MSc Project - Modelling a Double Barred Galaxy as a Double Binary

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1 Introduction

The aim of this project was to create a simple model of a double barred galaxy using a double binary.

2 Methods

Fortran 90 was selected as the language of choice in order to avoid the fixed line limitations of Fortran 77. The model was created using OdeInt from Numerical Recipes. I first converted OdeInt to Fortran 90 for this purpose. Please see the Compilers subsection for a few notes on compilers - may or may not be relevant to the project.
The classical equations of motion for an n-body problem are:

$$m_i \ddot{\mathbf{r}} = \sum_{i \neq j} \frac{m_i m_j}{r_{ij}^3} \mathbf{r_{ij}} \qquad i = 1, 2, 3... \tag{1}$$

where $\mathbf{r_i} = (x_i, y_i)$ and $\mathbf{r_{ij}} = \mathbf{r_j} - \mathbf{r_i}$. So for a four body problem this equation results in the following:

$$\ddot{\mathbf{r_1}} = \frac{m_2 \mathbf{r}_{12}}{r_{12}^3} + \frac{m_3 \mathbf{r}_{13}}{r_{13}^3} + \frac{m_4 \mathbf{r}_{14}}{r_{14}^3} \tag{2}$$

$$\ddot{\mathbf{r_2}} = \frac{m_1 \mathbf{r}_{21}}{r_{21}^3} + \frac{m_3 \mathbf{r}_{23}}{r_{23}^3} + \frac{m_4 \mathbf{r}_{24}}{r_{24}^3} \tag{3}$$

$$\ddot{\mathbf{r_3}} = \frac{m_1 \mathbf{r}_{31}}{r_{31}^3} + \frac{m_2 \mathbf{r}_{32}}{r_{32}^3} + \frac{m_4 \mathbf{r}_{34}}{r_{34}^3} \tag{4}$$

$$\ddot{\mathbf{r_4}} = \frac{m_1 \mathbf{r}_{41}}{r_{41}^3} + \frac{m_2 \mathbf{r}_{42}}{r_{42}^3} + \frac{m_3 \mathbf{r}_{43}}{r_{43}^3} \tag{5}$$

The above equations were encoded into Fortran. A full code listing is supplied in a separate file.

2.1 Initial Conditions

As a starting point both the outer binary and inner binary were placed in circular orbits and tested separately and then combined to ensure a stable starting point. The inner binary had initially a neglible mass. The outer binary initial conditions were:

```
m1=.5, m2=.5, x1=-.5, x2=.5, y1=0, y2=0,
vx1=0, vx2=0, vy1=-.5, vy2=.5
```

and for the inner binary:

```
m3=.001, m3=.001, x3=-.001, x4=.001, y3=0, y4=0,
vx3=0, vx4=0, vy3=-.5, vy4=.5
```

Plots of the two initial binaries are shown in Fig. 1 and 2. The scale of the inner binary is so small that it is difficult to see in GnuPlot.

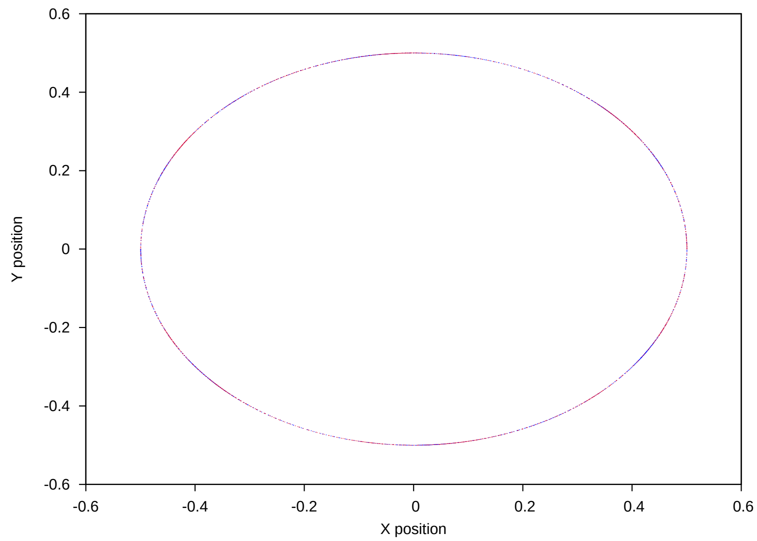

Figure 1: Outer Binary

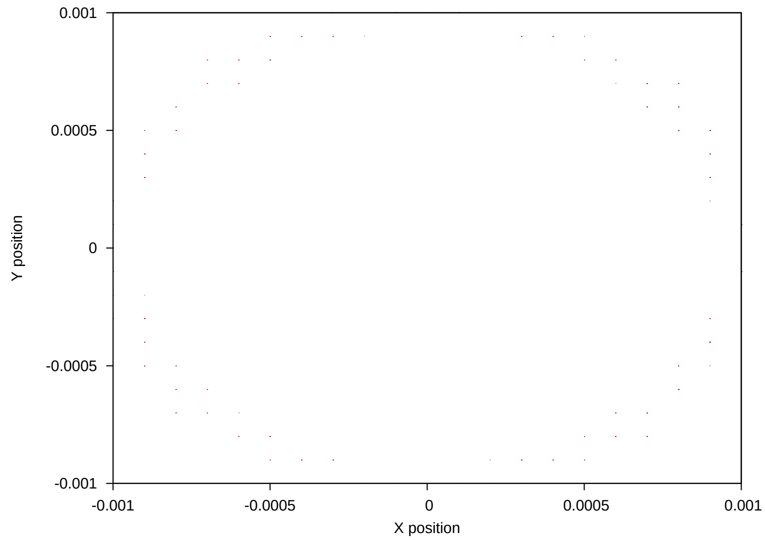

Figure 2: Inner Binary

I then proceeded to perturb the system by gradually increasing the mass of the inner binary. As the system became unstable I componensated for the increasing mass of the inner binary by increasing the velocity of the outer binary and also decreasing the velocity and increasing the binary separation of the inner binary. In many of the configurations the systems were inherently unstable and produced collisions with some or all of the bodies being ejected from the system. These were rejected and only 'stable' configurations were considered. Table 1 shows the perturbations made to the system - only the variables in this table changed and all the other initial conditions remained the same.

2.2 Compilers

As I had converted OdeInt into Fortran 90 I wanted to make sure I hadn't introduced any errors. I therefore compared the output of the Numerical Recipes version of OdeInt with mine. I was compiling with GFortran and the Fortran 77 version was compiled with g77. They were different! So naturally concerned I then compiled the Fortran 77 version with GFortran. The results were identical to mine, which satisified me that I hadn't introduced any errors.

Table 1: Configurations

Config.	m3	m4	vy1	vy2	x3	x4	vy3	vy4
1	.001	.001	-.5	.5	-.001	.001	-.5	.5
2	.002	.002	-.5	.5	-.001	.001	-.5	.5
3	.003	.003	-.5	.5	-.001	.001	-.5	.5
4	.004	.004	-.5	.5	-.004	.004	-.5	.5
5	.004	.004	-.55	.55	-.004	.004	-.5	.5
6	.004	.004	-.57	.57	-.004	.004	-.5	.5
7	.005	.005	-.58	.58	-.005	.005	-.3	.3
8	.005	.005	-.58	.58	-.005	.005	-.4	.4
9	.005	.005	-.58	.58	-.005	.005	-.35	.35
10	.006	.006	-.6	.6	-.006	.006	-.3	.3
11	.01	.01	-.7	.7	-.01	.01	-.2	.2
12	.02	.02	-.75	.75	-.02	.02	-.1	.1
13	.025	.025	-.65	.65	-.02	.02	-.5	.5
14	.05	.05	-.9	.9	-.05	.05	-.1	.1
15	.05	.05	-.9	.9	-.05	.05	-.15	.15
16	.06	.06	-.95	.95	-.06	.06	-.15	.15
17	.07	.07	-.1	.1	-.07	.07	-.15	.15
18	.08	.08	-.105	.105	-.08	.08	-.12	.12

3 Results

As can be seen from Fig. 3 even a neglible mass inner binary causes the outer binary to begin to separate and become less tightly bound. Figures Fig. 3 to 20 show plots of the double binary using the parameters given in Table 1. In many cases the inner binary is only visible as a point at the centre of the outer binary. As the mass of the inner binary increased the system became increasingly unstable and the effect on the outer binary became more pronounced. Even the 'stable' configurations listed in Table 1 hold for only a few orbits. I was unable to recreate a stable system for a mass $>$ .08 for the inner bar. The plots below reveal that increasing the mass of the inner binary causes the outer binary to separate and form wider orbits.

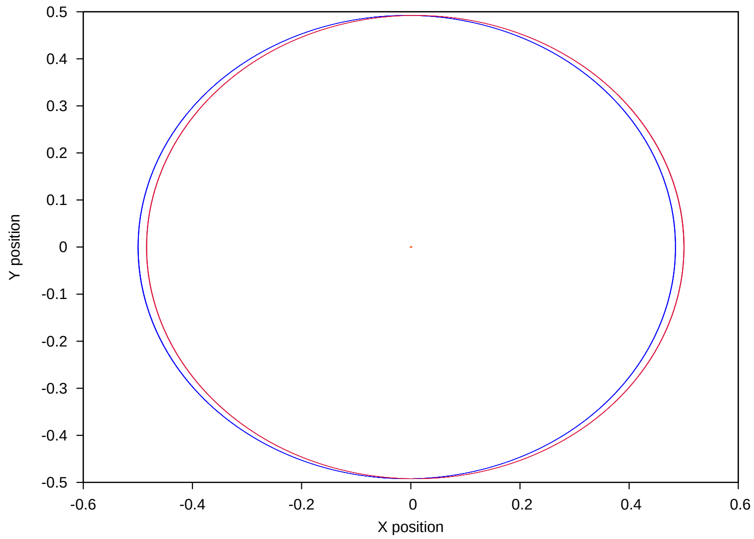

Figure 3: Configuration 1

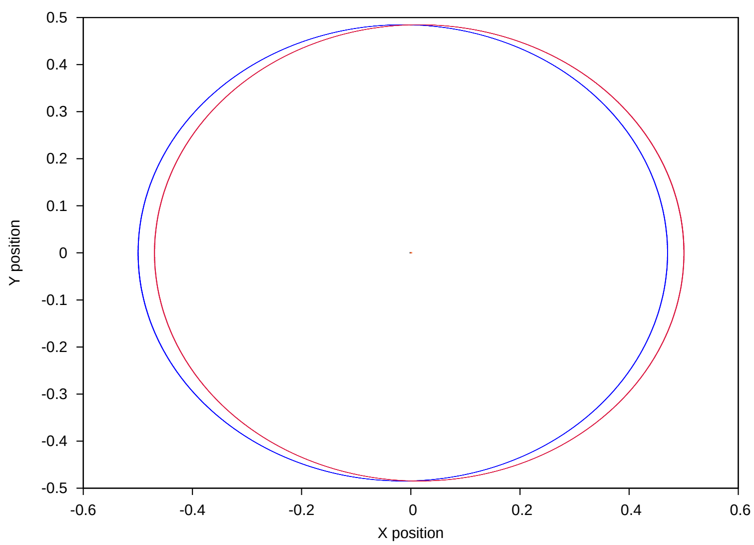

Figure 4: Configuration 2

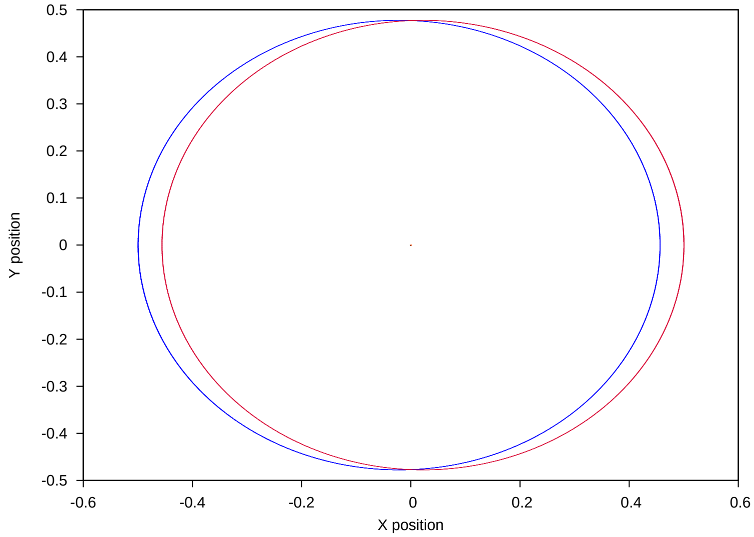

Figure 5: Configuration 3

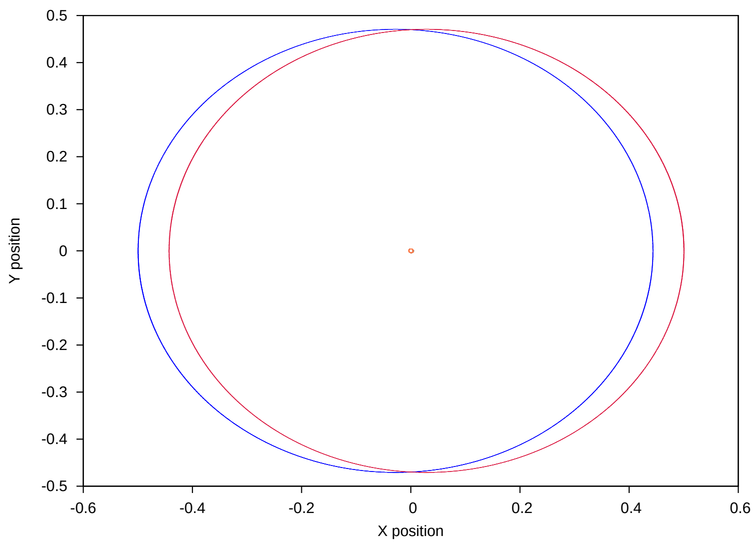

Figure 6: Configuration 4

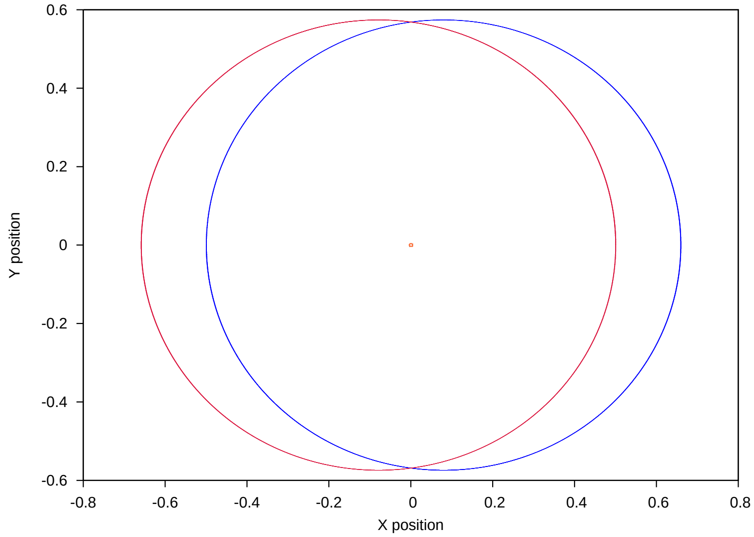

Figure 7: Configuration 5

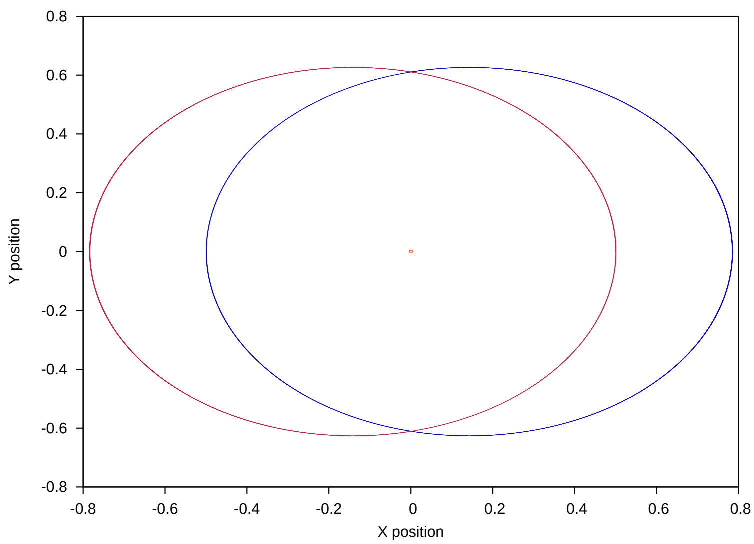

Figure 8: Configuration 6

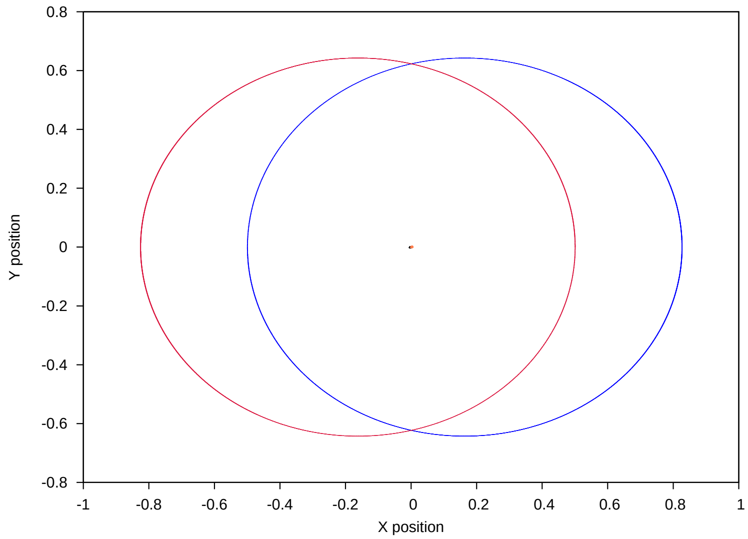

Figure 9: Configuration 7

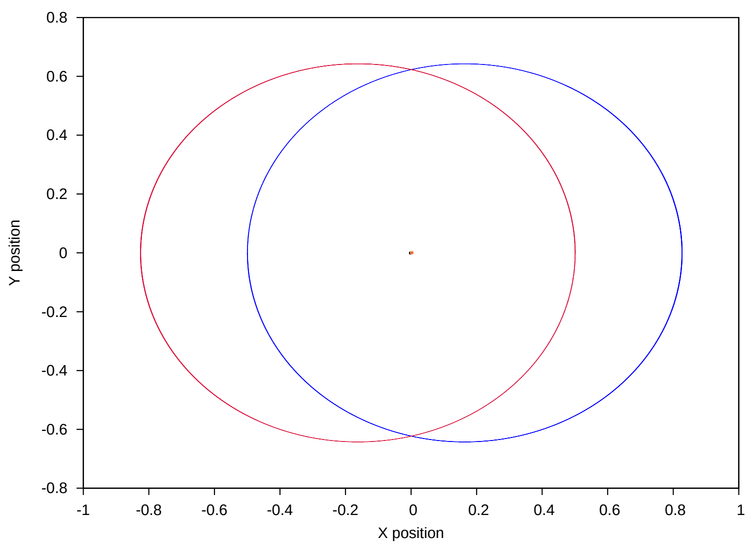

Figure 10: Configuration 8

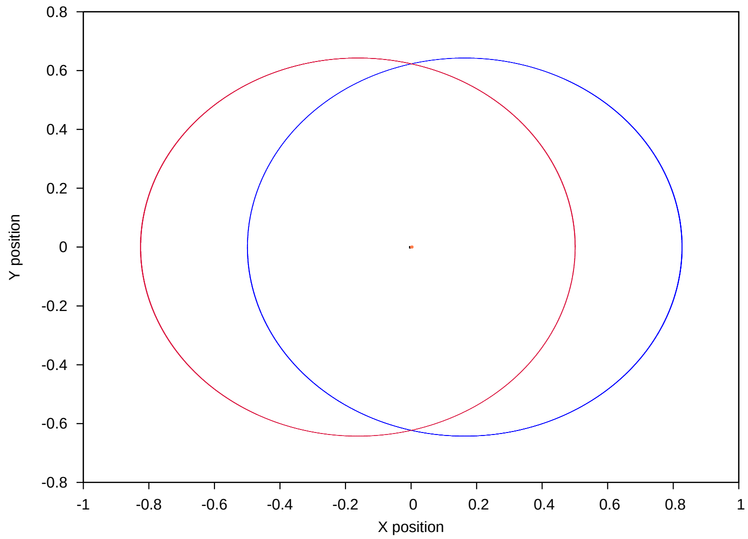

Figure 11: Configuration 9

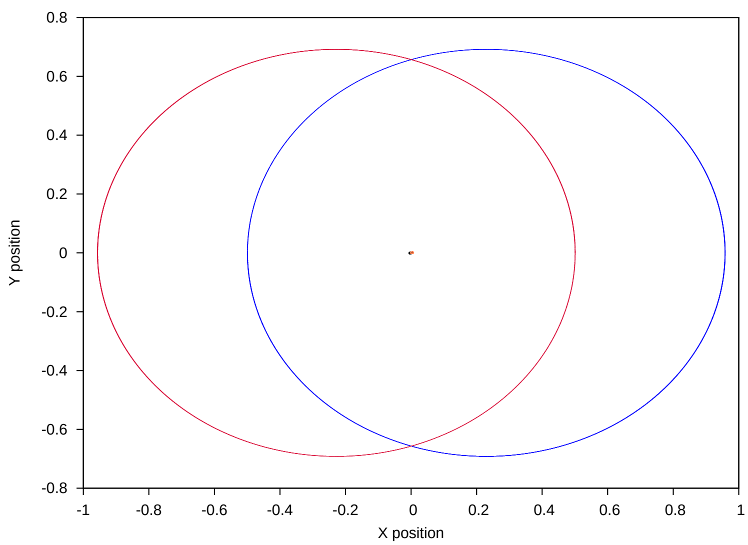

Figure 12: Configuration 10

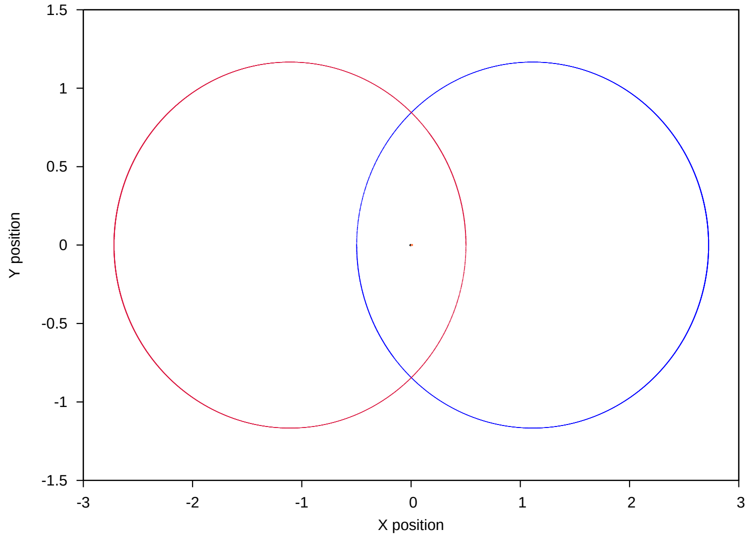

Figure 13: Configuration 11

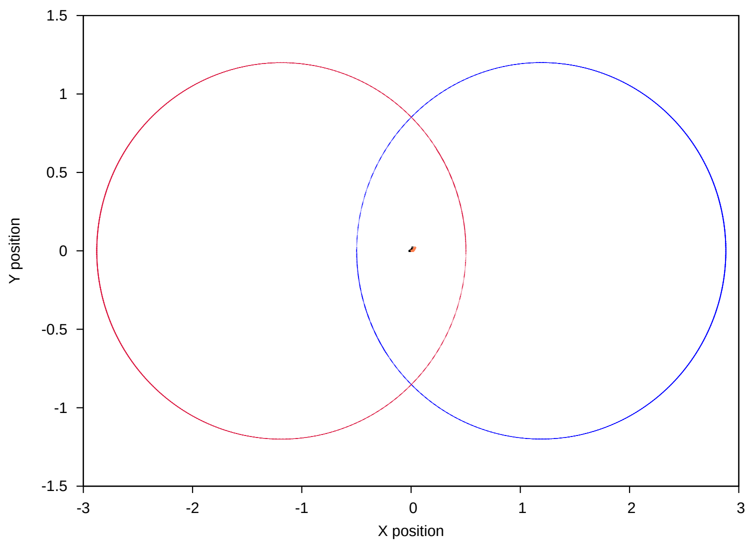

Figure 14: Configuration 12

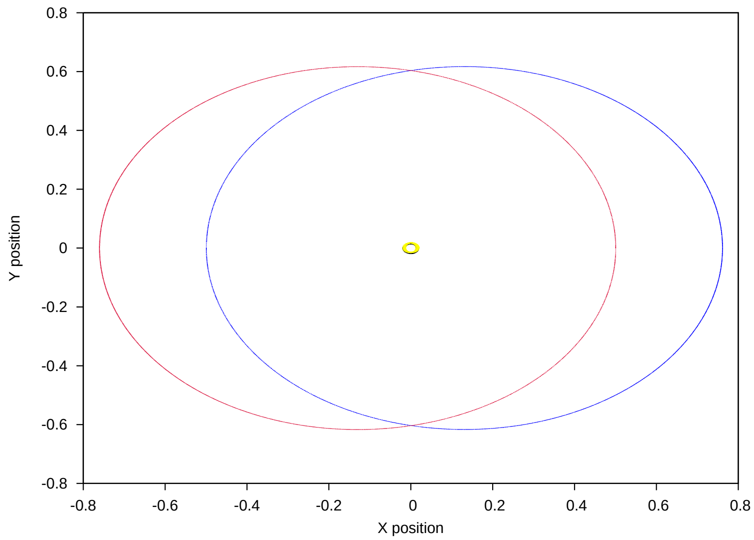

Figure 15: Configuration 13

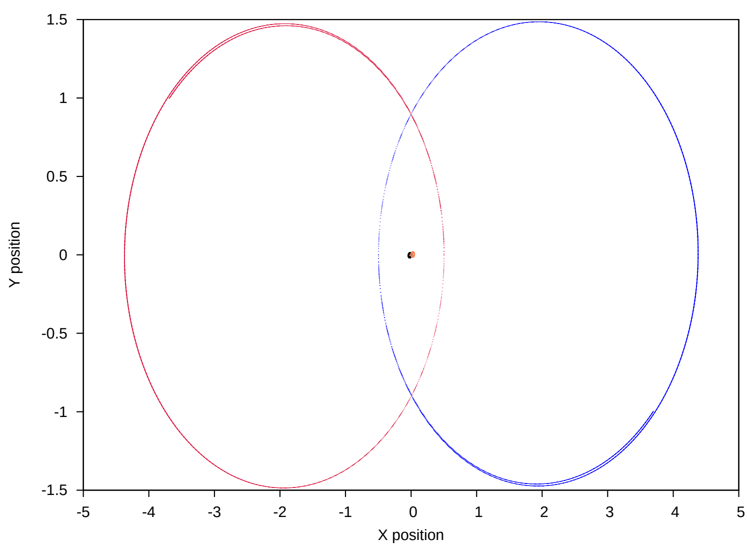

Figure 16: Configuration 14

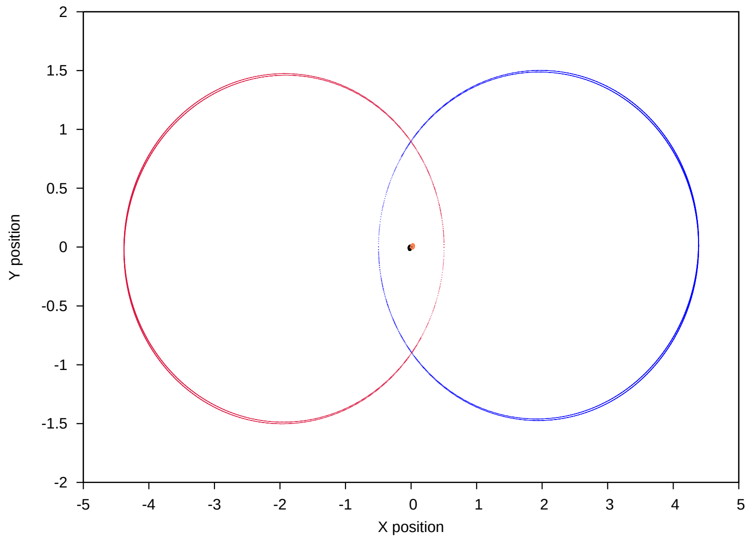

Figure 17: Configuration 15

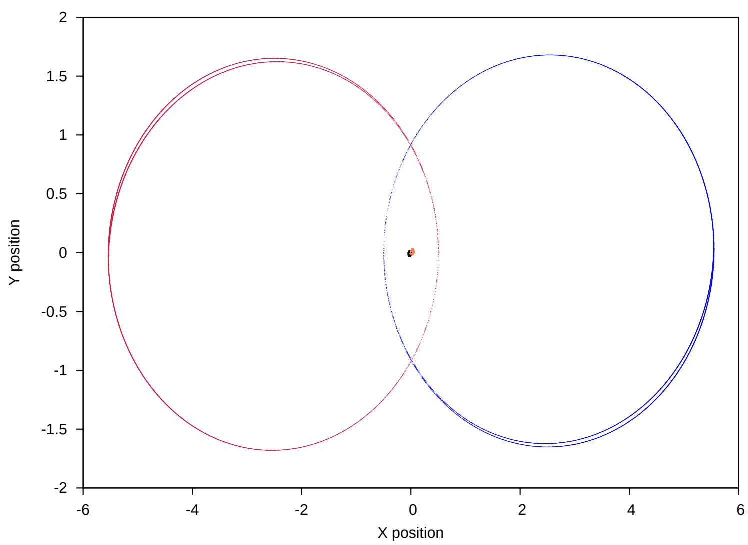

Figure 18: Configuration 16

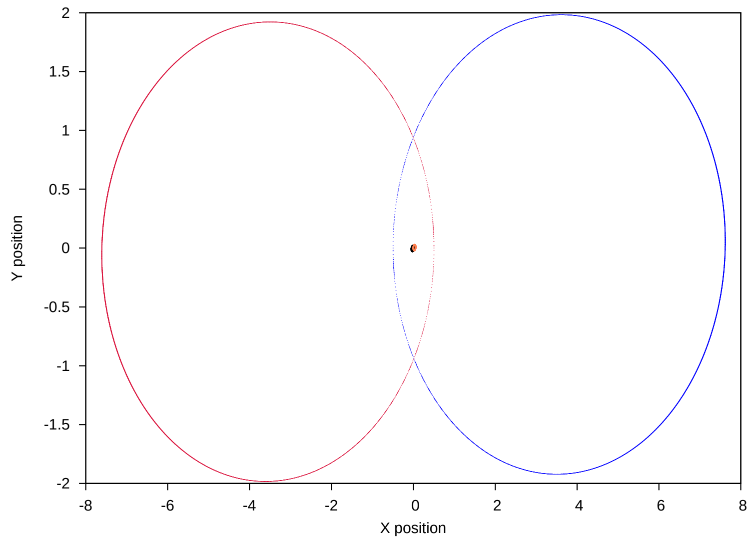

Figure 19: Configuration 17

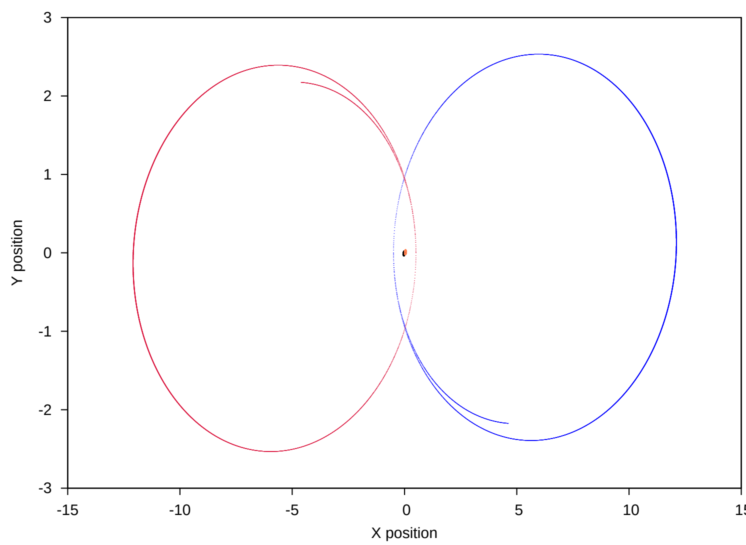

Figure 20: Configuration 18