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Phys265

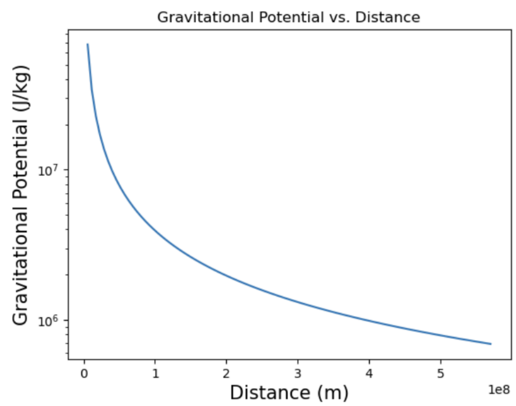
Lab 1: Apollo Missions

I. Introduction

This report will go into detail about the change in velocity experienced, burn time, and achieved altitude by the Saturn V rocket during the Apollo missions. Through the use of Python and the libraries numpy, matplotlib, and scipy, various calculations will also be performed for the gravitational potentials and gravitational forces of the Earth and Earth-Moon System. Plots for these calculations will also be shown using matplotlib.

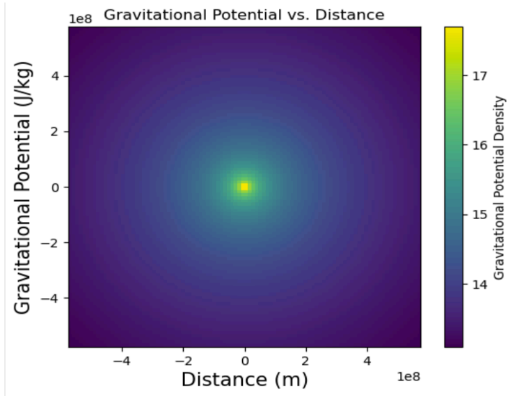
II. The Gravitational Potential of the Earth-Moon System

The gravitational potential Φ of the Earth will be found first through the equation $\Phi = \frac{-GM}{r}$, and then the absolute value of the Φ versus distance from the surface of the Earth will be plotted as a one-dimensional plot. The constants G, M, and r are defined for gravity (6.67 x $10^{-11}\ m^3/kg/s^2$), the mass of the Earth (5.9 x $10^{24}\ kg$), and the distance from the Earth to the moon (3.8 x $10^{8}\ m$). A range for x and y are created, with x being from 0 to 1.5 times the distance from the surface of the Earth to the moon (i.e., a range from 0 to 1.5 times distance r). The variable y remains 0 so that the plot is one dimensional. The origin for the Earth is 0 for the x and y coordinates. A function is then created to evaluate Φ versus x, with the constants G, M, and r also being passed in the function. Using the matplotlib Python library, the values for Φ are plotted versus x. Since the potential falls quickly versus r, the y-axis was made logarithmic to make it easier to visualize the data.



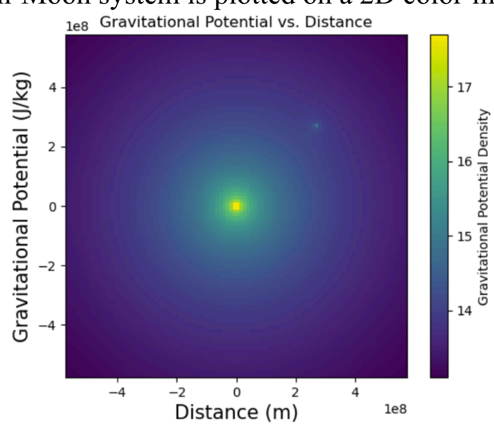
Fig, 1. 1D plot of gravitational potential Φ vs. distance x.

A two dimensional color-mesh plot is then created to visualize Φ over a large range of x and y values. The variables x and y are redefined to be from negative 1.5 times the distance from the surface of the Earth to the moon to positive 1.5 times the distance from the surface of the Earth to the moon. The meshgrid function from numpy is utilized to create a meshgrid from the x and y values. A logarithmic colorbar is displayed in order to visualize the density of Φ versus distance.

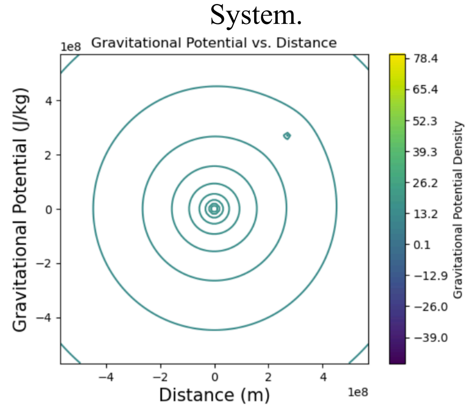


Fig, 2. 2D color-mesh plot of gravitational potential Φ vs. distance x and y.

The gravitational potential Φ of the Earth-Moon system is then observed. Φ for the moon is calculated using the same function described above, with the origin of the moon being $\frac{r}{\sqrt{2}}$ for the x and y coordinates. The x and y ranges from negative 1.5 times the distance from the surface of the Earth to the moon to positive 1.5 times the distance from the surface of the Earth to the moon remain the same. The potentials of the Earth and the Moon are added together. The value of that sum for the combined Earth-Moon system is plotted on a 2D color-mesh plot and a contour plot.



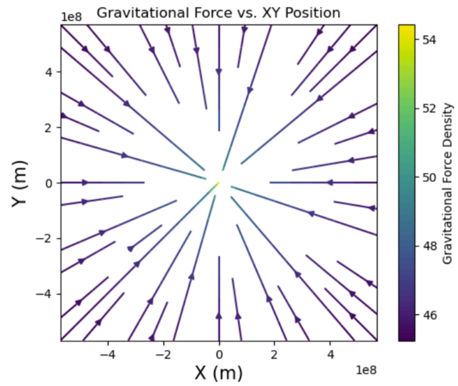
Fig, 3. 2D color-mesh plot of gravitational potential Φ vs. distance x and y for Earth-Moon System.



Fig, 4. Contour plot of gravitational potential Φ vs. distance x and y for Earth-Moon System.

III. The Gravitational Force of the Earth-Moon System

The gravitational force between two bodies will be found first through the equation $F = \frac{-GM1m2}{r}$. The constants G, M1, m2, and r are defined for gravity (6.67 x 10^{-11} $m^3/kg/s^2$), the mass of the Earth (5.9 x 10^{24} kg), the mass of the Moon (7.3 x 10^{22} kg), and the distance from the Earth to the moon (3.8 x 10^8 m). The x and y ranges remain negative 1.5 times the distance from the surface of the Earth to the moon to positive 1.5 times the distance from the surface of the Earth to the moon. The origin for the Earth is 0 for the x and y coordinates, and the origin for the Moon is $\frac{r}{\sqrt{2}}$ for the x and y coordinates. A function is created to evaluate the gravitational force in the x- and y-directions. These values are plotted as a streamplot. A logarithmic colorbar is displayed in order to visualize the density of the gravitational force versus distance.



Fig, 5. Stream plot of Gravitational Force vs. distance x and y for Earth-Moon System.

IV. Projected Performance of the Saturn Stage 1

The altitude of the Saturn V rocket is determined through a series of calculations. The change in the rocket's velocity $\Delta v = v_e ln(\frac{m0}{m(t)}) - gt$. The variable m0 refers to the initial "wet mass" (fuel + rocket parts + payload), which is 2.8 x 10^6 kg. The variable m(t) refers to the mass at time t, v_e is the fuel exhaust velocity (2.4 x 10^3 m/s), and g is the gravitational acceleration (9.81 m/s^2). The total burn time is T = $\frac{m0 - mf}{\dot{m}}$, where mf is the final mass after the fuel is burned off (7.5 x 10^5 kg) and ṁ is the burn rate of the fuel (1.3 x 10^4 m/s). The altitude h that is achieved before the fuel runs out is the change in velocity integrated from 0 to T. Δv is defined as a function utilizing the aforementioned constants, and it is integrated through scipy's quad integrate function. The burn time T found is 157.69 s and the altitude h found is 674093.98 m.

V. Discussion and Future Work

The burn time T found was 157.69 s and the altitude h found was 74093.98 m. Compared to the values found by NASA of 160 s and 70 km, the burn time is underestimated and the altitude is overestimated. This is possibly due to the constants being approximated causing the final values to be somewhat off, like gravitational acceleration being assumed to be constant when that is not necessarily true. The thrust power will also probably change as the fuel burns off. Drag is also not considered. These factors could be looked at when considering future calculations that could be made.