Cosmology Tutorial 4

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This was challenging. I attempted all questions but some results are wrong.

1

I attempted this without looking at notes but didn't get very far.

1.1

$\rho = 0, k < 0$
$\dot{R}^2 = kc^2$
$\dot{R} = \pm\sqrt{k}c$

integrating both sides gives

$R = \pm\sqrt{k}ct + C$
$R \propto t$

1.2

$\rho > 0, k = 0$
$\rho \propto R^{-3}$
$\rho(t) = \dfrac{\rho_0 R_0^3}{R}$
$\dot{R}^2 = \dfrac{8\pi G}{3}\dfrac{\rho_0 R_0^3}{R}$
$\dot{R} = \sqrt{\dfrac{8\pi G \rho_0 R_0^3}{3}}\dfrac{1}{R^{1/2}}$
$R^{1/2} dR = \pm \left(\dfrac{8\pi G \rho_0 R_0^3}{3}\right)^{1/2} dt$

integrating both sides gives

$\dfrac{2}{3} R^{3/2} = \pm \left(\dfrac{8\pi G \rho_0 R_0^3}{3}\right)^{1/2} t + C$
$\dfrac{2}{3} R = \pm \left(\dfrac{8\pi G \rho_0 R_0^3}{3}\right)^{1/3} t^{2/3} + C$
$R = \pm \left(4\pi G \rho_0 R_0^3\right)^{1/3} t^{2/3} + C$
$R \propto t^{2/3}$

Obviously something wrong with my maths (constant is wrong).

1.3

$k = 0, p_m >> p_r$
$\rho_r \propto R^{-4}$

$\rho(t) = \frac{\rho_0 R_0^4}{R}$

$\dot{R}^2 = \frac{8\pi G}{3} \frac{\rho_{r,0} R_0^4}{R^2}$

$\dot{R} = \pm \left(\frac{8\pi G}{3} \rho_{r,0} R_0^4 \right)^{1/2} \frac{1}{R}$

$R\dot{R} = \pm \left(\frac{8\pi G}{3} \rho_{r,0} R_0^4 \right)^{1/2}$

$RdR = \pm \left(\frac{8\pi G}{3} \rho_{r,0} R_0^4 \right)^{1/2} dt$

integrating both sides gives

$1/2R^2 = \pm \left(\frac{8\pi G}{3} \rho_{r,0} R_0^4 \right)^{1/2} t$

$R = \pm \left(\frac{16\pi G}{3} \rho_{r,0} R_0^4 \right)^{1/4} t^{1/2}$

$R \propto t^{1/2}$

1.4

$\dot{R}^2 = \frac{\Lambda}{3} R^2$

$\dot{R} = \pm \sqrt{\frac{\Lambda}{3}}$

$\frac{1}{R} dR = \pm \sqrt{\frac{\Lambda}{3}} dt$

$log R = \pm \sqrt{\frac{\Lambda}{3}} t$

$R = e^{\sqrt{\frac{\Lambda}{3}} t}$

2

2.1

$$R = \pm \left(6\pi G\rho_0 R_0^3\right)^{1/3} t^{2/3}$$

$$R = R_0 \left(\frac{t}{t_0}\right)^{2/3}$$

$$R_0 \left(\frac{t}{t_0}\right)^{2/3} = \pm \left(6\pi G\rho_0 R_0^3\right)^{1/3} t^{2/3}$$

$$R_0 t_0^{2/3} = \pm \left(\frac{1}{6}\pi G\rho_0 R_0^3\right)^{1/3} R_0$$

$$t_0 = \pm \left(\frac{1}{6}\pi G\rho_0 R_0^3\right)^{1/2}$$

2.2

$$R = \pm \left(\frac{16\pi G}{3}\rho_0 R_0^4\right)^{1/4} t^{1/2}$$

$$R = R_0 \left(\frac{t}{t_0}\right)^{1/2}$$

$$R_0 \left(\frac{t}{t_0}\right)^{1/2} = \pm \left(\frac{16\pi G}{3}\rho_0 R_0^3\right)^{1/4} t^{1/2}$$

$$R_0 t_0^{1/2} = \pm \left(\frac{16\pi G}{3}\rho_0 R_0^3\right)^{1/4} R_0$$

$$t_0 = \pm \left(\frac{16\pi G}{3}\rho_0\right)^{1/2}$$

Not quite.

3

$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$

$H^2 = \left(\frac{\dot{R}}{R}\right) = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$

$\frac{kc^2}{R^2} = \frac{8\pi G\rho}{3} - H^2$

$\frac{8\pi G\rho}{3} - H^2 = 0$

$8\pi G\rho = H^2$

$8\pi G\rho = 3H^2$

$\rho = \frac{3H^2}{8\pi G}$

$1Mpc = 3.09 \times 10^{19} km$ and $H_0 = 68000/3.09 \times 10^{19} = 2.2 \times 10^{-18} m/s$.

$$\frac{3 \times 4.84 \times 10^{-36}}{8\pi 6.67 \times 10^{-11}} = 8.66 \times 10^{-27} kg/m^3$$

4

$\omega = \frac{P}{\rho c^2}$

$PV = NkT$

$P = \frac{NkT}{V}$

$\omega = \frac{\frac{NkT}{V}}{\rho c^2}$

$\omega = \frac{kT}{c^2}$

$\omega = \frac{1.38 \times 10^{-23}}{8.94 \times 10^{16}} = 10^{-40}$

Ideal gas law is more acccurate with high temperatures due to more kinetic energy.

5

Not too sure about this one! I looked it up. Integrating the Planck function gives:

$E_{rad} = \alpha T^4$

$\alpha = \dfrac{\pi^2 k^4}{15\hbar^3 c^3}$

$n_\lambda \propto R^3 \propto (1+z)^3$

$E = h\nu \propto h\lambda^{-1} \propto (1+z)$

$E_{rad} = E n_\lambda \propto (1+z)^4$

$T \propto (1+z)$