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Lab 2: Mine Crafting

I. Introduction

The focus of this mission is to calculate the time it would take for a 1 kg test mass to reach the bottom of a mineshaft at the Earth's equator. The time is first calculated in the ideal case where there is no drag or a variable gravitational force. Increasingly complex assumptions are made to adjust the time calculated, including with a non-zero drag, a variable gravitational force, a Coriolis force acting on the test mass, an infinitely deep mine shaft, and a non-uniformly dense Earth. Using the equations provided in the various parts of the lab, the case where the mine shaft is dug on the Moon rather than the Earth is considered.

II. The Ideal Case

In the ideal case of no drag and a constant gravitational force, the time it would take for a test mass to fall to the end of a 4km mineshaft would be 28.6 s, which was found through the equation $\sqrt{\frac{2h}{g}}$. By integrating the second order differential equation $\frac{d^2y}{dt^2} =- g + \alpha\left|\frac{dy}{dt}\right|^{\gamma}$ using solve_ivp for the time span from 0 to 30 seconds, and then creating a stopping event for when the position is equal to the bottom of the mineshaft, it was found that the time this event occurred was at 28.6 s. This is equal to the time found analytically.

III. Case With Linear Gravity and Drag

Assuming that $g(r) = g_0\frac{r}{R}$, the gravitational force can be integrated with respect to r over the timespan from 0 to 1500 seconds, with r being the distance of the mass from the center of the Earth and R being the radius of the Earth. The drag remains 0, and it is found that the fall time with the variable gravitational force is approximately increased by 0.01 s. Drag is then introduced, which is calibrated to 0.004 by manually adjusting the value until the time for the hit_bottom event was approximately at 50 m/s. The fall time is now 84 s, which is around 56 s more than our previous fall time. Making the gravitational force variable and increasing the drag correlates with an increase in the fall time.

IV. Case With Coriolis Force

Since the Earth is rotating, a Coriolis force is exerted on the mass as it falls. The equation for the Coriolis force is $F_c =- 2m(\Omega\, x\, v)$. The differential equation for motion is adjusted to include the Coriolis force by breaking up the velocity and acceleration into x- and y-components. The differential equation is then integrated over the timespan from 0 to 200 s. The motion of the mass in the transverse direction (which has a depth of 5 km) is also considered with the x-component being added. The falling time found after these assumptions were added was 28.6 s, which is equal to what was found previously in the less complex cases. The mass does hit the 0 position in the transverse direction before the vertical direction, meaning it hits the wall before the bottom of the mine shaft. When drag was added at 0.004, the falling time increased to approximately 84.2 s.

V. Crossing Times of Trans-Earth-Moon

The case where the density of the Earth is nonuniform is then considered, with the equation $p(r) = p_n(1 - \frac{r^2}{R^2})^n$ being used to calculate the density as a function of r, the distance from the center of the Earth, and R being the radius of the Earth. The total mass of the Earth is calculated as $M = 4\pi\int_0^R p(r)r^2dr$. p_n is set to 1, and then the normalized density profile is calculated for n = 0, 1, 2, 9 by integrating the density function with respect to r for each n.

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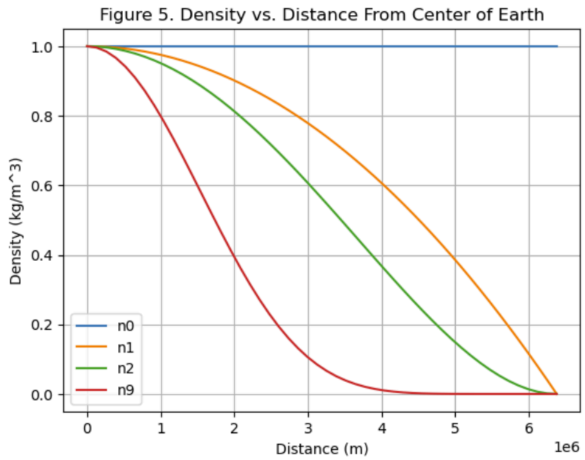
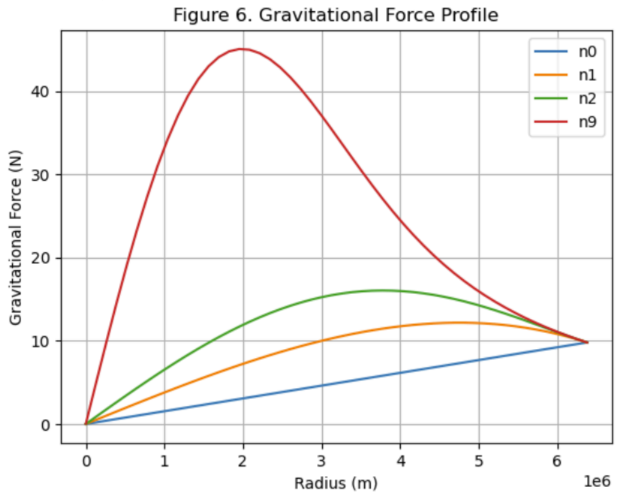
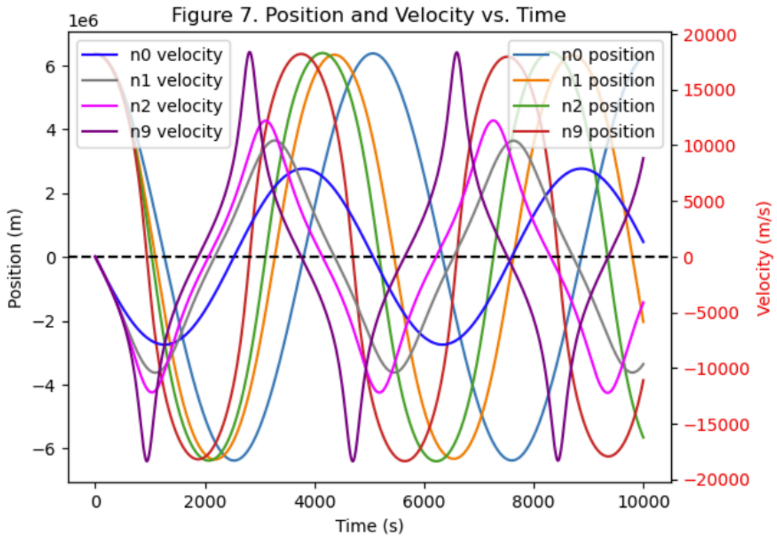


The force profile for each n is then found by computing the density constant for each n by dividing the mass of the Earth by the integrated result of the density function multiplied by 4π. The force is found by using the equation $\frac{G*M(r)}{r^2}$ for each n value, with the density constant for each n being found using the previously described density function.



The position and the velocity are then found by adding the differential equations from section I $\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\gamma}$ and then integrating using solve_ivp.

The travel time to the center of the Moon is found through the equation $t = \frac{\pi\sqrt{\frac{R^3}{GM}}}{2}$, which is ¼ of the orbital period since that is from where the surface of the Moon to the center of the Moon goes. The orbital period is T=$\frac{R}{v}$ with the velocity found analytically to be 7905.2 m/s and the orbital period being 806.8 s. The crossing time is 1625.1 s.



For Earth, the time to center (s) for each n is: n0: 1267.2, n1: 1096.9, n2: 1035.1, n9: 943.9. The velocity (m/s) at center for each n is: n0: -7905.9, n1: -10435.2, n2: -12200.7, and n9: -18391.9.

VI. Conclusions

It was found that by adding in the drag, Coriolsis force, and variable gravity, the falling time tended to increase. By adding in these assumptions, the case was more similar to the physical case that would actually occur on Earth. These assumptions and code used to determine the falling times and position/velocity were then considered with the Moon, where it was found that the time to cross to the center of the Moon is 1625.1 s versus the Earth's time to cross of 2195.0 seconds. To further enhance realism, a non-linear gravity model could be used and the Earth could be modeled not as a perfect sphere.