

# Supplier Search and Market Concentration \*

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## Abstract

This paper studies how easier access to intermediate input suppliers affects productivity and market concentration. I develop a quantitative model of supplier search in which firms incur fixed costs to discover and bargain with input suppliers. The model provides a microfoundation for how input trade influences aggregate productivity and resource allocation. Evidence from firm-level import data motivates the framework through four patterns: growing dispersion in imported varieties, rising inequality in importer sales, lower input prices for larger firms, and stronger supplier-network expansion in municipalities with better digital infrastructure. In general equilibrium, lower input search frictions reallocate resources toward more productive firms. Quantitatively, the mechanism accounts for about 30 percent of observed real GDP growth and between 29 and 64 percent of the rise in market concentration in Sweden. A 10 percent tariff on imported inputs offsets the GDP gain and lowers concentration, providing a policy counterpart to the supplier-search mechanism.

*Keywords:* Input markets; Firm-to-firm trade; Search costs; Market concentration; Tariffs

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# 1 Introduction

Market concentration has risen steadily since the 1990s in many advanced economies, sparking debate over its causes and consequences for productivity and allocative efficiency. Over the same period, it has become easier and cheaper for firms to source intermediate goods. Improvements in digital communication, transportation technology, together with growing sophistication of global supply networks and trade policies, have reduced search frictions and broadened the range of available suppliers. When input markets become more efficient, firms gain access to a larger and higher quality pool of suppliers, which lowers production costs and raises productivity (Amiti and Konings, 2007; Goldberg et al., 2010; Halpern et al., 2015). However, it is unclear if the gains are evenly distributed. This raises important questions: which firms benefit and by how much; what are the implications for aggregate productivity; and how do these changes influence firm entry into importing. In this paper, I argue that rising productivity and increasing market concentration are two sides of the same coin, both driven by improvements in input market efficiency.

I begin my analysis by establishing three key empirical patterns using detailed Swedish administrative data in the manufacturing sector. These facts provide new systematic evidence linking firm-level sourcing behavior to rising firm concentration. First, there is substantial dispersion in the number of imported varieties across firms. Further, firms at the upper tail now source about 60% more inputs than twenty years ago, making the dispersion even larger. Second, the sales distribution among importing firms has become considerably more unequal: the largest importers are 75–125% larger than they were in 1998, while the smallest importers are about 50% smaller. Third, firms that are 10% larger pay about 1.6% lower unit values.

To provide evidence for the mechanism linking input market efficiency and firm concentration, I use the staggered rollout of the high-speed broadband infrastructure across Swedish municipalities as plausibly exogenous variation in input search costs. I show that firms located in municipalities with lower search costs, proxied by better high-speed internet coverage, expand their supplier networks and particularly among the largest firms. High-speed internet coverage leads firms to import 5 more varieties on average, with the largest firms importing around 23 more varieties.

I rationalize these empirical findings through the lens of a model of monopolistic competition with frictional input markets. The model incorporates the necessary ingredients to explain the facts documented above and to study how frictions in input markets affect allocative efficiency as well as market concentration. Specifically, I embed

a random search and bargaining framework into a model of two-sided heterogeneity and monopolistic competition. Buyers (downstream firms) can produce in-house or source intermediate inputs from sellers (upstream firms). Upstream firms are heterogeneous in their productivity and produce with only labor. Downstream firms are also heterogeneous in their productivity and produce using a Cobb-Douglas aggregator between labor and a CES aggregator of intermediate goods that features love of variety.

Downstream firms can only source intermediate inputs from upstream firms in their supplier network. Buyers can enlarge their supplier network by sequentially searching for new sellers. A matched buyer and seller then bargain over quantities and prices for the intermediate good. The firm size distribution and supplier-network size distribution are thus endogenously determined in equilibrium.

The key mechanism of the model is that there is endogenous selection into both the extensive margin (the decision to start buying from an upstream firm) and the intensive margin (the number of upstream firms a buyer chooses to buy from). The mechanism allows the model to match the cross-sectional facts: more productive firms search more, build larger supplier networks, become larger, and pay lower unit prices due to their better outside options in bargaining. A reduction in search frictions allows productive firms to expand their networks and reduce the prices of intermediate inputs further, reallocating resources toward them. Aggregate efficiency rises as production shifts toward more productive firms and as broader supplier networks enhance input complementarities. The firm size distribution widens, and market concentration increases.

To quantify these mechanisms, I estimate the model to match the key moments documented in the Swedish manufacturing data. The key endogenous parameters in the estimation are the dispersion in firm productivity, which is estimated to reproduce the observed top sales share, and the decline in search costs, which is estimated to match the observed rise in the mean number of imported varieties. In the model, search frictions must fall by about 40 percent to match the observed increase in imported varieties. The decline in frictions leads to a 13 percent increase in real GDP and an increase in the Herfindahl-Hirschman index (HHI) by about 12.7 percent. For comparison, Sweden's real GDP per capita increased by approximately 42 percent over 1998–2021 (World Bank, 2025), and market concentration rose by between 20 and 44 percent over the same period. These figures imply that the mechanism accounts for roughly 30 percent of observed GDP growth and between 29 and 63 percent of the increase in concentration.

I next compare the decline in search frictions with an alternative counterfactual in which foreign suppliers become more efficient over time. In this exercise, I vary only the mean productivity of foreign suppliers while keeping search costs fixed. The magnitude of the

increase in productivity is disciplined to match the decrease in the demand shifter in the main exercise, which is a composite of aggregate price and production. Rising supplier efficiency delivers a similar increase in aggregate productivity and market concentration but mainly through the intensive margin. It does not match the expansion in the number of imported varieties observed in the data, while lower search costs do. This pattern suggests that both forces could have contributed, with declining search frictions playing a central role in driving the observed rise in market concentration.

Finally, I use the model to study a timely and topical policy question: the impact of a uniform tariff on imported intermediates. This exercise links technological changes in input market efficiency to tangible policy instruments that affect firms in comparable ways. Although tariffs are not identical to search costs, both raise the effective cost of accessing foreign inputs and discourage the formation of supplier relationships. As a result, a tariff compresses supplier networks and shifts activity toward smaller and less productive firms, replicating the economic effects of higher search frictions. Quantitatively, a 10 percent tariff offsets the output gains and the increase in market concentration generated by the decline in search costs over the past two decades. While such a policy lowers aggregate productivity, it narrows performance differences between large and small firms, which can make it politically attractive.<sup>1</sup>

This paper contributes to the literature in three ways. First, I examine the role of changes in input markets as a determinant of market concentration in downstream industries. In doing so, I complement existing studies that emphasize other drivers of concentration, such as rising entry barriers (Covarrubias et al., 2020), intensified import competition (Amiti and Heise, 2025), declining market-spanning costs due to advances in information technology (Aghion et al., 2023), and the rise of intangible capital (Crouzet and Eberly, 2019; De Ridder, 2024; Weiss, 2020; Bajgar et al., 2025).

The literature has studied how changes in trade patterns can reshape the firm size distribution in profound ways. For example, Amiti and Heise (2025) show that intensified import competition raises market concentration among local firms by displacing less productive domestic producers. This perspective, however, focuses on trade in final goods, while the role of intermediate inputs in shaping firm dynamics has received much less attention. Imported final goods directly compete with domestic producers and reduce demand for their output, a demand shock. By contrast, importing inputs generates a productivity shock for downstream firms by lowering effective production

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<sup>1</sup>Since large and highly productive firms are concentrated in major urban regions, restricting input trade slows their expansion and limits further spatial concentration of economic activity, a recurrent theme in European policy discussions (OECD, 2023; European Commission Directorate General for Regional and Urban Policy, 2024).

costs. This distinction shifts the focus from how trade reallocates demand across firms (Eaton et al., 2011) to how heterogeneity in access to international suppliers influences the production efficiency of firms. Understanding this margin is particularly important because intermediate inputs account for roughly 56 percent of global trade (Miroudot et al., 2009) and dominate the EU’s import basket. Yet we know relatively little about how improvements in input market efficiency translate into firm-level outcomes.

Second, this paper contributes to the growing firm-to-firm trade literature, which studies how individual buyer and supplier relationships shape trade patterns, prices, and productivity. Using detailed Swedish import data, I document substantial heterogeneity in firms’ sourcing behavior and input prices, consistent with Atalay (2014). Prior studies attribute such patterns to buyer market power (Morlacco, 2020; Rubens, 2023), input quality differences (Kugler and Verhoogen, 2012), and match-specific frictions (Burstein et al., 2024). These patterns motivate my theoretical framework that explains why more productive and more connected firms obtain systematically lower input prices.

I develop a model of buyer and supplier relationships that brings together the key ingredients emphasized in this literature, including bargaining (Alviarez et al., 2023), search (Eaton et al., 2022a), and matching (Eaton et al., 2022b). The model allows for endogenous relationship formation and bilateral bargaining over both quantities and prices, and is related to Oberfield (2018), who studies how production linkages shape aggregate productivity. The framework contributes to general search and bargaining theory by combining explicit search costs, two-sided heterogeneity, and multidimensional bargaining in a single model. To my knowledge, existing work incorporates at most two of these elements. Canonical Diamond–Mortensen–Pissarides (DMP) search-and-matching models include explicit search costs but feature one-sided heterogeneity and one-dimensional bargaining (Pissarides, 2017). Models with two-sided heterogeneity in a DMP environment have been developed (e.g., Lise et al., 2016; Shimer and Smith, 2000), and models with explicit search costs in two-sided settings exist (e.g., Eeckhout and Kircher, 2010; Eaton et al., 2022a), but all abstract from bargaining.

Within this broader firm-to-firm framework, the paper also connects to the literature on imported intermediates, which shows that access to foreign inputs can raise firm productivity by lowering costs or improving input quality (Amiti and Konings, 2007; Goldberg et al., 2010; Halpern et al., 2015; Gopinath and Neiman, 2014). This paper complements that work by modeling how input market efficiency, shaped by search frictions, supplier heterogeneity, and bilateral bargaining, endogenously determines firms’ access to imported inputs and the prices they face. In doing so, it links firm-level sourcing behavior to aggregate outcomes such as productivity and market concentration.

Third, I show that importing intermediate goods can generate distortions in the form of input price dispersion across firms. In contrast to standard misallocation models, where such wedges are exogenous, my framework derives them from input market frictions and bargaining. As search frictions decline, aggregate productivity increases and the market becomes more efficient, yet dispersion in both input prices and TFP also rises because more productive firms gain better access to high-quality or low-cost suppliers. This mechanism complements the findings of Boehm and Oberfield (2020), who link input trade to allocative efficiency, and connects to the misallocation frameworks of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). It also relates to Edmond et al. (2015) and Dhingra and Morrow (2019), who study the procompetitive and welfare-improving effects of trade in consumer goods, by showing that input trade can raise aggregate productivity while reshaping the distribution of firm-level distortions.

The remainder of the paper is organized as follows. Section 2 describes the data and Section 3 documents the empirical facts. Section 4 develops the model, while Sections 5 and 6 illustrate the main mechanisms using simplified versions. Section 7 maps model predictions to the data and quantifies the impact of falling search costs on productivity and concentration. Section 8 examines rising foreign supplier efficiency and compares it with the baseline mechanism. Section 9 presents the tariff counterfactual. Finally, Section 10 concludes.

## 2 Data

I assemble a comprehensive firm-level panel from high quality Swedish registry data that spans 1998 to 2021 by combining three sources. The first source contains detailed import transactions. The second source provides firm financial statements with standard size and productivity measures. The third source reports municipality level diffusion of fast internet. I merge these sources at the firm year level and attach municipality characteristics by firm location and year. In this study, I focus on manufacturing firms.

### 2.1 Import Data

The import dataset is organized at the firm–product–country–year level. For each observation, I observe both the customs value and the physical quantity, which allows me to construct unit prices as value divided by quantity. I define a variety as an

eight-digit product code by country, which corresponds to a supplier variety in the model. The panel covers manufacturing firms from 1998 to 2021.

The data come from the import component of the *Utrikesandel med varor* (Foreign Trade in Goods) dataset compiled by Statistics Sweden (SCB). These firm-level import records report, for each year, the country of shipment, the HS8 product code, the import value, and the physical quantity (usually weight). For some products, the dataset also includes an additional variable, “Other Quantities” (for example, number of pieces for pencils or  $m^2$  for curtains). A list of such products is available in the section *Varukoder (KN) för uppgiftslämnande till Intrastat* on the SCB website. I use these records to compute unit values, which I interpret as prices. For each firm  $f$ , variety  $v$ , and year  $t$ , I define the unit price as:

$$P_{f,v,t} = \frac{\text{Deflated Value}_{f,v,t}}{\text{Quantity}_{f,v,t}},$$

where the import value is deflated using the aggregate CPI to adjust for inflation.

To remove variation across products and countries, I compute the within-variety–year relative (residualized) price:

$$p_{f,v,t} = \log\left(\frac{P_{f,v,t}}{\bar{P}_{v,t}}\right),$$

where the benchmark price  $\bar{P}_{v,t}$  is the value-weighted average price for variety  $v$  in year  $t$ :

$$\bar{P}_{v,t} = \frac{\sum_f \text{Deflated Value}_{f,v,t}}{\sum_f \text{Quantity}_{f,v,t}}.$$

I also construct firm-level indices for input price and quantity. The input price index measures the average log deviation of firm-specific prices from variety-level averages, weighted by import value:

$$\text{InputPrice}_{f,t} = \frac{\sum_v \text{Value}_{f,v,t} \log\left(\frac{P_{f,v,t}}{\bar{P}_{v,t}}\right)}{\sum_v \text{Value}_{f,v,t}}.$$

Analogously, the input quantity index is defined as:

$$\text{InputQuantity}_{f,t} = \frac{\sum_v \text{Value}_{f,v,t} \log\left(\frac{\text{Quantity}_{f,v,t}}{\bar{\text{Quantity}}_{v,t}}\right)}{\sum_v \text{Value}_{f,v,t}}.$$

For example, suppose a firm imports 10,000 SEK worth of German cars at a 20% higher price than the average German car buyer and 3,000 SEK worth of Chinese headphones

at a 30% lower price. The firm-level price index is then:

$$\text{FirmPriceIndex} = \frac{(1.2 \times 10,000) + (0.7 \times 3,000)}{10,000 + 3,000}.$$

The dataset includes all imports originating from outside the European Union (including Switzerland and EEA countries) and intra-EU imports for firms whose total annual imports exceed a reporting threshold. This threshold has gradually increased over time, from about SEK 1.5 million in the early 2000s to roughly SEK 9 million in recent years (see SCB (2018) for details).<sup>2</sup>

For data cleaning, I exclude observations with total import value below 100 SEK (approximately 9 USD) and those where the unit price is more or less than 15 times the average variety price, as these likely reflect non-arm's-length transactions, typographical errors, or placeholder values.

A limitation of the data is that if a firm imports the same variety from multiple suppliers within a year, the dataset reports only one aggregated record.

## 2.2 Balance sheet data and other supporting datasets

To characterize importing firms, I link the import data to firms' balance sheets using a common firm identifier available in both datasets. I use the industry codes (SNI) reported in the balance sheet data to restrict the sample to manufacturing firms. This linkage also allows me to compare the characteristics of importing firms with those of the broader firm population, as documented in [Data Appendix](#).

From the balance sheet data, I obtain firm-level measures of size, input expenditures, and productivity. I focus primarily on sales and employment, and I construct a measure of total factor productivity (TFP). I first compute revenue-based productivity (TFPR) under a Cobb–Douglas production function:

$$\text{TFPR}_{f,t} = \frac{p_{f,t} y_{f,t}}{\left( w_{f,t} \ell_{f,t} \right)^{\alpha_\ell} K_{f,t}^{\alpha_k} \left( \sum_u T_{f,t,u} \right)^{1-\alpha_\ell-\alpha_k}}.$$

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<sup>2</sup>Intra-EU trade is subject to this reporting threshold. When the United Kingdom left the EU in 2021, imports below the threshold from the UK that had previously been excluded began to appear in the data. This results in a mechanical increase in the number of small import records in 2021. To address this issue, I exclude post-2020 UK observations from trend plots.

Here,  $p_{f,t}y_{f,t}$  denotes firm  $f$ 's nominal output (price times quantity) in year  $t$ ,  $w_{f,t}\ell_{f,t}$  is the wage bill,  $K_{f,t}$  is the capital stock, and  $T_{f,t,u}$  are intermediate input expenditures (indexed by input type  $u$ ). All variables are observed in firms' financial statements.

I set the rental rate of capital to  $r = 0.15$ , following standard values in the literature. The elasticities  $\alpha_\ell$  and  $\alpha_k$  are obtained using the Cobb–Douglas cost-share property: under cost minimization and competitive input markets, each elasticity equals the input's expenditure share. I assume these elasticities are constant within an industry and equal to the industry's average expenditure shares. Provided the firm uses strictly positive labor, intermediate inputs, and capital, the TFPR measure is well defined.

I then estimate physical productivity (TFPQ) as:

$$\text{TFPQ}_{f,t} = \frac{\text{TFPR}_{f,t}}{p_{f,t}}.$$

Here,  $p_{f,t}$  denotes the firm-level output price. I obtain  $p_{f,t}$  from the producer price index (PPI) or from an export database. A limitation is that PPI coverage is concentrated among large firms, which are often exporters, so this sample is biased toward larger firms. In the Appendix, I construct model-based output prices that adjust for product quality and allow for a broader set of firms to be included. The conclusions are robust to using these model-based prices.

## 2.3 Internet data

As a proxy for search costs, I use variation in internet infrastructure that facilitates communication between firms and suppliers. To measure digital infrastructure, I use municipality-level internet connection data from the Swedish Post and Telecom Authority (PTS).<sup>3</sup>

The dataset provides annual information on internet coverage by technology across municipalities, including the total number of firms and the number of firms with access to each type of connection. From these data, I construct the share of firms in each municipality with access to fiber broadband, which I interpret as a proxy for lower search costs. The variable is available annually from 2010 to 2023.

Each establishment in the administrative data is identified by a municipality code, which allows me to merge the internet coverage data to firms' geographic locations. This

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<sup>3</sup><https://statistik.pts.se/telekom-och-bredband/mobiltackning-och-bredband/dokument/>

linkage enables me to study how differences in local digital infrastructure affect firms' sourcing behavior and the number of suppliers they connect with.

## 3 Stylized Facts

In this section, I present the main empirical patterns that motivate the model and analysis. I use a set of empirical moments and regularities to illustrate how reductions in search costs can influence firms' behavior in input markets. In particular, I study how firms of different sizes and productivity levels respond to an environment with lower search costs. I also document long-run trends in the distribution of firm size and market concentration in input markets.

### 3.1 Trends in Input Markets

Search costs have declined substantially over the past three decades due to rapid improvements in transport and communication technologies. This trend is also reflected in Sweden's internet infrastructure. In 2010, only about 27% of commercial establishments were connected to the internet through fiber, whereas by 2021 this share had increased to 79%. Given the mechanisms discussed in the previous sections, such reductions in search costs are expected to induce important distributional changes within input markets. In this subsection, I show how key variables in the input market have evolved and discuss how these patterns may translate into changes in market concentration in the final goods market.

#### 3.1.1 Dispersion within Import Market

I define a variety as a unique country–product pair. To illustrate changes in sourcing patterns, I compare the number of varieties imported across firms at different points of the distribution. For comparability over time, I normalize the number of imported varieties in each percentile by its value in 1998.

I find that, except for the top percentiles, most firms have experienced little change in the number of imported varieties. In contrast, firms in the upper part of the distribution (particularly at the 90th and 99th percentiles) have expanded their variety of imports by up to 20%. Because the number of imported varieties reflects firms' search activities,

this pattern suggests that lower search costs have had differential effects across the firm size distribution.

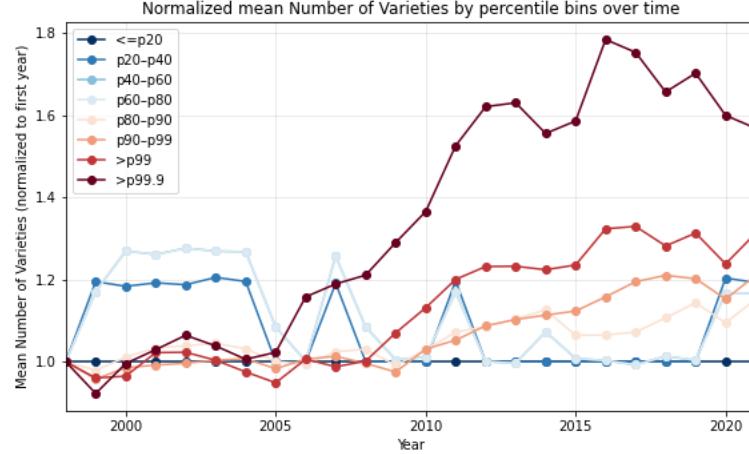
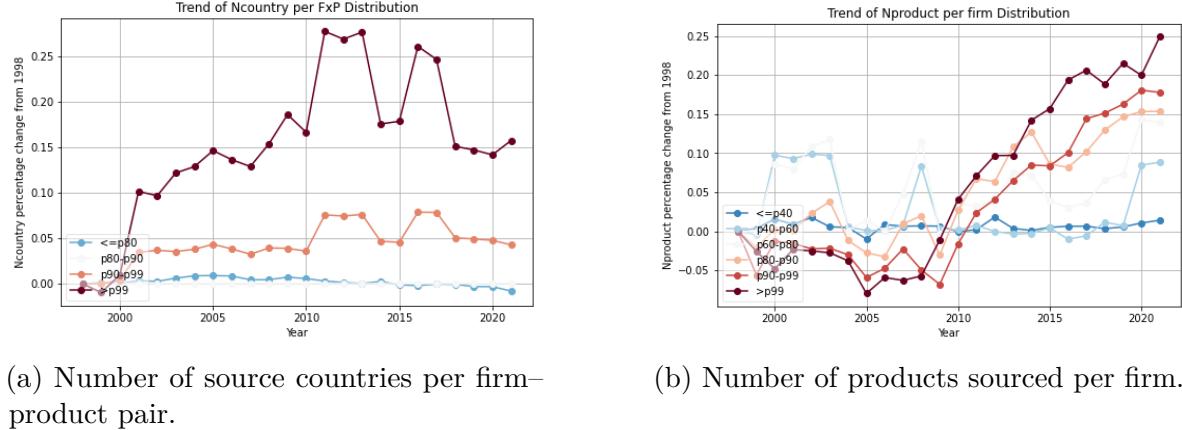


Figure 1: Changes in the no. of imported varieties across no. of varieties bins

A large part of this pattern is driven by the entry of smaller firms into importing. When I restrict the sample to a balanced panel of firms that import every year in the data, all firms show an increase in the number of imported varieties relative to 1998.



(a) Number of source countries per firm–product pair.

(b) Number of products sourced per firm.

Figure 2: Sourcing scope across firms: Panel A shows the number of source countries from which each firm sources a given eight-digit product (left panel) or the number of distinct eight-digit products sourced by each firm in a given year (right panel). In both cases, firms at the top of the distribution expand their sourcing scope more rapidly than others.

The same pattern emerges when examining the number of countries from which each firm sources a given eight-digit product (left panel) or the number of distinct eight-digit products sourced by each firm in a given year (right panel). In both cases, firms at the top of the distribution expand their sourcing scope more rapidly than others.

### 3.1.2 Importing Firm Size Dispersion

In this subsection, I examine changes in firm size, measured by annual sales, across percentiles from 1998 to 2021. Specifically, I analyze how firm sizes at different points of the distribution have evolved relative to their 1998 levels, normalizing each percentile by its 1998 value.

The results show a substantial reduction in firm size at the median and lower percentiles. For instance, the 20th percentile firm in 2021 is approximately 50% smaller than the 20th percentile firm in 1998. This large decline likely reflects the effect of lower search costs, which previously acted as a barrier to entry into importing. As search costs fell, less productive firms that had faced prohibitively high entry barriers could begin to import. This pattern indicates that the cost of entering international supply chains has declined considerably.

In contrast, firm sizes at the upper end of the distribution have grown substantially, by up to 75% at the 99th percentile and 125% at the 99.9th percentile. This expansion likely reflects reduced search costs that enable larger incumbent firms to expand their supplier base, as discussed in the previous subsection. Among firms that were already importers in 1998, sales increased across the entire distribution, suggesting that incumbents also benefited from the reduction in search frictions. The corresponding graph is shown in the [Data Appendix](#).

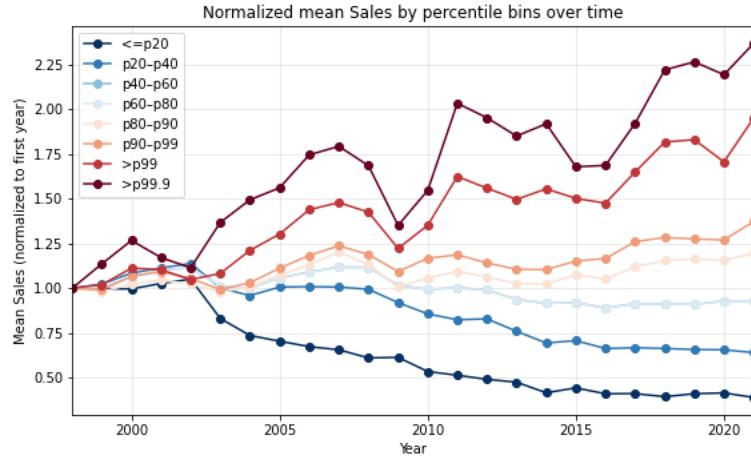


Figure 3: Changes in importing firm size across percentiles, 1998–2021

Overall, these findings point to increasing dispersion in firm size within the importing sector. A plausible explanation is that falling search costs simultaneously lower entry barriers for small firms and reduce expansion costs for large firms, thereby affecting both the extensive and intensive margins of the firm-size distribution.

## 3.2 Static Moments

This subsection documents key cross-sectional patterns that guide the calibration and motivate the model. I first examine how input prices vary with firm size, capturing differences in buyer–supplier bargaining conditions. I then show how firms with better internet connectivity tend to operate larger supplier networks, consistent with lower frictions in input sourcing.

### 3.2.1 Input Prices and Firm Size

In this section, I primarily examine what determines input prices, as we know that it is another important channel that input markets can affect productivity and allocative efficiency.

I examine whether larger firms pay lower input prices by estimating

$$\text{InputPrice}_{f,t} = \beta_0 + \beta_1 \log\left(\frac{\text{Size}_{f,t}}{\overline{\text{Size}}_{i,t}}\right) + \epsilon_{f,t}, \quad (1)$$

where  $\text{InputPrice}_{f,t}$  is the value-weighted average of log price deviations defined in Section 2.1, and  $\text{Size}_{f,t}$  is measured by either (i) the number of employees or (ii) the value-weighted average of import quantities. Both size measures are demeaned by their industry–year average  $\overline{\text{Size}}_{i,t}$  so that  $\beta_1$  is an industry–year–relative elasticity. I have also include estimates using other size measures, such as sales or TFPQ, in the [Data Appendix](#).

Table 1: Regression Results: Input Prices and Firm Size

	(1) Employment	(2) Import Quantity
log No. of Workers	-0.1629*** (0.0017)	—
Input Quantity	—	-0.2774*** (0.0014)
Observations	172,602	172,602
$R^2$	0.0534	0.2837

Notes: Dependent variable is log value–weighted import unit value at the firm–year level. Size measures are demeaned by industry  $\times$  year means. Standard errors in parentheses. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

I find a strong negative relationship: a 10% larger firm pays about 1.6% lower input prices when size is proxied by employment and about 2.8% lower when proxied by

import quantities. This is consistent with bargaining power or market power on the buyer's side. When buyer firms have better outside options, they can leverage it and obtain a better deal.

It is also important to note that the slope is similar every year. That indicates while there is some oligopolistic market power, the power has not changed much over time.

### 3.2.2 Connectivity, Search Costs, and Supplier-Network Expansion

Evidence from the import market shows growing dispersion in firms' supplier networks alongside rising concentration. Many factors could generate these patterns, but I focus on *buyer search costs* as a parsimonious mechanism linking firm-level supplier acquisition to aggregate concentration. When search frictions fall, firms can locate and bargain with additional suppliers more easily. This expands their input variety networks, especially for larger firms, and through both extensive and intensive margins, contributes to higher market concentration. To provide evidence consistent with this mechanism, I relate the size of a firm's supplier network to local improvements in digital connectivity.

From broadband data, I use the share of firms in a municipality with access to fiber internet as a proxy for the inverse of buyer search costs. Fiber is the fastest and most reliable broadband technology, supporting high-quality communication such as video calls and real-time coordination with suppliers. The diffusion of fiber connectivity across Swedish municipalities can be seen in the [Data Appendix](#).

Let  $\text{Varieties}_{f,t}$  denote the number of distinct imported input varieties used by firm  $f$  in year  $t$ . I estimate

$$\begin{aligned} \text{Varieties}_{f,t} = & \beta_1 \text{FiberCoverage}_{m,t} + \beta_2 [\text{FiberCoverage}_{m,t} \times \text{BigFirm}_{f,t}] \\ & + \text{FE}_t + \text{FE}_i + \text{FE}_m + \varepsilon_{f,t}, \end{aligned}$$

where  $\text{FiberCoverage}_{m,t}$  is the fraction of firms with fiber access in municipality  $m$  and year  $t$ .  $\text{BigFirm}_{f,t}$  is an indicator equal to one for firms with sales above the contemporaneous industry mean. All regressions include year, industry, and municipality fixed effects, and standard errors are clustered by municipality.

Table 2: Connectivity and supplier-network size (levels specification)

	(1)	(2)
Fiber coverage	5.229*** (1.004)	0.307 (0.200)
Fiber $\times$ BigFirm	—	23.314*** (3.978)
BigFirm	—	17.882*** (1.248)
Observations	349,400	349,400
$R^2$	0.024	0.070
Industry FE	Yes	Yes
Year FE	Yes	Yes
Municipality FE	Yes	Yes

*Notes:* Dependent variable is the level of imported input varieties for firm  $f$  in year  $t$ . “BigFirm” is an indicator equal to one for firms with sales above the contemporaneous industry mean. Standard errors are clustered by municipality.  
 \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

The estimates in Table 2 show that supplier network size expands more strongly for large firms in municipalities with higher fiber coverage. The coefficient on overall fiber coverage is small and statistically insignificant once the interaction with the big-firm indicator is included, while the interaction term is large and precisely estimated. This pattern suggests that improved digital connectivity disproportionately benefits firms already operating at a larger scale, consistent with a mechanism where lower search costs amplify existing asymmetries in supplier access. The resulting concentration of supplier relationships mirrors the model’s prediction that reductions in communication frictions expand networks at the top of the distribution.

Two additional specifications are presented in the Appendix. The first, described in [Robustness to Pretrend](#), replaces the continuous treatment with a binary indicator that equals one when municipal fiber coverage exceeds 50 percent and adds a *pre-event placebo dummy* that equals one in the three years before the event. This specification tests not only for anticipatory behavior but also for potential endogeneity in the timing of fiber rollout. A possible concern is that municipalities with stronger demand for digital connectivity—especially those with many large firms—might have received fiber earlier. If this were the case, supplier-network expansion could reflect pre-existing local dynamics rather than the causal effect of improved connectivity. The pre-event placebo dummy helps assess this alternative story. The main pre-event coefficient is close to zero and statistically insignificant. The coefficient on the interaction term

$Preevent \times BigFirm$  is positive (3.49) and statistically significant at the 5 percent level, but its magnitude is less than 15 percent of the corresponding post-treatment coefficient (23.3) and therefore economically minor. Overall, these estimates indicate that, while large firms in municipalities with higher fiber coverage may have experienced a modest pretrend in supplier-network expansion, the effect is too small to meaningfully challenge the parallel-trends assumption. This supports the interpretation that the main results are not driven by pre-existing differential trends.

The second specification, presented in [Robustness Using Growth Rate Specification](#), re-estimates the relationship using the percentage change in imported varieties as the dependent variable. This alternative outcome measure normalizes by firm size and confirms that the main results are not driven by differences in scale between large and small firms.

### 3.3 Summary

I begin by documenting three central empirical patterns that motivate the model and guide its quantitative analysis.

First, the distribution of imported input varieties across firms has become more unequal over time. Firms at the top now import many more varieties than two decades ago, while firms at the bottom import roughly the same number as before. At the same time, the sales distribution among importers has widened in a similar way, suggesting that changes in input sourcing are linked to the growing gap in firm size.

Second, larger and more productive firms pay lower unit prices for comparable imported inputs. Input unit values decline with firm size across several measures, consistent with stronger bargaining positions or better outside options among large buyers. Firms that face lower input prices also tend to earn higher profit shares, directly linking input cost heterogeneity to firm performance.

Third, using the staggered rollout of fiber broadband as a proxy for improved input market connectivity, I find that firms in municipalities with higher fiber coverage import more distinct input varieties. This expansion is concentrated among large firms, consistent with lower search costs allowing them to locate and contract with more suppliers.

Taken together, these patterns suggest that falling search frictions in input markets allow large firms to expand their supplier networks more rapidly, increasing productivity at the top and contributing to greater concentration.

These empirical findings motivate the key features of the model: the connection between supplier networks and firm scale, bargaining over input prices, and heterogeneous responses to lower search costs. The model uses these facts to quantify how improvements in input market efficiency reallocate resources toward more productive firms and affect aggregate concentration.

In addition, I incorporate two auxiliary regularities from the literature that support the model’s design. First, buyer–supplier relationships are short-lived, typically lasting about one year, as shown by Martin et al. (2023), which justifies modeling the supplier network as re-optimized each period. Second, inventory considerations are minor for most importers in an annual framework, since firms restock imported inputs roughly every 150 days on average (Alessandria et al., 2010).

## 4 Theory

Motivated by these empirical findings, I develop a model of buyer–supplier search and bargaining that links reductions in search frictions to supplier network formation and market concentration. The model builds on a standard framework with monopolistic competition in the final goods market. Downstream firms, interpreted as buyers, produce final goods using a composite of intermediate inputs. The novel contribution lies in introducing heterogeneity and frictions into the input procurement process. Specifically, intermediate inputs differ in variety, and downstream firms must search for upstream suppliers that produce differentiated inputs. Upon matching, buyers and suppliers engage in bilateral bargaining over both price and quantity. This structure allows the model to capture how search frictions and contractual bargaining jointly shape input variety, firm-to-firm link formation, and ultimately production outcomes.

The economy consists of upstream suppliers and downstream producers. Each upstream supplier  $u$  uses labor  $l$  to produce an intermediate input  $x_u$  according to a linear production technology:

$$x_u = z_u l,$$

where  $z_u$  denotes the supplier’s productivity. The cost of production is  $\frac{w_{\text{foreign}}}{z_u}$ , where  $w_{\text{foreign}}$  is the foreign wage. Without loss of generality, I henceforth assume  $w_{\text{foreign}}$  to be 1, so all the efficiency measure of the supplier is captured by  $z_u$ .<sup>4</sup> Productivity  $z_u$  are observable to the buyer once match is formed.

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<sup>4</sup>It will be same if I just named another efficiency variable  $e_u = \frac{w_f}{z_u}$ , but as I don’t distinguish between different foreign countries that doesn’t matter.

Downstream firms combine a continuum of differentiated intermediate inputs with labor to produce the final good. The production function of a representative downstream firm is given by

$$y = zX^\alpha l^{1-\alpha}, \quad X = \left( \sum_u x_u^\rho \right)^{\frac{1}{\rho}},$$

where  $y$  is output,  $z$  is the downstream firm's productivity, and  $l$  is the amount of labor used in final production. The term  $X$  represents the composite of all inputs sourced from upstream suppliers. The parameter  $\alpha \in (0, 1)$  captures the expenditure share on intermediate inputs, while  $\rho \in (0, 1]$  determines the elasticity of substitution across input varieties. A higher  $\rho$  implies that inputs are more easily substitutable, whereas a lower  $\rho$  corresponds to greater complementarity among input varieties.

The model timing is as follows: all downstream firms begin the period without any inputs. Each firm first decides whether to pay a search cost to look for suppliers or to stop searching altogether. A firm that chooses to search is randomly matched with a potential supplier. Upon meeting, the two parties bargain over a binding contract that specifies both the quantity of inputs to be delivered and the corresponding payment. After reaching an agreement, the downstream firm decides whether to terminate its search or to incur the search cost again to potentially find additional suppliers.

When all firms have completed their search and chosen to stop, production of the final good takes place simultaneously across all firms. The entire sequence of search, matching, bargaining, and production occurs instantaneously within the model period. To characterize equilibrium behavior, I solve the model by backward induction, starting from the production stage.

Now, consider a downstream firm with productivity  $z$  that has already completed its search process and is matched with  $N$  upstream suppliers indexed by  $u = 1, \dots, N$ . Let  $\mathbf{x} = (x_1, x_2, \dots, x_N)$  denote the vector of input quantities purchased from these suppliers. The composite input used in final production is given by

$$X(\mathbf{x}) = \left( \sum_{u=1}^N x_u^\rho \right)^{\frac{1}{\rho}}.$$

Given its existing input bundle  $\mathbf{x}$ , the downstream firm chooses price  $p$  and labor  $l$  to maximize profits. The value of operating without further search, denoted  $V^{NS}(z, \mathbf{x})$ , is

$$V^{NS}(z, \mathbf{x}) = \max_{p, l \geq 0} [p z X(\mathbf{x})^\alpha l^{1-\alpha} - wl]$$

subject to product-market demand

$$y = C \left( \frac{p}{P} \right)^{-\epsilon},$$

where  $w$  is the wage rate,  $C$  is aggregate consumption,  $P$  is the aggregate price index, and  $\epsilon > 1$  is the elasticity of substitution across differentiated final goods. Notice that this is the familiar monopolistic competition demand curve. This stage captures the production and pricing decisions of a firm that has concluded its supplier search and operates given its current network of input relationships.

There is a closed form solution that can express  $V^{NS}$  in terms of only state  $(z, \mathbf{x})$  and parameters. See details of derivations in the [Closed Form Solution for Important Variables](#).

Before making a decision to stop searching, a downstream firm with productivity  $z$  and an existing input portfolio  $\mathbf{x}$  decides whether to continue searching for additional suppliers or to stop and proceed directly to production. The firm's value function is

$$V(z, \mathbf{x}) = \max \{ V^S(z, \mathbf{x}), V^{NS}(z, \mathbf{x}) \},$$

where  $V^S(z, \mathbf{x})$  is the value of continuing to search for new suppliers, and  $V^{NS}(z, \mathbf{x})$  is the value of stopping, as defined in previous step.

If the firm chooses to search, it incurs a search cost  $\kappa$  measured in labor units and faces random matching with potential upstream suppliers. The expected value of searching is

$$V^S(z, \mathbf{x}) = \int V^m(z, z_u, \mathbf{x}) dF(z_u) - w \kappa,$$

where  $z_u$  denotes the productivity of a prospective supplier drawn from distribution  $F(z_u)$ , and  $w$  is the wage rate. The term  $\int V^m(z, z_u, \mathbf{x}) dF(z_u)$  represents the ex-ante expected value upon matching with random supplier of productivity  $z_u$  with probability given by  $F(z_u)$ . After paying the search cost, the firm proceeds to Stage 1, where it negotiates a binding contract specifying the input quantity  $x_{\text{new}}$  and the corresponding transfer payment  $T$ .

When a downstream firm with productivity  $z$  and existing input bundle  $\mathbf{x}$  meets a new upstream supplier with productivity  $z_u$ , the two parties negotiate a binding contract over the quantity of the new input  $x_u$  and the transfer payment  $T$ . The outcome of this bilateral negotiation maximizes a Nash product of buyer and seller surpluses:

$$\max_{T, x_u \geq 0} \left( V(z, \mathbf{x}_{\text{new}}) - T - V(z, \mathbf{x}) \right)^\theta \left( T - \frac{w_{\text{foreign}}}{z_u} x_u \right)^{1-\theta},$$

where  $\theta \in (0, 1)$  denotes the bargaining weight of the buyer,  $V(z, \mathbf{x}_{\text{new}})$  is the downstream firm's value after adding the new input, and  $V(z, \mathbf{x})$  is its pre-match value. The term  $T$  represents the payment from the buyer to the supplier, and  $\frac{w_{\text{foreign}}}{z_u} x_u$  is the supplier's production cost, which depends on the foreign wage  $w_{\text{foreign}}$  and the supplier's productivity  $z_u$ .

The first term in the Nash product captures the buyer's surplus from obtaining the new input, while the second term captures the seller's surplus from the transaction. The negotiated outcome determines both the optimal quantity  $x_u$  traded and the transfer  $T$ . The value of a successful match to the downstream firm is then given by

$$V^m(z, z_u, \mathbf{x}) = V(z, \mathbf{x}_{\text{new}}) - T,$$

which represents the firm's continuation value after accounting for the payment to the new supplier.

## 4.1 Recursive CES Aggregator and State Compression

Now, I have always use  $(z, \mathbf{x})$  as state variables, where  $\mathbf{x}$  is a vector that is growing. If we assume each upstream suppliers is one variety, the length of  $\mathbf{x}$  can go up to 1,000 for bigger firms. This large state space poses significant difficulties in solving the model. In this subsection, I will provide a solution to this computation problem.

Consider a downstream firm with productivity  $z$  that has already formed relationships with  $N$  upstream suppliers providing input quantities  $\mathbf{x} = (x_1, \dots, x_N)$ . Inputs are aggregated through a CES composite

$$X = \left( \sum_{u=1}^N x_u^\rho \right)^{1/\rho}, \quad \rho \in (-\infty, 1],$$

so that final output depends on  $X$  and labor as specified in the environment. When the firm successfully bargains with a new supplier for quantity  $x_{\text{new}} \geq 0$ , the input vector expands to  $\mathbf{x}_{\text{new}} = (x_1, \dots, x_N, x_{\text{new}})$  and the composite updates exactly as

$$X_{\text{new}} = \left( \sum_{u=1}^N x_u^\rho + x_{\text{new}}^\rho \right)^{1/\rho} = (X^\rho + x_{\text{new}}^\rho)^{1/\rho}.$$

This identity implies that the history of individual matches can be summarized by the

scalar state  $X$ . The continuation value can therefore be written as

$$V(z, \mathbf{x}) \equiv V(z, X),$$

and future negotiations with prospective suppliers only require the pair  $(z, X)$  together with the draw of the supplier's productivity. In particular, if a meeting yields productivity  $z_u$ , the contracting problem selects  $(x_{\text{new}}, T)$  to solve the Nash program given in the bargaining subsection, and the next period state becomes  $X_{\text{new}} = (X^\rho + x_{\text{new}}^\rho)^{1/\rho}$ .

*Notation.*  $x_u$  denotes the quantity bought from supplier  $u$ .  $X$  is the CES composite of all purchased inputs. The parameter  $\rho$  governs substitution across input varieties, with elasticity  $\sigma = 1/(1 - \rho)$  under the standard CES parameterization. The symbol  $x_{\text{new}}$  is the quantity negotiated with a newly met supplier, and  $X_{\text{new}}$  is the updated composite after adding this input.

*Methodological contribution.* The recursion

$$X_{\text{new}} = (X^\rho + x_{\text{new}}^\rho)^{1/\rho}$$

allows the dynamic buyer problem with an expanding set of differentiated input relationships to be solved on a low dimensional state that consists of  $(z, X)$  rather than the full vector  $\mathbf{x}$ . This reduction is exact under CES aggregation and does not rely on symmetry or distributional approximations. It delivers substantial computational gains for value function iteration and policy iteration because one replaces an ever growing vector of inputs with a single sufficient statistic for procurement history. To our knowledge, prior work uses CES aggregation mainly to obtain tractable demand systems, nested aggregator structures, or sufficient statistics, but not to implement recursive state compression for search and bargaining over input relationships. This subsection formalizes that compression and uses it to build a tractable quantitative model of firm to firm matching with differentiated inputs.

## 5 Simplified Model

I use a stripped down environment to highlight how a change in the search cost  $\kappa$  alters input choices and output. I remove supplier heterogeneity by setting  $z_u = \bar{z}_u = 1$  for all potential suppliers. I set the bargaining weight to  $\theta = 1$ , which gives the buyer full bargaining power. Under these assumptions, the negotiated input price equals the seller's unit production cost,  $1/\bar{z}_u$ .

## 5.1 Objective and Reduced Contracting Problem

With no seller surplus, the one match contracting problem is

$$\max_{x \geq 0} \left[ V(z, X_{\text{new}}) - V(z, X) - \frac{1}{z_u} x \right],$$

where  $X$  is the current CES composite of inputs and  $X_{\text{new}} = (X^\rho + x^\rho)^{1/\rho}$  after purchasing quantity  $x$  from the newly met supplier. To study comparative statics in a transparent way, I consider a collapsed version that chooses the full procurement path directly. The collapsed problem is equivalent as the original sequential setup because there are no randomness and no strategic interaction as buyer have full power. I impose symmetry across relationships, so  $x_i = x$  for all  $i \in \{0, \dots, N\}$ . The problem becomes

$$\max_{N \in \mathbb{N}_+, x \geq 0} \left\{ K_1 \left[ z \left( \sum_{i=0}^N x_i^\rho \right)^{\frac{\alpha}{\rho}} \right]^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} - \frac{1}{z_u} Nx - Nw\kappa \right\},$$

where  $K_1$  collects constants and demand shifters,  $\alpha \in (0, 1)$ , and  $\epsilon > 1$  is the demand elasticity in the final goods market. Symmetry implies  $\sum_{i=0}^N x_i^\rho = Nx^\rho$ .

## 5.2 Optimal Number of Searches

The optimal number of supplier searches satisfies

$$N^* = \left[ \frac{K_2 z^{\epsilon-1} \bar{z}_u^{\alpha(\epsilon-1)}}{w \kappa} \right]^\phi, \quad \phi = \frac{\rho}{\rho - \alpha(\epsilon - 1)(1 - \rho)}.$$

I use the following expression for  $K_2$  in the computations:

$$K_2 = \epsilon^{-\epsilon} \left( \frac{w}{(\epsilon - 1)(1 - \alpha)} \right)^{(1-\alpha)(1-\epsilon)} \left( \frac{C}{P^{-\epsilon}} \right) (\alpha(\epsilon - 1))^{\alpha(\epsilon-1)} \frac{\alpha(1 - \rho)}{\rho}.$$

The comparative static with respect to search costs is

$$\frac{\partial \ln N^*}{\partial \ln \kappa} = -\phi.$$

A proportional decline in  $\kappa$  raises  $N^*$  by the same proportion across all  $z$  when  $\phi > 0$ , which requires  $\rho - \alpha(\epsilon - 1)(1 - \rho) > 0$ . Higher substitutability across inputs (higher  $\rho$ ) increases the gain from additional relationships and raises the elasticity of  $N^*$  with respect to  $\kappa$ .

### 5.3 Discrete Choice and Dispersion

Because  $N$  is discrete, firms adjust the number of relationships in steps. Large firms, for which the marginal value of an extra link varies smoothly with  $z$ , display near continuous adjustments. Small firms face lumpy changes as they cross thresholds at which an additional relationship becomes profitable.

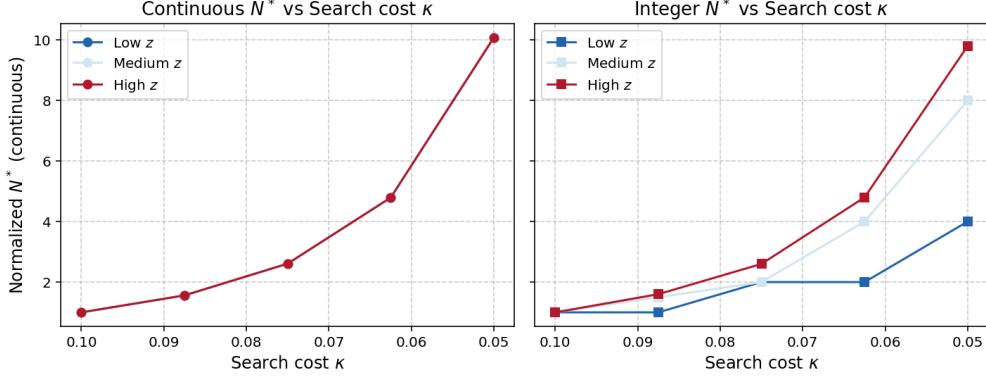


Figure 4: Discrete supplier choice generates lumpy adjustment for small firms and smoother adjustment for large firms.

### 5.4 Extensive Margin and Entry

A lower search cost also shifts the extensive margin. Some small firms that previously chose zero imported relationships now find it profitable to enter.

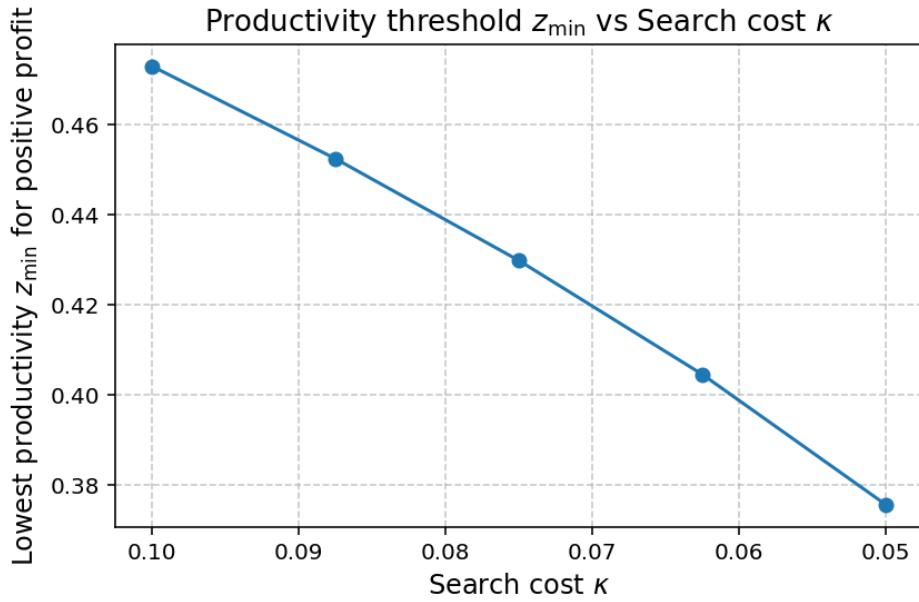


Figure 5: A decline in  $\kappa$  increases entry at the extensive margin.

## 5.5 Summary

A reduction in the search cost increases the optimal number of supplier relationships on the intensive margin through the elasticity  $\phi$ . The discreteness of  $N$  generates heterogeneous adjustment, with smooth changes for large firms and lumpy jumps for small firms. The same reduction in search costs also activates the extensive margin by inducing entry among small firms. These forces together reshape the distribution of firms across input relationships and output levels.

## 6 Price Dispersion with Identical Suppliers

I now relax the assumption that the buyer has full bargaining power. Sellers receive weight  $1 - \theta$  in the bargaining process, and the input quantity is chosen efficiently to maximize the joint surplus. To isolate the role of bargaining, I continue to assume that all potential suppliers have identical productivity  $\bar{z}_u$ .

The efficient bargaining problem is

$$\max_{x \geq 0} \left( V(z, X_{\text{new}}) - V(z, X) - \frac{1}{\bar{z}_u} x \right),$$

where  $V(z, X_{\text{new}})$  is the buyer's continuation value after forming a new supplier relationship, and  $V(z, X)$  is the buyer's outside option prior to the new match. The first order condition defines the efficient input  $x^*(z, \bar{z}_u)$ , which increases with buyer productivity,

$$\frac{dx^*(z, \bar{z}_u)}{dz} > 0.$$

When there is no randomness in supplier productivity, the buyer's outside option is the value of meeting another supplier with the same productivity  $\bar{z}_u$ , net of the search cost  $w\kappa$ :

$$V(z, X) = V(z, X_{\text{new}}) - w\kappa.$$

This expression follows from the deterministic setup in which every new search yields an identical supplier; Because suppliers are identical, the same efficient input level  $x^*(z, \bar{z}_u)$  applies to any potential match. The only difference between continuing and stopping is the additional search cost.

Under Nash bargaining with weights  $(\theta, 1 - \theta)$  for the buyer and the seller respectively,

the total transfer from the buyer to the supplier is

$$T = (1 - \theta) [V(z, X_{\text{new}}) - V(z, X)] + \theta \frac{1}{\bar{z}_u} x^*.$$

Substituting the outside option expression yields

$$T = (1 - \theta) \left\{ V(z, X_{\text{new}}) - [V(z, X_{\text{new}}) - w\kappa] \right\} + \theta \frac{1}{\bar{z}_u} x^*,$$

which simplifies to

$$T = (1 - \theta)w\kappa + \theta \frac{1}{\bar{z}_u} x^*.$$

The payment therefore consists of a fixed component  $(1 - \theta)w\kappa$  that shares the buyer's surplus and a variable component  $\theta \frac{1}{\bar{z}_u} x^*$  reflecting the supplier's production cost.

Dividing through by  $x^*$  gives the per unit input price,

$$p_u = \frac{T}{x^*} = (1 - \theta) \frac{w\kappa}{x^*} + \theta \frac{1}{\bar{z}_u}.$$

Because  $x^*$  increases with productivity  $z$ , the term  $\frac{w\kappa}{x^*}$  decreases with  $z$ , and more productive buyers pay a lower input price  $p_u$ . The degree of price heterogeneity across buyers depends on the relative magnitudes of  $\frac{1}{\bar{z}_u}$  and  $\kappa$ : when either  $\bar{z}_u$  or  $\kappa$  increases, dispersion in input prices also rises.

See Appendix [Sequential Search with Efficient Bargaining](#) for the full derivation of the sequential version of this model.

The example above illustrates one mechanism through which the model generates differential effects across firms of different sizes. However, this is not the only source of heterogeneity. Owing to the love of variety structure of the input aggregator  $X$ , firms with higher productivity  $z$  employ a larger number of input varieties. Consequently, even in the absence of explicit unit price dispersion  $(\sum_x^T)$  as analyzed in this section, their effective unit price, measured as total spending per composite input  $\frac{\sum_x^T}{X}$ , is lower. The CES aggregation therefore reinforces the cost advantage of more productive firms.

## 7 Quantitative Model and Results

### 7.1 Full Equilibrium Model

In the full model, I incorporate the possibility of in-house production as an outside option. It would be unrealistic to assume that firms are unable to produce without imported inputs. Instead, each firm can always produce internally using a basic technology that relies only on labor and it doesn't depend on its own productivity. The domestic labor market is perfectly competitive, so the in-house input  $M$  is available at the unit labor cost  $p_M = w$ .

After firms decide whether or not to search for foreign suppliers, their production technology is

$$y = z \max\{X, M\}^\alpha l^{1-\alpha},$$

where  $X$  denotes the composite of imported inputs and  $M$  represents in-house production.

The  $\max\{X, M\}$  operator is introduced purely for simplicity. It treats imported inputs  $X$  and in-house production  $M$  as perfect substitutes, so a firm either sources inputs from abroad or produces them internally. A smoother CES aggregator could allow firms to use both simultaneously, but it would introduce an additional continuous input-share choice and greatly increase computational complexity without affecting the qualitative insights. Using the max specification keeps the model transparent and emphasizes the key discrete choice between importing and in-house production.

### 7.2 General Equilibrium

In equilibrium, aggregate labor demand equals the fixed labor supply  $\bar{L}$ ,

$$\int (l_f + n_f \kappa) df = \bar{L},$$

where both production labor  $l_f$  and search effort  $n_f \kappa$  depend on the demand shifter  $CP^\epsilon$ . The general equilibrium solution is computed using a nested fixed point algorithm. In the outer loop, I update the aggregate demand term  $CP^\epsilon$  until the labor market clears. In the inner loop, I solve firms' dynamic problems by value function iteration, simulate the distribution of firms over productivity and input stocks  $(z, X)$ , compute implied labor demand, and feed the result back to the outer loop to update  $CP^\epsilon$ .

The household's budget constraint is given by

$$\max_{c_i} \quad C = \left( \int (y_i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad \text{s.t.} \quad PC \leq wL_t + \pi + \text{export},$$

where the export term reflects that, since the economy imports some intermediate inputs, an equivalent value of domestic goods must be exported to balance trade. I assume these exports are sold at the domestic price  $P$ .

The goods market clearing condition

$$\int \mathbb{E}[y|z] dz = C$$

is automatically satisfied and therefore does not impose an additional equilibrium restriction. In the CES aggregator, aggregate demand  $C$  is defined as the integral of firm-level demands, and each firm's output  $y$  exactly equals the quantity demanded in equilibrium. Because total expenditure across varieties equals total revenue by construction of the CES price index, aggregate supply equals aggregate demand identically once firms produce their equilibrium outputs.

### 7.3 Numerical Solution and Simulation

To compute the equilibrium described above, I implement the nested fixed-point algorithm numerically. The outer loop, coded in Python, updates the aggregate demand term  $CP^\epsilon$  until the labor market clears, while the inner loop, written in C++, solves the firm's dynamic problem and returns implied labor demand to the outer loop.

**Inner Loop: Solving the Firm Problem.** Each firm is characterized by productivity  $z$  and the accumulated measure of contracted inputs

$$X = \left( \sum_u x_{i,u}^\rho \right)^{1/\rho},$$

which summarizes its past contracting history. When a firm forms a new contract for input  $x_{\text{new}}$ , the next-period state becomes

$$X_{\text{new}} = (X^\rho + x_{\text{new}}^\rho)^{1/\rho}.$$

At each state  $(z, X)$ , the firm decides whether to search for a new supplier. Conditional on meeting a supplier with productivity  $z_u$ , the two parties bargain over  $(x, T)$  to

maximize the Nash product:

$$\max_{x,T} [V(z, X_{\text{new}}) - V(z, X) - T]^{1-\theta} [T - K(x)]^\theta,$$

where  $K(x) = x/z_u$  is the supplier's cost function.

Because this problem is strictly concave in the transfer  $T$  (see [Concavity of the Bargaining Problem](#) for a formal proof), the first-order condition with respect to  $T$  satisfies

$$(1 - \theta) [V(z, X_{\text{new}}) - V(z, X) - T] = \theta [T - K(x)].$$

Solving for  $T$  gives the closed-form transfer rule

$$T = (1 - \theta)V(x) + \theta K(x), \quad V(x) = V(z, X_{\text{new}}) - V(z, X).$$

Substituting this expression back into the Nash product eliminates  $T$  and reduces the problem to a one-dimensional maximization:

$$\max_{x \geq 0} \theta^\theta (1 - \theta)^{1-\theta} [V(z, X_{\text{new}}) - V(z, X) - K(x)].$$

This analytical reduction is crucial for tractability: it transforms a two-dimensional problem into a single-variable search over  $x$ . Because the objective is not globally concave in  $x$  due to the discrete search cost, I solve this problem by grid search rather than gradient-based optimization.

Given the optimal  $x^*$  and corresponding transfer  $T^*$ , the expected value of searching is updated as

$$V^S(z, X) = \mathbb{E}_{z_u}[V(z, X_{\text{new}}) - T^*(z, X, z_u)] - w\kappa,$$

and the overall value function is

$$V(z, X) = \max\{V^S(z, X), V^{NS}(z, X)\}.$$

**Algorithmic Implementation.** The inner loop is implemented in C++ for computational efficiency. Value function iteration proceeds on discretized grids for  $z$  and  $X$  with several features that enhance numerical performance:

1. *Adaptive over-relaxation ( $\omega > 1$ )*. To accelerate convergence, I apply an adaptive over-relaxation step when updating the value function:

$$V_{t+1}^S = (1 - \omega)V_t^S + \omega \tilde{V}_t^S, \quad \omega > 1.$$

Unlike standard Bellman operators with discounting, the recursion here takes an affine form  $V = \mathbb{E}[V'] - K$  with a constant search cost  $K$ . This mapping behaves like an identity transformation with a fixed shift, which converges monotonically but slowly. Setting  $\omega > 1$  effectively “leans forward” toward the next iteration, pushing the update closer to the steady state without breaking stability. The idea is related to *successive over-relaxation (SOR)* methods for accelerating convergence in fixed-point problems, though applied here to a nonlinear value function mapping. The weight  $\omega$  is adjusted dynamically when oscillations arise, typically near the discrete boundary where firms switch between searching and not searching.

2. *Precomputed interpolation maps.* For each grid point  $X$  and potential input  $x$ , the implied next state  $X_{\text{new}} = (X^\rho + x^\rho)^{1/\rho}$  is mapped to its neighboring grid points in advance. This precomputation removes repeated binary searches during iteration and accelerates evaluation of continuation values  $V(z, X_{\text{new}})$ .
3. *Parallelization.* Because firm problems are independent across productivity types  $z$ , updates are executed in parallel across all  $z$ -states, exploiting shared-memory parallelism.

The iteration continues until the sup-norm difference between successive value functions falls below a tight tolerance. The resulting policy functions  $(x^*, T^*, V^S, V^{NS})$  characterize the firm’s optimal contracting and search behavior given aggregate conditions.

**Simulation and Aggregation.** After convergence, I simulate a large cross-section of firms to obtain the stationary distribution of states. Each firm starts with  $X = 0$  and draws productivity  $z$  from  $F(z)$ . In each simulated step, firms decide whether to search, draw suppliers  $z_u$  if searching, and form contracts according to  $(x^*, T^*)$ . The input stock evolves as  $X' = (X^\rho + x^{*\rho})^{1/\rho}$ . The simulation produces firm-level histories of inputs, payments, and search activity, which are aggregated to compute total labor demand. The outer loop then updates  $CP^e$  and repeats until the labor market clears.

**Interpretation.** The computational procedure combines a Python outer loop enforcing general equilibrium with a C++ inner loop solving firms’ dynamic problems. The recursive state representation  $(z, X)$ , the analytical elimination of  $T$ , and the adaptive over-relaxation scheme together make the model both tractable and faithful to the underlying theory.

## 7.4 Calibration

In this subsection, I describe the strategy of calibrating the model. I take some of the parameters from external sources and other calibrate data moments and model moments. I use the following parameters:

Table 3: Externally set parameters

Variable	Definition	Value(s)	Source
$\theta$	Buyer bargaining power	0.83	Alviarez et al. (2023)
$\epsilon$	Final market elasticity	4	Broda and Weinstein (2006)
$\rho$	Input elasticity	.75	Broda and Weinstein (2006)
$\alpha$	Cobb–Douglas intermediate share	0.7	Intermediate share (manufacturing)
$\mu_{z_u}$	Mean of supplier $z_u$	0	Normalization
$\sigma_{z_u}$	Std. of supplier $z_u$	0.235	Normalization

Table 4: Calibrated parameters and matched moments

Variable	Definition	Value(s)	Moment	Data	Model
$\mu_z$	Mean of buyer $z$	0	Top 1% vs. median sales ratio	259 → 474	129 → 211
$\sigma_z$	Std. of buyer $z$	0.235	Top 1% sales share	0.44 → 0.49	0.424 → 0.452
$\kappa_t$	Search cost	0.25 → 0.15	Mean number of varieties	19.8 → 24.4	19.7 → 24.3

Table 3 lists the parameters I take as given in the quantitative exercise. Buyer bargaining power is set to  $\theta = 0.83$ , taken from Alviarez et al. (2023). They develop a theory of market power that link buyers' market share, supplier market share and prices. Then, they use firm-to-firm trade data to estimate the bargaining power of U.S. firms. On the demand side, I adopt standard benchmark elasticities from Broda and Weinstein (2006): the final-goods market elasticity  $\epsilon = 4$  and an input elasticity of substitution across varieties equal to 4. This is also consistent with the Swedish-specific elasticity from Boppart et al. (2023), where they find a median elasticity of 4.39 within industries. The Cobb–Douglas share on intermediates is  $\alpha = 0.7$ , chosen to match the intermediate-input expenditure share in manufacturing. For supplier productivity, I normalize the mean to the downstream scale, setting  $\mu_{z_u} = 0$ , and choose a dispersion  $\sigma_{z_u} = 0.235$  to mirror the downstream distribution. None of these parameters are targeted in the calibration moments below; they serve as conventional benchmarks and normalizations.

Table 4 reports the calibration linking model primitives to sales and sourcing moments. To calibrate the moments, I begin with buyer productivity  $z$ , which I assume is lognor-

mally distributed. This leaves two parameters to pin down, so I target two sales-based moments because physical productivity is difficult to measure accurately with available data. Specifically, within industries with at least five firms (to avoid mechanically concentrated sectors), I compute (i) the top-1% sales share and (ii) the ratio of top-1% sales to the median; I then take the median estimate across industries. Anchoring the location parameter at  $\mu_z = 0$  aligns the model's top-1% vs. median sales ratio with the data, rising from  $129 \rightarrow 211$  in the model versus  $259 \rightarrow 474$  in the data. While setting the dispersion to  $\sigma_z = 0.235$  matches the increase in the top-1% sales share ( $0.424 \rightarrow 0.452$  in the model vs.  $0.44 \rightarrow 0.49$  in the data). Given the lack of reliable quality- and price-adjusted productivity measures, using the sales distribution avoids additional structure (e.g., TFPR or labor-productivity corrections) and ties the two lognormal parameters to the level and the tail of the distribution, respectively. Finally, I discipline the time path of search frictions by lowering the per-period search cost from  $\kappa_t : 0.25 \rightarrow 0.15$ , which reproduces the observed growth in the mean number of imported varieties (data:  $19 \rightarrow 23$ ; model:  $19.7 \rightarrow 24.3$ ).<sup>5</sup>

## 7.5 Firm Size Dispersions

To study how search frictions affect the firm size distribution, I reduce the buyer search cost parameter by 40 percent in the calibrated model and recompute the stationary equilibrium. Lower search costs make it easier for firms to locate and bargain with additional suppliers. The resulting expansion in supplier networks is uneven across firms: more productive firms add more varieties, while some smaller firms begin importing. The distributional consequence is a widening of firm size dispersion and, in the aggregate, higher market concentration.

In the data, the firm size distribution becomes more dispersed, as shown in the first graph. Smaller firms begin to enter the market, while the most productive firms continue to expand rapidly. This pattern reflects the discrete nature of supplier acquisition, the bargaining process, and the concavity of the profit function. When search costs are high, suppose the most productive firm has 20 suppliers while the median firm has only one. Adding one more supplier makes little difference for the large firm, but increasing from one to two suppliers is a substantial jump for the smaller firm. As explained in the simplified model, the most productive firms expand first and the fastest for this reason. As search costs decline further, less productive firms also begin to expand, but their growth eventually slows because the concavity of profits reduces the incentive

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<sup>5</sup>In the simulations I truncate the productivity distribution at the 0.01 and 99.99 percentiles.

to add more suppliers. However, bargaining and the love-of-variety structure of the intermediate-input aggregator counteract this force. Large firms with extensive supplier networks pay lower effective per-unit input costs, which reinforces their incentive to continue expanding their networks.

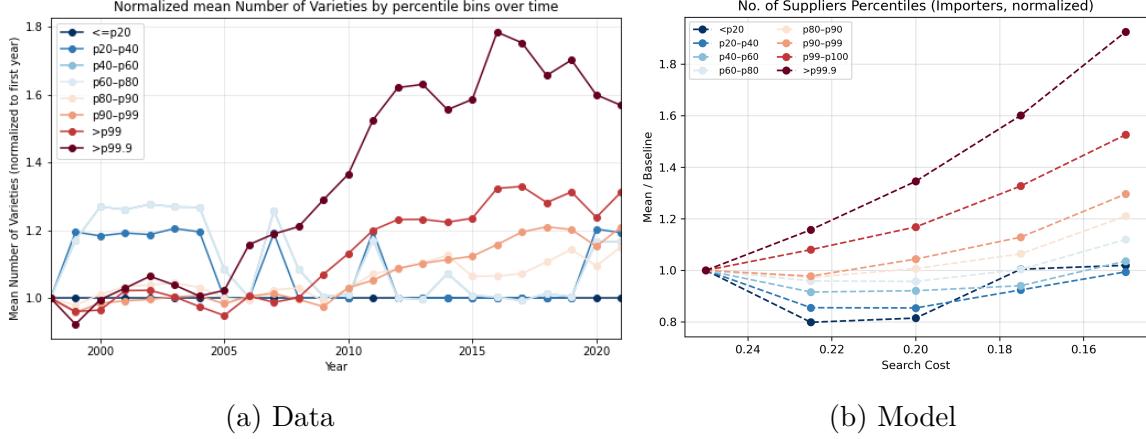


Figure 6: Number of imported input varieties: data vs. model. Both show stronger supplier expansion among larger and more productive firms.

Both the data and the model show a right-tail expansion in the number of supplier relationships. This increase in supplier networks directly affects firm output and sales. Firms with more suppliers produce more efficiently and grow faster, widening the overall distribution of sales. Figure 7 illustrates this relationship.

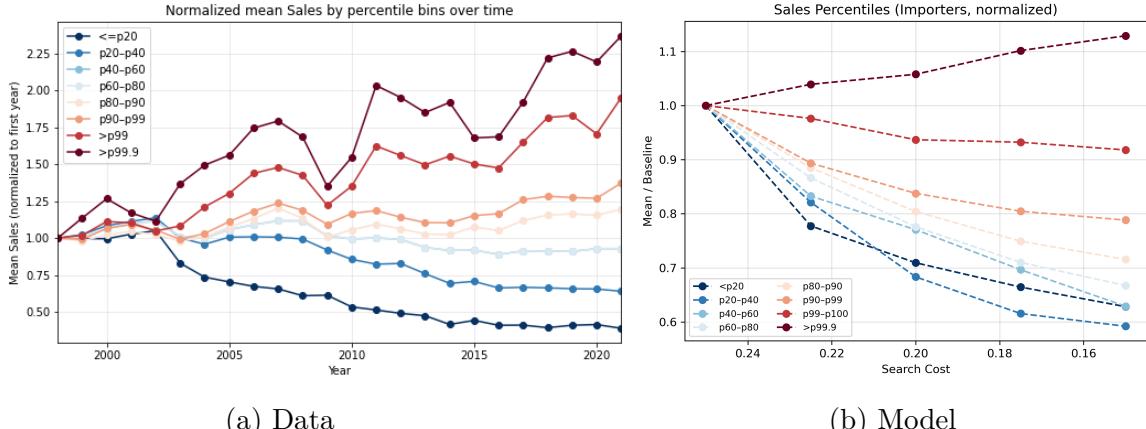


Figure 7: Firm-size dispersion (sales): data vs. model. Both show widening dispersion over time, consistent with the mechanism in the model.

## 7.6 Price level, GDP and Concentration

Given my parameterization, this experiment implies a **13%** decline in the aggregate price level and a **13%** increase in real GDP. On the concentration side (measured by the Herfindahl–Hirschman index of sales), the model delivers a **12.7%** increase. The corresponding rise in the data over 1998–2021 is **44%** overall, or **20%** when excluding the financial-crisis years. Taken together, reduced search costs account for roughly **29–64%** of the observed increase in concentration.

Table 5: Aggregate effects when buyer search costs fall by **40 percent**

Price level and output	
Price level	−13%
Real GDP	+13%
Market concentration (HHI of sales)	
Data (total, through 2021)	+44%
Data (excluding crisis years)	+20%
Model	+12.7%

*Notes:* Percent changes relative to the baseline calibrated equilibrium. Concentration is summarized by the Herfindahl–Hirschman index of firm sales.

## 8 Rising Foreign Supplier Efficiency

The previous section examined how declining search costs improve input market efficiency and reallocate production toward more productive firms. A closely related secular trend has been the rise in foreign supplier productivity, especially among manufacturers in China and other major exporting economies. These two forces—lower search frictions and higher supplier efficiency—represent complementary aspects of the same transformation in global input markets.

This section quantifies the effects of rising supplier productivity and compares them directly with those of falling search costs under a calibration that yields nearly identical aggregate outcomes.

### 8.1 Calibration

To isolate the supplier-side mechanism, I vary only the mean productivity of foreign suppliers, denoted  $\mu_{zu}$ . The experiment starts from a relatively low level and increases

until foreign suppliers reach a similar efficiency as domestic firms. The search cost parameter is fixed at  $\kappa = 0.15$ , the level used in the comparison described in Section 7.

The magnitude of the productivity shift is chosen so that both experiments generate comparable changes in the model’s demand shifter  $CP^\varepsilon$  and thus similar aggregate efficiency gains.

Table 6: Calibration for Rising Supplier Productivity Experiment

Parameter	Definition	Low Efficiency	High Efficiency
$\mu_{zu}$	Mean supplier productivity	-0.15	0
$\kappa$	Search cost parameter	0.15	0.15

## 8.2 Aggregate Results

The rise in supplier productivity raises real GDP by about 10.5 percent and lowers the aggregate price level by roughly 11.4 percent. Market concentration, measured by the Herfindahl–Hirschman Index of sales, increases by about 11.4 percent. These changes are very similar in magnitude to those generated by the fall in search costs considered earlier, indicating that both mechanisms produce comparable aggregate outcomes.

Table 7: Aggregate Effects when Supplier Productivity Rises to Domestic Level

Outcome	Change Relative to Low-Efficiency Benchmark
Price level	-11.4%
Real GDP	+10.5%
Market concentration (HHI of sales)	+11.4%

## 8.3 Firm-Level Outcomes

At the firm level, the simulated data show that the two mechanisms deliver similar changes in overall sales dispersion, but differ in the margin through which firms adjust. When supplier efficiency rises, the number of input varieties declines slightly on average and grows little for the most productive firms, consistent with efficiency improvements operating through existing relationships. The mean number of suppliers falls by about 22 percent, while the import share increases by 2.8 percentage points. Sales still rise broadly across the distribution. This decline in supplier count reflects substitution

toward more efficient foreign partners rather than a reduction in openness, as firms rely more on a smaller set of high-productivity suppliers.

When search costs fall, by contrast, the number of input varieties expands substantially, reflecting the creation of new supplier matches. In short, the data indicate that rising supplier efficiency operates primarily through the intensive margin (greater trade per match), whereas lower search costs operate through the extensive margin (more matches).

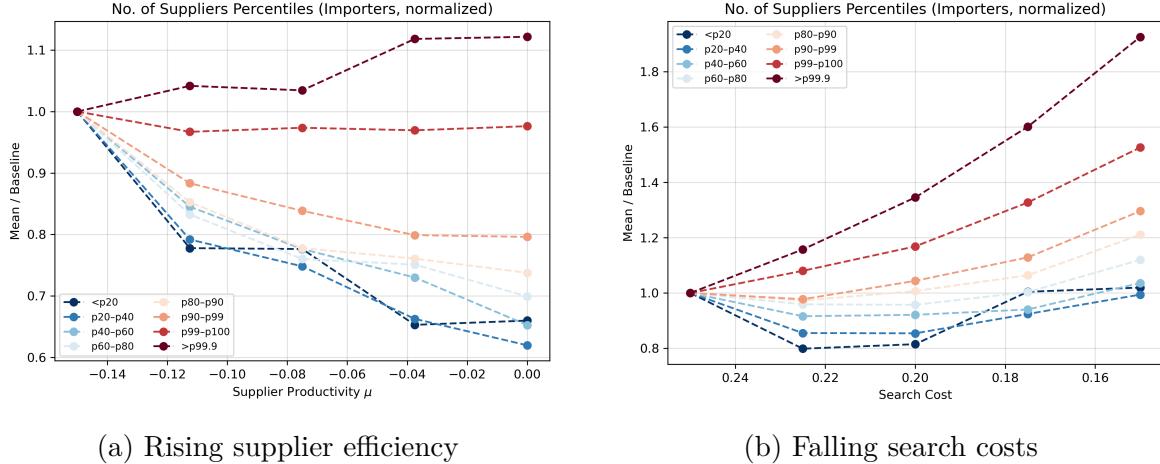


Figure 8: Number of input varieties across firm productivity percentiles. Rising supplier efficiency modestly changes variety counts, while falling search costs generate broad expansion through new matches.

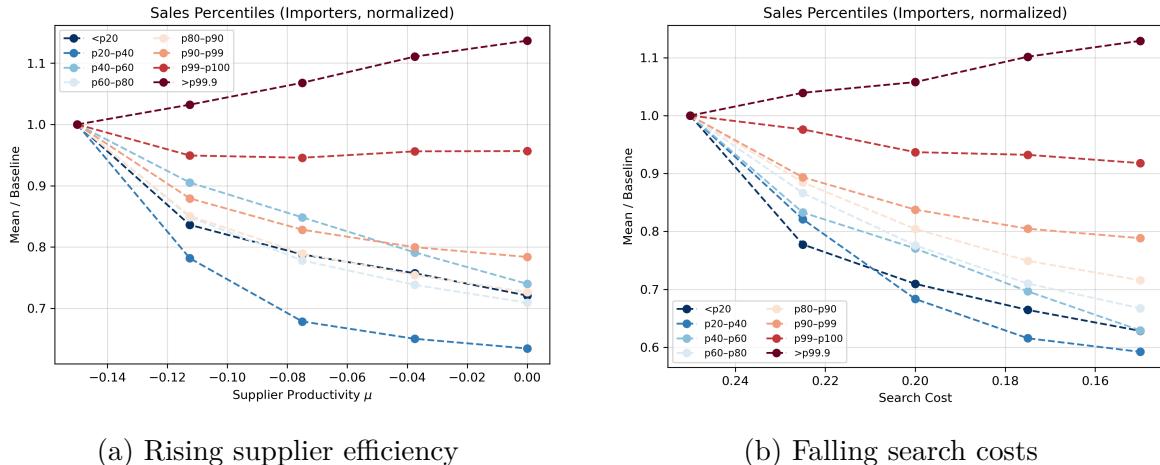


Figure 9: Firm sales across productivity percentiles. Both experiments raise sales across the distribution; the adjustment margin differs as described in the text.

## 8.4 Mechanism and Comparison

The simplified benchmark implies

$$N^* \propto \left( \frac{z z_u}{w \kappa} \right)^\phi,$$

so both higher supplier productivity ( $z_u \uparrow$ ) and lower search costs ( $\kappa \downarrow$ ) raise the returns to forming input relationships.<sup>6</sup> Calibrating each shock to yield similar aggregate gains highlights that, in the data from these experiments, the principal difference lies in the margin of adjustment rather than in average unit-cost or input-price dispersion.

Table 8: Comparison of Mechanisms at Matched Aggregate Gains

Outcome	Supplier Efficiency $\uparrow$	Search Cost $\downarrow$
Real GDP	+10.5%	+13%
Price level	-11.4%	-13%
Market concentration (HHI of sales)	+11.4%	+12.7%
Mean number of input varieties	-22%	+18%
Import share of inputs	+2.8 pp	+7.0 pp

Overall, both mechanisms deliver similar aggregate outcomes and similar sales dispersion. The distinguishing pattern in these simulations is the composition of adjustment: lower search costs expand the set of input varieties used by firms, while higher supplier efficiency raises the intensity of trade within a relatively stable or smaller set of varieties.

## 9 Counterfactual: Tariff

Another dimension of change in input markets comes from trade policy and institutional barriers to trade. Tariffs, export controls, and localization requirements have fluctuated over time, but recent years have seen a renewed rise in trade restrictions as part of a broader policy turn toward deglobalization or fragmentation (e.g. WTO, 2023; Aiyar et al., 2023). These developments directly affect firms' access to foreign suppliers and the relative cost of imported intermediates. In this section, I examine how such policy changes, in particular a tariff on imported inputs, alter key outcomes including productivity, market concentration, and aggregate welfare. The analysis connects technological changes in input market efficiency to tangible policy instruments that affect firms in similar ways.

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<sup>6</sup>This expression is a shorthand for the full benchmark derived in Section 5.2.

## 9.1 Policy and Implementation

I introduce an ad valorem tariff on imported intermediate inputs at rate  $\tau = 0.10$ . Tariff revenue is rebated to the representative household as a lump-sum transfer. The tariff scales the buyer's payment to foreign suppliers by the factor  $(1 + \tau)$ , so that the buyer's effective unit input price becomes

$$p_u^{\text{imp}} = (1 + \tau) p_u.$$

Given this wedge, the Nash bargaining problem between buyer and supplier becomes

$$\max_{T, x} \left( - (1 + \tau) T + V(z, X_{\text{new}}) - V(z, X) \right)^{\theta} \left( T - w_f \frac{x}{z_u} \right)^{1-\theta}.$$

The tariff therefore raises the cost of imported inputs proportionally to  $(1 + \tau)$  while leaving the rest of the environment, including the production structure and demand system, unchanged.

## 9.2 Aggregate Effects

The tariff increases the effective cost of imported inputs, which feeds through the CES aggregator into higher consumer prices. Figure 10 compares aggregate outcomes under the baseline and a 10% tariff across different search costs. In the baseline, lower search costs reduce prices and raise production, the domestic input share, and real GDP. With the tariff, these relationships remain qualitatively similar but are uniformly damped: prices are higher, production and real GDP are lower, and the shift toward domestic inputs is stronger. The tariff thus partially reverses the aggregate gains from improved supplier search efficiency.

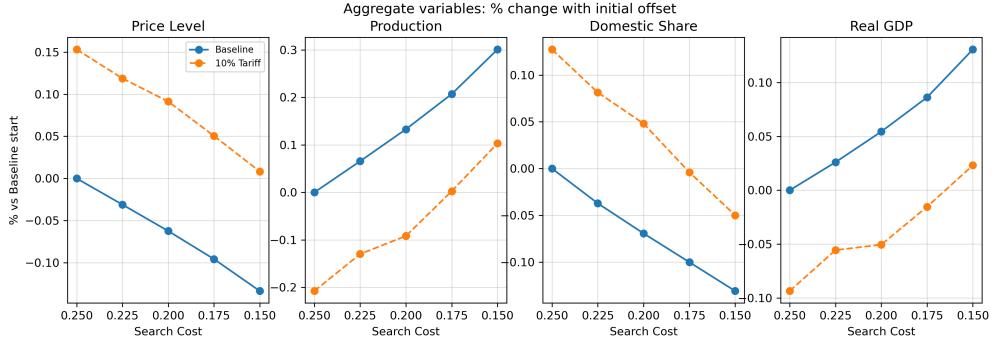


Figure 10: Aggregate outcomes under a 10% tariff. The tariff raises prices and weakens the production and real GDP response to lower search costs, while increasing the domestic input share.

### 9.3 Search incentives and network size

By scaling the variable cost of imported inputs, the tariff reduces the gain from adding a supplier. In the simplified benchmark with  $z_u = \bar{z}_u$  and buyer power  $\theta = 1$ , a tariff  $\tau$  scales the supplier's unit cost by  $(1 + \tau)$ , equivalent to replacing  $\bar{z}_u$  with  $\bar{z}_u/(1 + \tau)$ . Since in Section 5.2  $N^* \propto \bar{z}_u^{\alpha(\epsilon-1)\phi}$ , the number of supplier relationships under a tariff becomes

$$N_\tau^* = N_0^*(1 + \tau)^{-\alpha(\epsilon-1)\phi}, \quad \phi = \frac{\rho}{\rho - \alpha(\epsilon - 1)(1 - \rho)}, \quad (2)$$

where  $N_0^*$  denotes the no-tariff benchmark. The model therefore predicts a broad decline in network size when the tariff is introduced. This effect is confirmed in the full quantitative model, as shown below.

Table 9: Change in number of suppliers, baseline versus tariff, both at low search cost.

	Share (%)	Baseline	Tariff
Fewer	95.87	25.08	17.70
More	0.00	<i>n.a.</i>	<i>n.a.</i>
Equal	4.13	6.16	6.16

### 9.4 Distributional Effects on Sales and Profits

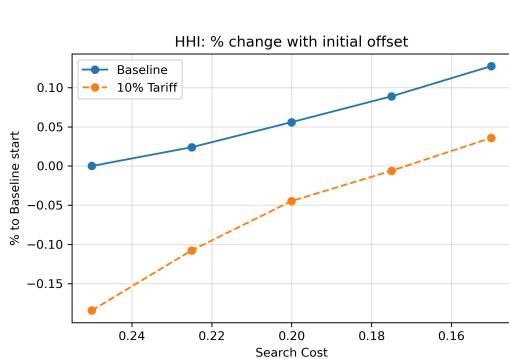
The tariff changes relative prices and demand. Some smaller non-importers benefit from higher output prices, while most importers experience lower margins. I report the share of firms with lower or higher outcomes and the corresponding averages.

Table 10: Sales and profits under the baseline and the tariff (low search cost).

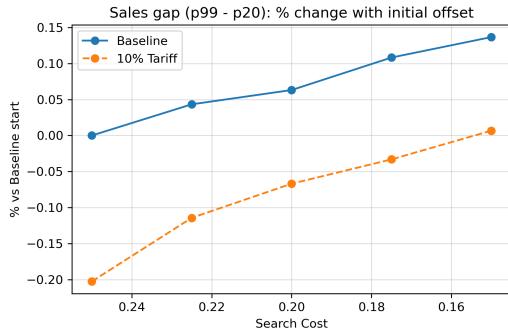
Panel A: Sales			Panel B: Profits		
	Share (%)	Baseline		Share (%)	Baseline
Fewer	5.58	27.81	25.45	3.73	2.08
More	94.42	0.69	0.83	96.27	0.16

### 9.5 Market Concentration

I measure concentration using the Herfindahl–Hirschman Index and the sales gap between the largest and smallest importing firms. The tariff compresses the firm size distribution in the model, both in general and among importers. Larger firms, which import the most, have less incentive to sustain a wide supplier network once imported inputs become more expensive.



(a) Herfindahl–Hirschman Index of sales.



(b) Sales gap between top and bottom importing firms.

Figure 11: Market concentration and dispersion of sales under the baseline and tariff scenarios.

To make the comparison explicit, I contrast the low-search-cost equilibrium with a 10% tariff to the high-search-cost baseline. Relative to the baseline ( $\kappa = 0.25$ ), the low-search-cost case ( $\kappa = 0.15$ ) with a 10% tariff delivers real GDP higher by 6.98% and an HHI higher by 3.60%. For reference, moving from high to low search cost with no tariff raises real GDP by 13.06% and HHI by 12.74%. Adding the 10% tariff at low search cost therefore reduces GDP by 5.37% and HHI by 8.11% relative to that low-search-cost, no-tariff case.

Overall, the tariff counterfactual shows that protectionist policies can mimic several features of higher search frictions by discouraging supplier formation and compressing firm networks. Although such policies reduce aggregate productivity, they may appear politically attractive because they narrow the performance gap between large and small firms and limit further spatial concentration of economic activity.

## 10 Conclusion

This paper highlights the role of input markets in shaping the firm size distribution. Using detailed Swedish administrative data, I document that more productive firms pay lower input prices, the dispersion in imported varieties and sales has widened, and regions with greater fiber-optic coverage experience faster supplier network expansion, especially among more productive firms.

To interpret these patterns, I develop a quantitative model with frictional input markets featuring random search and bilateral bargaining. The framework provides a unified

explanation for the observed dynamics: lower search costs disproportionately benefit importing firms by enabling them to expand supplier networks and raise productivity, while non-importers become relatively less competitive. Aggregate productivity rises, but market concentration increases as resources shift toward larger firms. A counterfactual exercise shows that a modest tariff on intermediate goods can undo much of the output gain while compressing the firm size distribution. Reduced search costs therefore reshape firm outcomes through intensive, extensive, and general equilibrium effects.

These results underscore the importance of input market frictions in shaping productivity, concentration, and inequality across firms. Policies that influence search costs—through infrastructure investment, digital connectivity, or trade regulation—can materially affect aggregate outcomes and market structure. Although the model abstracts from geography, the mechanisms identified here may also contribute to the spatial concentration of economic activity, since larger and more productive firms tend to cluster in major urban regions across the EU, where such patterns have intensified over time (OECD, 2023; European Commission Directorate General for Regional and Urban Policy, 2024).

## References

- Philippe Aghion, Antonin Bergeaud, Timo Boppart, Peter J Klenow, and Huiyu Li. A Theory of Falling Growth and Rising Rents. *The Review of Economic Studies*, 90(6): 2675–2702, November 2023. ISSN 0034-6527. doi: 10.1093/restud/rdad016. URL <https://doi.org/10.1093/restud/rdad016>.
- Shekhar Aiyar, Anna Ilyina, Jiaqian Chen, Alvar Kangur, Juan Trevino, Christian Ebeke, Tryggvi Gudmundsson, Gabriel Soderberg, Tatjana Schulze, Tansaya Kunaratskul, Michele Ruta, Roberto Garcia-Saltos, and Sergio Rodriguez. Geo-Economic Fragmentation and the Future of Multilateralism. *Staff Discussion Notes*, 2023 (001):1, January 2023. ISSN 2617-6750. doi: 10.5089/9798400229046.006. URL <http://dx.doi.org/10.5089/9798400229046.006>. Publisher: International Monetary Fund (IMF).
- George Alessandria, Joseph P. Kaboski, and Virgiliu Midrigan. Inventories, Lumpy Trade, and Large Devaluations. *American Economic Review*, 100(5):2304–2339, December 2010. ISSN 0002-8282. doi: 10.1257/aer.100.5.2304. URL <https://www.aeaweb.org/articles?id=10.1257/aer.100.5.2304>.
- Vanessa I. Alviarez, Michele Fioretti, Ken Kikkawa, and Monica Morlacco. Two-Sided Market Power in Firm-to-Firm Trade, May 2023. URL <https://www.nber.org/papers/w31253>.

Mary Amiti and Sebastian Heise. U.S. Market Concentration and Import Competition. *The Review of Economic Studies*, 92(2):737–771, March 2025. ISSN 0034-6527. doi: 10.1093/restud/rdae045. URL <https://doi.org/10.1093/restud/rdae045>.

Mary Amiti and Jozef Konings. Trade Liberalization, Intermediate Inputs, and Productivity: Evidence from Indonesia. *American Economic Review*, 97(5):1611–1638, December 2007. ISSN 0002-8282. doi: 10.1257/aer.97.5.1611. URL <https://www.aeaweb.org/articles?id=10.1257/aer.97.5.1611>.

Enghin Atalay. Materials Prices and Productivity. *Journal of the European Economic Association*, 12(3):575–611, 2014. ISSN 1542-4766. URL <https://www.jstor.org/stable/90023276>. Publisher: Oxford University Press.

Matej Bajgar, Chiara Criscuolo, and Jonathan Timmis. Intangibles and Industry Concentration: A Cross-Country Analysis. *Oxford Bulletin of Economics and Statistics*, n/a(n/a), 2025. ISSN 1468-0084. doi: 10.1111/obes.12659. URL <https://onlinelibrary.wiley.com/doi/abs/10.1111/obes.12659>. \_eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/obes.12659>.

Johannes Boehm and Ezra Oberfield. Misallocation in the Market for Inputs: Enforcement and the Organization of Production\*. *The Quarterly Journal of Economics*, 135(4):2007–2058, November 2020. ISSN 0033-5533. doi: 10.1093/qje/qjaa020. URL <https://doi.org/10.1093/qje/qjaa020>.

Timo Boppart, Mikael Carlsson, Markus Kondziella, and Markus Peters. Micro PPI-Based Real Output Forensics. 2023.

Christian Broda and David E. Weinstein. Globalization and the Gains From Variety\*. *The Quarterly Journal of Economics*, 121(2):541–585, May 2006. ISSN 0033-5533. doi: 10.1162/qjec.2006.121.2.541. URL <https://doi.org/10.1162/qjec.2006.121.2.541>.

Ariel Burstein, Javier Cravino, and Marco Rojas. Input Price Dispersion Across Buyers and Misallocation, November 2024. URL <https://www.nber.org/papers/w33128>.

Matias Covarrubias, Germán Gutiérrez, and Thomas Philippon. From Good to Bad Concentration? US Industries over the Past 30 Years. *NBER Macroeconomics Annual*, 34:1–46, January 2020. ISSN 0889-3365. doi: 10.1086/707169. URL <https://www.journals.uchicago.edu/doi/full/10.1086/707169>. Publisher: The University of Chicago Press.

Nicolas Crouzet and Janice C. Eberly. Understanding Weak Capital Investment: the Role of Market Concentration and Intangibles. Technical Report w25869, National Bureau of Economic Research, May 2019. URL <https://www.nber.org/papers/w25869>.

Maarten De Ridder. Market Power and Innovation in the Intangible Economy. *American Economic Review*, 114(1):199–251, January 2024. ISSN 0002-8282. doi: 10.1257/aer.20201079. URL <https://www.aeaweb.org/articles?id=10.1257/aer.20201079>.

Angus Deaton. Quality, Quantity, and Spatial Variation of Price. *The American Economic Review*, 78(3):418–430, 1988. ISSN 0002-8282. URL <https://www.jstor.org/stable/1809142>. Publisher: American Economic Association.

Swati Dhingra and John Morrow. Monopolistic Competition and Optimum Product Diversity under Firm Heterogeneity. *Journal of Political Economy*, 127(1):196–232, February 2019. ISSN 0022-3808. doi: 10.1086/700732. URL <https://www.journals.uchicago.edu/doi/full/10.1086/700732>. Publisher: The University of Chicago Press.

Jonathan Eaton, Samuel Kortum, and Francis Kramarz. An Anatomy of International Trade: Evidence From French Firms. *Econometrica*, 79(5):1453–1498, 2011. ISSN 1468-0262. doi: 10.3982/ECTA8318. URL <https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA8318>. \_eprint: <https://onlinelibrary.wiley.com/-doi/pdf/10.3982/ECTA8318>.

Jonathan Eaton, David Jinkins, James R. Tybout, and Daniel Xu. Two-Sided Search in International Markets, January 2022a. URL <https://www.nber.org/papers/w29684>.

Jonathan Eaton, Samuel S Kortum, and Francis Kramarz. Firm-to-Firm Trade: Imports, Exports, and the Labor Market. 2022b.

Chris Edmond, Virgiliu Midrigan, and Daniel Yi Xu. Competition, Markups, and the Gains from International Trade. *American Economic Review*, 105(10):3183–3221, October 2015. ISSN 0002-8282. doi: 10.1257/aer.20120549. URL <https://www.aeaweb.org/articles?id=10.1257/aer.20120549>.

Jan Eeckhout and Philipp Kircher. Sorting and Decentralized Price Competition. *Econometrica*, 78(2):539–574, 2010. ISSN 1468-0262. doi: 10.3982/ECTA7953. URL <https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA7953>. \_eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA7953>.

European Commission Directorate General for Regional and Urban Policy. *Ninth report on economic, social and territorial cohesion*. Publications Office, LU, 2024. URL <https://data.europa.eu/doi/10.2776/264833>.

Pinelopi Koujianou Goldberg, Amit Kumar Khandelwal, Nina Pavcnik, and Petia Topalova. Imported Intermediate Inputs and Domestic Product Growth: Evidence from India\*. *The Quarterly Journal of Economics*, 125(4):1727–1767, November 2010. ISSN 0033-5533. doi: 10.1162/qjec.2010.125.4.1727. URL <https://doi.org/10.1162/qjec.2010.125.4.1727>.

Gita Gopinath and Brent Neiman. Trade Adjustment and Productivity in Large Crises. *American Economic Review*, 104(3):793–831, March 2014. ISSN 0002-8282. doi: 10.1257/aer.104.3.793. URL <https://www.aeaweb.org/articles?id=10.1257/aer.104.3.793>.

László Halpern, Miklós Koren, and Adam Szeidl. Imported Inputs and Productivity. *American Economic Review*, 105(12):3660–3703, December 2015. ISSN 0002-8282. doi: 10.1257/aer.20150443. URL [https://www.aeaweb.org/articles?id=10.1257%2Faer.20150443&utm\\_source=chatgpt.com](https://www.aeaweb.org/articles?id=10.1257%2Faer.20150443&utm_source=chatgpt.com).

Chang-Tai Hsieh and Peter J. Klenow. Misallocation and Manufacturing TFP in China and India\*. *The Quarterly Journal of Economics*, 124(4):1403–1448, November 2009. ISSN 0033-5533. doi: 10.1162/qjec.2009.124.4.1403. URL <https://doi.org/10.1162/qjec.2009.124.4.1403>.

Maurice Kugler and Eric Verhoogen. Prices, Plant Size, and Product Quality. *The Review of Economic Studies*, 79(1):307–339, January 2012. ISSN 0034-6527. doi: 10.1093/restud/rdr021. URL <https://doi.org/10.1093/restud/rdr021>.

Jeremy Lise, Costas Meghir, and Jean-Marc Robin. Matching, sorting and wages. *Review of Economic Dynamics*, 19:63–87, January 2016. ISSN 1094-2025. doi: 10.1016/j.red.2015.11.004. URL <https://www.sciencedirect.com/science/article/pii/S109420251500071X>.

Julien Martin, Isabelle Mejean, and Mathieu Parenti. Relationship stickiness and economic uncertainty. 2023. URL [http://www.isabellemejean.com/MMP\\_2023.pdf](http://www.isabellemejean.com/MMP_2023.pdf).

J. J. McCall. Economics of Information and Job Search. *The Quarterly Journal of Economics*, 84(1):113–126, 1970. ISSN 0033-5533. doi: 10.2307/1879403. URL <https://www.jstor.org/stable/1879403>. Publisher: Oxford University Press.

Sébastien Miroudot, Rainer Lanz, and Ragoussis Ragoussis. Trade in Intermediate Goods and Services, November 2009. URL [https://www.oecd.org/en/publications/trade-in-intermediate-goods-and-services\\_5kmlcxtdlk8r-en.html](https://www.oecd.org/en/publications/trade-in-intermediate-goods-and-services_5kmlcxtdlk8r-en.html).

Monica Morlacco. Market Power in Input Markets: Theory and Evidence from French Manufacturing, 2020. URL [https://www.dropbox.com/s/4ic4ylvgngzn947/BP\\_draft\\_01102020.pdf?dl=0](https://www.dropbox.com/s/4ic4ylvgngzn947/BP_draft_01102020.pdf?dl=0).

Ezra Oberfield. A Theory of Input-Output Architecture. *Econometrica*, 86(2):559–589, 2018. ISSN 1468-0262. doi: 10.3982/ECTA10731. URL <https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA10731>. eprint: <https://onlinelibrary.wiley.com/-doi/pdf/10.3982/ECTA10731>.

OECD. *OECD Regional Outlook 2023: The Longstanding Geography of Inequalities*. OECD Regional Outlook. OECD Publishing, October 2023. ISBN 978-92-64-81264-2 978-92-64-32839-6 978-92-64-34116-6. doi: 10.1787/92cd40a0-en. URL [https://www.oecd.org/en/publications/oecd-regional-outlook-2023\\_92cd40a0-en.html](https://www.oecd.org/en/publications/oecd-regional-outlook-2023_92cd40a0-en.html).

Ministry of Enterprise and Innovation. A Completely Connected Sweden by 2025, a Broadband Strategy. Technical Report N2016/08008/D, Ministry of Enterprise and Innovation, Sweden, December 2016.

Christopher A. Pissarides. *Equilibrium Unemployment Theory*. MIT Press, Cambridge, MA, USA, 2 edition, February 2017. ISBN 978-0-262-53398-0.

James E. Rauch. Networks versus markets in international trade. *Journal of International Economics*, 48(1):7–35, June 1999. ISSN 0022-1996. doi: 10.1016/S0022-1996(98)00009-9. URL <https://www.sciencedirect.com/science/article/pii/S0022199698000099>.

Diego Restuccia and Richard Rogerson. Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707–720, October 2008. ISSN 1094-2025. doi: 10.1016/j.red.2008.05.002. URL <https://www.sciencedirect.com/science/article/pii/S1094202508000203>.

Michael Rubens. Market Structure, Oligopsony Power, and Productivity. *American Economic Review*, 113(9):2382–2410, September 2023. ISSN 0002-8282. doi: 10.1257/aer.20210383. URL <https://www.aeaweb.org/articles?id=10.1257/aer.20210383>.

SCB. STATISTIKENS FRAMSTAELLNING, Utrikeshandel med varor, 2018. February 2018.

Robert Shimer and Lones Smith. Assortative Matching and Search. *Econometrica*, 68(2):343–369, 2000. ISSN 0012-9682. URL <https://www.jstor.org/stable/2999430>. Publisher: [Wiley, Econometric Society].

Joshua Weiss. Intangible Investment and Market Concentration. 2020. URL <https://jweissecon.github.io/JobMarket/WeissPaper.pdf>.

World Bank. GDP per capita (constant LCU). World Development Indicators, 2025. URL <https://data.worldbank.org/indicator/NY.GDP.PCAP.KN?locations=SE>.

WTO. World Trade Report 2023 - Re-globalization for a secure, inclusive and sustainable future. Technical report, World Trade Organization, 2023. URL [https://www.wto.org/english/res\\_e/publications\\_e/wtr23\\_e.htm](https://www.wto.org/english/res_e/publications_e/wtr23_e.htm).

# Data Appendix

## Summary Statistic

I calculated for an average year in my dataset, the summary statistics:

Table 11: Summary Statistics - Imports

Variable	Mean	Std. Dev.	P10	Median	P95
No. Varieties per Firm	20.8	78.7	1.0	4.0	84.7
No. Product per Firm	13.2	31.6	1.0	3.3	54.8
No. Country per Firm	4.6	6.3	1.0	2.0	17.0
No. Country per Product	7.7	8.6	1.0	5.0	24.9
No. Firms per Variety	3.1	7.3	1.0	1.0	10.0
No. Country per FirmXProduct	1.6	1.8	1.0	1.0	4.0
No. of Firms			8396		
No. of Product			7400		
No. of Varieties			56608		
No. of Countries			179		
Percentage of Domestic Firms			0.28		
Imported Intermediate share			0.02383		

And a comparison between Importing and all firms:

Table 12: Summary Statistics - Importing Firm vs All Firm

Variable	Importers	All Firms	Ratio
No. of Firms per Year	8396	52811	0.16
Total No. of Firms	36483	150046	0.24
Sales	1.63e+08	2.83e+07	5.77
Employment	59.06	11.15	5.30
Labor Productivity	13.72	12.82	1.07
Intermediate share of Production	0.44	0.36	1.23

## Within EU Data Selection and Non-EU subsample

Imports originating from other EU countries will not go through the customs because Sweden has been part of EU from 1995 on. Therefore, data on EU imports are collected from mandatory self-reports by firms. However, if the firm import below a certain value threshold, then they have no obligation to report. This cutoff is 1.5 million SEK worth

of goods in 1998-2004, 2.2 mil SEK in 2005-2008, 4.5 million SEK in 2009-2014 and 9.0 million SEK onwards (See SCB (2018) for more). Due to these rules, when Britain exit the EU in 2020, it also cause some irregularities in the data of year 2021. However, if we limit the sample to only non-EU imports or exclude Britian for all years, the conclusion of the this paper hasn't change that much. In opposite, focusing on the Non-EU sample let us understand the bottom part of the distribution of importing firms better. Non-EU import consist around 50% of the full sample.

I have include robustness tests for only non-EU imports for some of my empirical facts. I define non-EU imports as imports from countries that have never been in the EU throughout the sample period. Therefore, for example, import records from Bulgaria (joined EU in 2007) in 2001 will be excluded from those robustness tests.

## **Robustness Check - Arm length**

To test if my results holds for non-arm-length trade (i.e. Trade that happens not within the same multi-national corporation group), I have carry out robustness test on the sub-sample of firm that neither 1.) do not belong to a foreign-based corporate group nor 2) belong to a corporate group that owns subsidiary abroad. In this case, any international trade of this firm have to be arm-length. The reason for using such conservative measure is that I still lack the indicator of the dataset (will be delivered). This subsample contains around 25% of the observations in the original dataset.

For arm length trade, 1) the negative relationship between TFPQ-Input Price is unchanged 2) the evolution of number of variety imported is slightly different, the 99th percentile firm still increase significantly, 90-99th percentile drop slightly while all other ones are flat. I believe the same conclusion can be drawn. 3) Nsupply increase much less in TFPQ when we throw out all this companies, but it is not surprising and a bit unclear if this result are affected by excluding non-arm-length trade or excluding the biggest/most productive multi-national firms. 4) Price Dispersion result also holds, the absolute value of variance in all years increase. Interestingly, there is a big drop in 2020 (COVID-19), this is not seen in the original graph.

# Robustness Tests on Number of Variety Percentiles Evolution

In the main parts of this paper, I have been looking at the Number of Variety Distribution. It is also interesting to look at how big firms and small firms behave differently in this aspect. I still define a "variety" as a unique country-product pair and measure "firm size" based on annual firm sales. In the graph presented, The graph below illustrates the number of varieties (product  $\times$  country) imported by each Swedish manufacturing firm. To facilitate comparison across firm-size percentiles, I normalize the annual number of varieties by their respective values from 1998. While the median and lower-percentile firms have experienced no change or even slight decrease in the number of imported varieties from 1998 to 2021, the largest firms have increased their imported varieties by approximately 10% to 30%. Given that the number of varieties imported directly relates to search activities, this observation suggests differential effects of search costs across firm sizes.

## By sales percentile

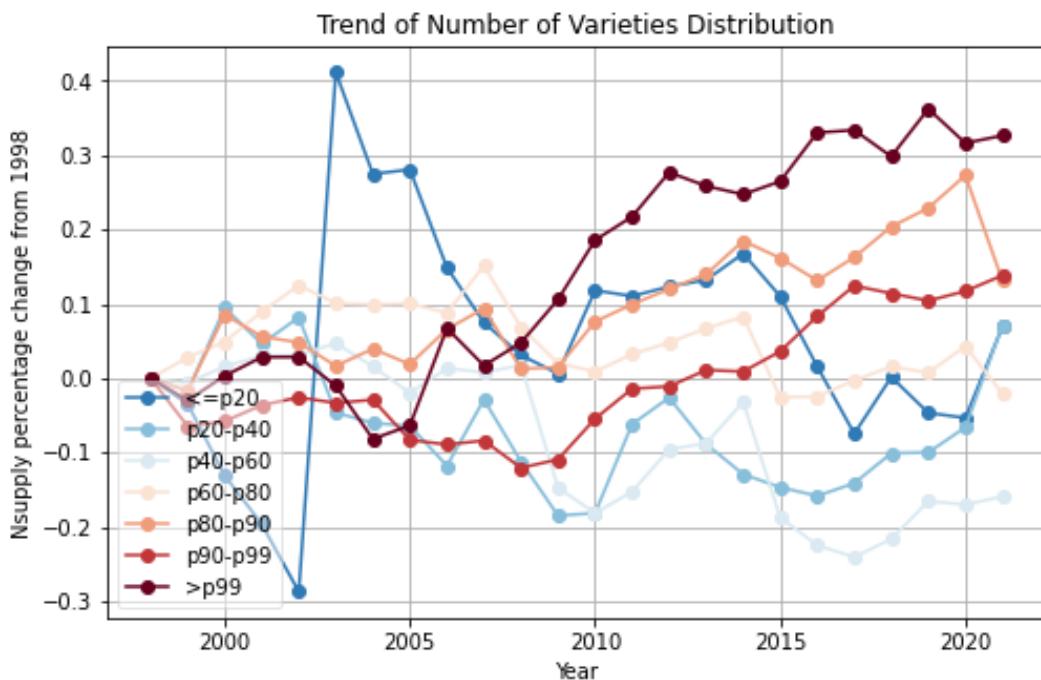


Figure 12: Number of varieties by sales percentile

## Balanced panel

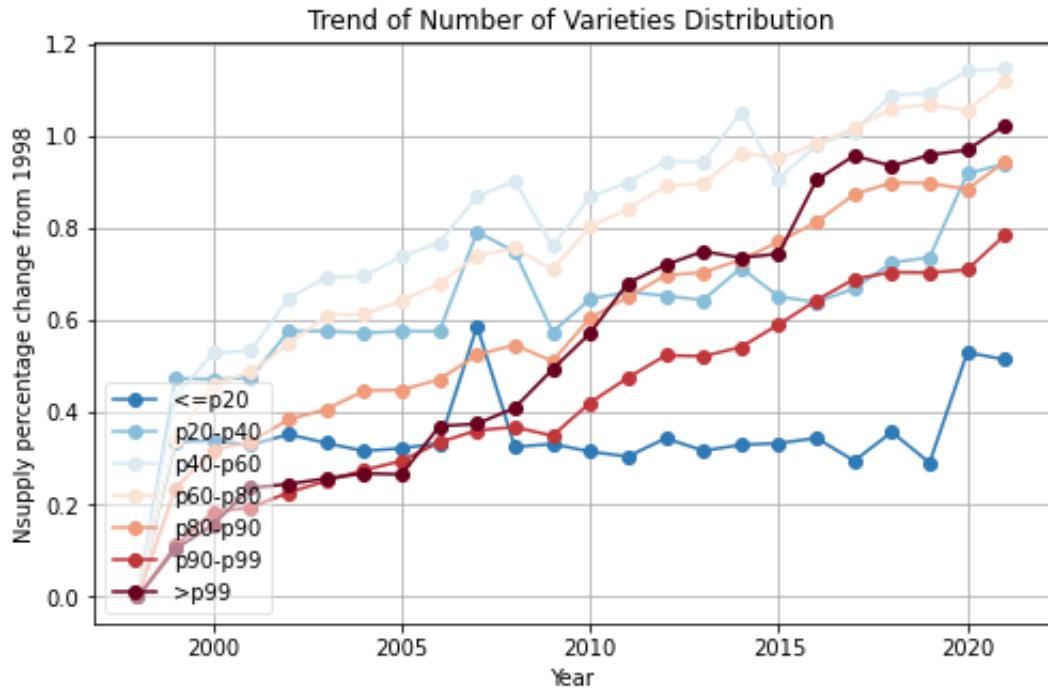


Figure 13: Number of varieties in a balanced panel

## Industry by year fixed effects

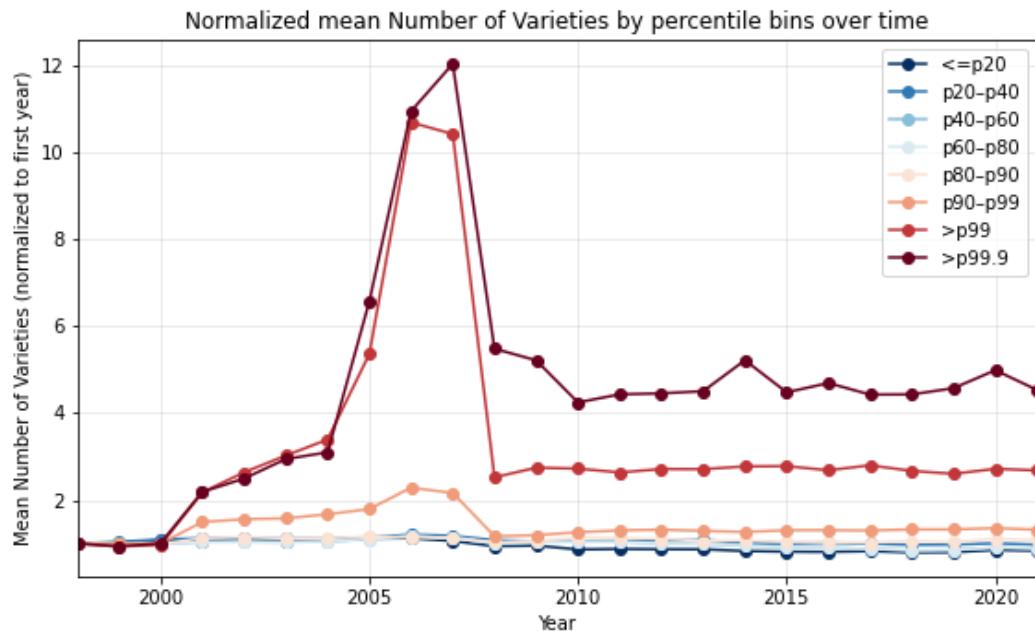


Figure 14: Number of varieties with industry by year fixed effects

## Originating countries for a firm product pair

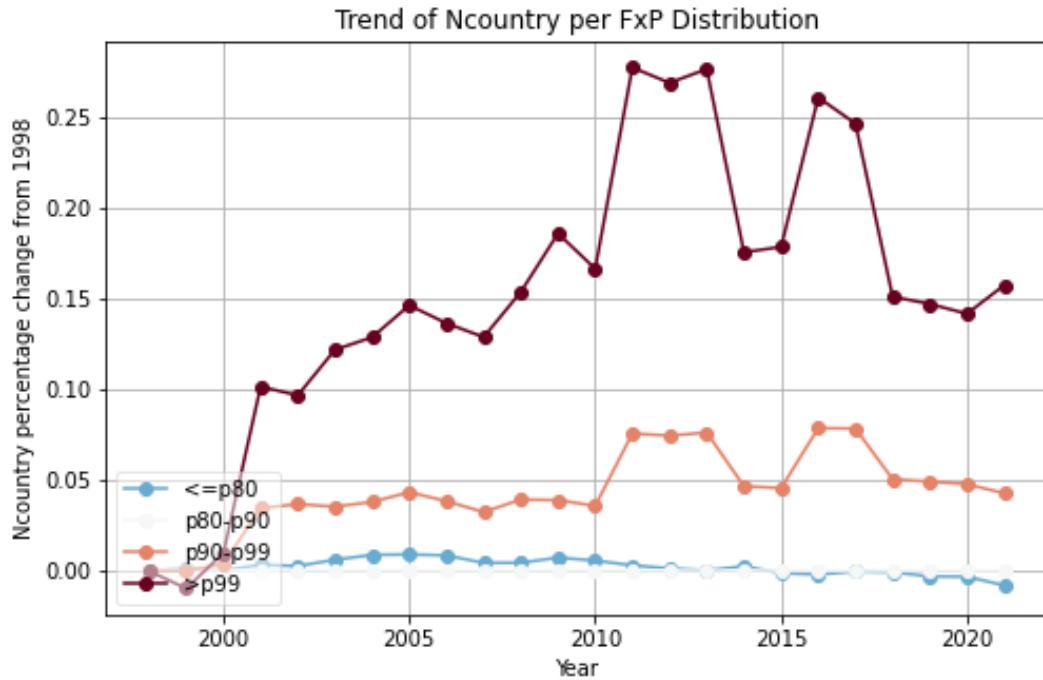


Figure 15: Number of originating countries for a firm product pair

## Only non EU imports

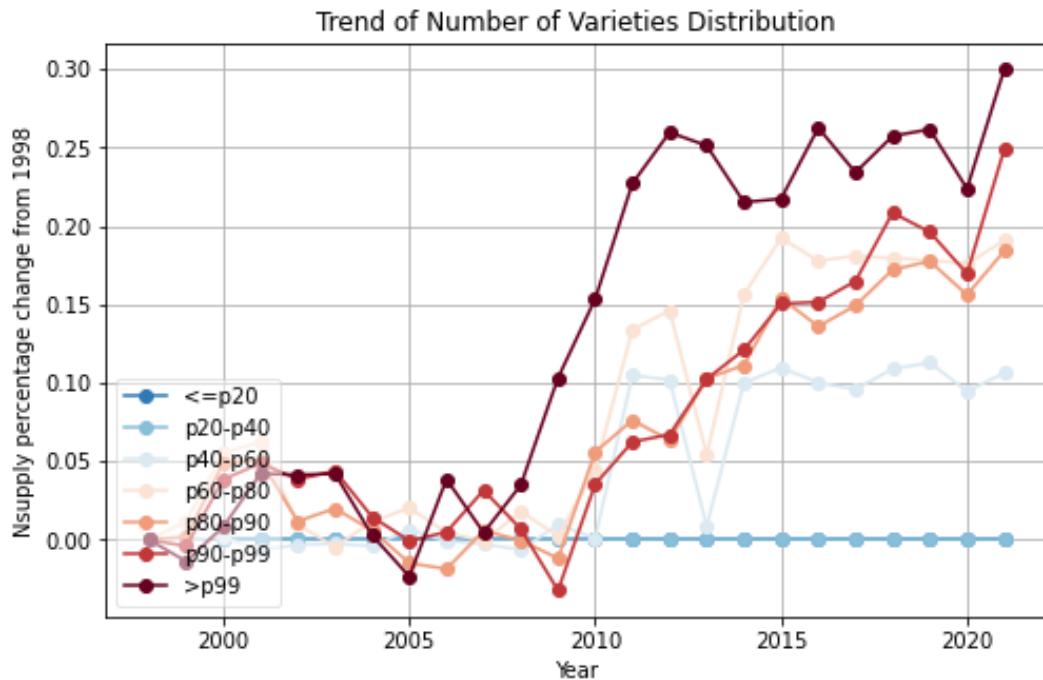


Figure 16: Number of varieties for non EU imports

## Firms without foreign subsidiary

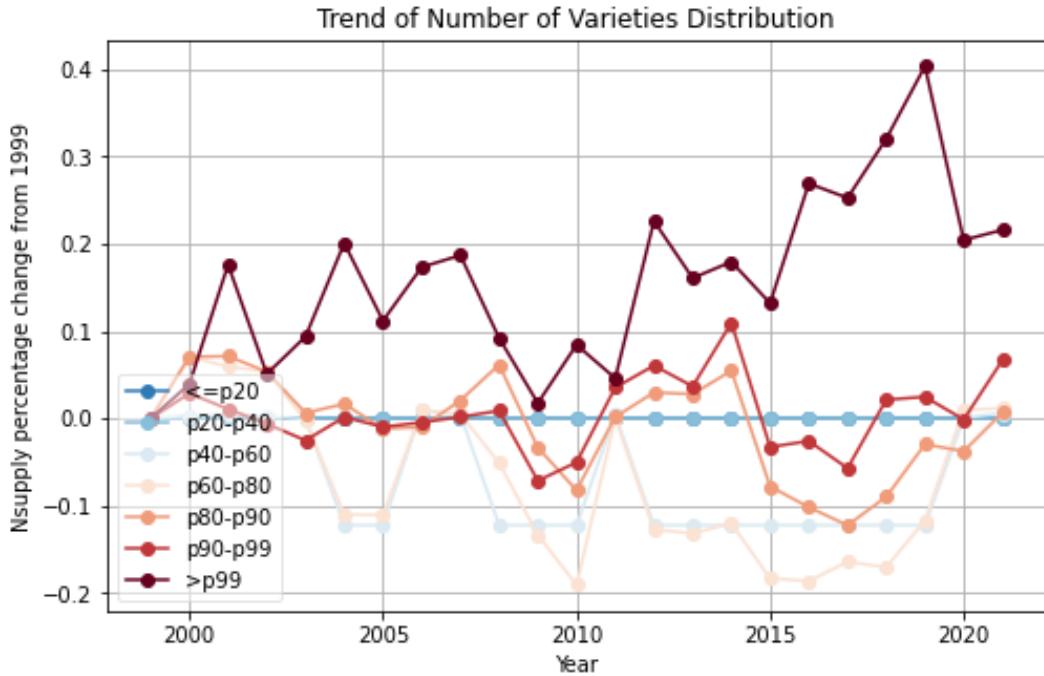


Figure 17: Number of varieties for firms without a foreign subsidiary

## Data and model: balanced panel

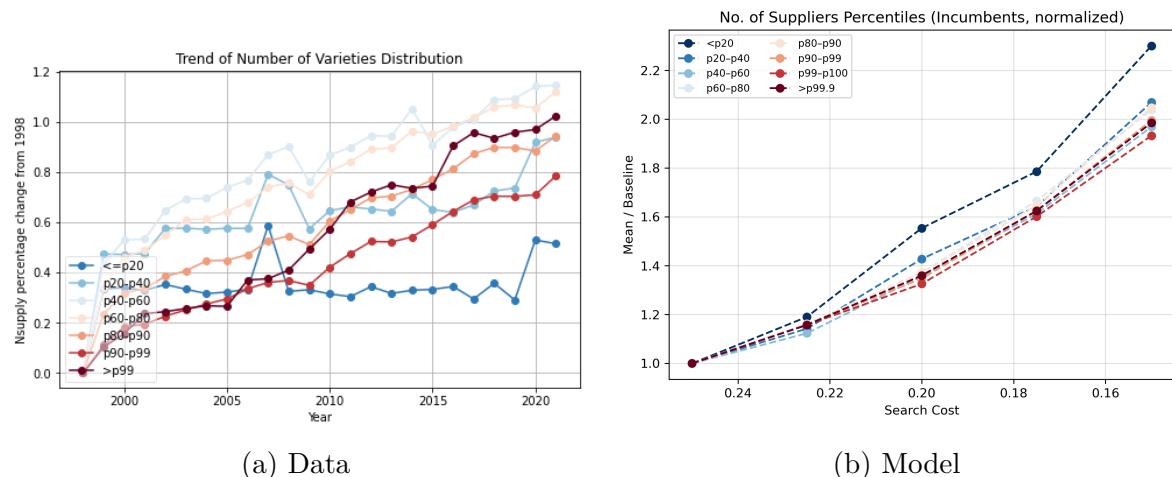


Figure 18: Number of varieties dispersion in a balanced panel

## Data and model: by sales percentile

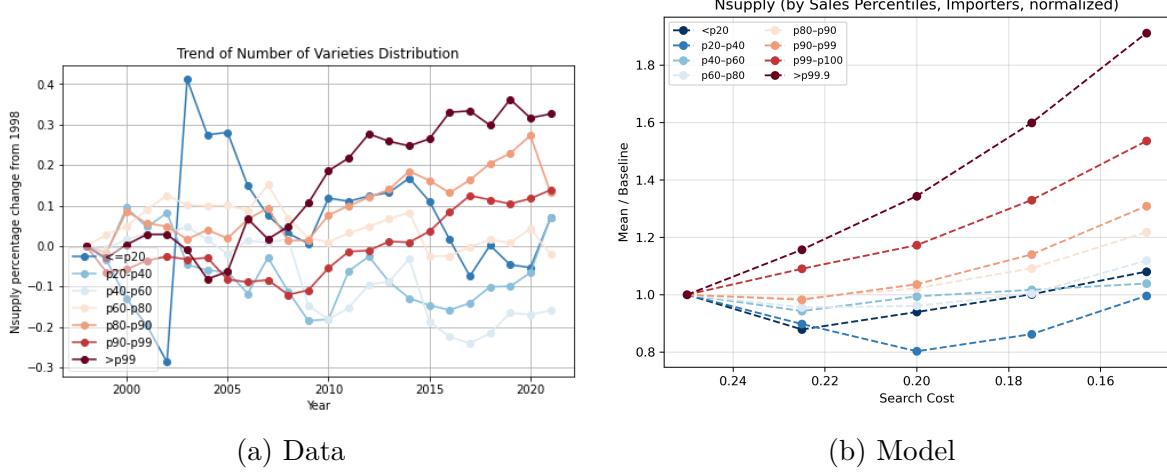


Figure 19: Number of varieties dispersion by sales percentile

## Robustness Tests on Sales Percentiles Evolution

There are alternative explanations for the widening firm size distribution. One possibility is that the composition of importing firms has changed for reasons unrelated to input market efficiency, such as structural shifts or sector-specific technological shocks. To address this, I conduct four robustness exercises that track the evolution of the firm size distribution across different subsamples.

First, I examine a balanced panel of firms that appear in all periods of the dataset. In this subsample, the largest firms (at the 99th percentile) exhibit the fastest growth, while other firms show similar and comparatively stable growth rates. This pattern mirrors the dispersion observed in the number of imported varieties, where the upper tail of the distribution pulls away from the rest.

Second, I demean firm sales by the industry-year mean to account for potential changes in industry composition. Although demeaning introduces some year-to-year volatility, the underlying dispersion pattern remains evident.

Third, I restrict the sample to firms without foreign subsidiaries and that are not owned by foreign parents. This ensures that the results are not driven by intra-firm trade. The dispersion pattern persists in this arm's-length trade subsample.

Finally, I focus on imports from non-EU countries. Since within-EU imports are subject to higher reporting thresholds, there is concern that measurement errors could bias the

results toward larger firms. Repeating the exercise on extra-EU imports yields the same qualitative findings.

## Balanced panel

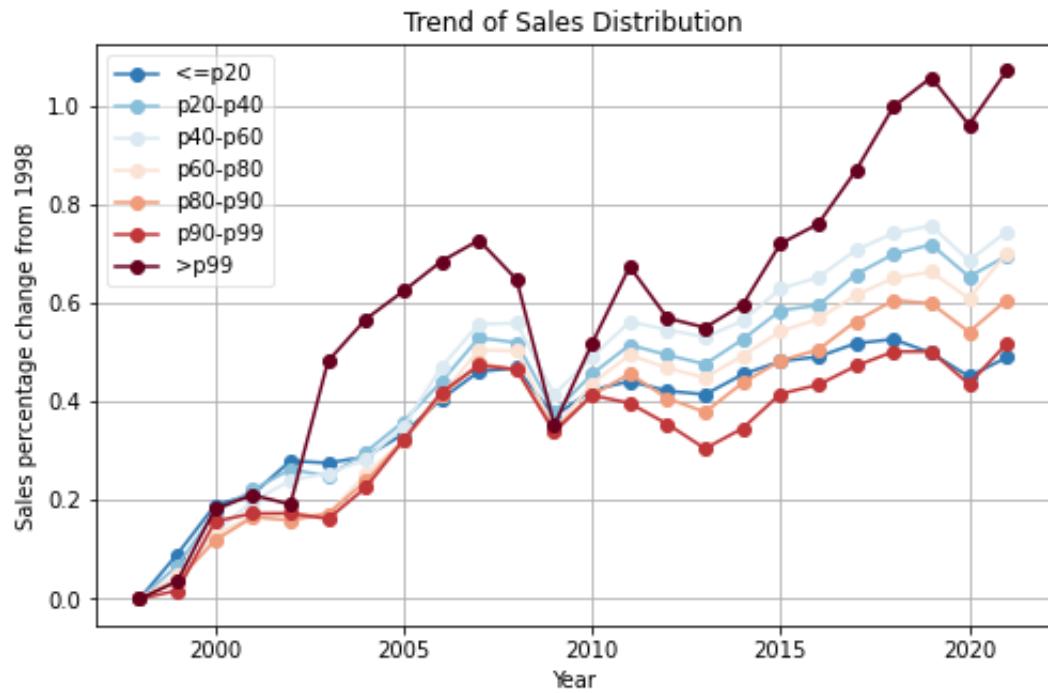


Figure 20: Sales dispersion in a balanced panel

## All manufacturing

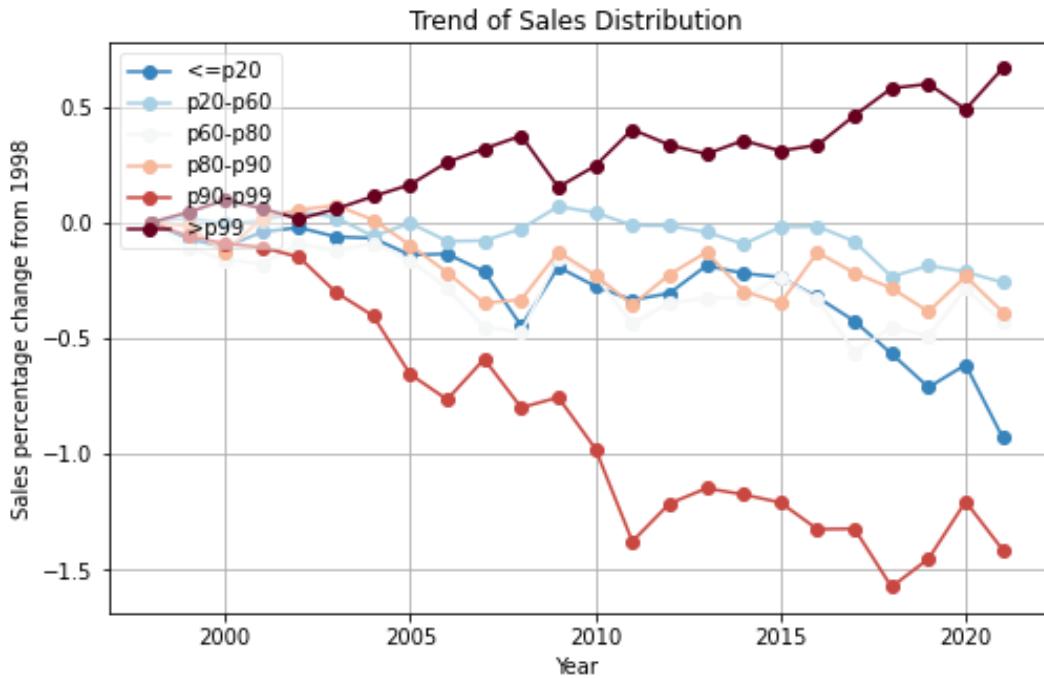


Figure 21: Sales dispersion for all manufacturing firms

## Industry by year fixed effects

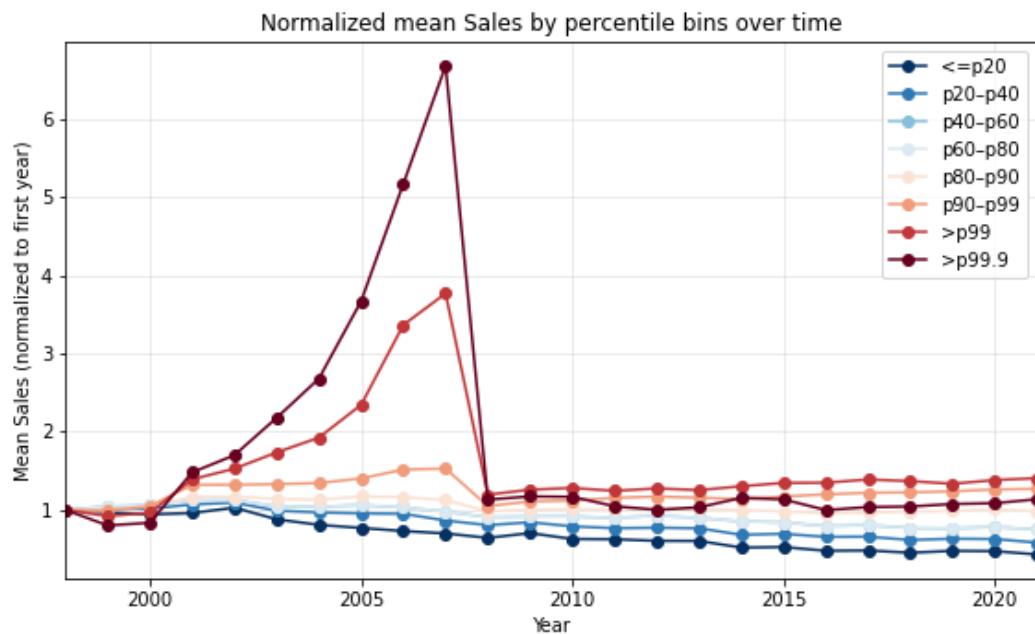


Figure 22: Sales dispersion with industry by year fixed effects

## Firms without foreign subsidiary

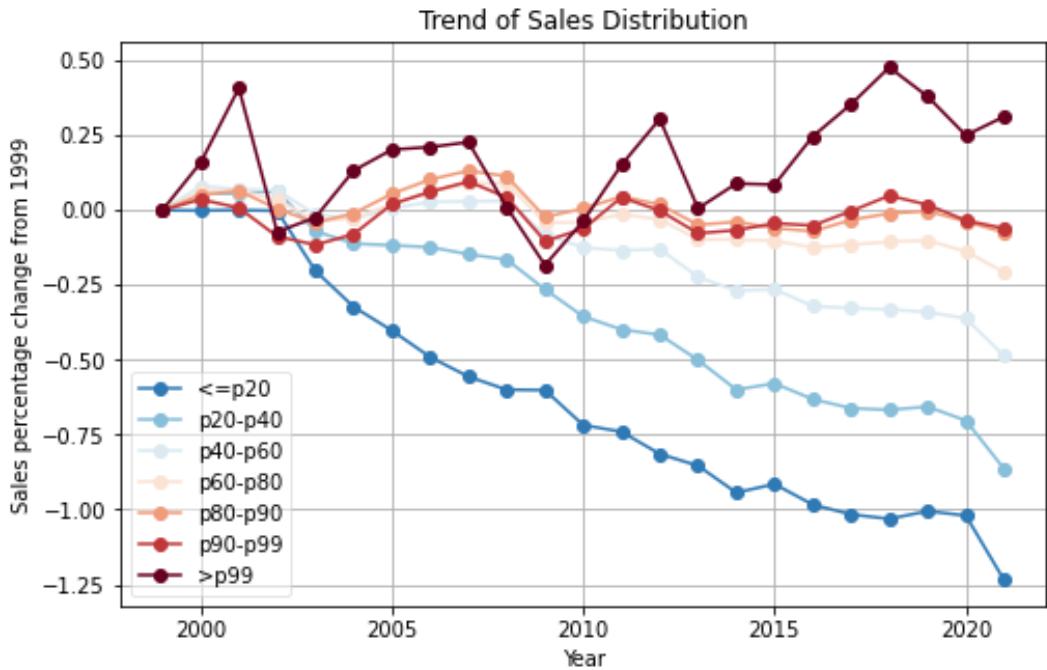


Figure 23: Sales dispersion for firms without a foreign subsidiary

## Only non EU imports

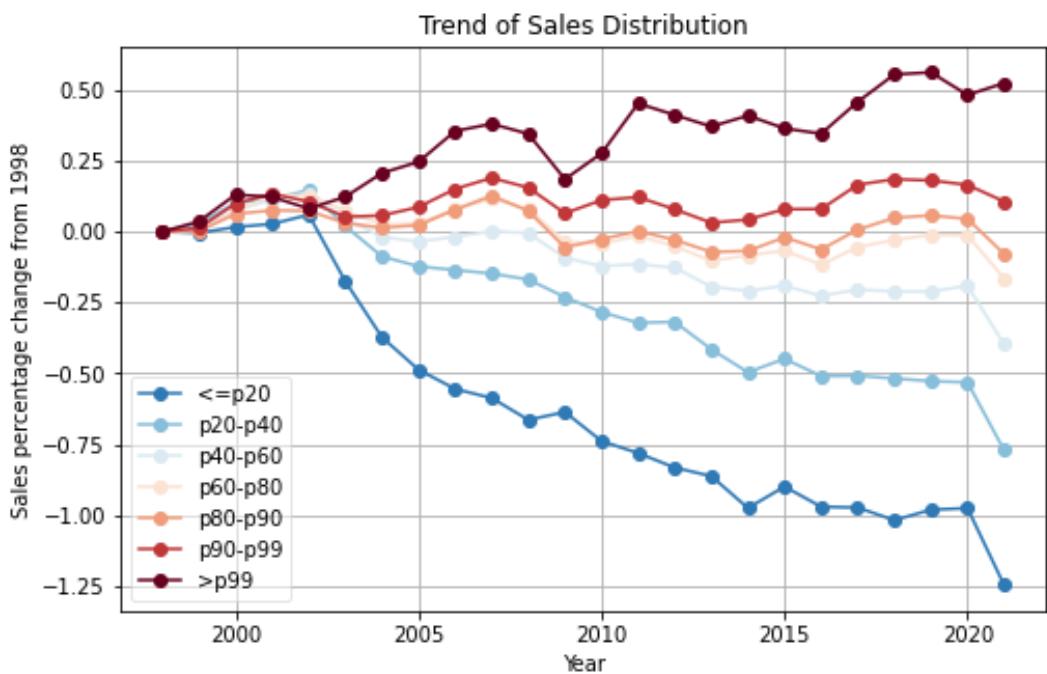


Figure 24: Sales dispersion for firms that import only from outside the EU

## Data and model: balanced panel

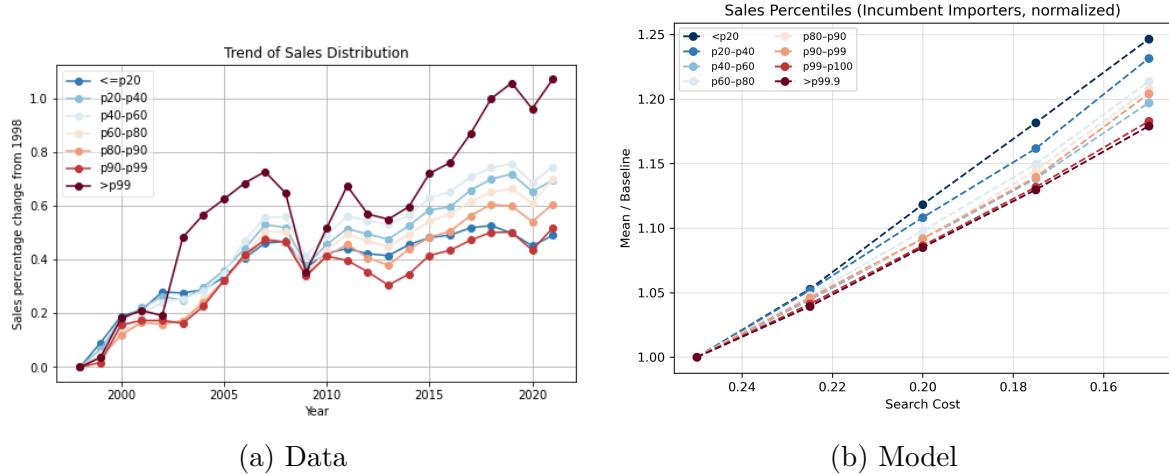


Figure 25: Sales dispersion in a balanced panel

## Data and model: all firms

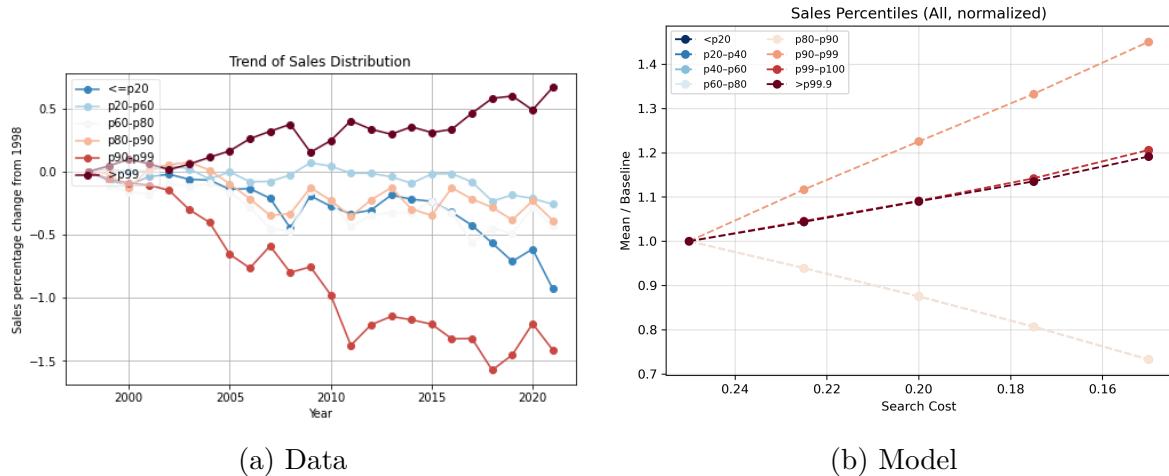


Figure 26: Sales dispersion for all firms

# Robustness Tests on Price and Size

## Regressing size and quantity together

Table 13: Regression results on input price

	Coefficient	(Std. error)
log number of workers	0.0048***	(0.0017)
Input quantity	-0.2790***	(0.0015)
Observations	172,602	
$R^2$	0.2838	
Industry fixed effects	Yes	
Time fixed effects	Yes	

\*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

When employment and input quantity are included in the same regression, the coefficient on log employment becomes small and slightly positive (0.0048), while the coefficient on input quantity remains strongly negative (-0.279). This pattern does not overturn the main result. Because employment and input quantity are highly correlated, the regression attributes most of the price-size relationship to input quantity. The small positive employment coefficient reflects residual differences in firm composition—such as labor intensity—after accounting for sourcing scale.

## Alternative measures and other samples

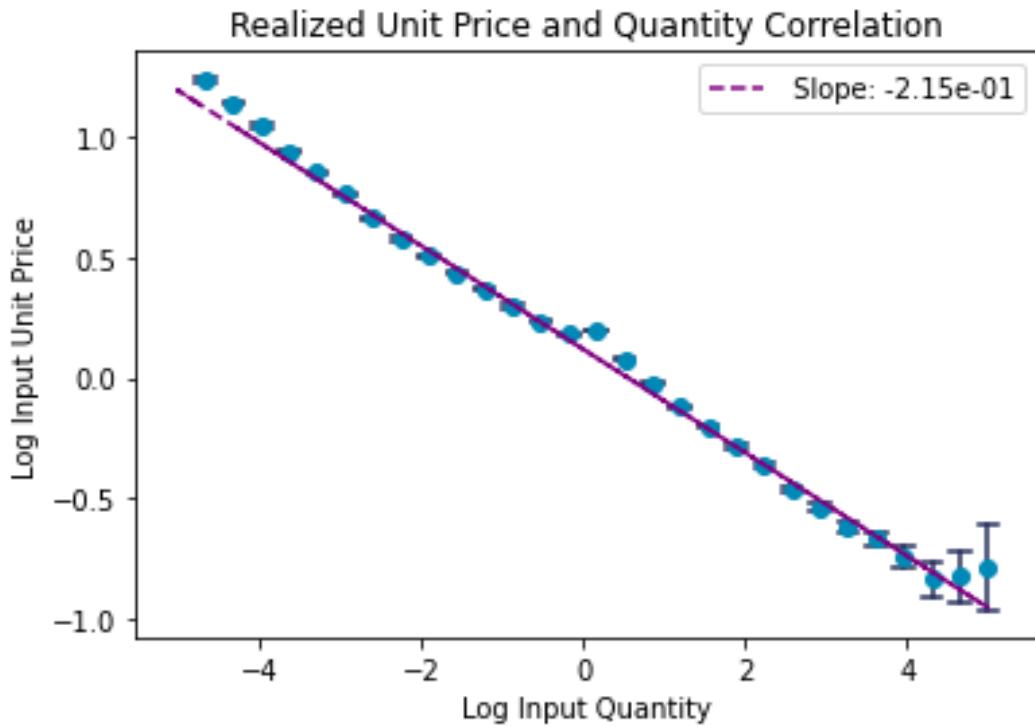
Table 14: Regression results on input price: alternative measures

	Full	non EU	Arm length
TFPQ	-0.056*** (0.003)	-0.042*** (0.004)	-0.071*** (0.005)
TFPR	-0.336*** (0.007)	-0.356*** (0.008)	-0.363*** (0.008)
Output per worker	-0.234*** (0.013)	-0.247*** (0.016)	-0.295*** (0.019)
Output per wage	-0.150*** (0.006)	-0.162*** (0.006)	-0.163*** (0.007)
Value added per worker	-0.191*** (0.012)	-0.216*** (0.015)	-0.226*** (0.022)
Sales	-0.123*** (0.001)	-0.125*** (0.001)	-0.129*** (0.002)
Employment	-0.166*** (0.002)	-0.173*** (0.002)	-0.113*** (0.004)
Total assets	-0.105*** (0.001)	-0.107*** (0.001)	-0.119*** (0.001)

\*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

## Realized price and quantity relationship on transaction level

The negative correlation between price and quantity is also observed at the firm–variety–year level. This pattern can arise for several reasons, including non-linear pricing, market power, or heterogeneity in product quality within a variety—firms tend to import larger quantities of lower-priced goods classified under the same product code. However, it is a complement evidence to my claim in the main text. More productive firms will in expectation buy more, therefore find more efficient suppliers and pay lower unit price.



A major concern is measurement error of quantity can mechanical create correlation between price and quantity (Deaton (1988)), as I use value and quantity to back out unit price. In the dataset I use, around 25% of observations have both variable. I assume that it is unlikely that both quantity measures have measurement error at the same time. Under this assumption, I compute unit price with one measure and regress it on the other quantity as a robustness check against such potential error. In this exercise, the shape and coefficient slightly changes, but the most important negative correlation is robust against measurement error.

## Robustness Tests for Fiber Rollout

From the report Ministry of Enterprise and Innovation (2016) , the Swedish government have installed a national-wide internet strategy in 2009. The goal is to promote fast internet for household, business and public services. Also, although there are profitability issue such that rural areas will be slower, the goal is to close the gap between rural and cities eventually. Therefore, it shouldn't be expected the treatment is directly related to trade.

## Municipality fiber map

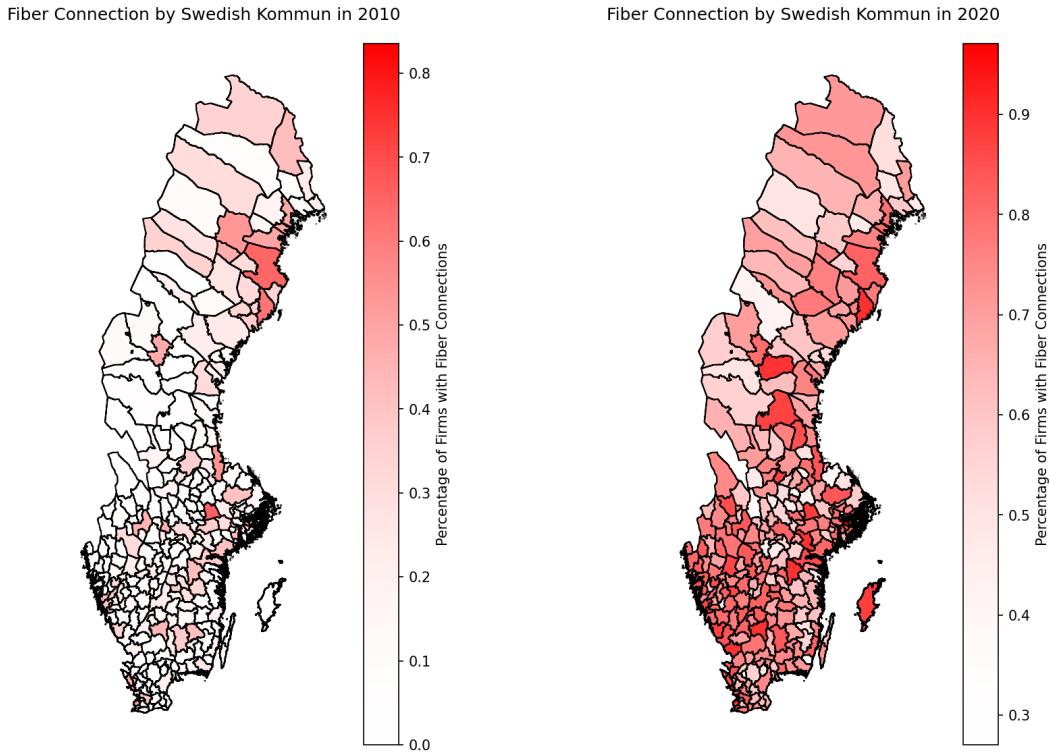


Figure 27: Municipality level fiber internet rollout

Figure 27 shows the share of firms with fiber broadband connections across Swedish kommuner in 2010 and 2020. In 2010, fiber connectivity was limited across the whole country, with relatively higher adoption in a few urban and northern regions. By 2020, coverage had expanded substantially across the country, with most municipalities showing fiber connection rates above 60–70 percent. The maps illustrate both the rapid diffusion of broadband infrastructure and the remaining regional disparities in adoption.

## Robustness to Pretrend

This appendix reports a binary pretrend robustness that complements the continuous coverage specification in the main text. I set  $Event_{m,t} = 1$  when municipal fiber coverage exceeds 50 percent, and define *PreEvent* as one in the three years preceding the event. The dependent variable is measured in levels of supplier network size rather than in log or percentage changes.

The big firm indicator equals one for firms with sales above the contemporaneous industry mean.

$$\begin{aligned}
\text{Varieties}_{f,t} = & \beta_1 \text{Event}_{m,t} + \beta_2 [\text{Event}_{m,t} \times \text{BigFirm}_{f,t}] \\
& + \gamma_1 \text{PreEvent}_{m,t} + \gamma_2 [\text{PreEvent}_{m,t} \times \text{BigFirm}_{f,t}] \\
& + \text{FE}_t + \text{FE}_i + \text{FE}_m + \varepsilon_{f,t}.
\end{aligned}$$

Table 15: Connectivity and supplier network size with pre-event control (sales based big)

	(1)	(2)	(3)	(4)
Treated	1.9191*** (0.5809)	2.3428*** (0.6420)	0.0404 (0.0856)	0.1406 (0.1879)
Treated $\times$ BigFirm			9.328*** (2.576)	11.527*** (2.871)
PreEvent		0.6608 (0.4473)		0.1541 (0.1823)
PreEvent $\times$ BigFirm				3.4857** (1.642)
BigFirm (main effect)			21.934*** (1.053)	19.736*** (1.228)
Observations	349,775	349,775	349,775	349,775
R <sup>2</sup>	0.0237	0.0237	0.0675	0.0675
Industry FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Municipality FE	Yes	Yes	Yes	Yes

*Notes:* Dependent variable is the level of imported input varieties for firm  $f$  in year  $t$ . “Treated” equals one when municipal fiber coverage exceeds 50 percent. “PreEvent” equals one in the three pre-event years. “BigFirm” is an indicator equal to one for firms with sales above the contemporaneous industry mean. Standard errors clustered by municipality in parentheses.

\*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

The estimates indicate that supplier network size expands more strongly for large firms following fiber rollout, while pretrend coefficients are small and statistically insignificant.

## Robustness Using Growth Rate Specification

As a complementary exercise, I estimate the relationship between connectivity and the *percentage change* in a firm’s imported varieties. This outcome allows for comparability across firms of different sizes, although the theoretical predictions refer to levels rather than growth rates. The specification parallels the baseline regression but uses the change in imported varieties relative to the previous year as the dependent variable:

$$\begin{aligned} \% \Delta \text{Varieties}_{f,t} = & \beta_1 \text{FiberCoverage}_{m,t} + \beta_2 [\text{FiberCoverage}_{m,t} \times \text{BigFirm}_{f,t}] \\ & + \text{FE}_t + \text{FE}_i + \text{FE}_m + \varepsilon_{f,t}. \end{aligned}$$

The big firm indicator equals one for firms with sales above the contemporaneous industry mean.

Table 16: Connectivity and the growth rate of imported varieties

	(1)	(2)
Fiber coverage	0.1485*** (0.0160)	0.1064*** (0.0184)
Fiber $\times$ BigFirm	—	0.1817*** (0.0325)
Observations	112,022	112,022
R <sup>2</sup>	0.0013	0.0020
Industry FE	Yes	Yes
Year FE	Yes	Yes
Municipality FE	Yes	Yes

*Notes:* Dependent variable is the percentage change in a firm's number of imported input varieties. “BigFirm” is an indicator equal to one for firms with sales above the contemporaneous industry mean. Standard errors clustered by municipality in parentheses. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

The estimates show that variety growth is faster in municipalities with higher fiber coverage, and the effect is substantially stronger for large firms. While the R<sup>2</sup> values are small, consistent with firm-level heterogeneity and municipality-level aggregation, the pattern supports the idea that reduced search costs expand supplier networks more strongly among large firms.

## Other Empirical Specifications

### Quality-adjusted TFPQ

In the main part of the paper, I used a direct measure of size and productivity. There are 2 potential drawbacks of that approach. First, it select a small subsample where output price are observed. These are usually bigger firms and therefore the results can

be biased. Second, in my model estimation, there is heterogeneous quality across firms, but quality is omitted by using direct output price measure.

I define the revenue productivity the same way as above, but then I estimate the quality-adjusted TFPQ by:

$$\text{TFPQ}_f = \frac{p_f}{Q_f} \text{TFPR}_f$$

, where  $p$  is the price and  $Q$  is the quality of the firm's production. Unfortunately, quality  $Q$  is generally not observable. To obtain an estimate of  $Q$ , I have to rely on the theory. Assuming the monopolistic competition in final good demand, the household optimization gives:

$$\frac{p_i y_i}{p_j y_j} = \left( \frac{p_i / Q_i}{p_j / Q_j} \right)^{1-\epsilon}$$

where Sales  $py$  are observables in the data. I can then back out  $\frac{p}{Q}$  by applying the standard elasticity  $\epsilon = 5$ . Now that I get an estimate for quality-adjusted TFPQ, without substantially reducing sample size and includes quality. I do the same analysis as above:

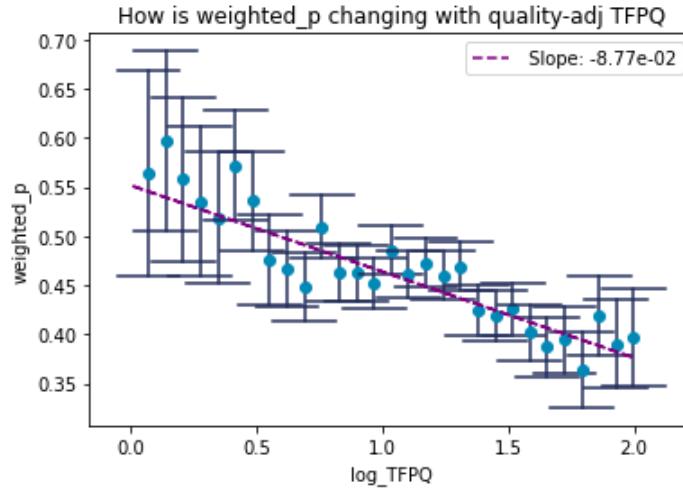


Figure 28: Relationship between quality-adjusted TFPQ and weighted prices

As predicted, the coefficient is also negative with a smaller magnitude than the non-quality adjusted version. As explained in Kugler and Verhoogen (2012), firms with higher productivity usually use higher quality inputs, which dampens the quality-adjusted coefficient.

Also different aggregation level (industry digit) or including capital/not, doesn't change the result much.

## Within-across firm decomposition of price

To illustrate that more productive firms do not consistently pay lower input prices across all products, I estimate

$$\text{InputPrice}_{fpcy} = \alpha_f + \varepsilon_{fpcy},$$

where  $\alpha_f$  are firm fixed effects and  $\varepsilon_{fpcy}$  captures within-firm variation. The variance decomposition then separates across-firm and within-firm components of input-price differences.

The result is surprising.

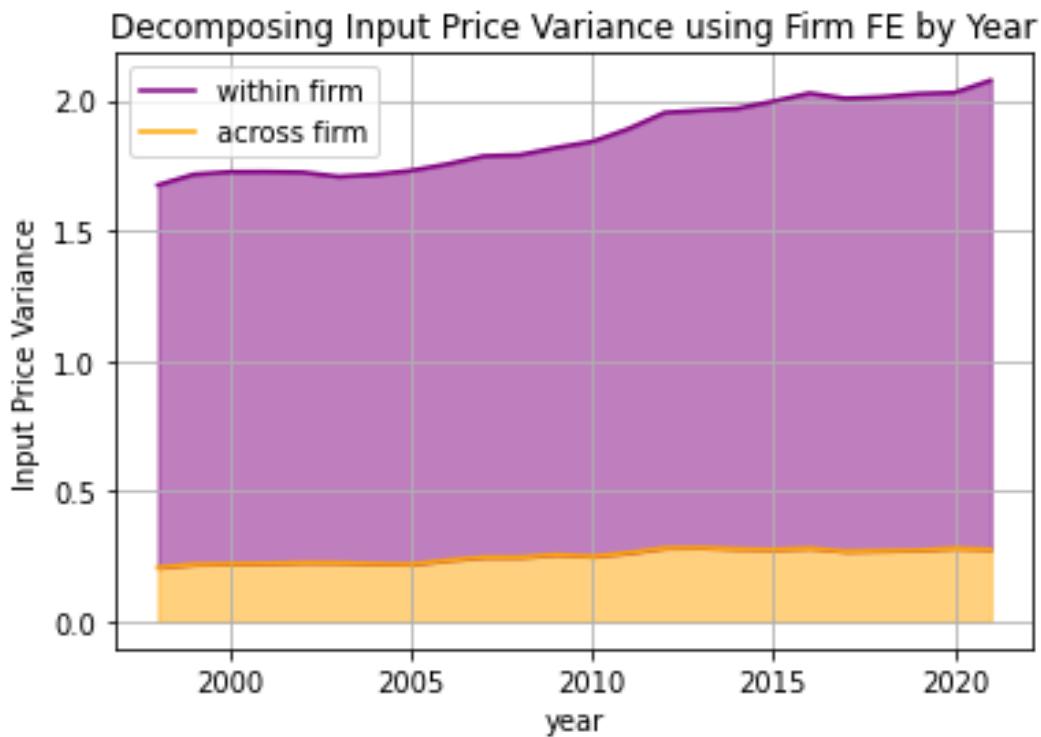


Figure 29: Within- and across-firm decomposition of input price variance

At odds with many model, that features firm level heterogeneity of markup or markdown, the across-firm effect is only around 15% of the total variance. It means that a same firm can purchase one input very cheaply, but another expensively.

## TFPQ and Variety

Another productivity-related empirical fact is that more productive firms purchase more variety. It is another piece of evidence that it is likely that productive firms also benefit

from the decreasing search cost because they search more. When the productive firms expect to produce more, they also have more incentive to search for the most efficient suppliers and also have multiple of them. Where else, the less productive suppliers even they finally gain access to the market,

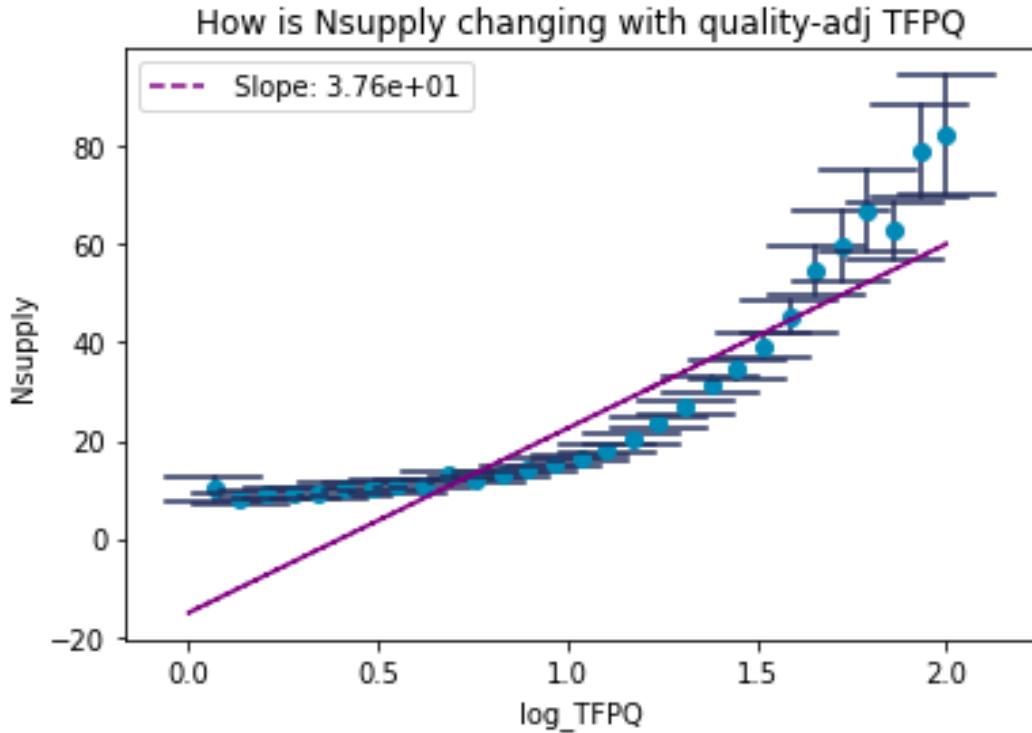


Figure 30: Relationship between TFPQ and number of imported varieties

## Input Price and Profit

Input price also directly affect the firm's profitability directly. I find a negative correlation between firm-level input price and profit share, even controlling for industry Fixed Effects. It means that the standard CES model with constant markup probably misses important features of the intermediate good market.

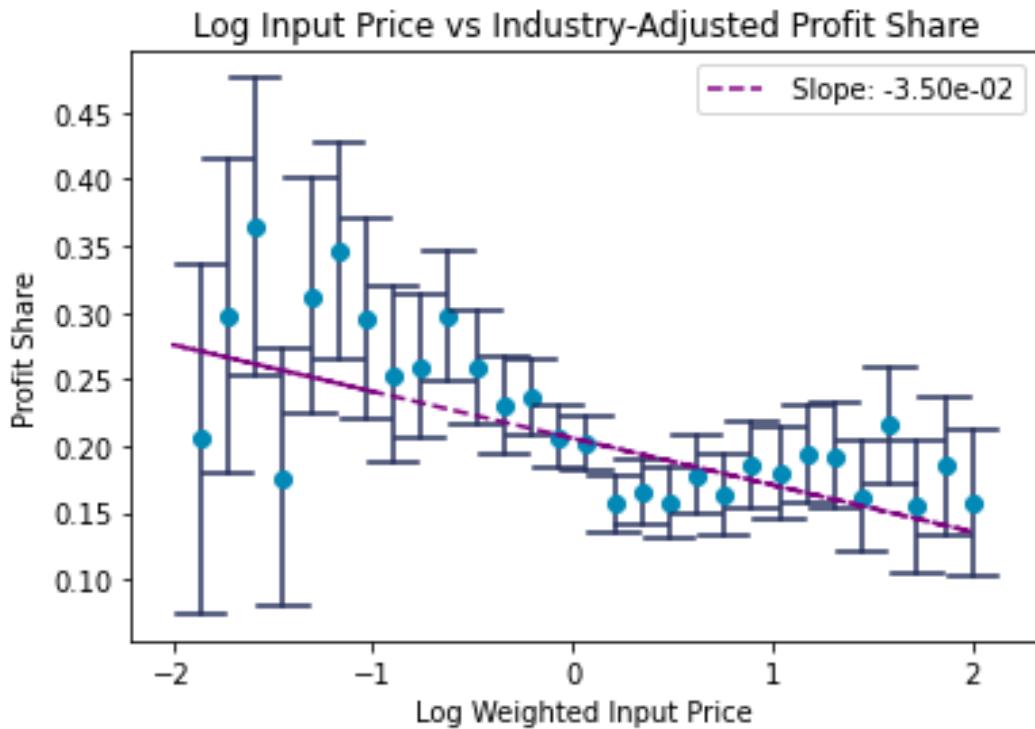


Figure 31: Relationship between input price and profit share

## Quality Differences

As researchers such as Kugler and Verhoogen (2012) show that it is an important channel of input price differences, I estimate the dispersion of input price that is generated from quality differences.

I use a reduced form approach that utilizes the Rauch (1999) classification, where he separates goods into 3 classes: traded in organized exchange, traded with referenced price and differentiated goods. Organized exchange goods also included products that have an organized exchange but can also be traded decentralized. Some examples are banana, wool or other basic agriculture products. Those products, however, have very little quality differences and therefore are suitable reference goods to estimate quality differentiation.

To link the Rauch's classifications that is based on SITC Rev. 2 the main dataset which uses CN code, I refer to the HS-SITC conversion tables provided by UNSD<sup>7</sup>. The CN code shares the first 6 digits with the HS code system and I assume all products that share the first 6 digits should get the same Rauch classifications. I look at the price standard deviation of the 3 groups of products:

<sup>7</sup><https://unstats.un.org/unsd/classifications/Econ>

Table 17: Price dispersion by Rauch classification

Category	Standard deviation
Organized exchange	0.90
Referenced price	1.18
Differentiated goods	1.44

Notes: Standard deviation of log unit prices by Rauch (1999) classification.

Although differentiated goods exhibit higher standard deviations, part of the price dispersion also persists within the “organized exchange” category. This exercise shows that quality differences alone cannot account for all of the observed variation in input prices.

The key point is that there is no systematic trend in quality differences over time. Since non-differentiated goods have minimal quality variation, I define the quality gap as the difference in price variance between differentiated and non-differentiated goods. Over the observed years, this quality gap remains largely unchanged. Using weighted averages, there is virtually no variation at all. Moreover, the share of differentiated goods in total imports is stable, ranging between 74Taken together, these patterns indicate that changes in product quality cannot explain the observed increase in price variance.

### Decreasing Search Cost for Suppliers and Input Price Dispersion

Related to the decreasing search cost and the influx of smaller firms into the import market is the input price dispersion. As the distribution of importer size become more disperse, variance of input price is also increasing. I define input price dispersion as:

$$\text{Var}(P) = \text{Var}(\log(\frac{P_{fvy}}{\bar{P}_{vy}}))$$

It is also robust when I limit the analysis to within-EU import or imports from most of Sweden’s major trading partner.

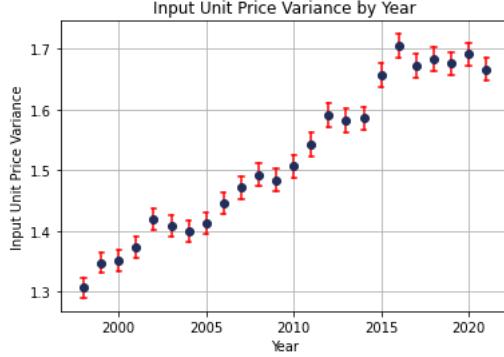


Figure 32: Input price dispersion over time

One potential link between the two dispersing distribution is that input unit price is inversely correlated to firm size. When there are a lot of small firms goes into the import market, they buy very little and they are not familiar with the market, therefore they will pay a relatively high price. On the other hand, big firms become bigger, gain market power and face a even lower price. In this case, the price gap will grow. I will present more evidence below.

It is also important to note that quality differences is likely not the reason. I measure the price variance driven by quality difference by comparing price variance within differentiated products and non-differentiated ones, as defined by Rauch (1999). It is a very conservative measure that likely exaggerate the important because this gap likely include other channel such as bargaining and information frictions. But what is important is that the quality gap hasn't been increasing during the period, neither does the composition of differentiated products in all products. See more in Appendix.

## Theory Appendix

### Simplified Benchmark - Single Supplier Case

In this section, I examine another simplified model where firms can only have one supplier (variety), no quality differentiation and both upstream and downstream productivity follows a simple  $U(0, 1)$  distribution. This model resembles a familiar model of McCall (1970) random job search model. With this class of model, I can solve the optimal stopping problem by getting the reservation productivity  $z_u^R(z)$  and  $z^R$ . I can get an idea how search cost  $\kappa$  affect firm size distribution from a fixed point solution.

The reservation supplier productivity is solved by 2 equations. The first equation is the definition of the outside option  $D$ :

$$D = \int_{z_u^R}^{\bar{z}_u} [\pi(z_u) - T(z_u)] dF(z_u) + \int_{\underline{z}_u}^{z_u^R} D dF(z_u) - w\kappa$$

which consists of 3 terms: The first one is expected profit given the probability of meeting a satisfactory supplier ( $z_u \geq z_u^R$ ), the second one is falling back to outside option if the supplier is worse than the reservation productivity and the last term is minus the search cost.

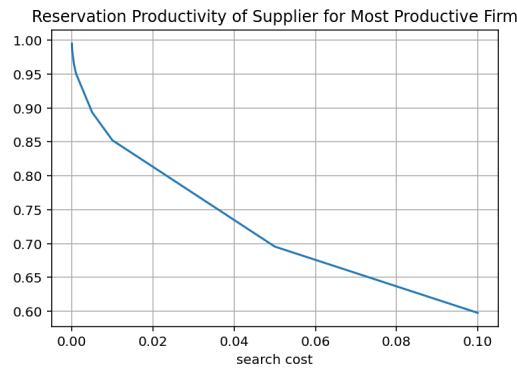
The second equation is defined by the indifference condition, where for any buyer, she should be indifferent between the profit for meeting the reserved supplier and going into outside option:

$$\pi(z, z_u^R) - T(z, z_u^R) = D$$

Combining the equations we get a expression for solving  $z_u^R$  for each  $z$ :

$$\frac{\kappa}{Kz^{\epsilon-1}} = [1 + \alpha(\epsilon - 1)z_u^{R[\alpha(\epsilon-1)+1]} - (\alpha(\epsilon - 1) + 1)z_u^{R\alpha(\epsilon-1)}]$$

This expression provides a relationship between  $z_u^R$  and  $\kappa$  for any firm with productivity  $z$ . And solving this fixed point formula gives:

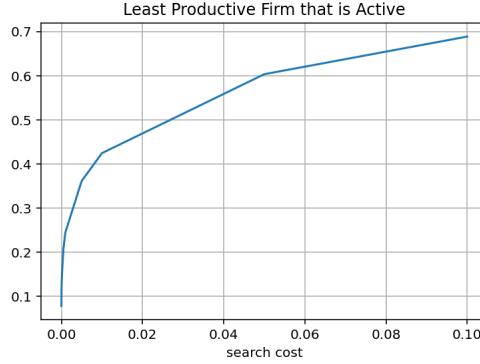


This shows that when search cost decrease, the reservation productivity, given  $z$ , is increasing. In other words, when search cost decrease, the expected productivity of the supplier increase. It is obvious that matching with a more productivity supplier means producing more. Therefore, downstream firm produce more when search cost decrease. It showcase, in this restrictive model, search cost also functions as expansion cost.

On top of that, I can show that the lowest productivity  $z^R$  where this downstream firm still remain active can be calculated by solving:

$$\frac{\kappa}{1 - F[z_u^R(z^R)]} = \int_{z_u^R(z^R)}^{\bar{z}_u} [\pi(z_u, z^R) - T(z_u, z^R)] dF(z_u)$$

And keeping everything but search cost  $\kappa$  constant,  $z^R$  increase with  $\kappa$ :



It shows that when search cost decrease, less productive firm can also enter the intermediate good market and produce. This showcase search cost can act as entry barrier.

## Sequential Search with Efficient Bargaining

This appendix solves the sequential version of the buyer's problem with identical upstream productivity and constant search cost. Upstream unit cost is fixed at  $c = 1/\bar{z}_u$ . At any state  $(z, X)$  the buyer meets a new supplier and bargains efficiently over  $x_i$ . After contracting, total input becomes

$$X_{\text{new}} = (X^\rho + x_i^\rho)^{1/\rho}.$$

Define

$$A \equiv \frac{\epsilon - 1}{1 + \alpha(\epsilon - 1)}, \quad K_1 \equiv [1 + \alpha(\epsilon - 1)] \epsilon^{-\frac{\epsilon}{1+\alpha(\epsilon-1)}} \left( \frac{w}{(\epsilon-1)(1-\alpha)} \right)^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}}.$$

In the main text the value function  $V(z, X)$  is sequential and includes both the direct production benefit and the continuation value from future supplier matches. For the derivation it is convenient to write

$$V(z, X) = K_1[zX^\alpha]^A + W(z, X),$$

so that

$$V(z, X_{\text{new}}) - V(z, X) = K_1 \left( [zX_{\text{new}}^\alpha]^A - [zX^\alpha]^A \right) + [W(z, X_{\text{new}}) - W(z, X)].$$

**Per match bargaining problem** At  $(z, X)$  the efficient surplus from contracting  $x_i$  is

$$\max_{x_i \geq 0} \underbrace{K_1 \left[ z(X^\rho + x_i^\rho)^{\alpha/\rho} \right]^A - K_1 \left[ zX^\alpha \right]^A}_{\text{direct benefit}} + \underbrace{W(z, (X^\rho + x_i^\rho)^{1/\rho}) - W(z, X)}_{\text{continuation effect}} - c x_i.$$

Let  $x_i^*(z, X)$  denote the maximizer of this problem and write  $x^* \equiv x_i^*(z, X^*)$  at the stopping boundary introduced below. The buyer's surplus from the match is

$$G(z, X) = \theta \left\{ K_1 \left[ z(X^\rho + x_i^\rho)^{\alpha/\rho} \right]^A - K_1 \left[ zX^\alpha \right]^A + W(z, (X^\rho + x_i^\rho)^{1/\rho}) - W(z, X) - c x_i \right\}.$$

**First order condition** Let  $X_{\text{new}} = (X^\rho + x_i^\rho)^{1/\rho}$ . The first order condition with respect to  $x_i$  is

$$\left[ \alpha A K_1 z^A (X^\rho + x_i^\rho)^{\frac{\alpha A}{\rho} - 1} + W_2(z, X_{\text{new}}) (X^\rho + x_i^\rho)^{\frac{1}{\rho} - 1} \right] x_i^{\rho-1} = c,$$

or, equivalently,

$$\left[ \alpha A K_1 z^A X_{\text{new}}^{\alpha A - 1} + W_2(z, X_{\text{new}}) \right] x_i^{\rho-1} X_{\text{new}}^{1-\rho} = c.$$

**Search rule and stopping boundary** Search continues while  $G(z, X) > w\kappa$ . The endogenous stopping boundary  $X^*(z)$  satisfies

$$G(z, X^*) = w\kappa.$$

At  $X^*$  the continuation effect from the marginal match vanishes,  $W_2(z, X_{\text{new}}) = 0$ , and the first order condition reduces to

$$\alpha A K_1 z^A (X^{*\rho} + x_i^{*\rho})^{\frac{\alpha A}{\rho} - 1} x_i^{*\rho-1} = c.$$

**Solving in ratios** Define  $r \equiv x^*/X^*$  and note  $X_{\text{new}} = X^*(1+r^\rho)^{1/\rho}$ . The first order condition implies

$$X^{*\alpha A-1} = \frac{c}{\alpha A K_1 z^A (1+r^\rho)^{\frac{\alpha A - \rho}{\rho}} r^{\rho-1}}, \quad X^* = \left[ \frac{c}{\alpha A K_1 z^A (1+r^\rho)^{\frac{\alpha A - \rho}{\rho}} r^{\rho-1}} \right]^{\frac{1}{\alpha A - 1}}.$$

At the boundary, the surplus condition becomes

$$K_1 [z(X^{*\rho} + x_i^{*\rho})^{\alpha/\rho}]^A - K_1 (zX^{*\alpha})^A - c x^* = \frac{w\kappa}{\theta},$$

which, after substituting  $r = x^*/X^*$  and the expression for  $X^{*\alpha A-1}$ , reduces to

$$X^* c \left[ \frac{(1+r^\rho) - (1+r^\rho)^{1-\frac{\alpha A}{\rho}}}{\alpha A r^{\rho-1}} - r \right] = \frac{w\kappa}{\theta}.$$

Solving this nonlinear equation for  $r$  yields  $X^*(z)$  and  $x^*(z) = r X^*(z)$  in closed form, since  $X^*$  is explicit in  $r$  above. The model can then be solved recursively by backward induction starting from the stopping boundary.

## Closed Form Solution for Important Variables

In the baseline model with imported inputs, the firm's state is represented by the pair  $(z, X)$ . The variables required for the value function iteration, which are the optimal price  $p(z, X)$ , output  $y(z, X)$ , and profits  $\pi(z, X)$ , can all be expressed as functions of this state. The problem is therefore fully characterized by the pair  $(z, X)$ .

### Output price and output.

$$\max_{p,y} p y - w \left( \frac{y}{z X^\alpha} \right)^{\frac{1}{1-\alpha}} \quad \text{s.t.} \quad y = C \left( \frac{p}{P} \right)^{-\epsilon}$$

Using the constraint to eliminate  $y$ :

$$\max_p p^{1-\epsilon} \frac{C}{P^{-\epsilon}} - w p^{\frac{\epsilon}{\alpha-1}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1-\alpha}} \left( \frac{C}{z X^\alpha} \right)^{\frac{1}{1-\alpha}}$$

FOC in  $p$ :

$$(1 - \epsilon) \frac{C}{P^{-\epsilon}} p^{-\epsilon} = \frac{w\epsilon}{\alpha - 1} p^{\frac{\epsilon-\alpha+1}{\alpha-1}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1-\alpha}} \left( \frac{C}{zX^\alpha} \right)$$

Solve for  $p$ :

$$p^{\frac{1+\alpha(\epsilon-1)}{1-\alpha}} = \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{\alpha}{1-\alpha}} (zX^\alpha)^{-\frac{1}{1-\alpha}},$$

$$p = \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{1-\alpha}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{\alpha}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{-\frac{1}{1+\alpha(\epsilon-1)}}$$

Then  $y$ :

$$y = \frac{C}{P^{-\epsilon}} p^{-\epsilon}$$

$$= \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon(1-\alpha)}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1-\alpha}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon}{1+\alpha(\epsilon-1)}}$$

**Profit function or not search value  $V^{NS}$**

$$\pi(z, X) = py - w \left( \frac{y}{zX^\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$= \frac{C}{P^{-\epsilon}} p^{1-\epsilon} - w \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon-1-\alpha(\epsilon-1)}{1+\alpha(\epsilon-1)} \cdot \frac{1}{1-\alpha}}$$

$$= \left\{ \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}} - w \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}} \right\} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}}$$

$$= \left[ \frac{1+\alpha(\epsilon-1)}{\epsilon} \right] \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}}$$

Note that the profit function  $\pi$  is also the not-search value  $V^{NS}$

**Labor cost.**

$$w \left( \frac{y}{zX^\alpha} \right)^{\frac{1}{1-\alpha}} = \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} w \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}}$$

$$= \frac{(\epsilon-1)(1-\alpha)}{\epsilon} py \quad \text{or} \quad \left[ \frac{(\epsilon-1)(1-\alpha)}{1+\alpha(\epsilon-1)} \right] \pi$$

## Concavity of the Bargaining Problem

Given all possible  $x$ , I can use the first-order condition (FOC) to find  $T$ , because the following function is strictly concave (as long as it is defined). To simplify notation, I define

$$V(x) := V(z, X_N) - V(z, X), \quad K(x) := \frac{w}{z_u}x.$$

Consider

$$B(\delta T_1 + (1 - \delta)T_2) = (V(x) - \delta T_1 - (1 - \delta)T_2)^\theta (\delta T_1 + (1 - \delta)T_2 - K(x))^{1-\theta}.$$

Taking log of both sides,

$$\log B(\delta T_1 + (1 - \delta)T_2) = \theta \log(V(x) - \delta T_1 - (1 - \delta)T_2) + (1 - \theta) \log(\delta T_1 + (1 - \delta)T_2 - K(x)).$$

Because the logarithm is strictly concave, for any  $\delta \in (0, 1)$ ,

$$\begin{aligned} \log(V(x) - \delta T_1 - (1 - \delta)T_2) &\geq \delta \log(V(x) - T_1) + (1 - \delta) \log(V(x) - T_2), \\ \log(\delta T_1 + (1 - \delta)T_2 - K(x)) &\geq \delta \log(T_1 - K(x)) + (1 - \delta) \log(T_2 - K(x)). \end{aligned}$$

Substituting these inequalities into the expression for  $\log B$  and grouping terms gives

$$\begin{aligned} \log B(\delta T_1 + (1 - \delta)T_2) &\geq \delta \left[ \theta \log(V(x) - T_1) + (1 - \theta) \log(T_1 - K(x)) \right] \\ &\quad + (1 - \delta) \left[ \theta \log(V(x) - T_2) + (1 - \theta) \log(T_2 - K(x)) \right] \\ &= \delta \log B(T_1) + (1 - \delta) \log B(T_2). \end{aligned}$$

Hence  $\log B(T)$  is strictly concave on  $(K(x), V(x))$ . Since  $\log$  is strictly increasing,  $B(T)$  is strictly log-concave. As long as  $V(x) \geq K(x)$ , the maximizer of  $B(T)$  exists and is unique. We can therefore define a function *Bargain* that gives the best value given  $x$ , and then find  $\arg \max_x \text{Bargain}$ .

**Proof.** Let  $T(x^o) = \arg \max_T B(x^o, T)$ , which is unique. Assume  $(x^*, T^*)$  is the maximizer of  $B$  such that  $B(x^*, T^*) \geq B(x', T')$  for all  $(x', T')$ . If  $T^* \neq T(x^*)$ , then  $B(x^*, T^*) \geq B(x^*, T(x^*))$ , which contradicts the uniqueness of  $T(x^*)$ . Therefore, the solution is unique.

I can then obtain *Bargain* (the one-dimensional version of the bargaining problem) by substituting the FOC into  $B$ :

$$\begin{aligned}
\text{Bargain}(x) &= (-T + V(z, X_N) - V(z, X))^{\theta} (T - w \frac{x}{z_u})^{1-\theta} \\
&= \left( -(1-\theta)V(x) - \theta K(x) + V(x) \right)^{\theta} \left( (1-\theta)V(x) + \theta K(x) - K(x) \right)^{1-\theta} \\
&= \left( \theta V(x) - \theta K(x) \right)^{\theta} \left( (1-\theta)V(x) - (1-\theta)K(x) \right)^{1-\theta} \\
&= \theta^{\theta} (1-\theta)^{1-\theta} \left( V(x) - K(x) \right) \\
&= \theta^{\theta} (1-\theta)^{1-\theta} \left( V(z, X^N Q^N, n+1) - V(z, X Q, n+1) - w \frac{x}{z_u} \right).
\end{aligned}$$