Supplier Search and Market Concentration

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Abstract

This paper examines how lower search costs in input markets reshape the distribution of firm sizes. Using Swedish administrative data, I document four facts: (i) more productive firms pay lower input prices, (ii) the dispersion in imported varieties across firms is widening, (iii) the sales distribution among importers has become more unequal, and (iv) municipalities with greater fiber-optic coverage experience faster supplier network growth, especially among more productive firms. To interpret these patterns, I develop a quantitative model of monopolistic competition with frictional input markets, in which firms incur fixed costs to search and bargain with suppliers. The model shows that falling search costs reallocate resources toward importers, raising aggregate productivity but also increasing market concentration. A counterfactual exercise indicates that a 10 percent tariff on inputs would largely offset these gains.

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1 Introduction

Advances in communication and transportation technologies have made it much easier for firms to expand their supplier networks internationally. For example, online meetings and significantly cheaper flights, which were not widely available two decades ago, now allow firms to connect with suppliers located thousands of kilometers away from their physical location. This development holds for both small and large firms. Smaller firms can now enter international supply chains that were previously too costly to access, while larger firms that were already importing can expand their supplier networks more easily. Such technological advancements benefit firms of all sizes, but are the benefits equally distributed? If not, how do they reshape the distribution of firm sizes and market concentration?

In this paper, I study the question: How does reducing search costs in input markets reshape market concentration? There is broad consensus that trade liberalization affects the firm size distribution. Notably, increasing market concentration is partly explained by intensified import competition (Amiti and Heise (2025)), where foreign products out-compete less productive domestic firms. Most of this literature focuses on imports of final goods. Yet, in major economies, a large share of imports consists of intermediate inputs. For instance, an OECD report documents that 56% of global goods trade is in intermediate goods[1]. In 2023, 60.5% of EU imports were intermediate goods, compared with only 17.3% for consumption goods[2]. Importing final outputs directly competes with domestic varieties, generating effects similar to a demand shock. By contrast, importing inputs alters the measured productivity of domestic firms, more akin to a productivity shock.

To investigate this mechanism, I use Swedish administrative data to document four key empirical facts about importing manufacturing firms. First, I find that more productive firms pay lower input prices. Second, there is wide dispersion in the number of imported varieties across firms. While the lower end of the distribution remains stable, the upper end has been pulling away over time. Third, I show that the sales distribution among Swedish importing firms has also become increasingly dispersed over the last two decades. Fourth, I use roll-out of high-speed internet as a proxy for decreasing search cost and find that firms located in municipalities that is more fiber-ready grow their supplier network around 7% quicker and firms that are productive will grow on top of that 19% quicker.

To interpret these facts, I develop a quantitative model that embeds a frictional intermediate-inputs market within a canonical monopolistic competition framework. In the input market, heterogeneous domestic firms must pay a fixed search cost to acquire inputs from foreign suppliers. I compare equilibria under different levels of search costs. The model generates predictions on how search costs affect the distribution of suppliers, sales, and productivity across firms. I then map simulated model

[1] https://www.oecd.org/en/publications/trade-in-intermediate-goods-and-services_5kmlcxtdlk8r-en.html

[2] https://ec.europa.eu/eurostat/statistics-explained/index.php?title=International_trade_in_goods_by_type_of_good

moments to the empirical patterns observed in the administrative data. Through the lens of this model, I identify search costs in input markets as a channel that helps explain increasing market concentration and derive relevant policy implications.

The model highlights how reducing search costs benefits firms asymmetrically. Firms that were already importing expand their networks further, while firms at the margin gain access to international supply chains that were previously out of reach. However, for firms that never engage in supplier searches, lower search costs have little effect. In general equilibrium, these non-importing firms become relatively less productive compared to their importing counterparts, leading to declining sales and profits. I find that reducing search cost can increase both market concentration rises aggregate domestic consumption.

I also conduct a counterfactual exercise in which the government imposes a uniform tariff on all imported intermediate inputs. The results show that a 10% tariff is sufficient to reverse both the output gains and the market concentration effects associated with the decline in search costs over the past 20 years.

This paper contributes to the literature in three ways.

First, I uncover the role of search costs in input markets as a determinant of market concentration. In doing so, I complement existing studies that emphasize other drivers of concentration, such as rising entry barriers (Covarrubias et al. (2020)), intensified import competition (Amiti and Heise (2025)), or falling overhead costs due to advances in information technology (Aghion et al. (2023)).

Second, my paper relates closely to the growing literature on firm-to-firm trade. Using Swedish import data, I document increasing dispersion in both input prices and the number of imported varieties. Substantial input price dispersion has been documented in other contexts as well (Atalay (2014)). Explanations include buyer market power (Morlacco (2020); Rubens (2023)), quality differences (Kugler and Verhoogen (2012)), and match-specific frictions (Burstein et al. (2024)). I summarize these potential channels, document correlations between price gaps and firm characteristics, and highlight additional empirical regularities.

Third, I show that importing intermediate goods can increase markup distortions, contrasting with the findings of Edmond et al. (2015), who show the opposite effect for imports of consumer goods. This places my work within the misallocation literature pioneered by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), and further developed by Dhingra and Morrow (2019). Specifically, I study how search costs in supply chains affect allocative efficiency for downstream firms. My theoretical framework, which models search and bargaining between buyers and suppliers, generates endogenous price gaps and allows me to assess their impact on aggregate output. This introduces a novel mechanism, whereby the wedge depends directly on firms' productivity.

The remainder of the paper is organized as follows. Section 2 describes the data and documents the four facts. Section 3 develops the model. Section 4 maps model predictions to the data and quantifies the impact of falling search costs on productivity and concentration. Section 5 presents the tariff counterfactual. Section 6 concludes.

2 Data and Stylized Facts

My theorectical framework can be used to study general supply chain structures, domestic and international ones alike, but I limit the empirical analysis to international imports of Swedish manufacturing firms. Customs data in Sweden is comprehensive It includes import records on the firm, year, country of shipment and HS8 product level during the period 1998-2021. Importantly, there is also

I mainly use Swedish administrative data provided by the Statistic Sweden. The main data block is the import part of the *Utrikeshandel med varor*, Foreign Trade in Goods dataset. For example, if a company import german cars in 2018 from two different suppliers, I can only see one aggregated record.

It includes all imports records during the period 1998-2021 for imports from outside of EU (Switzerland or EEA countries included) and the intra-EU importing records of companies which import at least SEK 9.0 million worth of goods(See SCB (2018) for more). Each record entry includes the importer ID, origin country of the purchase, value, weight and the 8 digit level product code of the imported product. The customs data also includes, for a subset of products, an additional variable "Other Quantities" (e.g. pieces for pencils or m^2 for curtains in 2024 etc.)[3]. I also use firm level balance sheet data to obtain importers' characteristics, such as sales, wage bill and productivity etc.

- Imported inputs are very important to the Swedish economy, I use the Trade in Value Added (TiVA) 2023 edition , year 2020 dataset provided by OECD and calculate 2 statistics - percentage of Foreign Value Added in Swedish manufactured goods, and the percentage of intermediates in import. The domestic value added in Swedish manufactured product: 69%, which means 31% is foreign value added. Also, around 36% of all imports are intermediate goods. Both figures show that imported inputs are non-trivial for Sweden.

Each entry includes value and quantity of the imported product. I define the product-country pair *pc* as an variety. And I define price as the log residualized price

$$P = log(\frac{P_{fvy}}{\bar{P_{vy}}})$$

where the numerator is the deflated value over quantity of one data entry and the denominator is the within-variety-year weighted-average price

$$\bar{P_{pcy}} = \frac{\sum_{pcy} \text{Deflated Value}}{\sum_{pcy} \text{Quantity}}$$

I deflate the value using aggregate CPI data to adjust for inflation.

[3]See the subset of products in the Övrigt om varukoder section on https://www.scb.se/lamna-uppgifter/undersokningar/intrastat-in--och-utforsel-av-varor/varukoder-kn-for-uppgiftslamnande-till-intrastat/

For some analysis, I construct a firm-level price index, which is the weighted average across all variety the firm buy within a year:

$$\frac{\sum \text{Price} * \text{Deflated Value}}{\sum_f \text{Deflated Value}}$$

For example, if a company buy 10000SEK worth of german cars at 20% higher unit price than average german cars buyer and 3000SEK worth of Chinese headphones at 30% lower price than average. Then, their firm level price index is:

$$\frac{1.2 * 10000 + .7 * 3000}{10000 + 3000}$$

I did basic data cleaning by excluding observations where the total value is below 100 SEK (around 9USD). I also exclude observations where its unit price is more or less than 15 times of the average variety price. I suspect those records are either not arm-length trade, a typo or just a placeholder value.

As mentioned above, there is a cutoff for intra-EU trade. In 2021, Britain left EU and below-curoff imports from Britain that would not be in the sample before will now show up. It create a mechanical effect where suddenly in 2021, there are more small imports than previous years. To address this, I exclude observations that is after 2020 and from Britain for the trends plots.

3 Stylized Facts

In this section, I describe the empirical results. I will use the empirical moments and regularities to motivate how can reduction of search cost incluence firms behaviour in input markets. Particularly, how firms of different sizes and productivities differ in their reaction to a environment with lower search cost. Then, I will provide secular trends about firm size ditributions in the input markets.

3.1 Static Moments

In this section, I describe how can change in search cost in the input market affect firm size distribution. In the first part of the this section, I use internet connection as a measure for search cost to see how it affect firms behaviour for acquiring inputs. In the second part, I look at how prices are different for different buyers and explains why.

3.1.1 Determinants of Input Prices

In this section, I primarily examine what determines input prices, as we know that it is another important channel that input markets can affect

First off - how a firm's output productivity influences the prices it face in the input market. To understand this, I have to first define a measure for productivity. I define productivity as the quality adjusted physical productivity TFPQ. First, I use balance sheet data to estimate revenue-based productivity TFPR under Cobb-Douglas assumption:

$$\text{TFPR}_{f,t} = \frac{(p_{f,t}y_{f,t})}{(w_{f,t}l_{f,t})^{\alpha_l} K_{f,t}^{\alpha_k} (\sum T_{f,t,u})^{1-\alpha_l-\alpha_k}}$$

where py is sales, wl is the wage bill, K is capital and T is intermediate cost, which are all observables on the balance sheet. I assume the capital rental rate $r = 0.15$, which is standard estimates from the literature. The remaining unknown parameters αs can be easily back-out using the Cobb-Douglas property where α equal to the expenditure share of that input under cost minimization. I assume α is constant within industry and is equal to the industry average expenditure share. As long as the firm use positive wage, intermediate cost and capital

I then estimate the quality-adjusted TFPQ by:

$$\text{TFPQ}_f = p_f \text{TFPR}_f$$

, where p is the price of the firm's production. p can be obtained from the producer price index (PPI) or export database. One caveat is that the PPI database covers only the largest firms, and larger companies are typically those that tend to export. Consequently, the sample analyzed in this section is biased toward larger firms. In the Appendix, I have used an model-based estimation to calculate the output price, so that most of the sample can be included. The conclusion is not contradicting.

I run the following regression:

$$\text{Input Price}_{f,t} \sim \text{TFPQ}_{f,t} + \text{Input Quantity}_{f,t} + \text{FE}_{\text{industry}} + \text{FE}_t$$

where the firm-level price and quantity is defined as:

$$\text{Input Price}_{f,t} = \frac{\sum_{\text{All Imports}_{f,t}} \text{Value}_{f,t} log(\frac{P_{f,v,t}}{\bar{P}_{v,t}})}{\text{Total Value of Imports}_{f,t}}$$

$$\text{Input Quantity}_{f,t} = \frac{\sum_{\text{All Imports}_{f,t}} \text{Value}_{f,y} log(\frac{\text{Quantity}_{f,v,t}}{\overline{\text{Quantity}}_{v,t}})}{\text{Total Value of Imports}_{f,t}}$$

The firm level price index is negatively correlated to TFPQ. That means more productive firms pay lower unit price compared to their less productive competitors for the same input bundle.

Table 1: Regression Results on Price of Input Bundle

	(1)	(2)	(3)
Intercept	0.9069^{***}	0.3496^{***}	0.5208^{***}
	(0.0099)	(0.0031)	(0.0079)
log TFPQ	-0.2333^{***}	–	-0.1235^{***}
	(0.0084)		(0.0066)
Weighted Input Quantity	–	-0.2395^{***}	-0.2314^{***}
		(0.0039)	(0.0038)
Observations	72801	72801	72801
R^2	0.0244	0.1867	0.1934
Industry FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes

Notes: Standard errors in parentheses. Dashes indicate the variable is not included in that specification.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

It is interesting because this correlation means that on top of their productivity advantage, more productive gain further step ahead to their less productive counterpart in the input market. This correlation widened the gap between productive and unproductive firms. This potentially make the variance of firm size distribution larger and can have implication of increasing concentration.

Note that this relationship doesn't hold when we replace TFPQ by sales or TFPR. The reason is simple. Sales or TFPR is positively correlated with the output price p and output price p is positively correlated to marginal cost and input cost, which is the independent variable. Therefore, the negative correlation we see in the figure above maybe cancelled out/dampened or even dominated by this positive correlation. However, this result is robust when TFPQ is not quality-adjusted (See Appendix).

It is also important to note that the slope is similar every year.

3.1.2 Internet

I use the municipality level internet connection data provided by PTS, the The Swedish Post and Telecom Authority[4]. The dataset covers historical internet coverage by different technology in Sweden. For each municipality, there is data of number of firms in total and number of firms with internet access through a specific technology. I use the percentage of firm that have access to internet through fiber as a proxy for (inverse) search cost. This variable is availiable from 2010-2023. I choose the this

[4]\url{https://statistik.pts.se/telekom-och-bredband/mobiltackning-och-bredband/dokument/}

technology as the proxy instead of xDSL/cable because fiber is the fastest and most realiable internet technology, which is optimal for video calls. Therefore, having fiber connection can significantly reduce communication costs with suppliers. I run the following regression:

$$\begin{aligned}\log\left(\frac{\text{No. of Varieties Imported}_{f,t+1}}{\text{No. of Varieties Imported}_{f,t}}\right) \sim & \%\text{Fiber}_{m,t} \\ & + \mathbf{1}\{\text{TFPQ}_{f,t} > \bar{\text{TFPQ}}_{i,t}\} \times \%\text{Fiber}_{f,t} \\ & + \text{FE}_t + \text{FE}_i + \text{FE}_m\end{aligned}$$

Table 2: Regression Results - Fast Internet Connection

	(1)	(2)
Fiber	0.149^{***}	0.0733^{***}
	(0.016)	(0.0199)
Fiber x 1(Productive)	–	0.187^{***}
		(0.0246)
Observations	112022	112022
R^2	0.0013	0.0024
Industry FE	Yes	Yes
Time FE	Yes	Yes
Municipality FE	Yes	Yes

Notes: Standard errors in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

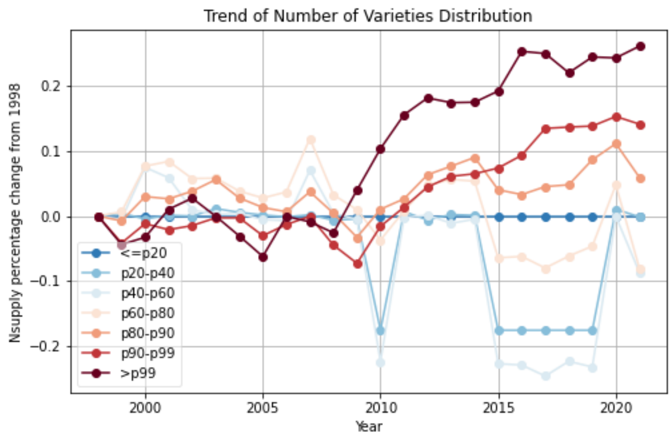
It is not surprising that having fast internet connection increase

3.2 Trends in Input Market

It is true that search costs have been decreasing with transport and communications technology advances swiftly in the last 30 years. It can also be reflected in the fiber connection in Sweden. In 2010, there are only 27% of commercial establishments that are connect to internet by optical fiber, the number is 79% in 2021. Therefore, given the mechanisms in section above, it can be expected that there are big distributional changes within the input market. In this section, I will show changes of different variables within the input markets and discuss how can it be translated to final market concentration.

3.2.1 Dispersions within the Import Market

I define a "variety" as a unique country-product pair. In the graph presented, I compare the number of varieties imported ou To facilitate comparison across variety percentiles, I normalize the annual number of varieties by their respective values from 1998. I observe that, except for the top percentiles, most variety percentiles have not experienced significant changes in the number of imported varieties. In contrast, the firms that import the most (specifically, the 90th and 99th percentiles) have increased the variety of imports by up to 20%. Given that the number of varieties imported directly relates to search activities, this observation suggests differential effects of search costs across distribution.



Large part of this pattern is driven by the entry of smaller firms. When I focus on only a balanced panel that imports every year in the data, all firms have import more varieties compared to in 1998.

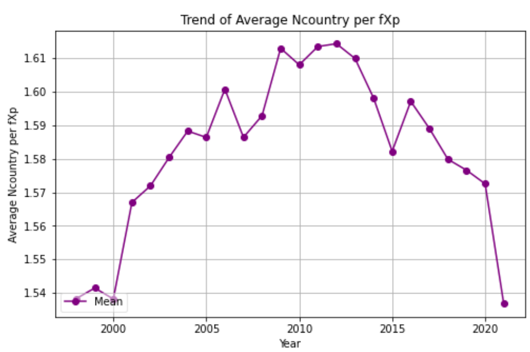


Figure 1: Average No. of Source Country by FirmXProduct Pair

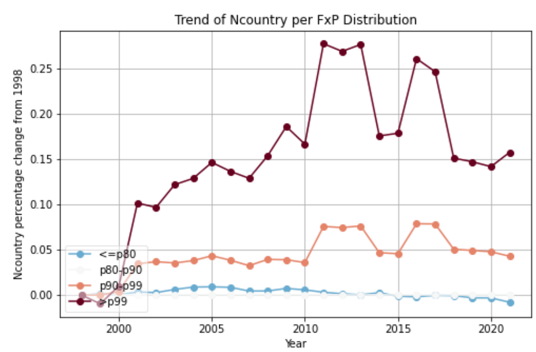


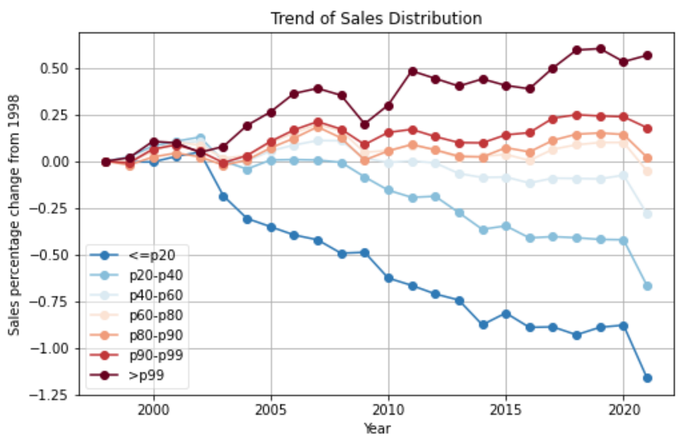
Figure 2: Distribution of No. of Source Country by FirmXProduct Pair

3.2.2 Importing Firm Size Dispersion

In this section, I examine changes in firm size, measured by annual sales, at various percentiles over the period from 1998 to 2021. Specifically, I analyze how firm sizes at different percentiles evolve relative to their 1998 levels by normalized by their respective values from 1998.

The graph clearly shows a significant reduction in firm size at the median and lower percentiles. For instance, the 5th percentile firm in 2021 is approximately 80% smaller than the 5th percentile firm in 1998. This substantial decrease likely results from reduced search costs, which acted as a barrier to entry into the import market. Lower search costs may have allowed less productive firms, which previously faced prohibitively high barriers, to enter the market. It is an indicator that entry cost to the international supply chain has reduced drastically.

In contrast, firm sizes at higher percentiles have substantially increased, by up to 40% at the 99th percentile, demonstrating the rapid growth potential of the largest firms. This growth can likely be attributed to reduced search costs facilitating expansion through the addition of new suppliers for incumbent, larger firms, which is explained in the previous section. As a result, we can observed that incumbent importers, whose already import from 1998, at all parts of the distribution experiences sustantial sales growth. (Graph in Appendix)



Overall, these findings indicate increasing dispersion in firm sizes within the market. A potential contributing factor can be falling search costs, which simultaneously influence entry barriers and expansion costs, thereby affecting both the intensive and extensive margins of the firm-size distribution.

3.3 Short Summary

On top of the 4 main empirical facts, I document also:

- Firms that receive a lower weighted-average input price tends to have higher profit share.
- More productive firms import more varieties
- Price and quantity are inversely correlated

These data moments are used in model calibration and are further explained in appendix.

In conclusion, I have look into the data to uncover some important characteristic that intermediate input market exhibit. First of all, there is an inverse relationship between productivity and weighted average unit price, but no relationship between productivity and just unit price.Also, for realized transactions, price and quantity are inversely correlated. There are also big within firm price difference and large quality differences. Among all these, search costs seem to play an important role.

On top of my own findings, I also adopt 2 additional features in my model from recent trade literature. The first one concerns the time dependency of the supplier network. Martin et al. (2023) suggests that an median buyer-supplier relationship last around an year. Therefore, as my model period is an year, having a repeated static model is not unreasonable and simplifies the numerical exercises by a lot. The second is related to inventory. Alessandria et al. (2010) point out that average company import internationally every 150 days, which indicate that inventory also should not matter for most firms in a yearly model.

4 Theory

Based on my empirical findings, I build a intermediate good market that features search and bargaining on top of a standard monopolistic competitive model.

The model timing is as followings: All productivities (and quality for seller) realize and are observed by the buyer d and seller u themselves according to some commonly known distribution. Buyers start from intermediate good $XQ = 0$, where Q is the corresponding quality. Given the intermediate good contracted XQ and self-productivity, buyers pay search cost $\kappa(n)$ or drop out from searching. The searching buyers will randomly meet a seller. Sellers produce with linear technology:

$$x_u q_u = z_u l$$

with labor as its only input. The cost of production is $\frac{w_f}{z_u}$, where w_f is the foreign wage. Without loss of generality, I henceforth assume w_f to be 1, so all the efficiency measure of the supplier is captured by z_u.[5] Productivity z_u and quality q are observable to the buyer. The matched firms will (Nash) bargain over a contract which specify intermediate good quantity xq (can be nothing) and total price T. After a contract is signed, buyer can stop or pay another search cost κ.

After all firms stop searching, the buyer will be endowed with intermediate good $Q_d X_d = (\int q_u^\rho x_u^\rho du)^{\frac{1}{\rho}}$, where Q_d is the quality. Buyers produce consumption good

[5]It will be same if I just named another efficiency variable $e_u = \frac{w_f}{z_u}$, but as I don't distinquish between different foreign countries that doesn't matter.

with Cobb-Douglas technology:

$$Q_d^\alpha y_d = z_d(Q_d X_d)^\alpha l^{1-\alpha}$$

by choosing labor and household consumes final good $Y = (\int (Q_d^\alpha y_d)^{\frac{\epsilon}{\epsilon-1}} dd)^{\frac{\epsilon-1}{\epsilon}}$. When next period starts, X is reset. Search cost κ_t is decreasing every period.

The model can be summarized into 4 value functions, which represent search decision:

$$V(z, XQ, n) = \max\{V^s(z, XQ, n), V^{NS}(z, XQ)\}$$

The buyer with productivity z with contracted inputs XQ have to decide if she want to have her nth search by comparing value function V^S and V^{NS} for searching and not searching respectively. She will choose the decision that brings her bigger expected value.

The search value is:

$$V^s(z, XQ, n) = \int V^m(z, z_u, XQ)dF(z_u) - w\kappa_t(n)$$

where $\lambda > 0$.

The not-search value is:

$$\begin{aligned}
V^{NS}(z, XQ) &= \pi(z, XQ) \\
&= p^* y^* - w(\frac{y^*}{z(XQ)^\alpha})^{\frac{1}{1-\alpha}} \\
&= [\frac{1+\alpha(\epsilon-1)}{\epsilon}](\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}}(\frac{C}{P^{-\epsilon}})^{\frac{1}{1+\alpha(\epsilon-1)}}(zX^\alpha Q^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}}
\end{aligned}$$

The not-searching value is simply the profit that can be obtained based on the exiting input XQ. Given XQ and the demand system, the firm solve the deterministic profit maximizing function:

$$\begin{aligned}
&\max_{p,y} py - w(\frac{y}{zX^\alpha Q^\alpha})^{\frac{1}{1-\alpha}} \\
&s.t. yQ^{\alpha(1-\epsilon)} = C(\frac{p}{P})^{-\epsilon}
\end{aligned}$$

Note that even though input X is alsoThe output price, output quantity, labor usage and thus the profit function all have closed form solutions. Note that the intermediate cost and search costs are not in the

The matching value is:

$$
\begin{aligned}
V^m(z, z_u, XQ) &= -T(z, z_u, XQ) + V(z, X^N Q^N) \\
X^N &= (X^\rho + x^\rho)^{\frac{1}{\rho}} \\
Q^N &= \frac{1}{X^N}(\sum_{u \in U_i} x_u^\rho q_u^\rho)^{\frac{1}{\rho}}
\end{aligned}
$$

The bargaining problem:

$$
\max_{T,xq}(-T + V(z, X^N Q^N) - V(z, XQ))^\theta (T - w\frac{xq}{z_u})^{1-\theta}
$$

In this Nash bargaining framework, the buyer and supplier negotiate over the payment T (from the buyer to the supplier) and the quality-adjusted input quantity xq. The buyer possesses a bargaining power of θ, and its surplus is defined as the value gained from a successful negotiation minus the payment. The seller's surplus, on the other hand, is the payment received minus the production costs required to fulfill the contract. Due to the linear nature of the production technology, the bargaining process does not affect the seller's outside option, which can therefore be set to zero.

In the general equilibrium model, good and labor markets should be clear and budget constraint for the household should be satisfied.

The problem for the 2 types of household :

$$
\begin{aligned}
&\max_{c_i} C = \int (y_i Q_i^\alpha)^{\frac{\epsilon-1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1}} \\
&s.t. PC \leq wL_t + \pi
\end{aligned}
$$

Factor Market Clearing:

$$
\int \mathbb{E}[l + \kappa | z] dz = \bar{L}_t
$$

Good Market Clearing:

$$
\int \mathbb{E}[y|z] dz = C
$$

I solve the model by a standard VFI procedure. In the PE model, I first make an initial guess on the search value function V^S and calculate not-search V^{NS} based on the closed form solution above. Based on V^s and V^{NS}, I solve the firm search policy and get a guess V. Assuming this guess of V is true, I solve the policy functions for payment T and quality-adjusted quantity xq by grid searching. Then, I calculate the matching value V^m by plugging in the optimal T and new state $X^N Q^N$ and update the expected search value V^S and V. The process stops when the value functions converge.

To speed up the grid searching, I make a proof that the bargaining problem is strictly concave in T (See Appendix). In this way, I reduce the 2D optimization to 1D.

4.1 Restrictive Model

I start with looking at an restrictive model where firms can only have one supplier (variety), no quality differentiation and both upstream and downstream productivity follows a simple $U(0,1)$ distribution. It become a quite familiar model of McCall (1970) random job search model. With this class of model, we can solve the optimal stopping problem by getting the reservation productivity $z_u^R(z)$ and z^R. I can get an idea how search cost κ affect firm size distribution from a fixed point solution.

The reservation supplier productivity is solved by 2 equations. The first equation is the definition of the outside option D:

$$D = \int_{z_u^R}^{\bar{z_u}} [\pi(z_u) - T(z_u)] dF(z_u) + \int_{\underline{z}_u}^{z_u^R} D dF(z_u) - w\kappa$$

which consists of 3 terms: The first one is expected profit given the probability of meeting a satisfactory supplier ($z_u \geq z_u^R$), the second one is falling back to outside option if the supplier is worse than the reservation productivity and the last term is minus the search cost.

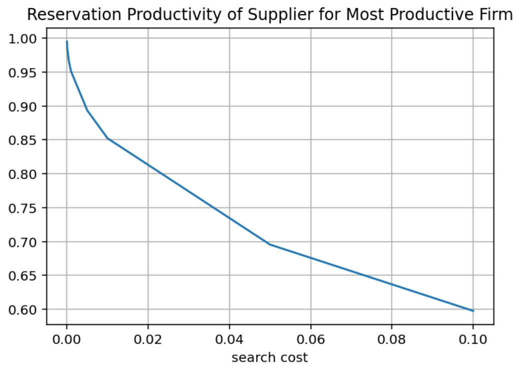
The second equation is defined by the indifference condition, where for any buyer, she should be indifferent between the profit for meeting the reserved supplier and going into outside option:

$$\pi(z, z_u^R) - T(z, z_u^R) = D$$

Combining the equations we get a expression for solving z_u^R for each z:

$$\frac{\kappa}{Kz^{\epsilon-1}} = [1 + \alpha(\epsilon-1) z_u^{R[\alpha(\epsilon-1)+1]} - (\alpha(\epsilon-1)+1) z_u^{R\alpha(\epsilon-1)}]$$

This expression provides a relationship between z_u^R and κ for any firm with productivity z. And solving this fixed point formula gives:

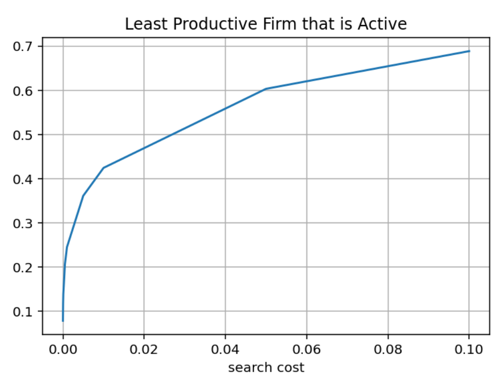


This shows that when search cost decrease, the reservation productivity, given z, is increasing. In other words, when search cost decrease, the expected productivity of the supplier increase. It is obvious that matching with a more productivity supplier means producing more. Therefore, downstream firm produce more when search cost decrease. It showcase, in this restrictive model, search cost also functions as expansion cost.

On top of that, I can show that the lowest productivity z^R where this downstream firm still remain active can be calculated by solving:

$$\frac{\kappa}{1 - F[z_u^R(z^R)]} = \int_{z_u^R(z^R)}^{\bar{z_u}} [\pi(z_u, z^R) - T(z_u, z^R)] dF(z_u)$$

And keeping everything but search cost κ constant, z^R increase with κ:



It shows that when search cost decrease, less productive firm can also enter the intermediate good market and produce. This showcase search cost can act as entry barrier.

5 Quantitative Model and Results

5.1 Full Equilibrium Model Simulation

The full model have no closed form solutions, so I resolve to numerical method. I first calculate the policies of firms given states z and X .

I simulate the full model with various different parameters to understand 1) how does search cost change allocative efficiency and concentration 2) how does economy in different condition subject to this friction.

I first initiate a set of downstream firms with productivity z_d according to the distribution $F(z)$. Given a set of parameters and the policies, I simulate forward. The downstream firms choose to search according to the search policy and its state. The downstream firms will get a random draw of supplier z_u and gain intermediate good and pay according to the policy functions that I solved previously. I then obtained a panel of simulated inputs xq, payments to suppliers T and search cost κ.

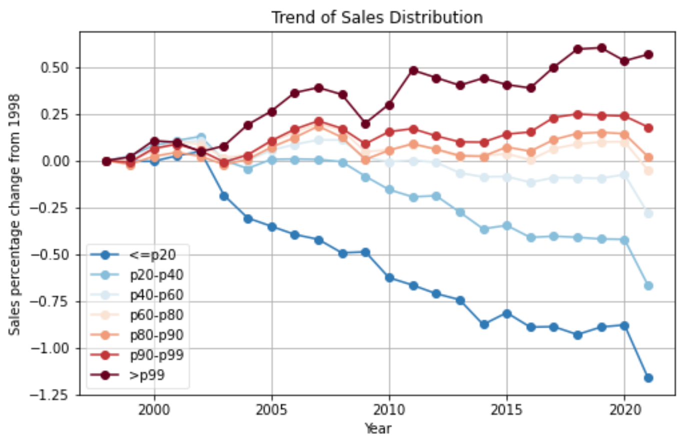
I put limit on extrapolation. Therefore, firms with very high productivity will be dropped out from the simulation. It is on one hand, problematic, as I want to study concentration where super large firms are important. However, if it is unclear if the extrapolation will be very correct, if it is very far from the policy grids.

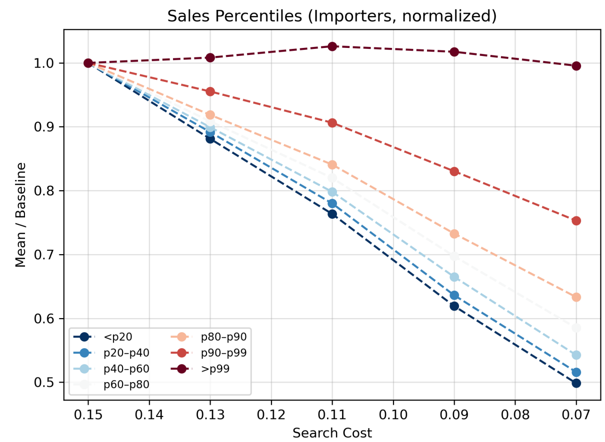
I use the following parameters:

Variable	**Values**	**Source**
Buyer's Bargaining Power θ	.83	Alviarez et al. (2023)
Final Market Elasticity ϵ	3	Aggregate Markup
Intermediate Elasticity ρ	4	Standard
Cobb-Douglas Intermediate α	.43	Intermediate Share in Data
Distribution of z	log N(-2,.55)	Direct Measure in Data
Distribution of z_u	log N(-2,.55)	Assume to be same as Domestic Firms
search cost κ_0	[.07, .15]	TFPQ time Series

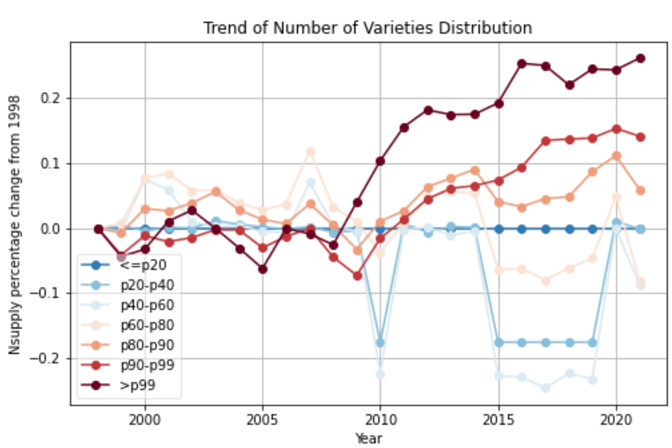
Table 3

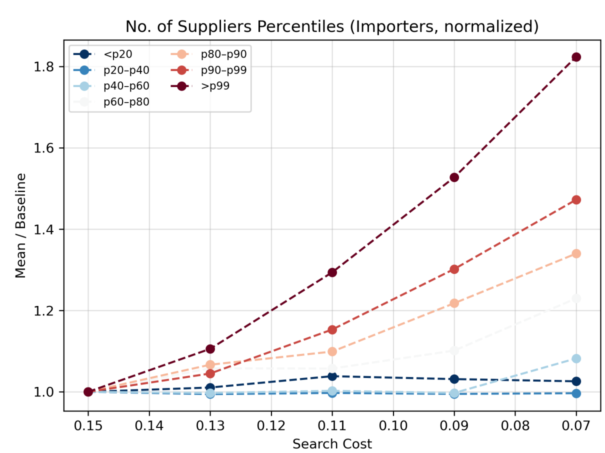
5.2 Market Concentration



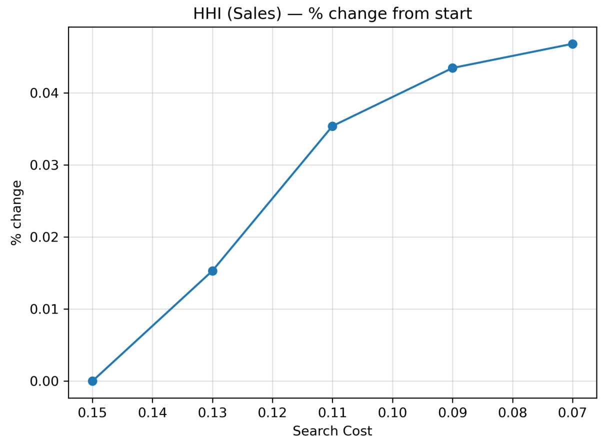


In the data, firm size distribution is increasing, that assembles the first part of the graph. On one hand, smaller firms get to enter the market, but the firms that expand the fastest are the most productive firms. However, under this parameterization, very quickly the all potential downstream entrants are active, this allows me to focus on the expansion channel. In the model, when the search cost further decrease, the growth rate of median firms catch up with the most productive firms. And then eventually, the growth rate of even smaller firms catch up. The reason is the discrete increase of suppliers and the concavity of profit function. When search cost is high, let's assume the most productive firm have 20 suppliers while the median firm has 1. While adding one extra supplier is not a lot for a firm with 20 suppliers, a increase from 1 to 2 is a big jump. That's why at the start big firms expand the quickest. However, when the search cost decrease enough that less productive firms also start to expand, they expand quicker because the big firms have less incentive to further grow because of the concavity of the profit function.





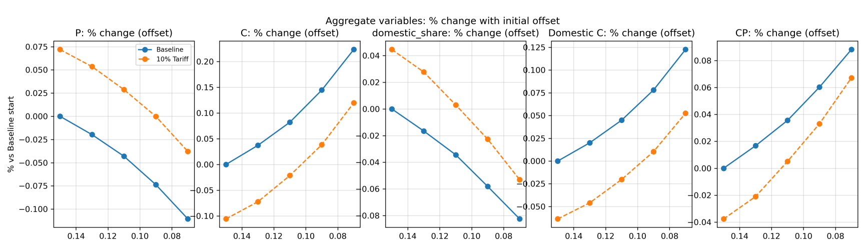
As a result, the market concentration, measured by HHI, is increasing with search cost, as shown in the following graph.

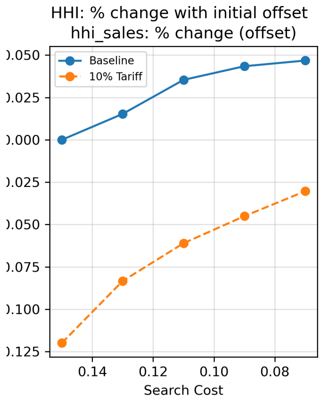


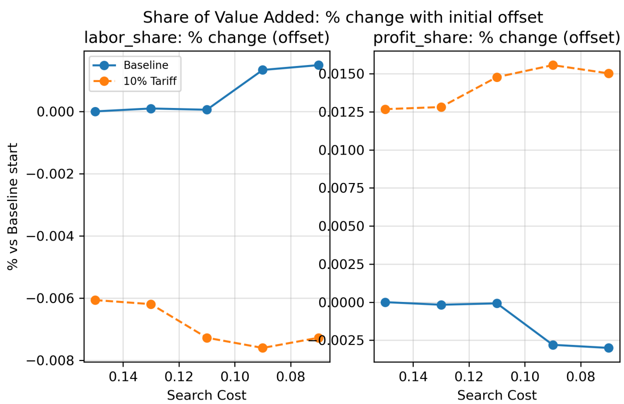
5.3 Policy Counterfactual - Tariff

In this subsection, I conduct a policy experiment introducing a 10% tariff and compare the outcomes to a baseline environment without trade protection. In the baseline, declining search costs systematically widen the sales gap, reduce markups, and increase sales-based concentration. By contrast, the tariff scenario flattens these slopes: the sales gap expands more slowly, markups remain persistently higher, and sales concentration stabilizes at a lower level. Thus, while the direction of change with lower search costs is similar across regimes, tariffs consistently moderate the magnitude of adjustment rather than eliminating it.

The tariff also alters distributional and aggregate outcomes. Labor's share of value added remains largely unchanged in the baseline but declines under the tariff, while profit shares diverge upward, reflecting a shift in income distribution. At the aggregate level, lower search costs in the baseline reduce prices and stimulate both overall and domestic consumption. These upward responses remain present with tariffs but are uniformly dampened, with prices even shifting upward relative to the baseline. Taken together, the results indicate that tariffs do not offset the effects of falling search costs entirely but instead introduce systematic level shifts that blunt their impact.







6 Conclusions

This paper has examined how reductions in search costs in input markets reshape the distribution of firm sizes. Using detailed Swedish administrative data, I documented four empirical facts: more productive firms pay lower input prices, there is widening dispersion in imported varieties, the sales distribution among importers has become increasingly dispersed, and municipalities with greater fiber-optic coverage experience faster supplier network expansion, especially for more productive firms.

To interpret these findings, I developed a quantitative model that embeds a frictional input market into a canonical monopolistic competition framework. The model shows that lower search costs benefit importing firms more strongly, allowing them to expand supplier networks and raise productivity, while non-importers become relatively less competitive. As a result, aggregate productivity rises but market concentration increases. A counterfactual exercise further showed that even a modest tariff on intermediate goods could offset these gains.

These results highlight the importance of input market frictions in shaping firm dynamics, productivity, and concentration. They suggest that policies affecting search costs, whether through infrastructure investments, digital technologies, or trade regulation, can have substantial effects on firm performance and aggregate outcomes.

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Data Appendix

Summary Statistic

I calculated for an average year in my dataset, the summary statistics:

Table 4: Summary Statistics - Imports

Variable	Mean	Std. Dev.	P10	Median	P95
No. Varieties per Firm	20.8	78.8	1.0	4.0	84.9
No. Product per Firm	13.2	31.6	1.0	3.3	54.7
No. Country per Firm	4.6	6.4	1.0	2.0	17.0
No. Country per Product	7.7	8.7	1.0	5.0	25.0
No. Firms per Variety	3.1	7.3	1.0	1.0	10.0
No. Country per FirmXProduct	1.6	1.8	1.0	1.0	4.0
No. of Firms			8432		
No. of Product			7406		
No. of Varieties			56778		
No. of Countries			179		
Percentage of Domestic Firms			0.29		
Imported Intermediate share			0.00063		

And a comparison between Importing and all firms:

Table 5: Summary Statistics - Importing Firm vs All Firm

Variable	Importers	All Firms	Ratio
No. of Firms per Year	8432	52811	0.16
Total No. of Firms	36864	150046	0.25
Sales	1.63e+11	2.83e+10	5.75
Employment	58.83	11.15	5.28
Labor Productivity	19.95	19.72	1.01
Intermediate share of Production	0.44	0.36	1.23

Within EU Data Selection and Non-EU subsample

Imports originating from other EU countries will not go through the customs because Sweden has been part of EU from 1995 on. Therefore, data on EU imports are collected from mandatory self-reports by firms. However, if the firm import below a certain value threshold, then they have no obligation to report. This cutoff is 1.5 million SEK worth of goods in 1998-2004, 2.2 mil SEK in 2005-2008, 4.5 million SEK in 2009-2014 and 9.0 million SEK onwards (See SCB (2018) for more). Due to these rules, when Britain exit the EU in 2020, it also cause some irregularities in the data of year 2021. However, if we limit the sample to only non-EU imports or exclude Britian for all years, the conclusion of the this paper hasn't change that much. In opposite, focusing on the Non-EU sample let us understand the bottom part of the distribution of importing firms better. Non-EU import consist around 50% of the full sample.

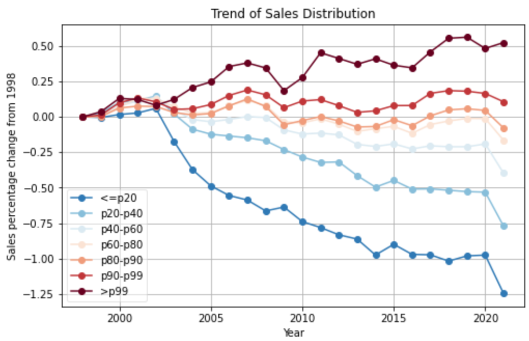


Figure 3: Sales Percentiles

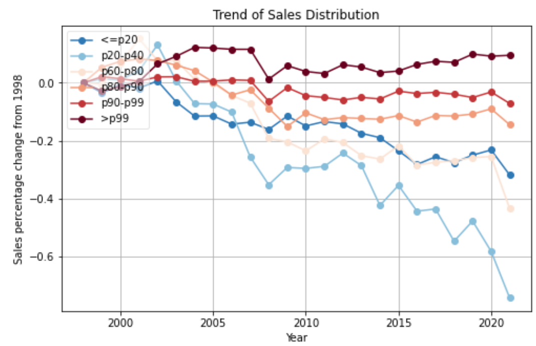


Figure 5: Sales Percentiles, with industry FE

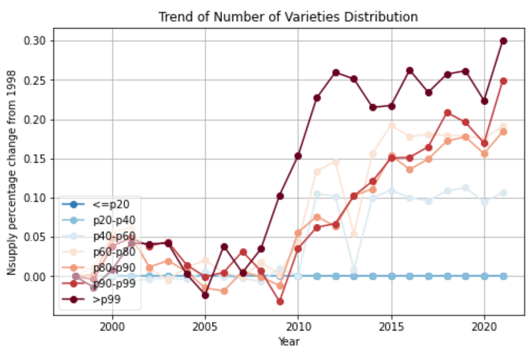


Figure 4: No. of Variety Percentiles

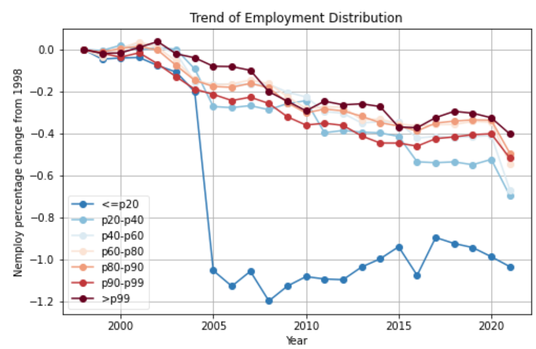


Figure 6: Employment Percentiles

Explain the bottom part.

Robustness Tests on Sales Percentiles Evolution

There can be alternative theories why firm size distribution is dispersing. One clear alternative is the composition of firms that are importing change due to other reasons, for example structural changes or sector-specific technological shocks. To address this, I test the following 4 robustness test. I do the same exercise to follow the evolution of firm size distribution in the following sub-sample. In the first graph I plot the distribution of the balanced panel, where the subsample is firms that exist in all periods in the dataset. We can see that in this subsample, the top firms (99th percentile) is actually leading in growth rate, while other firms have no particular order and similar growth rate, which points to the same style of dispersion in the number of variety graphs, where the top of the distribution is getting away from the rests.

However, when we look at the sample subsample, but we follow the same basket of firms every year, it tells another story.

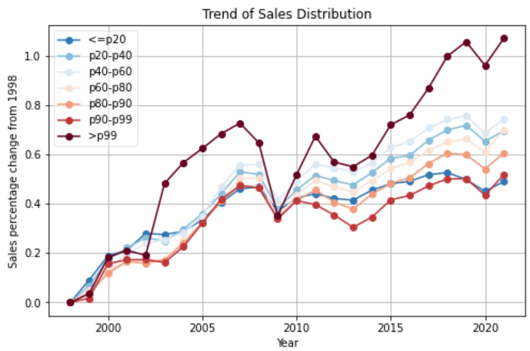


Figure 7: Balanced Panel

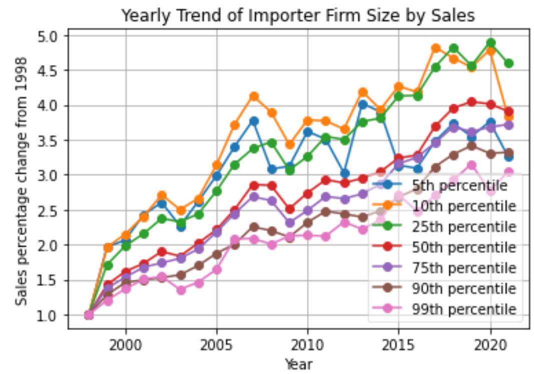


Figure 9: Incumbent that imports already at 1998

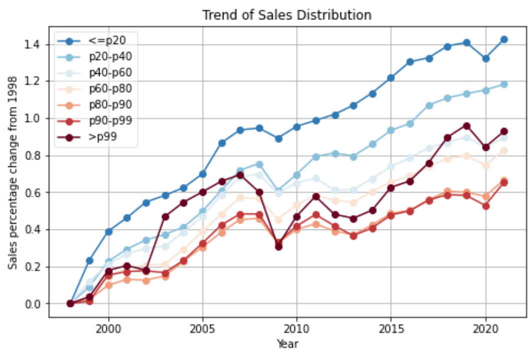


Figure 8: Balanced Panel, Fixed Basket

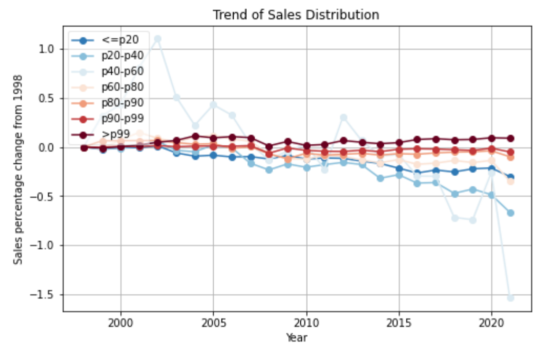
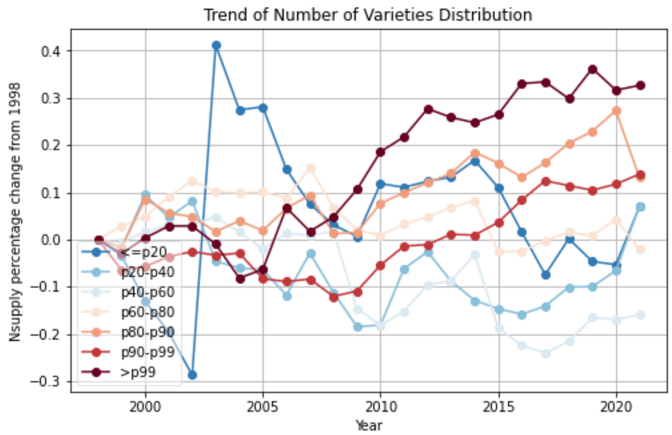


Figure 10: Industry Fixed Effects

Robustness Tests on Number of Variety Percentiles Evolution

In the main parts of this paper, I have been looking at the Number of Variety Distribution. It is also interesting to look at how big firms and small firms behave differently in this aspect. I still define a "variety" as a unique country-product pair and measure "firm size" based on annual firm sales. In the graph presented, The graph below illustrates the number of varieties (product × country) imported by each Swedish manufacturing firm. To facilitate comparison across firm-size percentiles, I normalize the annual number of varieties by their respective values from 1998. While the median and lower-percentile firms have experienced no change or even slight decrease in the number of imported varieties from 1998 to 2021, the largest firms have increased their imported varieties by approximately 10% to 30%. Given that the number of varieties imported directly relates to search activities, this observation suggests differential effects of search costs across firm sizes.



Quality-adjusted TFPQ (need to redraw graph with consistent TFPR)

In the main part of the paper, I used a direct measure output price from PPI and export price. There are 2 potential drawbacks of that approach. First, it select a small subsample where output price are observed. These are usually bigger firms and therefore the results can be biased. Second, in my model esitimation, there is heterogeneous quality across firms, but quality is omitted by using direct output price measure.

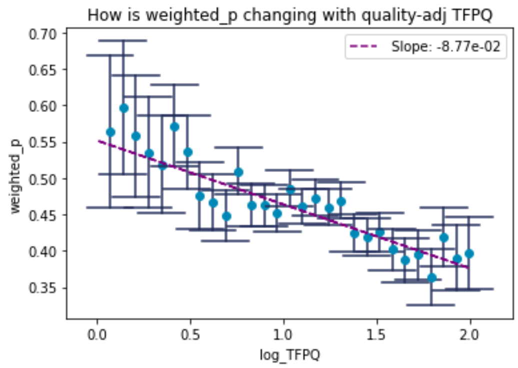
I define the revenue productivity the same way as above, but then I estimate the quality-adjusted TFPQ by:

$$\text{TFPQ}_f = \frac{p_f}{Q_f}\text{TFPR}_f$$

, where p is the price and Q is the quality of the firm's production. Unfortunately, quality Q is generally not observable. To obtain an estimate of Q, I have to rely on the theory. Assuming the monopolistic competition in final good demand, the household optimization gives:

$$\frac{p_i y_i}{p_j y_j} = (\frac{p_i/Q_i}{p_j/Q_j})^{1-\epsilon}$$

where Sales py are observables in the data. I can then back out $\frac{p}{Q}$ by applying the standard elasticity $\epsilon = 5$. Now that I get an estimate for quality-adjusted TFPQ, without substantially reducing sample size and includes quality. I do the same analysis as above:

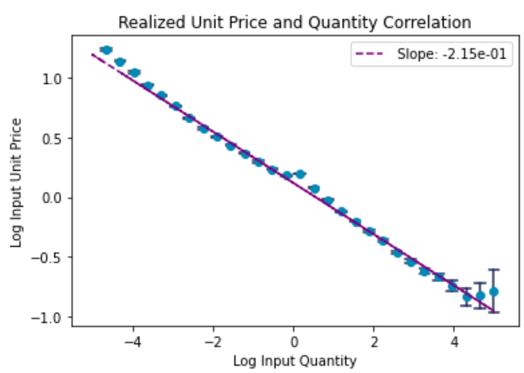


As predicted, the coefficient is also negative with a smaller magnitude than the non-quality adjusted version. As explained in Kugler and Verhoogen (2012), firms with higher productivity usually use higher quality inputs, which dampens the quality-adjusted coefficient.

Also different aggregation level (industry digit) or including capital/not, doesn't change the result much.

Realized Price and Quantity Relationship

Another distinct feature of the intermediate good market is the negative correlation between price and quantity. It can be a result by many causes, such as non-linear pricing, market power or simply downward sloping demand curve as it is yearly data. However, it is a complement evidence to my claim in last subsection. More productive firms will in expectation buy more, therefore find more efficient suppliers and pay lower unit price.



A major concern is measurement error of quantity can mechanical create correlation between price and quantity (Deaton (1988)), as I use value and quantity to back out unit price. In the dataset I use, around 25% of observations have both variable. I assume that it is unlikely that both quantity measures have measurement error at the same time. Under this assumption, I compute unit price with one measure and regress it on the other quantity as a robustness check against such potential error.

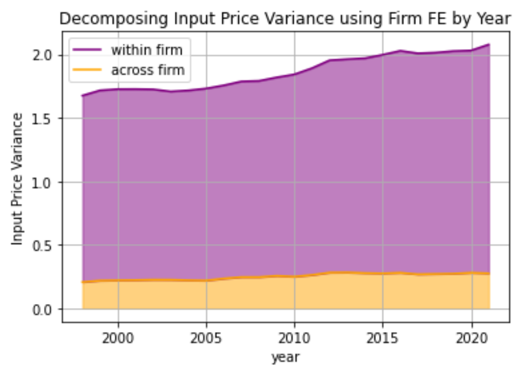
In this exercise, the shape and coefficient slightly changes, but the most important negative correlation is robust against measurement error.

Within-Across Firm Decomposition

Also, note that it is not true that productive firm get a lower price consistently across all products they purchase. To illustrate that, I run a regression on price against only the firm fixed effect:

$$p_{fpcy} = 1 + FE_f$$

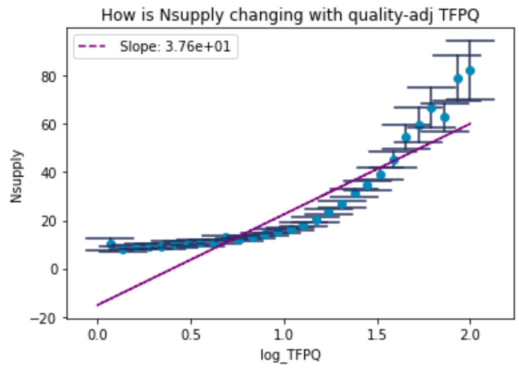
And then I decompose the variance. That part of the variance that the Fixed Effects explain is the across-firm effect and the rest is the within-firm effect. The result is rather surprising.



At odds with many model, that features firm level heterogeneity of markup or markdown, the across-firm effect is only around 15% of the total variance. It means that a same firm can purchase one input very cheaply, but another expensively.

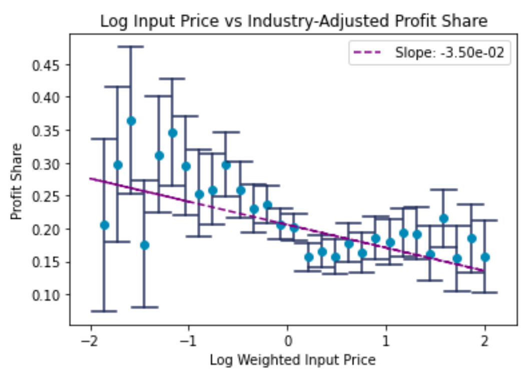
TFPQ and Variety

Another productivity-related empirical fact is that more productive firms purchase more variety. It is another piece of evidence that it is likely that productive firms also benefit from the decreasing search cost because they search more. When the produtive firms expect to produce more, they also have more incentive to search for the most efficient suppliers and also have multiple of them. Where else, the less productive suppliers even they finally gain access to the market,



Input Price and Economic Outcome

Input price also directly affect the firm's profitability directly. I find a negative correlation between firm-level input price and profit share, even controlling for industry Fixed Effects. It means that the standard CES model with constant markup probably misses important features of the intermediate good market.



Quality Differences

As researchers such as Kugler and Verhoogen (2012) show that it is a important channel of input price differences, I estimate the dispersion of input price that is generated from quality differences.

I use a reduce form approach that utilize the Rauch (1999) classification, where he separate goods into 3 classes: traded in organized exchange, traded with referenced price and differentiated goods. Organized exchange goods also included products that have a organize exchange but can also be traded decentralized. Some examples are banana, wool or other basic agriculture products. Those products, however, have very little quality differences and therefore are suitable reference goods to estimate quality differentiation.

To link the Rauch's classifications that is based on or SITC Rev. 2 the main dataset which use CN code, I refer to the HS-SITC conversion tables provided by UNSD[6].

[6] https://unstats.un.org/unsd/classifications/Econ

With resale	Std Dev
Organized Exchange	.9
Referenced Price	1.18
Differentiated Goods	1.44

The CN code shares the first 6 digit with the HS code system and I assume all products that shares the first 6 digits should get the same Rauch classifications. I look at the price standard deviation of the 3 groups of products:

We can see that while differentiated goods have higher standard deviation, some part of the dispersion persist also in the "organized exchange" category. This exercise shows that quality difference cannot explain all the differences in input prices.

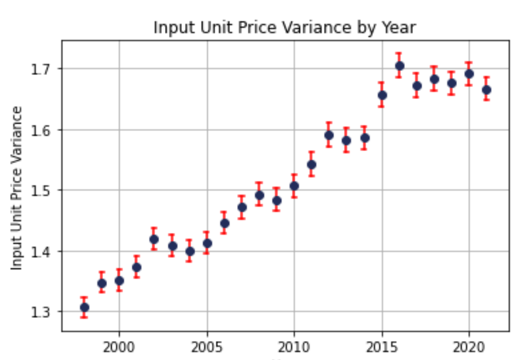
The important thing is there is no trend of quality differences year by year? As non-differentiated goods have minimal quality variation, I can define the quality gap as the price variance gap between differentiated and non-differentiated goods. Within my observed years, there are no big changes of quality gap. By weighted average, there are no change at all. Also, the composition of differentiated goods as a fraction of all imported goods is stable at 74% $\sim$ 76%. One way or the other, it is not sufficient to explain the price variance increase.

Decreasing Search Cost for Suppliers and Input Price Dispersion

Related to the decreasing search cost and the influx of smaller firms into the import market is the input price dispersion. As the distribution of importer size become more disperse, variance of input price is also increasing. I define input price dispersion as:

$$\text{Var}(P) = \text{Var}(log(\frac{P_{fvy}}{\bar{P_{vy}}}))$$

It is also robust when I limit the analysis to within-EU import or imports from most of Sweden's major trading partner.



One potential link between the two dispersing distribution is that input unit price is inversely correlated to firm size. When there are a lot of small firms goes into the import market, they buy very little and they are not familiar with the market, therefore they will pay a relatively high price. On the other hand, big firms become bigger, gain market power and face a even lower price. In this case, the price gap will grow. I will present more evidence below.

It is also important to note that quality differences is likely not the reason. I measure the price variance driven by quality difference by comparing price variance within differentiated products and non-differentiated ones, as defined by Rauch (1999). It is a very conservative measure that likely exaggerate the important because this gap likely include other channel such as bargaining and information frictions. But what is important is that the quality gap hasn't been increasing during the period, neither does the composition of differentiated products in all products. See more in Appendix.

Background Volatility

To check the how much of the price differences are unrelated to buyer characteristic, I look at currency volatility as one representative background noise. I construct a currency-year volatility index (against Swedish Krona) using data from the Swedish Riksbank. I then linked the currency volatility to the volatility.

I then run a regression against the variance of price:

$$\mathrm{Var}(\log(\frac{P_{pc}}{P_p})) \sim \mathrm{volatility}_{cy}$$

I find very little relationship between the 2 variables. (Need check)

Robustness Check - Arm length

To test if my results holds for non-arm-length trade (i.e. Trade that happens not within the same multi-national corporation group), I have carry out robustness test on the sub-sample of firm that neither 1.) do not belong to a foreign-based corporate group nor 2) belong to a corporate group that owns subsidiary abroad. In this case, any international trade of this firm have to be arm-length.The reason for using such conservative measure is that I still lack the indicator of the dataset (will be delivered). This subsample contains around 25% of the observations in the original dataset.

For arm length trade, 1) the negative relationship between TFPQ-Input Price is unchanges 2) the evolution of number of variety imported is slightly different, the 99th percentile firm still increase significantly, 90-99th percentile drop slightly while all other ones are flat. I believe the same conclusion can be drawn. 3) Nsupply increase much less in TFPQ when we throw out all this companies, but it is not surprising and a bit unclear if this result are affected by excluding non-arm-length trade or excluding the biggest/most productive multi-national firms. 4) Price Dispersion result also holds, the absolute value of variance in all years increase. Interestingly, there is a big drop in 2020 (Corona), this is not seen in the original graph.

Broadband Rollout

Theory Appendix

Structural Estimation - Quality

Before I get to the counterfactual, note that even though quality-adjusted inputs XQ have been treated as one variables through out the model solving stage. However, they have different implications when I need to calculate welfare, misallocation etc. The reason is quality Q goes into the consumer's welfare with a power α. Decomposing X and Q also help me to understand the real price dispersion.

However, input quality is in general unobservable in the data, but I have backed out output quality in the empirical section regarding productivity. Assuming the model is correct and quality distribution is constant across suppliers of all efficiency z_u, there is a mapping between input and output quality. This has been documented in extensive literature such as XXX and XXX. From the data, I get calculate the moments (μ_Q, σ_Q) of the downstream quality distribution. One caveat is that I mostly observe output quality of bigger firms. I make the log normal assumptions and calculate by XXX.

I then try to solve for the moments (μ_q, σ_q) of the upstream quality distribution, which I assume to be log normal. I guess and moments, then simulate a panel of quality q_u. As I have already simulated the corresponding XQ and xq in the previous section, I can solve for the simulated downstream Q by the following equations:

$$q = \frac{xq}{x}$$
$$XQ = (\sum_{u \in U_i} x_u^\rho q_u^\rho)^{\frac{1}{\rho}}$$
$$X = (\sum_{u \in U_i} x_u^\rho)^{\frac{1}{\rho}}$$
$$Q_i = \frac{1}{X}(\sum_{u \in U_i} x_u^\rho q_u^\rho)^{\frac{1}{\rho}}$$

I then estimate the simulated moments of Q, compare it to the empirical moments and update the upstream moments (μ_q, σ_q) until the simulated and empirical moments are close enough.

Closed Form Solution for Important Variables

Output Price and output:

$$
\begin{aligned}
&\max_{p,y} py - w(\frac{y}{zX^\alpha})^{\frac{1}{1-\alpha}} \\
&s.t. yQ^{\alpha(1-\epsilon)} = C(\frac{p}{P})^{-\epsilon} \\
\implies &\max_{p} p^{1-\epsilon}\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}} - wp^{\frac{\epsilon}{\alpha-1}}(\frac{\frac{C}{P^{-\epsilon}}}{Q^{\alpha(1-\epsilon)}zX^\alpha})^{\frac{1}{1-\alpha}} \\
&D_p :(1-\epsilon)\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}}p^{-\epsilon} = \frac{w\epsilon}{\alpha-1}p^{\frac{\epsilon-\alpha+1}{\alpha-1}}(\frac{\frac{C}{P^{-\epsilon}}}{Q^{\alpha(1-\epsilon)}zX^\alpha})^{\frac{1}{1-\alpha}} \\
p^{\frac{1+\alpha(\epsilon-1)}{1-\alpha}} =&\frac{w\epsilon}{(\epsilon-1)(1-\alpha)}(\frac{C}{P^{-\epsilon}})^{\frac{\alpha}{1-\alpha}}(\frac{1}{Q^{\alpha^2(1-\epsilon)}zX^\alpha})^{\frac{1}{1-\alpha}} \\
p =&(\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{1-\alpha}{1+\alpha(\epsilon-1)}}(\frac{C}{P^{-\epsilon}})^{\frac{\alpha}{1+\alpha(\epsilon-1)}}(\frac{Q^{\alpha^2(\epsilon-1)}}{zX^\alpha})^{\frac{1}{1+\alpha(\epsilon-1)}} \\
y =&\frac{C}{Q^{\alpha(1-\epsilon)}}(\frac{p}{P})^{-\epsilon} \\
=&\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}}(\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{-\epsilon(1-\alpha)}{1+\alpha(\epsilon-1)}}(\frac{C}{P^{-\epsilon}})^{\frac{-\epsilon\alpha}{1+\alpha(\epsilon-1)}}(\frac{1}{Q^{\alpha^2(1-\epsilon)}zX^\alpha})^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}} \\
=&(\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{-\epsilon(1-\alpha)}{1+\alpha(\epsilon-1)}}(\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}})^{\frac{1-\alpha}{1+\alpha(\epsilon-1)}}(zX^\alpha)^{\frac{\epsilon}{1+\alpha(\epsilon-1)}}
\end{aligned}
$$

Profit Function:

$$
\begin{aligned}
\pi(z,QX) =&py - w(\frac{y}{zX^\alpha})^{\frac{1}{1-\alpha}} \\
=&\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}}p^{1-\epsilon} - w(\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}}(\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}})^{\frac{1}{1+\alpha(\epsilon-1)}}(zX^\alpha)^{\frac{\epsilon-1-\alpha(\epsilon-1)}{1+\alpha(\epsilon-1)}\frac{1}{1-\alpha}} \\
=&\{(\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}} - w(\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}}\}(\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}})^{\frac{1}{1+\alpha(\epsilon-1)}}(zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} \\
=&[\frac{1+\alpha(\epsilon-1)}{\epsilon}](\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}}(\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}})^{\frac{1}{1+\alpha(\epsilon-1)}}(zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} \\
=&[\frac{1+\alpha(\epsilon-1)}{\epsilon}](\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}}(\frac{C}{P^{-\epsilon}})^{\frac{1}{1+\alpha(\epsilon-1)}}(zX^\alpha Q^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}}
\end{aligned}
$$

The labor cost is:

$$
\begin{aligned}
w(\frac{y}{zX^\alpha})^{\frac{1}{1-\alpha}} =&(\frac{C}{Q^{\alpha(1-\epsilon)}P^{-\epsilon}})^{\frac{1}{1+\alpha(\epsilon-1)}}w(\frac{w\epsilon}{(\epsilon-1)(1-\alpha)})^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}}(zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} \\
=&\frac{(\epsilon-1)(1-\alpha)}{\epsilon}py \text{ or } [\frac{(\epsilon-1)(1-\alpha)}{1+\alpha(\epsilon-1)}]\pi
\end{aligned}
$$

Concavity of Bargaining Problem

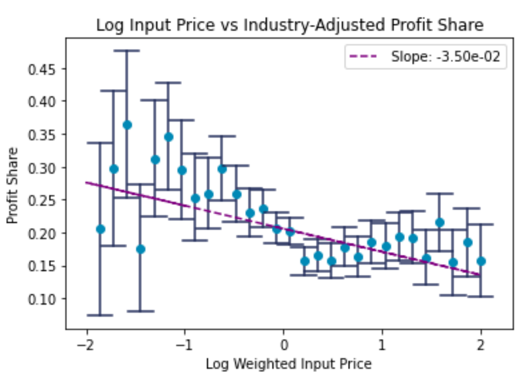
Given all possible x, I can use FOC to find T because the following function is strictly concave (as long as it is defined):

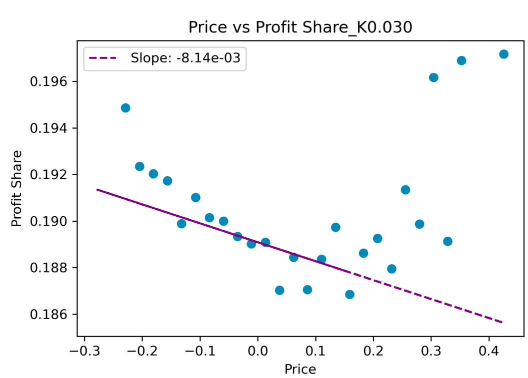
$$\begin{aligned}
B(\delta T_1 + (1-\delta)T_2) =& (V(x) - \delta T_1 - (1-\delta)T_2)^{\theta}(\delta T_1 + (1-\delta)T_2 - K(x))^{1-\theta} \\
\exp \sim& \ \theta \log[(\delta + (1-\delta)V(x) - \delta T_1 - (1-\delta)T_2] \\
& + (1-\theta)\log[\delta T_1 + (1-\delta)T_2 - (\delta + 1 - \delta)K(x)] \\
\exp >& \ \theta\{\log[\delta(V(x) - T_1)] + \log[(1-\delta)(V(x) - T_2)]\} \\
& + (1-\theta)\{\log[\delta(T_1 - K(x))] + \log[(1-\delta)(T_2 - K(x))]\} \\
\exp =& \ [\log(\delta^{\theta}(V(x) - T_1)^{\theta}\delta^{(1-\theta)}(T_1 - K(x))^{(1-\theta)})] \\
& + [\log((1-\delta)^{\theta}(V(x) - T_2)^{\theta}(1-\delta)^{(1-\theta)}(T_2 - K(x))^{(1-\theta)})] \\
& = e^{\log(\delta B(T_1))}e^{\log((1-\delta)B(T_1))} \\
& = \delta B(T_1) + (1-\delta)B(T_2)
\end{aligned}$$

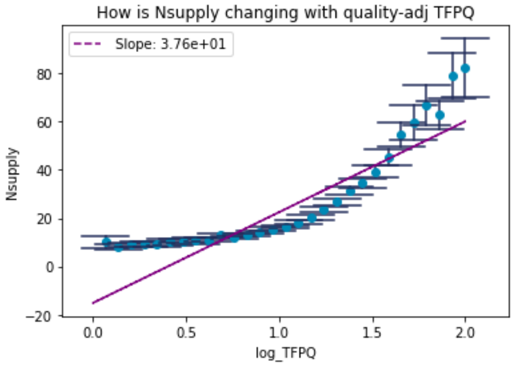
Because log is a strictly concave function.. As long as $V(x) \geq K(x)$, the solution exist and is unique. I can get a function $Bargain(x)$, which is the best value given x and then I just find the arg max. Proof: Call $T(x^o) = \arg\max_T B(x^o, T)$,which is proved to be unique. Now assume (x^*, T^*) is the maximizer of function B and $B(x^*, T^*) \geq B(x', T')$ for all (x', T') pairs. Let's say $T^* \neq T(x^*)$, then $B(x^*, T^*) \geq B(x^*, T(x^*)) \implies T(x^*)$ is not the unique $\arg\max_T B(x^*, T)$, which cannot be true. Thus contradiction.

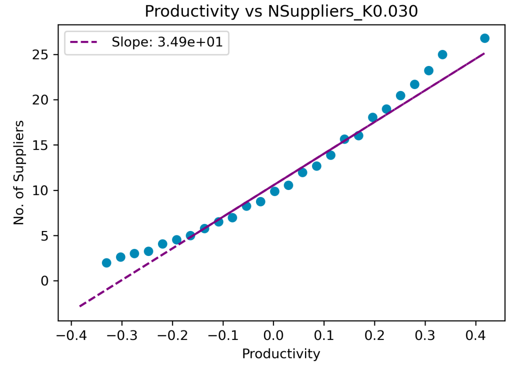
Static Moments Matching (Maybe change to table form? - just the slope)

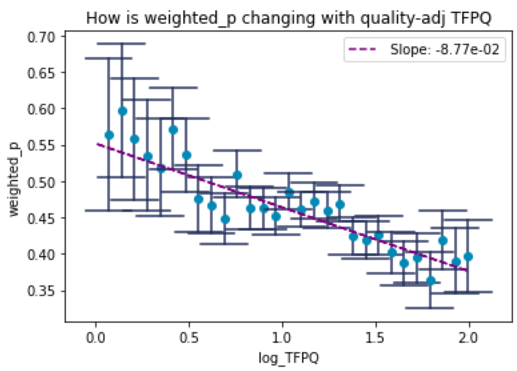
Here is the data moments matching for PE model:

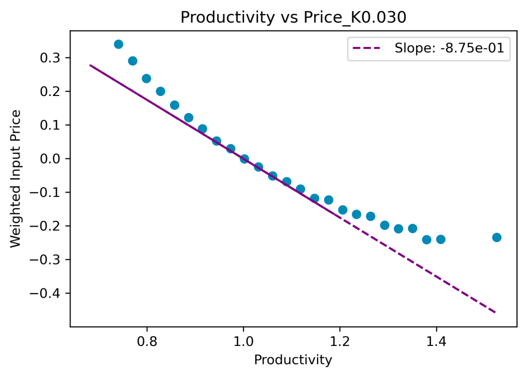


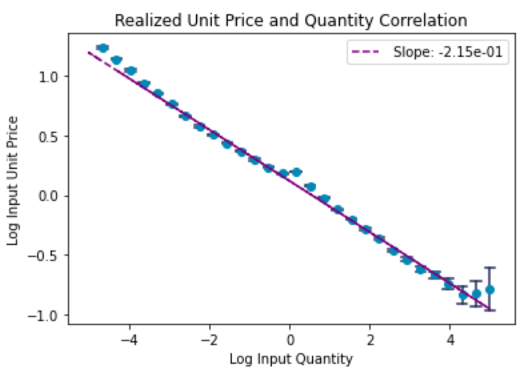


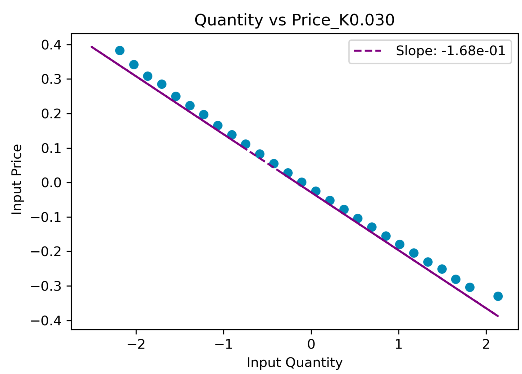

How is Nsupply changing with quality-adj TFPQ
Slope: 3.76e+01
Nsupply
log_TFPQ
80
60
40
20
0
−20
0.0
0.5
1.0
1.5
2.0


Productivity vs NSuppliers_K0.030
Slope: 3.49e+01
No. of Suppliers
Productivity
25
20
15
10
5
0
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4


How is weighted_p changing with quality-adj TFPQ
Slope: -8.77e-02
weighted_p
log_TFPQ
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.0
0.5
1.0
1.5
2.0


Productivity vs Price_K0.030
Slope: -8.75e-01
Weighted Input Price
Productivity
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0.8
1.0
1.2
1.4


Realized Unit Price and Quantity Correlation
Slope: -2.15e-01
Log Input Unit Price
Log Input Quantity
1.0
0.5
0.0
−0.5
−1.0
−4
−2
0
2
4


Quantity vs Price_K0.030
Slope: -1.68e-01
Input Price
Input Quantity
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−2
−1
0
1
2