concentration side (measured by the Herfindahl–Hirschman index of sales), the model delivers a **6.6%** increase. The corresponding rise in the data over 1998–2021 is **44%** overall, or **20%** when excluding the financial-crisis years. Taken together, reduced search costs account for roughly **15–33%** of the observed increase in concentration.

Table 5: Aggregate effects when buyer search costs fall by one-third

Price level and output	
Price level	−10%
Real GDP	+9%
Market concentration (HHI of sales)	
Data (total, through 2021)	+44%
Data (excluding crisis years)	+20%
Model	+6.6%

Notes: Percent changes relative to the baseline calibrated equilibrium. Concentration is summarized by the Herfindahl–Hirschman index of firm sales.

8 Counterfactual: tariff

8.1 Policy and implementation

I introduce an ad valorem tariff on imported intermediate inputs at rate $\tau = 0.10$. The government rebates the tariff revenue to the representative household as a lump-sum transfer. The tariff scales the buyer’s payment to foreign suppliers by the factor $(1 + \tau)$, so that the buyer’s effective unit input price becomes

$$p_u^{\text{imp}} = (1 + \tau) p_u.$$

Given this wedge, the Nash bargaining problem between buyer and supplier becomes

$$\max_{T, x} \left(- (1 + \tau)T + V(z, X_{\text{new}}, n + 1) - V(z, X, n + 1) \right)^\theta \left(T - w_f \frac{x}{z_u} \right)^{1-\theta}.$$

The tariff therefore raises the cost of imported inputs proportionally to $(1 + \tau)$ while leaving the rest of the environment, including the production structure and demand system, unchanged.

8.2 Aggregate effects

The tariff increases the effective cost of imported inputs, which feeds through the CES aggregator into higher consumer prices. Figure 9 compares aggregate outcomes under the baseline and a 10% tariff across different search costs. In the baseline, lower search costs reduce prices and raise production, domestic input share, and real GDP. With the tariff, these relationships remain qualitatively similar but are uniformly dampened: prices are higher, production and real GDP are lower, and the shift toward domestic inputs is stronger. The tariff therefore reduces the overall gains from improved supplier search efficiency.

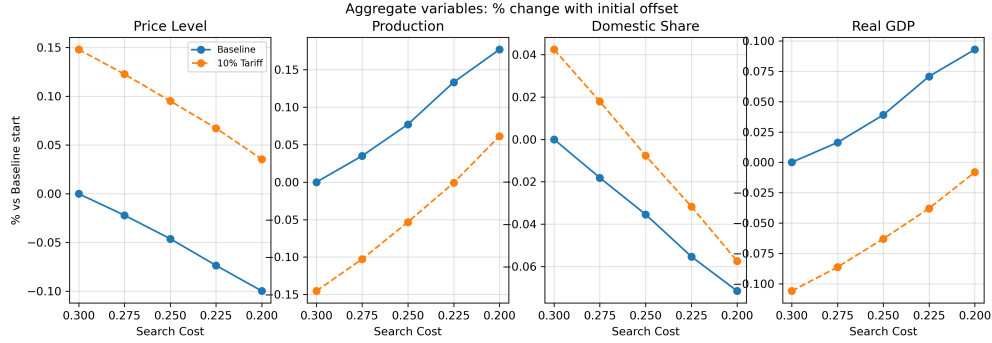


Figure 9: Aggregate outcomes under a 10% tariff. The tariff raises prices and weakens the production and real GDP response to lower search costs, while increasing the domestic input share.

8.3 Search incentives and network size

By scaling the variable cost of imported inputs, the tariff reduces the gain from adding a supplier. In the simplified benchmark with $z_u = \bar{z}_u$ and buyer power $\theta = 1$, the collapsed problem replaces $1/\bar{z}_u$ with $(1 + \tau)/\bar{z}_u$. Using the same algebra that yields N^* in Section 5.2, I obtain

$$N_\tau^* = N_0^* (1 + \tau)^{-\phi}, \quad \phi = \frac{\rho}{\rho - \alpha(\epsilon - 1)(1 - \rho)},$$

where N_0^* denotes the low tariff benchmark. The model implies broad declines in network size when I introduce the tariff. This effect is also observed in the full model, more drastically:

Table 6: Change in number of suppliers, baseline versus tariff, both at low search cost.

	Share (%)	Baseline	Tariff
Fewer	88.17	32.76	24.44
More	0.00	<i>n.a.</i>	<i>n.a.</i>
Equal	11.83	9.05	9.05

8.4 Distributional effects on sales and profits

The tariff changes relative prices and demand. Some smaller non importers benefit from higher output prices, while most importers experience lower margins. I report the share of firms with lower or higher outcomes and the corresponding averages.

Table 7: Sales, baseline versus tariff, both at low search cost.

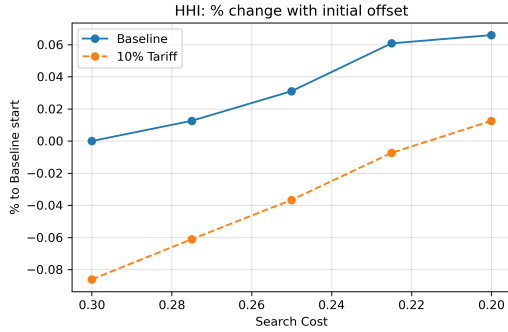
	Share (%)	Baseline	Tariff
Fewer	2.49	40.26	37.17
More	97.51	1.35	1.50

Table 8: Profits, baseline versus tariff, both at low search cost.

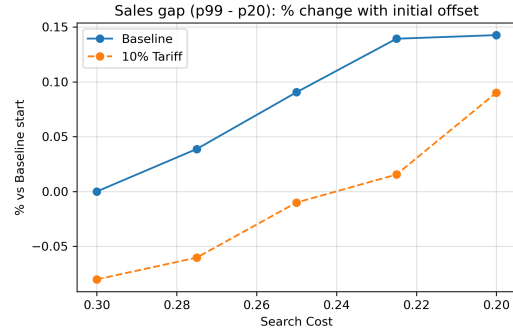
	Share (%)	Baseline	Tariff
Fewer	2.38	3.58	3.06
More	97.62	0.15	0.19

8.5 Market concentration

I measure concentration by the Herfindahl Hirschman Index and by the sales gap between the largest and smallest firms which imports. The tariff compresses the firm size distribution in the model, both in general and among importers. The reason is that the biggest firms which import the most will have significantly less incentive to sustain a high number of suppliers.



(a) Herfindahl Hirschman Index for sales under the baseline and the tariff.



(b) Sales gap between top and bottom importing firms under the baseline and the tariff.

9 Conclusion

This paper has highlights the role of input market in reshaping the firm size distribution. In particular, I look at the reduced search costs as the main mechanism. Using detailed Swedish administrative data, I documented four empirical facts: more productive firms pay lower input prices, there is widening dispersion in imported varieties and sales, and municipalities with greater fiber-optic coverage experience faster supplier network expansion, especially for more productive firms.

To interpret these findings, I developed a quantitative model that features a frictional input market with random search and bilateral bargaining. The model shows that lower search costs benefit importing firms more strongly, allowing them to expand supplier networks and raise productivity, while non-importers become relatively less competitive. Also, there are heterogenous effects among importing firms because of bargaining power. As a result, aggregate productivity rises but market concentration increases. A counterfactual exercise further showed that a modest tariff on intermediate goods could offset the output gains, while compressing the firm size distribution. Through the lens of my model, I find that reduced search costs in input market reshape firm size distribution through intensive, extensive margins and general equilibrium effects.

These results highlight the importance of input market frictions in shaping firm dynamics, productivity, and concentration. They suggest that policies affecting search costs, whether through infrastructure investments, digital technologies, or trade regulation, can have substantial effects on firm performance and aggregate outcomes.

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Data Appendix

Summary Statistic

I calculated for an average year in my dataset, the summary statistics:

Table 9: Summary Statistics - Imports

Variable	Mean	Std. Dev.	P10	Median	P95
No. Varieties per Firm	20.8	78.8	1.0	4.0	84.9
No. Product per Firm	13.2	31.6	1.0	3.3	54.7
No. Country per Firm	4.6	6.4	1.0	2.0	17.0
No. Country per Product	7.7	8.7	1.0	5.0	25.0
No. Firms per Variety	3.1	7.3	1.0	1.0	10.0
No. Country per FirmXProduct	1.6	1.8	1.0	1.0	4.0
No. of Firms			8432		
No. of Product			7406		
No. of Varieties			56778		
No. of Countries			179		
Percentage of Domestic Firms			0.29		
Imported Intermediate share			0.00063		

And a comparison between Importing and all firms:

Table 10: Summary Statistics - Importing Firm vs All Firm

Variable	Importers	All Firms	Ratio
No. of Firms per Year	8432	52811	0.16
Total No. of Firms	36864	150046	0.25
Sales	1.63e+11	2.83e+10	5.75
Employment	58.83	11.15	5.28
Labor Productivity	19.95	19.72	1.01
Intermediate share of Production	0.44	0.36	1.23

Within EU Data Selection and Non-EU subsample

Imports originating from other EU countries will not go through the customs because Sweden has been part of EU from 1995 on. Therefore, data on EU imports are collected from mandatory self-reports by firms. However, if the firm import below a certain value threshold, then they have no obligation to report. This cutoff is 1.5 million SEK worth

of goods in 1998-2004, 2.2 mil SEK in 2005-2008, 4.5 million SEK in 2009-2014 and 9.0 million SEK onwards (See SCB (2018) for more). Due to these rules, when Britain exit the EU in 2020, it also cause some irregularities in the data of year 2021. However, if we limit the sample to only non-EU imports or exclude Britain for all years, the conclusion of the this paper hasn't change that much. In opposite, focusing on the Non-EU sample let us understand the bottom part of the distribution of importing firms better. Non-EU import consist around 50% of the full sample.

I have include robustness tests for only non-EU imports for some of my empirical facts. I define non-EU imports as imports from countries that have never been in the EU throughout the sample period. Therefore, for example, import records from Bulgaria (joined EU in 2007) in 2001 will be excluded from those robustness tests.

Robustness Check - Arm length

To test if my results holds for non-arm-length trade (i.e. Trade that happens not within the same multi-national corporation group), I have carry out robustness test on the sub-sample of firm that neither 1.) do not belong to a foreign-based corporate group nor 2) belong to a corporate group that owns subsidiary abroad. In this case, any international trade of this firm have to be arm-length. The reason for using such conservative measure is that I still lack the indicator of the dataset (will be delivered). This subsample contains around 25% of the observations in the original dataset.

For arm length trade, 1) the negative relationship between TFPQ-Input Price is unchanged 2) the evolution of number of variety imported is slightly different, the 99th percentile firm still increase significantly, 90-99th percentile drop slightly while all other ones are flat. I believe the same conclusion can be drawn. 3) Nsupply increase much less in TFPQ when we throw out all this companies, but it is not surprising and a bit unclear if this result are affected by excluding non-arm-length trade or excluding the biggest/most productive multi-national firms. 4) Price Dispersion result also holds, the absolute value of variance in all years increase. Interestingly, there is a big drop in 2020 (COVID-19), this is not seen in the original graph.

Robustness Tests on Number of Variety Percentiles Evolution

In the main parts of this paper, I have been looking at the Number of Variety Distribution. It is also interesting to look at how big firms and small firms behave differently in this aspect. I still define a "variety" as a unique country-product pair and measure "firm size" based on annual firm sales. In the graph presented, The graph below illustrates the number of varieties (product \times country) imported by each Swedish manufacturing firm. To facilitate comparison across firm-size percentiles, I normalize the annual number of varieties by their respective values from 1998. While the median and lower-percentile firms have experienced no change or even slight decrease in the number of imported varieties from 1998 to 2021, the largest firms have increased their imported varieties by approximately 10% to 30%. Given that the number of varieties imported directly relates to search activities, this observation suggests differential effects of search costs across firm sizes.

By sales percentile

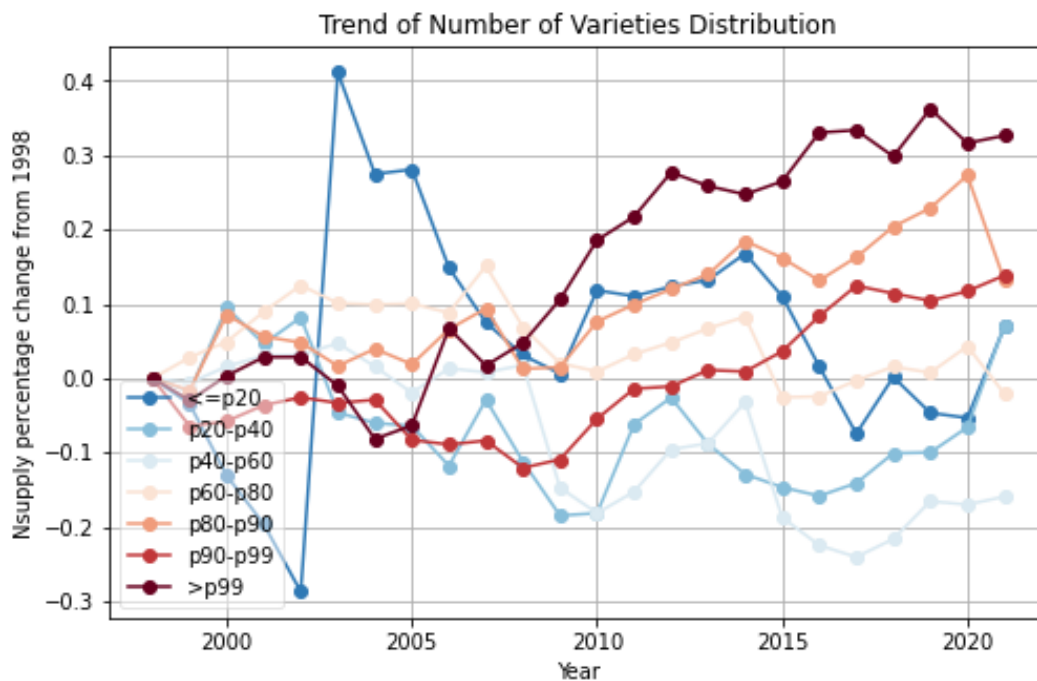


Figure 11: Number of varieties by sales percentile

Balanced panel

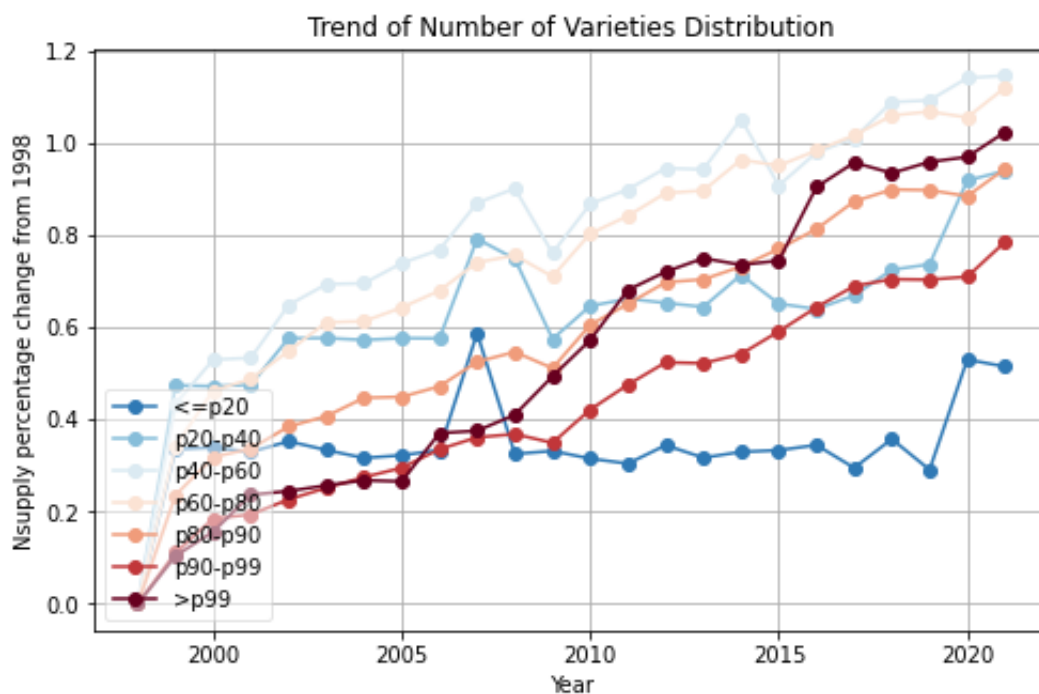


Figure 12: Number of varieties in a balanced panel

Industry by year fixed effects

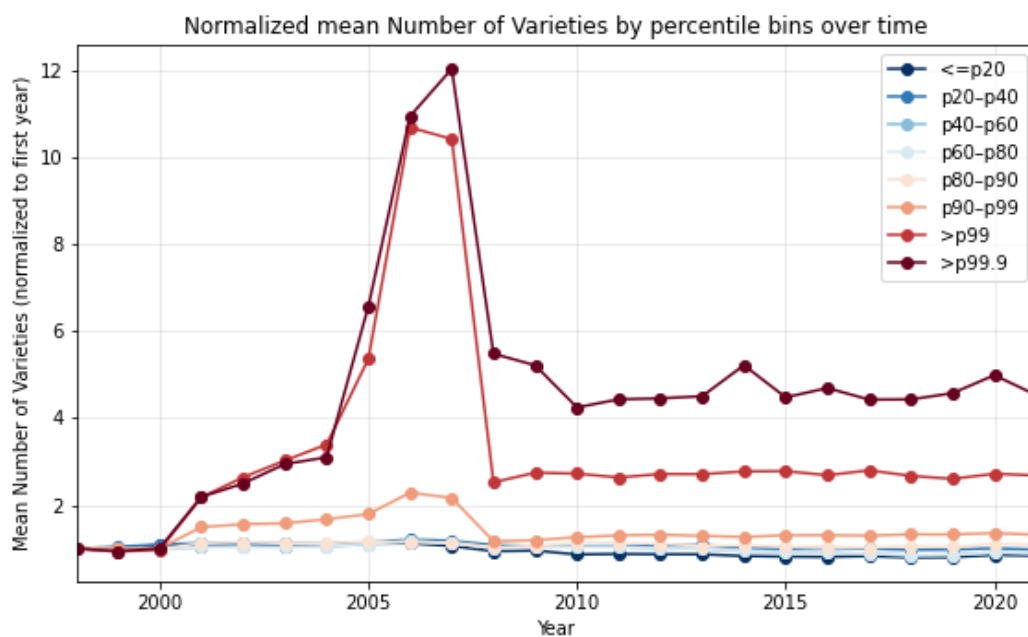


Figure 13: Number of varieties with industry by year fixed effects

Originating countries for a firm product pair

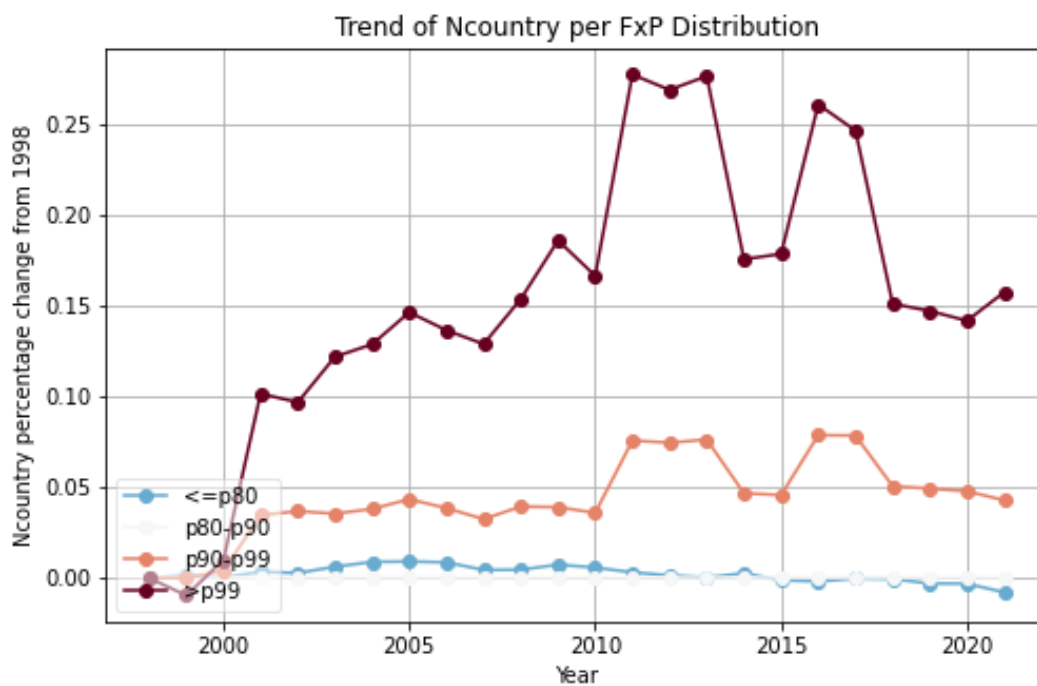


Figure 14: Number of originating countries for a firm product pair

Only non EU imports

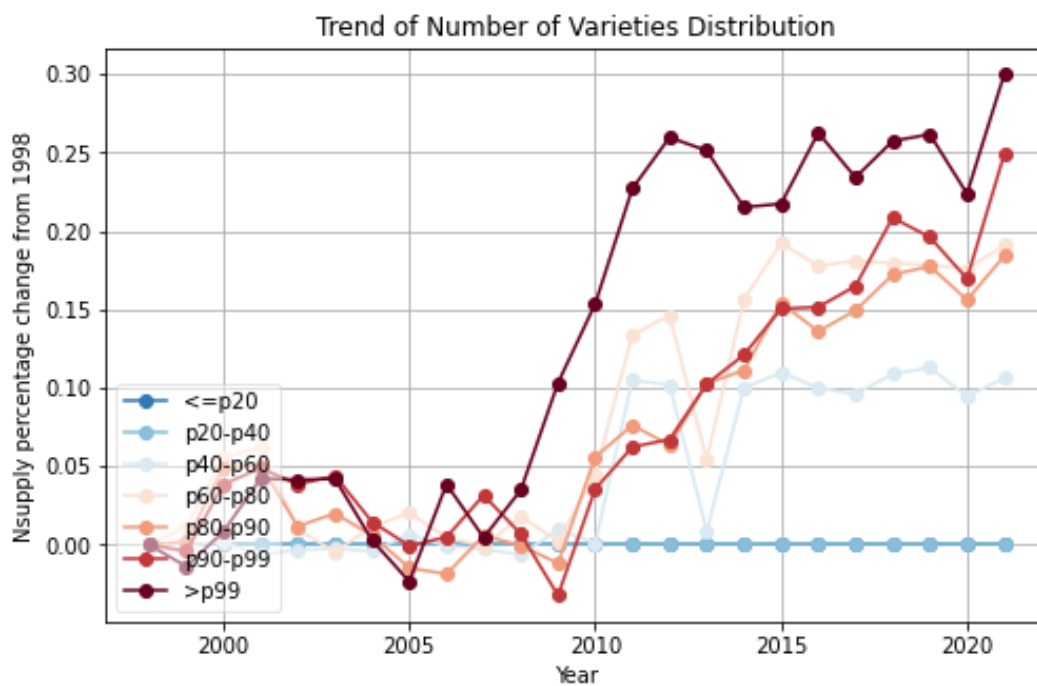


Figure 15: Number of varieties for non EU imports

Firms without foreign subsidiary

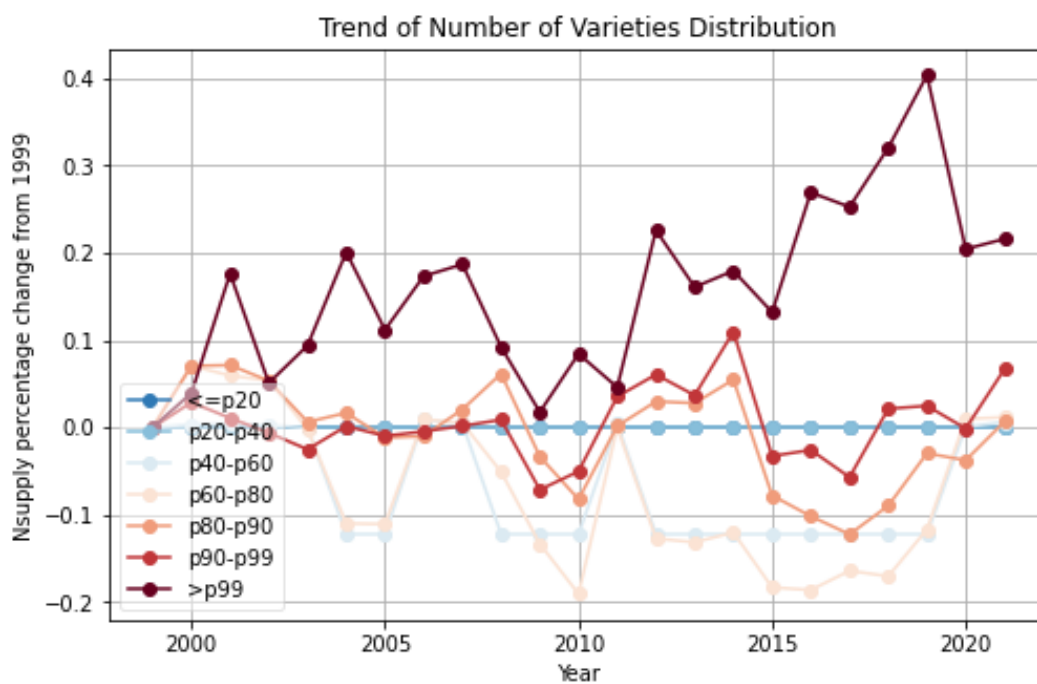


Figure 16: Number of varieties for firms without a foreign subsidiary

Data and model: balanced panel

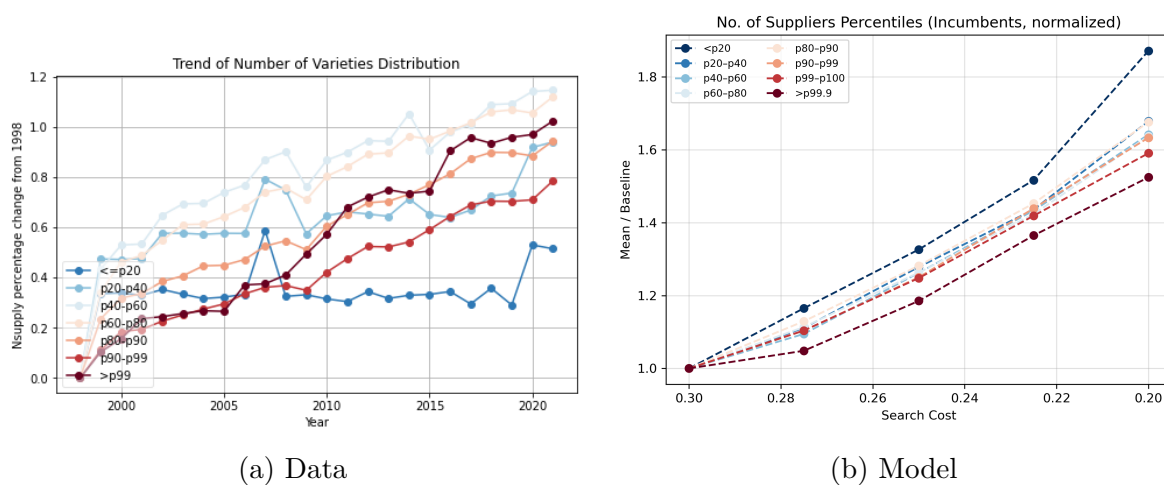


Figure 17: Number of varieties dispersion in a balanced panel

Data and model: by sales percentile

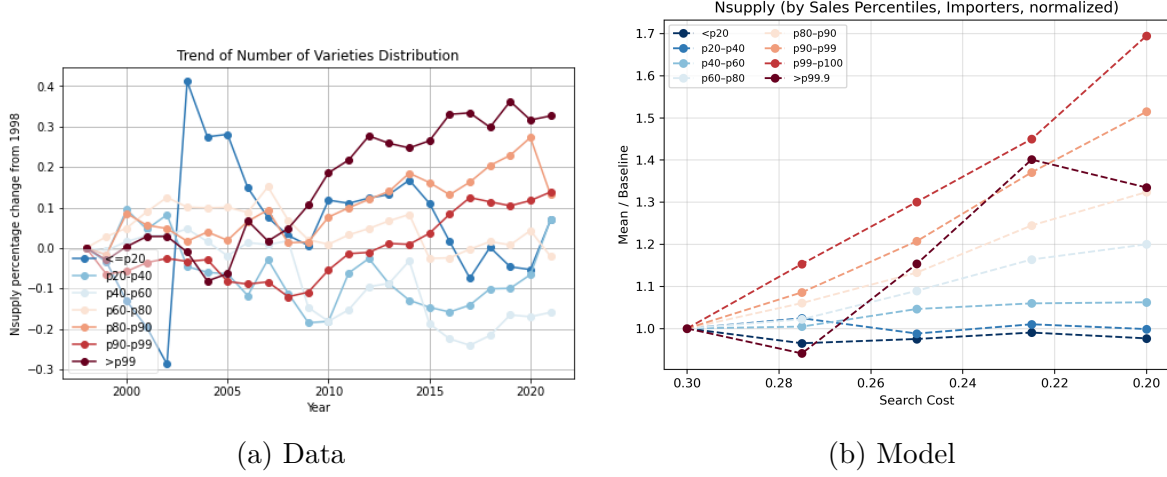


Figure 18: Number of varieties dispersion by sales percentile

Robustness Tests on Sales Percentiles Evolution

There can be alternative theories why firm size distribution is dispersing. One clear alternative is the composition of firms that are importing change due to other reasons, for example structural changes or sector-specific technological shocks. To address this, I test the following 4 robustness test. I do the same exercise to follow the evolution of firm size distribution in the following sub-sample. In the first graph I plot the distribution of the balanced panel, where the subsample is firms that exist in all periods in the dataset. We can see that in this subsample, the top firms (99th percentile) is actually leading in growth rate, while other firms have no particular order and similar growth rate, which points to the same style of dispersion in the number of variety graphs, where the top of the distribution is getting away from the rests.

However, when we look at the sample subsample, but we follow the same basket of firms every year, it tells another story.

Balanced panel

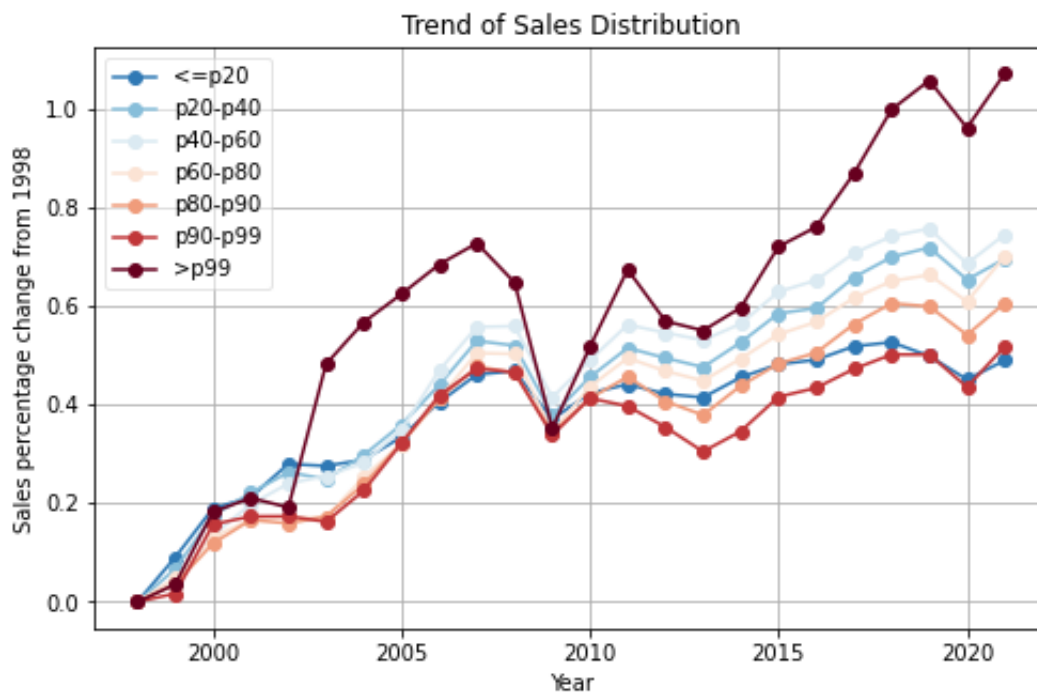


Figure 19: Sales dispersion in a balanced panel

All manufacturing

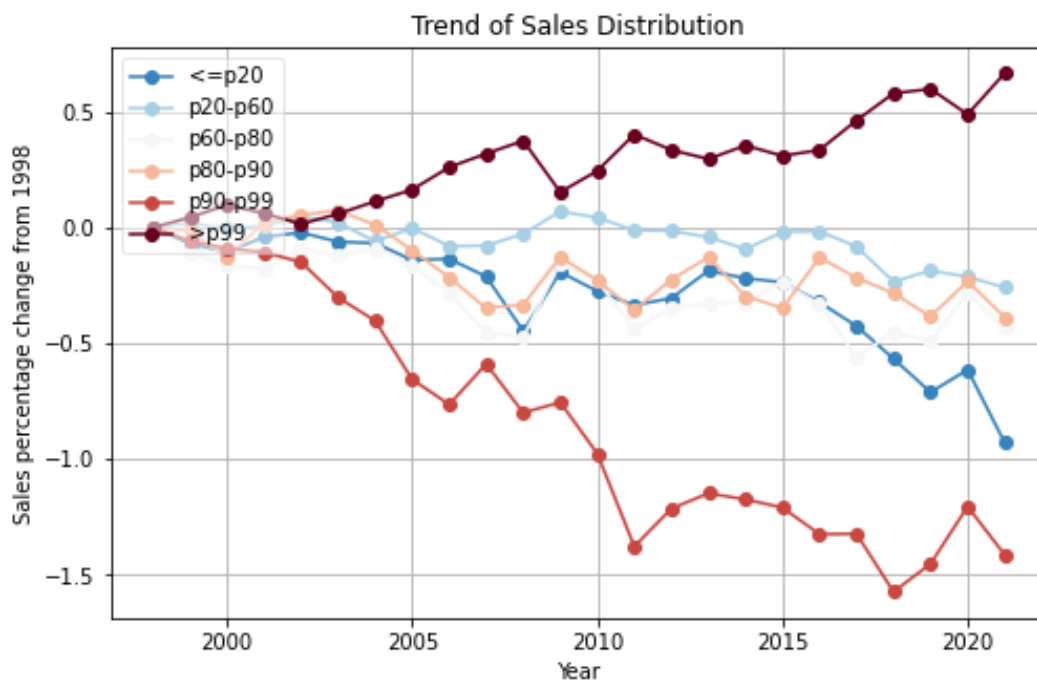


Figure 20: Sales dispersion for all manufacturing firms

Industry by year fixed effects

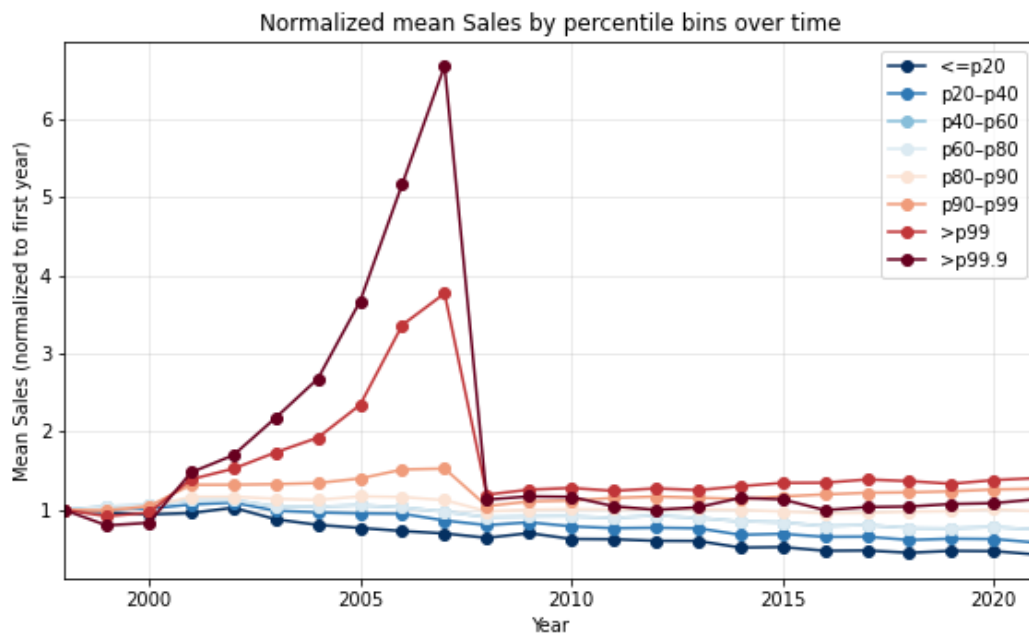


Figure 21: Sales dispersion with industry by year fixed effects

Firms without foreign subsidiary

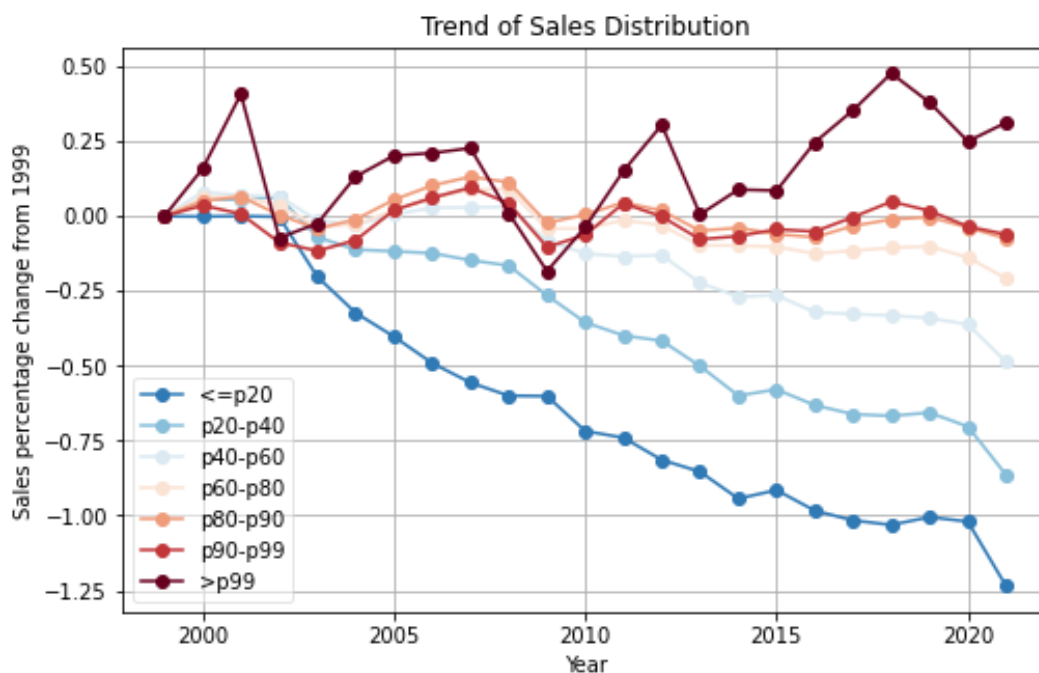


Figure 22: Sales dispersion for firms without a foreign subsidiary

Only non EU imports

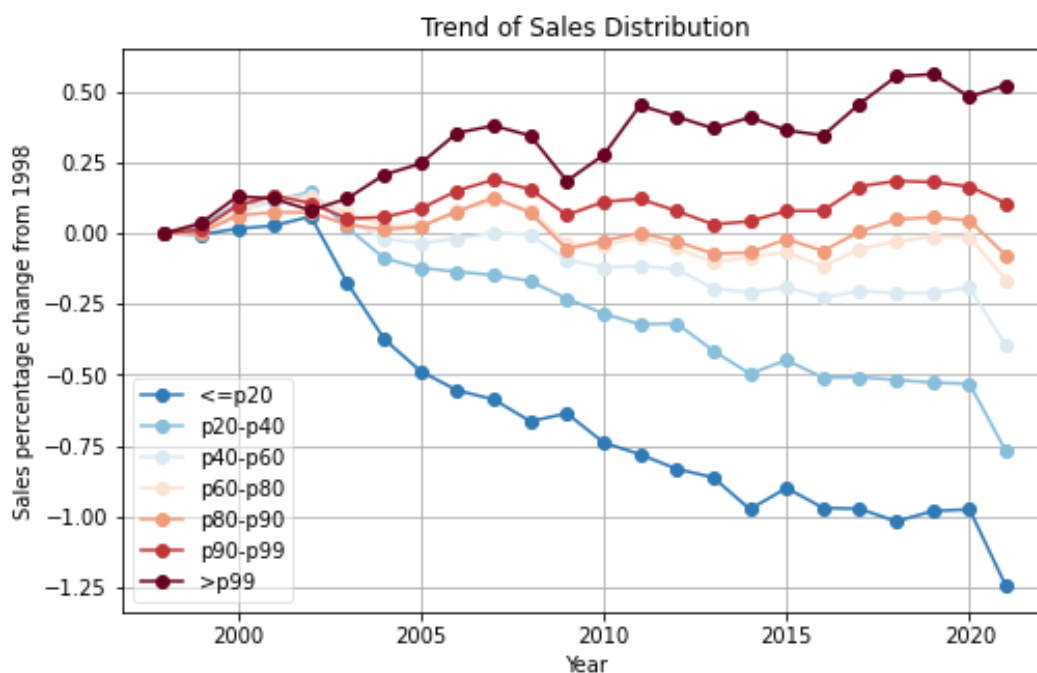


Figure 23: Sales dispersion for firms that import only from outside the EU

Data and model: balanced panel

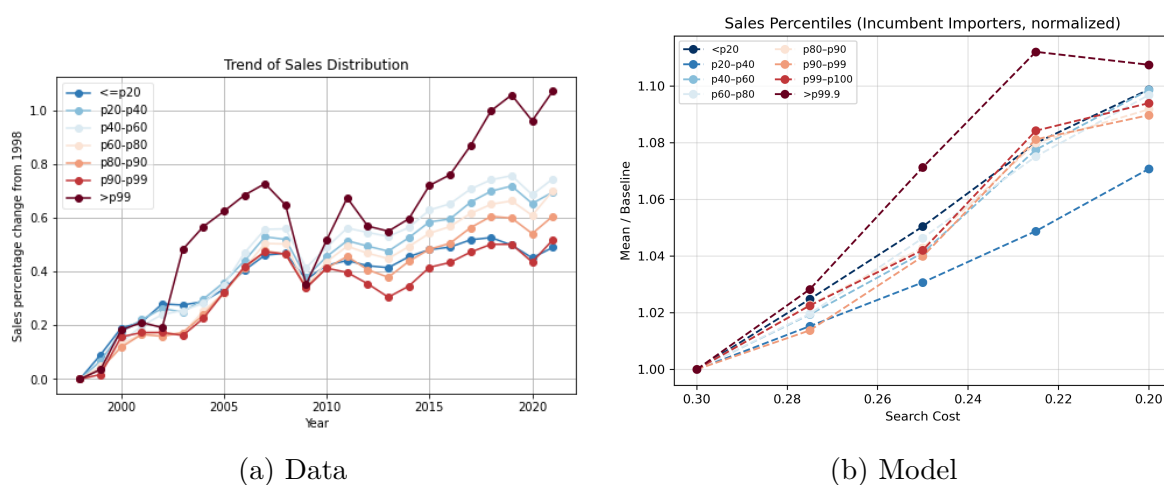


Figure 24: Sales dispersion in a balanced panel

Data and model: all firms

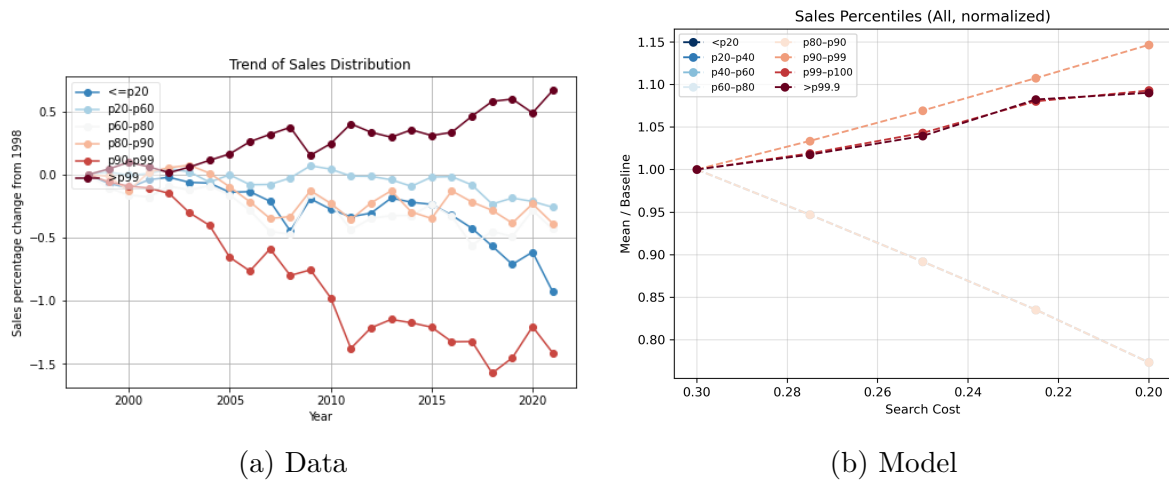


Figure 25: Sales dispersion for all firms

Robustness Tests for Fact 3

Regressing size and quantity together

Table 11: Regression results on input price

	Coefficient	(Std. error)
log number of workers	0.0048***	(0.0017)
Input quantity	-0.2790***	(0.0015)
Observations	172,602	
R^2	0.2838	
Industry fixed effects	Yes	
Time fixed effects	Yes	

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Alternative measures and other samples

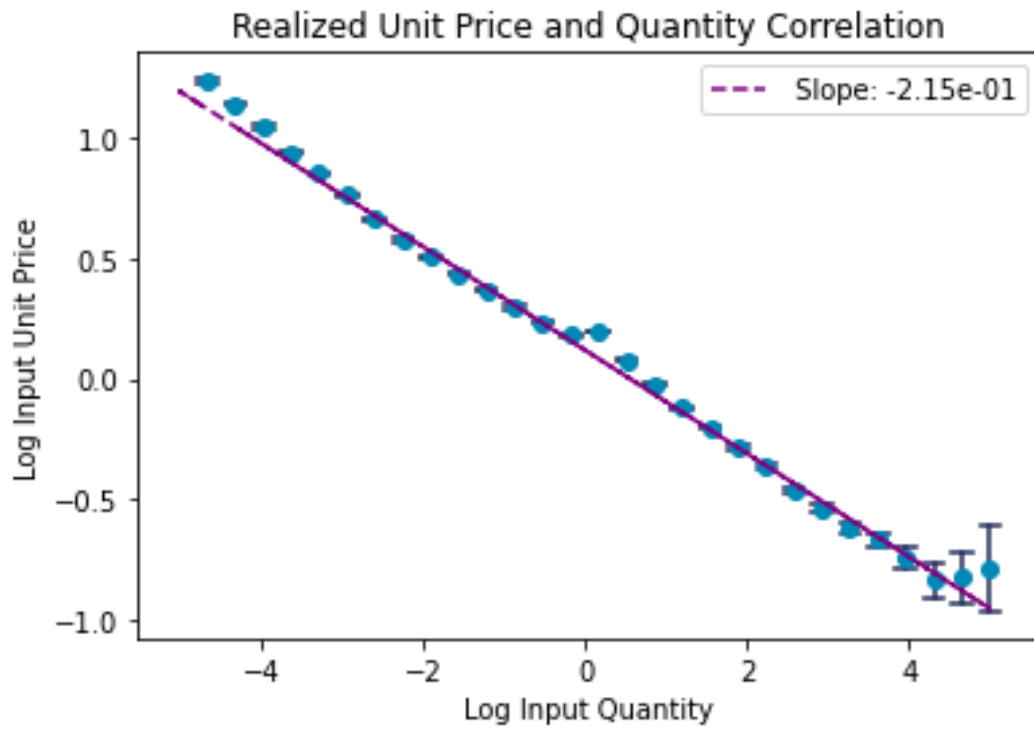
Table 12: Regression results on input price: alternative measures

	Full	non EU	Arm length
TFPQ	−0.056*** (0.003)	−0.042*** (0.004)	−0.071*** (0.005)
TFPR	−0.336*** (0.007)	−0.356*** (0.008)	−0.363*** (0.008)
Output per worker	−0.234*** (0.013)	−0.247*** (0.016)	−0.295*** (0.019)
Output per wage	−0.150*** (0.006)	−0.162*** (0.006)	−0.163*** (0.007)
Value added per worker	−0.191*** (0.012)	−0.216*** (0.015)	−0.226*** (0.022)
Sales	−0.123*** (0.001)	−0.125*** (0.001)	−0.129*** (0.002)
Employment	−0.166*** (0.002)	−0.173*** (0.002)	−0.113*** (0.004)
Total assets	−0.105*** (0.001)	−0.107*** (0.001)	−0.119*** (0.001)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Realized price and quantity relationship on transaction level

The negative correlation between price and quantity is also observed on the transaction level. It can be a result by many causes, such as non-linear pricing, market power or simply downward sloping demand curve as it is yearly data. However, it is a complement evidence to my claim in last subsection. More productive firms will in expectation buy more, therefore find more efficient suppliers and pay lower unit price.



A major concern is measurement error of quantity can mechanically create correlation between price and quantity (Deaton (1988)), as I use value and quantity to back out unit price. In the dataset I use, around 25% of observations have both variable. I assume that it is unlikely that both quantity measures have measurement error at the same time. Under this assumption, I compute unit price with one measure and regress it on the other quantity as a robustness check against such potential error. In this exercise, the shape and coefficient slightly changes, but the most important negative correlation is robust against measurement error.

Internet rollout

Municipality fiber map

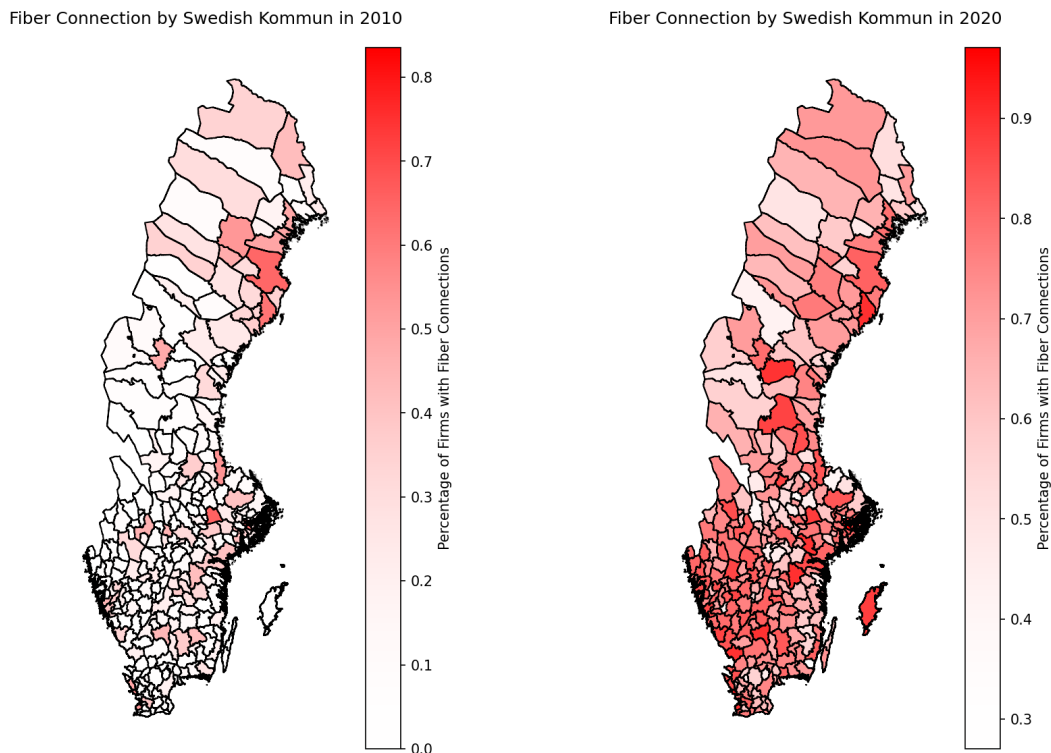


Figure 26: Municipality level fiber internet rollout

Figure 26 shows the share of firms with fiber broadband connections across Swedish kommuner in 2010 and 2020. In 2010, fiber connectivity was limited across the whole country, with relatively higher adoption in a few urban and northern regions. By 2020, coverage had expanded substantially across the country, with most municipalities showing fiber connection rates above 60–70 percent. The maps illustrate both the rapid diffusion of broadband infrastructure and the remaining regional disparities in adoption.

Broadband Rollout

From the report Ministry of Enterprise and Innovation (2016) , the Swedish government have installed a national-wide internet strategy in 2009. The goal is to promote fast internet for household, business and public services. Also, although there are profitability issue such that rural areas will be slower, the goal is to close the gap between rural and

cities eventually. Therefore, it shouldn't be expected the treatment is directly related to trade.

Other Empirical Specifications

Quality-adjusted TFPQ

In the main part of the paper, I used a direct measure of size and productivity. There are 2 potential drawbacks of that approach. First, it select a small subsample where output price are observed. These are usually bigger firms and therefore the results can be biased. Second, in my model estimation, there is heterogeneous quality across firms, but quality is omitted by using direct output price measure.

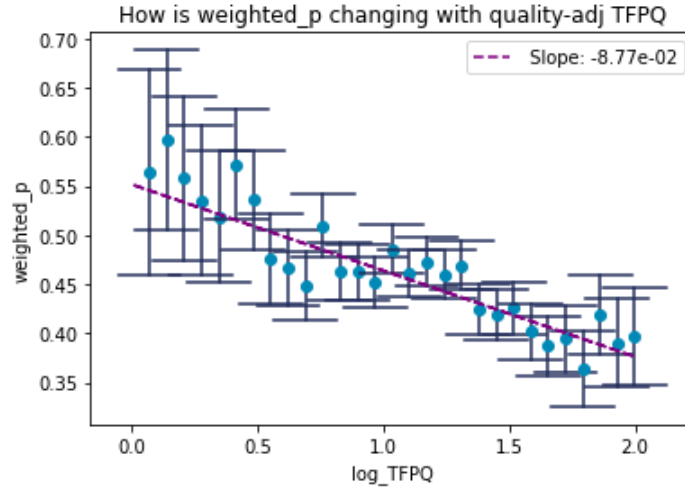
I define the revenue productivity the same way as above, but then I estimate the quality-adjusted TFPQ by:

$$\text{TFPQ}_f = \frac{p_f}{Q_f} \text{TFPR}_f$$

, where p is the price and Q is the quality of the firm's production. Unfortunately, quality Q is generally not observable. To obtain an estimate of Q , I have to rely on the theory. Assuming the monopolistic competition in final good demand, the household optimization gives:

$$\frac{p_i y_i}{p_j y_j} = \left(\frac{p_i / Q_i}{p_j / Q_j} \right)^{1-\epsilon}$$

where Sales py are observables in the data. I can then back out $\frac{p}{Q}$ by applying the standard elasticity $\epsilon = 5$. Now that I get an estimate for quality-adjusted TFPQ, without substantially reducing sample size and includes quality. I do the same analysis as above:



As predicted, the coefficient is also negative with a smaller magnitude than the non-quality adjusted version. As explained in Kugler and Verhoogen (2012), firms with higher productivity usually use higher quality inputs, which dampens the quality-adjusted coefficient.

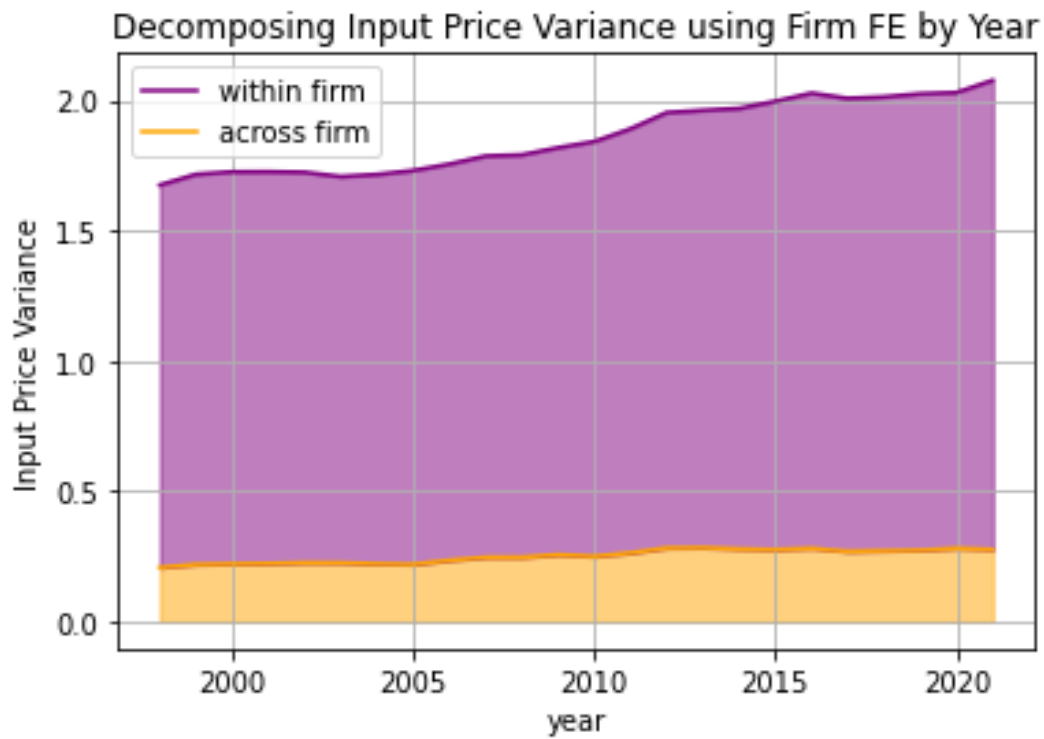
Also different aggregation level (industry digit) or including capital/not, doesn't change the result much.

Within-across firm decomposition of price

Also, note that it is not true that productive firm get a lower price consistently across all products they purchase. To illustrate that, I run a regression on price against only the firm fixed effect:

$$p_{fpcy} = 1 + FE_f$$

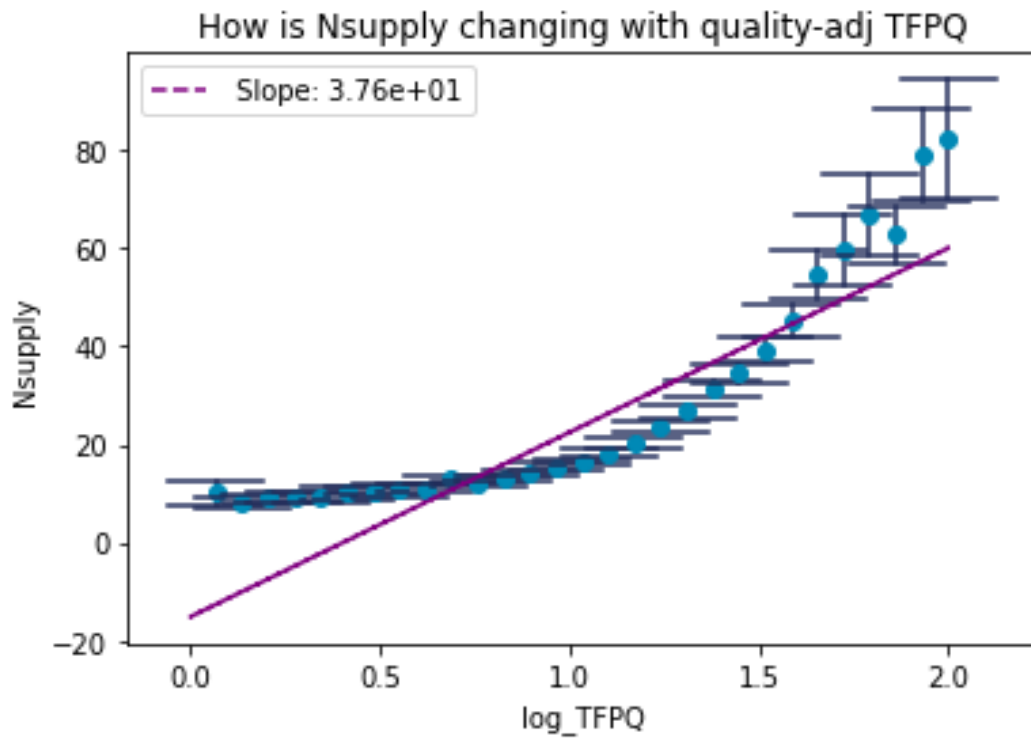
And then I decompose the variance. That part of the variance that the Fixed Effects explain is the across-firm effect and the rest is the within-firm effect. The result is rather surprising.



At odds with many model, that features firm level heterogeneity of markup or markdown, the across-firm effect is only around 15% of the total variance. It means that a same firm can purchase one input very cheaply, but another expensively.

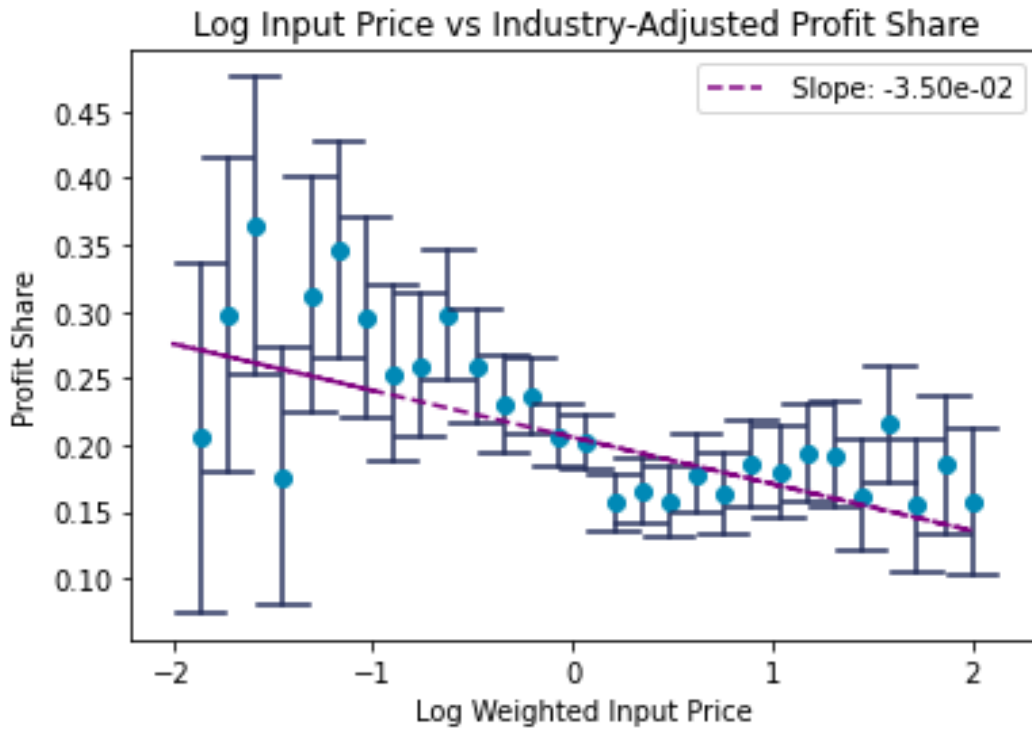
TFPQ and Variety

Another productivity-related empirical fact is that more productive firms purchase more variety. It is another piece of evidence that it is likely that productive firms also benefit from the decreasing search cost because they search more. When the productive firms expect to produce more, they also have more incentive to search for the most efficient suppliers and also have multiple of them. Where else, the less productive suppliers even they finally gain access to the market,



Input Price and Profit

Input price also directly affect the firm's profitability directly. I find a negative correlation between firm-level input price and profit share, even controlling for industry Fixed Effects. It means that the standard CES model with constant markup probably misses important features of the intermediate good market.



Quality Differences

As researchers such as Kugler and Verhoogen (2012) show that it is an important channel of input price differences, I estimate the dispersion of input price that is generated from quality differences.

I use a reduce form approach that utilizes the Rauch (1999) classification, where he separates goods into 3 classes: traded in organized exchange, traded with referenced price and differentiated goods. Organized exchange goods also include products that have an organized exchange but can also be traded decentralized. Some examples are banana, wool or other basic agriculture products. Those products, however, have very little quality differences and therefore are suitable reference goods to estimate quality differentiation.

To link the Rauch's classifications that is based on SITC Rev. 2 to the main dataset which uses CN code, I refer to the HS-SITC conversion tables provided by UNSD⁶. The CN code shares the first 6 digits with the HS code system and I assume all products that share the first 6 digits should get the same Rauch classifications. I look at the price standard deviation of the 3 groups of products:

⁶<https://unstats.un.org/unsd/classifications/Econ>

With resale	Std Dev
Organized Exchange	.9
Referenced Price	1.18
Differentiated Goods	1.44

We can see that while differentiated goods have higher standard deviation, some part of the dispersion persist also in the “organized exchange” category. This exercise shows that quality difference cannot explain all the differences in input prices.

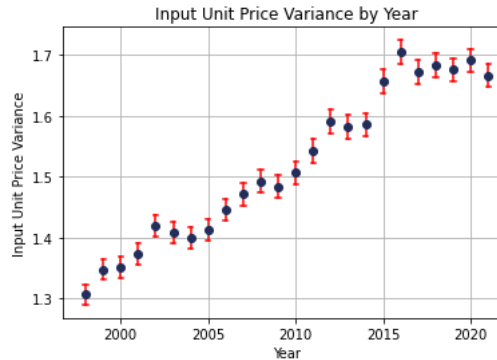
The important thing is there is no trend of quality differences year by year? As non-differentiated goods have minimal quality variation, I can define the quality gap as the price variance gap between differentiated and non-differentiated goods. Within my observed years, there are no big changes of quality gap. By weighted average, there are no change at all. Also, the composition of differentiated goods as a fraction of all imported goods is stable at 74% ~ 76%. One way or the other, it is not sufficient to explain the price variance increase.

Decreasing Search Cost for Suppliers and Input Price Dispersion

Related to the decreasing search cost and the influx of smaller firms into the import market is the input price dispersion. As the distribution of importer size become more disperse, variance of input price is also increasing. I define input price dispersion as:

$$\text{Var}(P) = \text{Var}(\log(\frac{P_{fvy}}{P_{vy}}))$$

It is also robust when I limit the analysis to within-EU import or imports from most of Sweden’s major trading partner.



One potential link between the two dispersing distribution is that input unit price is inversely correlated to firm size. When there are a lot of small firms goes into the import

market, they buy very little and they are not familiar with the market, therefore they will pay a relatively high price. On the other hand, big firms become bigger, gain market power and face a even lower price. In this case, the price gap will grow. I will present more evidence below.

It is also important to note that quality differences is likely not the reason. I measure the price variance driven by quality difference by comparing price variance within differentiated products and non-differentiated ones, as defined by Rauch (1999). It is a very conservative measure that likely exaggerate the important because this gap likely include other channel such as bargaining and information frictions. But what is important is that the quality gap hasn't been increasing during the period, neither does the composition of differentiated products in all products. See more in Appendix.

Background Volatility

To check the how much of the price differences are unrelated to buyer characteristic, I look at currency volatility as one representative background noise. I construct a currency-year volatility index (against Swedish Krona) using data from the Swedish Riksbank. I then linked the currency volatility to the volatility.

I then run a regression against the variance of price:

$$\text{Var}(\log(\frac{P_{pc}}{P_p})) \sim \text{volatility}_{cy}$$

I find very little relationship between the 2 variables.

Theory Appendix

Simplified Benchmark - Single Supplier Case

In this section, I examine another simplified model where firms can only have one supplier (variety), no quality differentiation and both upstream and downstream productivity follows a simple $U(0, 1)$ distribution. This model resembles a familiar model of McCall (1970) random job search model. With this class of model, I can solve the optimal stopping problem by getting the reservation productivity $z_u^R(z)$ and z^R . I can get an idea how search cost κ affect firm size distribution from a fixed point solution.

The reservation supplier productivity is solved by 2 equations. The first equation is the definition of the outside option D :

$$D = \int_{z_u^R}^{\bar{z}_u} [\pi(z_u) - T(z_u)] dF(z_u) + \int_{z_u}^{z_u^R} D dF(z_u) - w\kappa$$

which consists of 3 terms: The first one is expected profit given the probability of meeting a satisfactory supplier ($z_u \geq z_u^R$), the second one is falling back to outside option if the supplier is worse than the reservation productivity and the last term is minus the search cost.

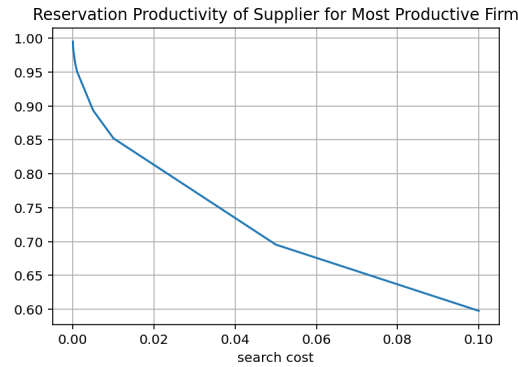
The second equation is defined by the indifference condition, where for any buyer, she should be indifferent between the profit for meeting the reserved supplier and going into outside option:

$$\pi(z, z_u^R) - T(z, z_u^R) = D$$

Combining the equations we get a expression for solving z_u^R for each z :

$$\frac{\kappa}{Kz^{\epsilon-1}} = [1 + \alpha(\epsilon - 1)z_u^{R[\alpha(\epsilon-1)+1]} - (\alpha(\epsilon - 1) + 1)z_u^{R\alpha(\epsilon-1)}]$$

This expression provides a relationship between z_u^R and κ for any firm with productivity z . And solving this fixed point formula gives:

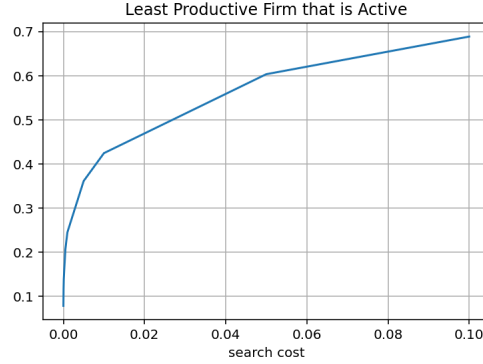


This shows that when search cost decrease, the reservation productivity, given z , is increasing. In other words, when search cost decrease, the expected productivity of the supplier increase. It is obvious that matching with a more productivity supplier means producing more. Therefore, downstream firm produce more when search cost decrease. It showcase, in this restrictive model, search cost also functions as expansion cost.

On top of that, I can show that the lowest productivity z^R where this downstream firm still remain active can be calculated by solving:

$$\frac{\kappa}{1 - F[z_u^R(z^R)]} = \int_{z_u^R(z^R)}^{\bar{z}_u} [\pi(z_u, z^R) - T(z_u, z^R)] dF(z_u)$$

And keeping everything but search cost κ constant, z^R increase with κ :



It shows that when search cost decrease, less productive firm can also enter the intermediate good market and produce. This showcase search cost can act as entry barrier.

Sequential search with efficient bargaining

This appendix solves the sequential version of the buyer's problem with identical upstream productivity and constant search cost. Upstream unit cost is fixed at $c = 1/\bar{z}_u$. At any state (z, X) the buyer meets a new supplier and bargains efficiently over x_i . After contracting, total input becomes

$$X_{\text{new}} = (X^\rho + x_i^\rho)^{1/\rho}.$$

Define

$$A \equiv \frac{\epsilon - 1}{1 + \alpha(\epsilon - 1)}, \quad K_1 \equiv [1 + \alpha(\epsilon - 1)] \epsilon^{-\frac{\epsilon}{1 + \alpha(\epsilon - 1)}} \left(\frac{w}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{(1 - \alpha)(1 - \epsilon)}{1 + \alpha(\epsilon - 1)}} \left(\frac{C}{P - \epsilon} \right)^{\frac{1}{1 + \alpha(\epsilon - 1)}}.$$

In the main text the value function $V(z, X)$ is sequential and includes both the direct production benefit and the continuation value from future supplier matches. For the derivation it is convenient to write

$$V(z, X) = K_1 [zX^\alpha]^A + W(z, X),$$

so that

$$V(z, X_{\text{new}}) - V(z, X) = K_1 \left([zX_{\text{new}}^\alpha]^A - [zX^\alpha]^A \right) + [W(z, X_{\text{new}}) - W(z, X)].$$

Per match bargaining problem At (z, X) the efficient surplus from contracting x_i is

$$\max_{x_i \geq 0} \underbrace{K_1 [z(X^\rho + x_i^\rho)^{\alpha/\rho}]^A - K_1 [zX^\alpha]^A}_{\text{direct benefit}} + \underbrace{W(z, (X^\rho + x_i^\rho)^{1/\rho}) - W(z, X)}_{\text{continuation effect}} - c x_i.$$

Let $x_i^*(z, X)$ denote the maximizer of this problem and write $x^* \equiv x_i^*(z, X^*)$ at the stopping boundary introduced below. The buyer's surplus from the match is

$$G(z, X) = \theta \left\{ K_1 [z(X^\rho + x_i^\rho)^{\alpha/\rho}]^A - K_1 [zX^\alpha]^A + W(z, (X^\rho + x_i^\rho)^{1/\rho}) - W(z, X) - c x_i \right\}.$$

First order condition Let $X_{\text{new}} = (X^\rho + x_i^\rho)^{1/\rho}$. The first order condition with respect to x_i is

$$\left[\alpha A K_1 z^A (X^\rho + x_i^\rho)^{\frac{\alpha A}{\rho} - 1} + W_2(z, X_{\text{new}}) (X^\rho + x_i^\rho)^{\frac{1}{\rho} - 1} \right] x_i^{\rho-1} = c,$$

or, equivalently,

$$\left[\alpha A K_1 z^A X_{\text{new}}^{\alpha A - 1} + W_2(z, X_{\text{new}}) \right] x_i^{\rho-1} X_{\text{new}}^{1-\rho} = c.$$

Search rule and stopping boundary Search continues while $G(z, X) > w\kappa$. The endogenous stopping boundary $X^*(z)$ satisfies

$$G(z, X^*) = w\kappa.$$

At X^* the continuation effect from the marginal match vanishes, $W_2(z, X_{\text{new}}) = 0$, and the first order condition reduces to

$$\alpha A K_1 z^A (X^{*\rho} + x_i^{*\rho})^{\frac{\alpha A}{\rho} - 1} x_i^{*\rho-1} = c.$$

Solving in ratios Define $r \equiv x^*/X^*$ and note $X_{\text{new}} = X^*(1 + r^\rho)^{1/\rho}$. The first order condition implies

$$X^{*\alpha A-1} = \frac{c}{\alpha A K_1 z^A (1 + r^\rho)^{\frac{\alpha A - \rho}{\rho}} r^{\rho-1}}, \quad X^* = \left[\frac{c}{\alpha A K_1 z^A (1 + r^\rho)^{\frac{\alpha A - \rho}{\rho}} r^{\rho-1}} \right]^{\frac{1}{\alpha A - 1}}.$$

At the boundary, the surplus condition becomes

$$K_1 \left[z (X^{*\rho} + x_i^{*\rho})^{\alpha/\rho} \right]^A - K_1 (z X^{*\alpha})^A - c x^* = \frac{w\kappa}{\theta},$$

which, after substituting $r = x^*/X^*$ and the expression for $X^{*\alpha A-1}$, reduces to

$$X^* c \left[\frac{(1 + r^\rho) - (1 + r^\rho)^{1 - \frac{\alpha A}{\rho}}}{\alpha A r^{\rho-1}} - r \right] = \frac{w\kappa}{\theta}.$$

Solving this nonlinear equation for r yields $X^*(z)$ and $x^*(z) = r X^*(z)$ in closed form, since X^* is explicit in r above. The model can then be solved recursively by backward induction starting from the stopping boundary.

Closed Form Solution for Important Variables

In the baseline model with imported inputs, the firm's state is represented by the pair (z, X) . The variables required for the value function iteration, which are the optimal price $p(z, X)$, output $y(z, X)$, and profits $\pi(z, X)$, can all be expressed as functions of this state. The problem is therefore fully characterized by the pair (z, X) .

Output price and output.

$$\max_{p, y} py - w \left(\frac{y}{z X^\alpha} \right)^{\frac{1}{1-\alpha}} \quad \text{s.t.} \quad y = C \left(\frac{p}{P} \right)^{-\epsilon}$$

Using the constraint to eliminate y :

$$\max_p p^{1-\epsilon} \frac{C}{P^{-\epsilon}} - w p^{\frac{\epsilon}{\alpha-1}} \left(\frac{\frac{C}{P^{-\epsilon}}}{z X^\alpha} \right)^{\frac{1}{1-\alpha}}$$

FOC in p :

$$(1 - \epsilon) \frac{C}{P^{-\epsilon}} p^{-\epsilon} = \frac{w\epsilon}{\alpha - 1} p^{\frac{\epsilon - \alpha + 1}{\alpha - 1}} \left(\frac{C}{zX^\alpha} \right)^{\frac{1}{1 - \alpha}}$$

Solve for p :

$$p^{\frac{1 + \alpha(\epsilon - 1)}{1 - \alpha}} = \frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \left(\frac{C}{P^{-\epsilon}} \right)^{\frac{\alpha}{1 - \alpha}} (zX^\alpha)^{-\frac{1}{1 - \alpha}},$$

$$p = \left(\frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{1 - \alpha}{1 + \alpha(\epsilon - 1)}} \left(\frac{C}{P^{-\epsilon}} \right)^{\frac{\alpha}{1 + \alpha(\epsilon - 1)}} (zX^\alpha)^{-\frac{1}{1 + \alpha(\epsilon - 1)}}$$

Then y :

$$y = \frac{C}{P^{-\epsilon}} p^{-\epsilon}$$

$$= \left(\frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{-\epsilon(1 - \alpha)}{1 + \alpha(\epsilon - 1)}} \left(\frac{C}{P^{-\epsilon}} \right)^{\frac{1 - \alpha}{1 + \alpha(\epsilon - 1)}} (zX^\alpha)^{\frac{\epsilon}{1 + \alpha(\epsilon - 1)}}$$

Profit function.

$$\pi(z, X) = py - w \left(\frac{y}{zX^\alpha} \right)^{\frac{1}{1 - \alpha}}$$

$$= \frac{C}{P^{-\epsilon}} p^{1 - \epsilon} - w \left(\frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{-\epsilon}{1 + \alpha(\epsilon - 1)}} \left(\frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1 + \alpha(\epsilon - 1)}} (zX^\alpha)^{\frac{\epsilon - 1 - \alpha(\epsilon - 1)}{1 + \alpha(\epsilon - 1)} \cdot \frac{1}{1 - \alpha}}$$

$$= \left\{ \left(\frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{(1 - \alpha)(1 - \epsilon)}{1 + \alpha(\epsilon - 1)}} - w \left(\frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{-\epsilon}{1 + \alpha(\epsilon - 1)}} \right\} \left(\frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1 + \alpha(\epsilon - 1)}} (zX^\alpha)^{\frac{\epsilon - 1}{1 + \alpha(\epsilon - 1)}}$$

$$= \left[\frac{1 + \alpha(\epsilon - 1)}{\epsilon} \right] \left(\frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{(1 - \alpha)(1 - \epsilon)}{1 + \alpha(\epsilon - 1)}} \left(\frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1 + \alpha(\epsilon - 1)}} (zX^\alpha)^{\frac{\epsilon - 1}{1 + \alpha(\epsilon - 1)}}$$

Note that the profit function π is also the not-search value V^{NS}

Labor cost.

$$w \left(\frac{y}{zX^\alpha} \right)^{\frac{1}{1 - \alpha}} = \left(\frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1 + \alpha(\epsilon - 1)}} w \left(\frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{-\epsilon}{1 + \alpha(\epsilon - 1)}} (zX^\alpha)^{\frac{\epsilon - 1}{1 + \alpha(\epsilon - 1)}}$$

$$= \frac{(\epsilon - 1)(1 - \alpha)}{\epsilon} py \quad \text{or} \quad \left[\frac{(\epsilon - 1)(1 - \alpha)}{1 + \alpha(\epsilon - 1)} \right] \pi$$

Concavity of the Bargaining Problem

Given all possible x , I can use the first-order condition (FOC) to find T , because the following function is strictly concave (as long as it is defined). To simplify notation, I define

$$V(x) := V(z, X_N) - V(z, X), \quad K(x) := \frac{w}{z_u}x.$$

Consider

$$B(\delta T_1 + (1 - \delta)T_2) = \left(V(x) - \delta T_1 - (1 - \delta)T_2 \right)^\theta \left(\delta T_1 + (1 - \delta)T_2 - K(x) \right)^{1-\theta}.$$

Taking $\exp(\log(\cdot))$ of both sides (which leaves the expression unchanged), I only show the logged terms for readability:

$$\log B(\delta T_1 + (1 - \delta)T_2) = \theta \log(V(x) - \delta T_1 - (1 - \delta)T_2) + (1 - \theta) \log(\delta T_1 + (1 - \delta)T_2 - K(x)).$$

Because the logarithm is strictly concave, I have

$$\begin{aligned} \log B(\delta T_1 + (1 - \delta)T_2) &> \theta [\log(\delta(V(x) - T_1)) + \log((1 - \delta)(V(x) - T_2))] \\ &\quad + (1 - \theta) [\log(\delta(T_1 - K(x))) + \log((1 - \delta)(T_2 - K(x)))] . \end{aligned}$$

Combining terms,

$$\begin{aligned} \log B(\delta T_1 + (1 - \delta)T_2) &> \log \left(\delta^\theta (V(x) - T_1)^\theta \delta^{1-\theta} (T_1 - K(x))^{1-\theta} \right) \\ &\quad + \log \left((1 - \delta)^\theta (V(x) - T_2)^\theta (1 - \delta)^{1-\theta} (T_2 - K(x))^{1-\theta} \right). \end{aligned}$$

Taking the exponential of both sides gives

$$B(\delta T_1 + (1 - \delta)T_2) > \delta B(T_1) + (1 - \delta)B(T_2).$$

which shows that B is strictly concave. Since \log is strictly concave, B is also strictly concave. As long as $V(x) \geq K(x)$, the solution exists and is unique. Therefore, I can define a function *Bargain*, which gives the best value given x , and then find $\arg \max_x \text{Bargain}$.

Proof. Let $T(x^o) = \arg \max_T B(x^o, T)$, which is unique. Assume (x^*, T^*) is the maximizer of B such that $B(x^*, T^*) \geq B(x', T')$ for all (x', T') . If $T^* \neq T(x^*)$, then

$B(x^*, T^*) \geq B(x^*, T(x^*))$, which contradicts the uniqueness of $T(x^*)$. Therefore, the solution is unique.

I can then obtain *Bargain* (the one-dimensional version of the bargaining problem) by substituting the FOC into B :

$$\begin{aligned}
\text{Bargain}(x) &= (-T + V(z, X_N) - V(z, X))^\theta (T - w_{z_u} \frac{x}{z_u})^{1-\theta} \\
&= \left(-(1-\theta)V(x) - \theta K(x) + V(x) \right)^\theta \left((1-\theta)V(x) + \theta K(x) - K(x) \right)^{1-\theta} \\
&= \left(\theta V(x) - \theta K(x) \right)^\theta \left((1-\theta)V(x) - (1-\theta)K(x) \right)^{1-\theta} \\
&= \theta^\theta (1-\theta)^{1-\theta} (V(x) - K(x)) \\
&= \theta^\theta (1-\theta)^{1-\theta} \left(V(z, X^N Q^N, n+1) - V(z, XQ, n+1) - w_{z_u} \frac{x}{z_u} \right).
\end{aligned}$$