

# MSc Project - Modelling a Double-Barred Galaxy as a Double Binary

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## Abstract

TODO

## 1 Introduction

### 1.1 Double-Barred Galaxies

Double-barred galaxies, where a smaller inner bar is nested inside a larger outer bar, were first observed in 1975 by de Vaucouleurs, however it was only in the 1990s that double-barred galaxies were recognised as being a distinct category of galaxies. The current estimates of double-barred galaxy frequencies are  $\approx 30\%$  of barred galaxies and  $\approx 20\%$  of all galaxies (Erwin, 2003). Erwin (2011) surveyed 38 barred galaxies and found a total of 10 double-barred galaxies with no preference for Hubble type. They found that inner bars were about 12% the size of the outer bars with semi-major axes from  $\approx 250\text{pc}$  to 1 kpc. Secondary bars probably rotate independently of, but in the same direction as their associated primary bars Erwin (2011). Erwin and Sparke (2002) believe that inner bars probably rotate independently of their outer bars and suggest that they are relatively long-lived structures. Inner bar size has been found to correlate slightly with outer bar size (Erwin, 2003). The median size ratio is  $\approx 0.12$  with an upper limit of  $\approx 0.25$  which is consistent with the theory that inner bars cannot be too large without disrupting the orbit of the outer bar.

It has been suggested that nested bars may provide a mechanism for fueling active galactic nuclei (Shlosman 1989) however more recently (Erwin, 2003) believe that inner bars only play a minor role in this activity. (Erwin and Sparke, 2002) compared single and double-barred galaxies and found that there was no significant difference in nuclear activity. To investigate

this theory further Lorenzo-Caceres et. al. (2002) carried out a study of NGC 357. This study aimed to present the first detailed morphological, kinematical and stellar population analysis of the bulge, inner and outer bars of a double-barred galaxy. They concluded that the bulge and inner bar show similar stellar population properties whilst the outer bar was less metal-rich. This result implied that, for the case of NGC 357, the outer bar was formed in a shorter timescale than the inner structures and so disagrees with traditional idea that gas flown along the outer structure triggers star formation causing the formation of the inner structure. This was strengthened by a further study of four double-barred galaxies by Lorenzo-Caceres et. al. (2013) where they concluded that the stellar populations of inner bars are younger and more metal-rich than the outer bars. This suggests that at present inner bars play a moderate or even minor role in the morphological evolution of double-barred galaxies (Lorenzo-Caceres et. al., 2013).

Through the use of computer modeling Saha and Maciejewski (2013) showed that double bars can form without gas in a dark matter dominated halo. They produced a model where the inner bar was rotating at almost as slowly as the outer bar. The route to the formation of double bars maybe very different to that of a single bar.

Photos!

(Moiseev, 2001), , (Sellwood and Wilkinson, 2006)

## 1.2 N-Body Simulations and the Four-Body Problem

Studies of three or more bodies interacting gravitationally dates back to the time of Newton. However it is only with the widespread use of computers that orbits of more than three bodies have been extensively studied. Today N-body simulations are an essential tool in the study of solar system dynamics and galactic dynamics (Binney and Tremaine, 2008). Celestial mechanics during the preceding centuries has mainly focussed on the three body problem due to the complexities involved in adding more bodies. It is estimated that within the Milky Way approximately two thirds of all stars exist in binary systems and a further one fifth of these are in triple systems whilst a

The classical equations of motion for an n-body problem are:

$$m_i \ddot{\mathbf{r}}_i = \sum_{i \neq j} \frac{m_i m_j}{r_{ij}^3} \mathbf{r}_{ij} \quad i = 1, 2, 3, \dots \quad (1)$$

where  $\mathbf{r}_i = (x_i, y_i)$  and  $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ . So for a four body problem this equation results in the following:

$$\ddot{\mathbf{r}}_1 = \frac{m_2 \mathbf{r}_{12}}{r_{12}^3} + \frac{m_3 \mathbf{r}_{13}}{r_{13}^3} + \frac{m_4 \mathbf{r}_{14}}{r_{14}^3} \quad (2)$$

$$\ddot{\mathbf{r}_2} = \frac{m_1\mathbf{r}_{21}}{r_{21}^3} + \frac{m_3\mathbf{r}_{23}}{r_{23}^3} + \frac{m_4\mathbf{r}_{24}}{r_{24}^3} \quad (3)$$

$$\ddot{\mathbf{r}_3} = \frac{m_1\mathbf{r}_{31}}{r_{31}^3} + \frac{m_2\mathbf{r}_{32}}{r_{32}^3} + \frac{m_4\mathbf{r}_{34}}{r_{34}^3} \quad (4)$$

$$\ddot{\mathbf{r}_4} = \frac{m_1\mathbf{r}_{41}}{r_{41}^3} + \frac{m_2\mathbf{r}_{42}}{r_{42}^3} + \frac{m_3\mathbf{r}_{43}}{r_{43}^3} \quad (5)$$

The units have been selected so that the gravitational constant (G) is equal to one.

(Aarseth, 2003), (AlvarezRamirez and Medina, 2014), (Heggie and Hut, 2002), (Binney and Tremaine, 2008), (Collins, 2004), (Moulton, 1914), (Steves and Roy, 1998), (Szell et. al., 2004), (Trenti and Hut, 2008)

### 1.3 Orbits, Loops, Resonances and Double-Barred Galaxy Simulations

(Debattista and Shen, 2007), (Du et. al., 2015), (Du et. al., 2016), (Erwin and Sparke, 2002), (Maciejewski, 2002), (Maciejewski, 2003), (Maciejewski, 2008), (Maciejewski and Athanassoula, 2008) (Maciejewski and Small, 2010), (Maciejewski and Sparke, 1998), (Maciejewski and Sparke, 1999), (Malhotra, 1998), (Manos and Athanassoula, 2011), (Shen and Debattista, 2008), (Shen and Debattista, 2009), (Wozniak, 2015)

## 2 Methods

Fortran 90 was selected as the language of choice in order to avoid the fixed line limitations of Fortran 77. The model was created using OdeInt from Numerical Recipes. I first converted OdeInt to Fortran 90 for this purpose. Please see the Compilers subsection for a few notes on compilers - may or may not be relevant to the project.

The above equations were encoded into Fortran. A full code listing is supplied in a separate file.

### 2.1 Initial Conditions

As a starting point both the outer binary and inner binary were placed in circular orbits and tested separately and then combined to ensure a stable starting point. The inner binary had initially a negligible mass. The outer binary initial conditions were:

```
m1=.5, m2=.5, x1=-.5, x2=.5, y1=0, y2=0,
vx1=0, vx2=0, vy1=-.5, vy2=.5
```

and for the inner binary:

$m3=.001$ ,  $m4=.001$ ,  $x3=-.001$ ,  $x4=.001$ ,  $y3=0$ ,  $y4=0$ ,  
 $vx3=0$ ,  $vx4=0$ ,  $vy3=-.5$ ,  $vy4=.5$

Plots of the two initial binaries are shown in Fig. 1 and 2. The scale of the inner binary is so small that it is difficult to see in GnuPlot.

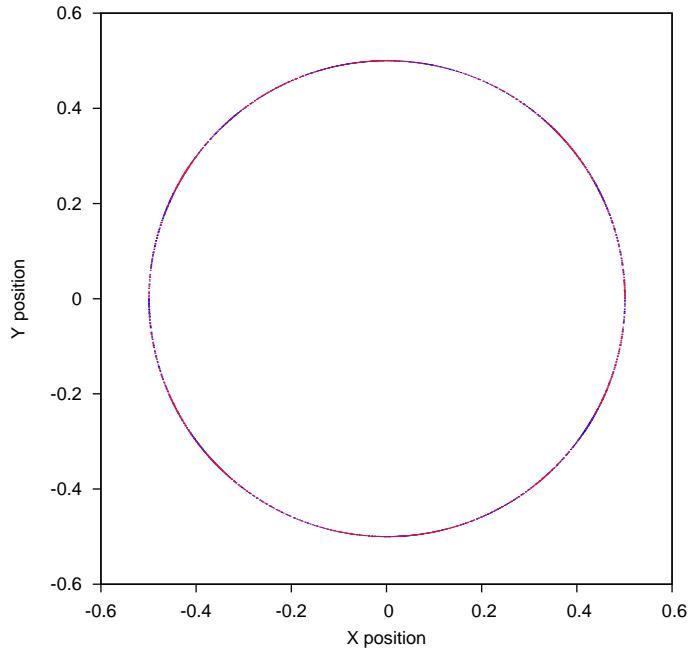


Figure 1: Outer Binary

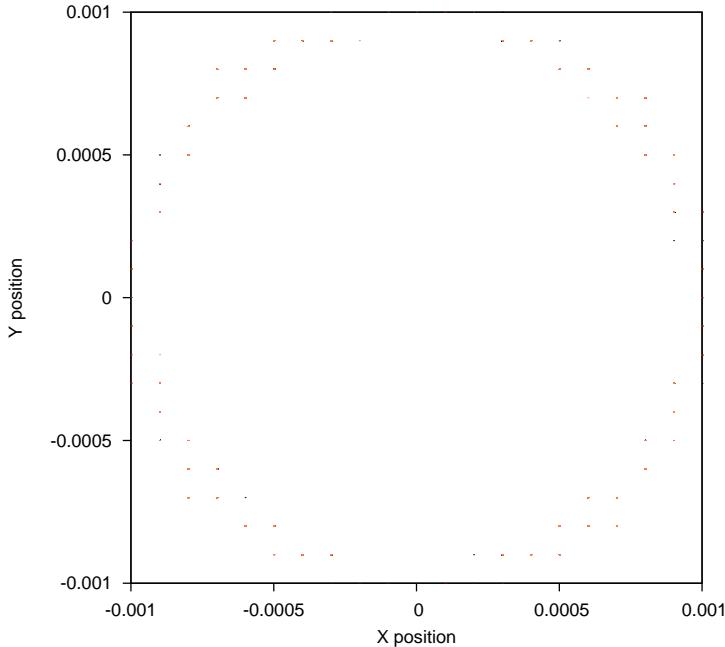


Figure 2: Inner Binary

I then proceeded to perturb the system by gradually increasing the mass of the inner binary. As the system became unstable I compensated for the increasing mass of the inner binary by increasing the velocity of the outer binary and also decreasing the velocity and increasing the binary separation of the inner binary. In many of the configurations the systems were inherently unstable and produced collisions with some or all of the bodies being ejected from the system. These were rejected and only 'stable' configurations were considered. Table 1 shows the perturbations made to the system - only the variables in this table changed and all the other initial conditions remained the same.

## 2.2 Compilers

As I had converted OdeInt into Fortran 90 I wanted to make sure I hadn't introduced any errors. I therefore compared the output of the Numerical Recipes version of OdeInt with mine. I was compiling with GFortran and the Fortran 77 version was compiled with g77. They were different! So naturally concerned I then compiled the Fortran 77 version with GFortran. The results were identical to mine, which satisfied me that I hadn't introduced any errors.

Table 1: Configurations

Config.	m3	m4	vy1	vy2	x3	x4	vy3	vy4
1	.001	.001	-.5	.5	-.001	.001	-.5	.5
2	.002	.002	-.5	.5	-.001	.001	-.5	.5
3	.003	.003	-.5	.5	-.001	.001	-.5	.5
4	.004	.004	-.5	.5	-.004	.004	-.5	.5
5	.004	.004	-.55	.55	-.004	.004	-.5	.5
6	.004	.004	-.57	.57	-.004	.004	-.5	.5
7	.005	.005	-.58	.58	-.005	.005	-.3	.3
8	.005	.005	-.58	.58	-.005	.005	-.4	.4
9	.005	.005	-.58	.58	-.005	.005	-.35	.35
10	.006	.006	-.6	.6	-.006	.006	-.3	.3
11	.025	.025	-.65	.65	-.02	.02	-.5	.5
12	.025	.025	-.7	.7	-.025	.025	-.5	.5
13	.03	.03	-.7	.7	-.03	.03	-.5	.5
14	.035	.035	-.75	.75	-.04	.04	-.4	.4
15	.04	.04	-.75	.75	-.045	.045	-.4	.4
16	.05	.05	-.75	.75	-.045	.045	-.4	.4
17	.05	.05	-.75	.75	-.05	.05	-.35	.35
18	.05	.05	-.75	.75	-.05	.05	-.3	.3
19	.06	.06	-.8	.8	-.06	.06	-.25	.25
20	.08	.08	-.9	.9	-.08	.08	-.15	.15
21	.09	.09	-.95	.95	-.09	.09	-.1	.1
22	.1	.1	-1	1	-.1	.1	-.05	.05
23	.1	.1	-1	1	-.1	.1	-.04	.04
24	.1	.1	-1	1	-.1	.1	-.03	.03
25	.1	.1	-1	1	-.1	.1	-.02	.02
26	.12	.12	-1.05	1.05	-.11	.11	-.015	.015
27	.12	.12	-1.1	1.1	-.11	.11	-.015	.015
28	.12	.12	-1.1	1.1	-.115	.115	-.015	.015
29	.12	.12	-1.1	1.1	-.12	.12	-.015	.015
30	.1	.1	-1	1	-.1	.1	-.1	.1
31	.1	.1	-1	1	-.102	.102	-.1	.1
31	.1	.1	-1	1	-.1021	.1021	-.1	.1

### 3 Results

As can be seen from Fig. 3 even a negligible mass inner binary causes the outer binary to begin to separate and become less tightly bound. Figures Fig. 3 to 38 show plots of the double binary using the parameters given in Table

1. In many cases the inner binary is only visible as a point at the centre of the outer binary. As the mass of the inner binary increased the system became increasingly unstable and the effect on the outer binary became more pronounced. Even the 'stable' configurations listed in Table 1 hold for only a few orbits. I was unable to recreate a stable system for a mass  $> .08$  for the inner bar. The plots below reveal that increasing the mass of the inner binary causes the outer binary to separate and form wider orbits.

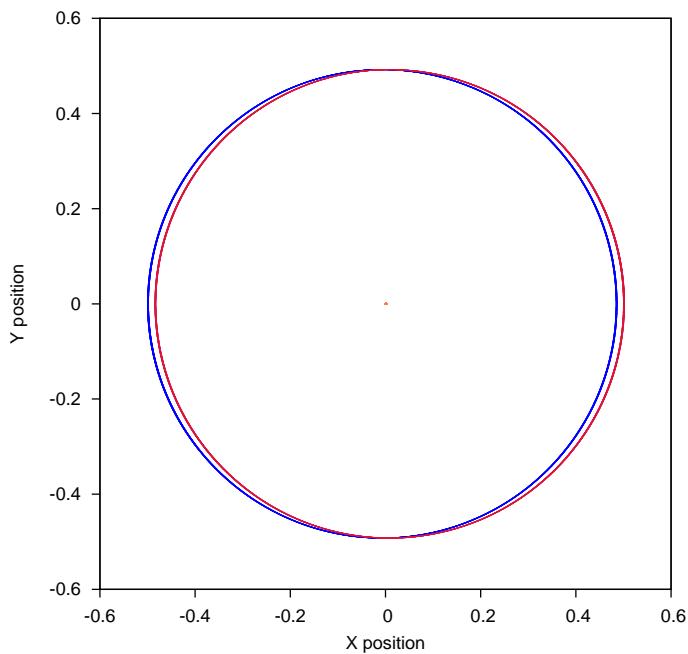


Figure 3: Configuration 1

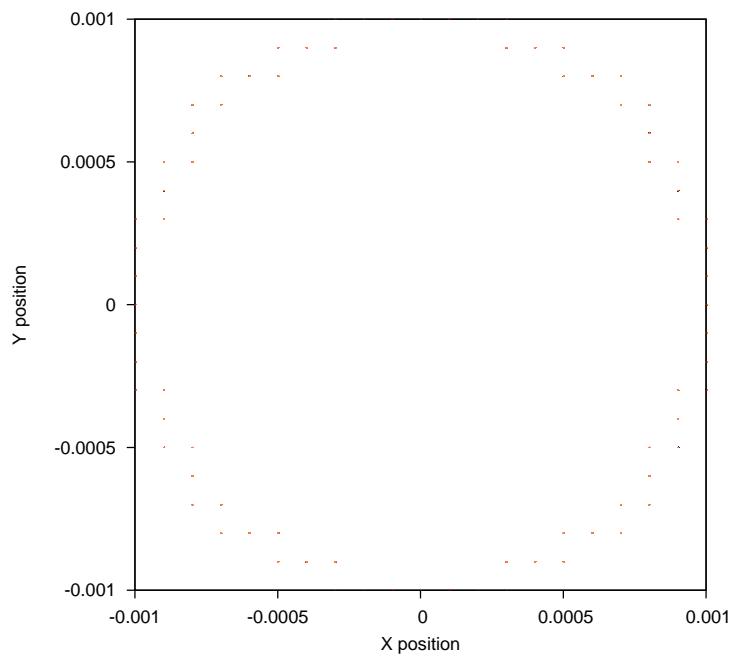


Figure 4: Configuration 1 - Inner Bar

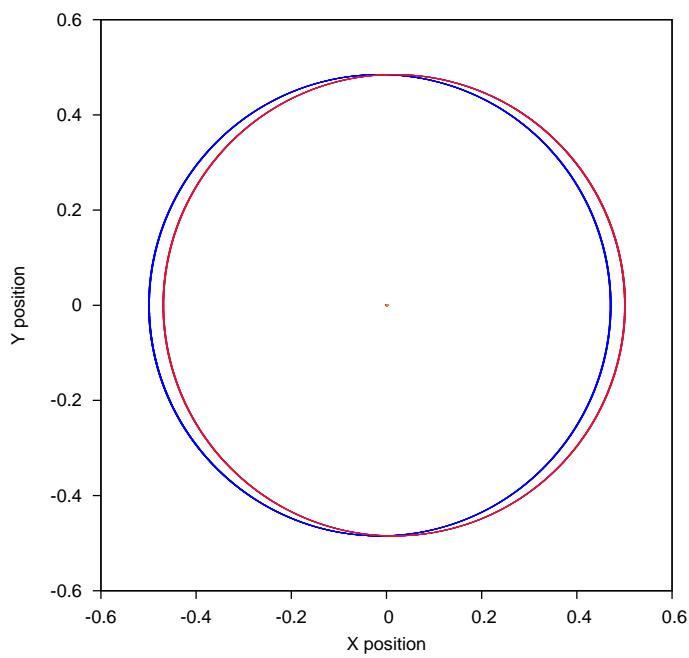


Figure 5: Configuration 2

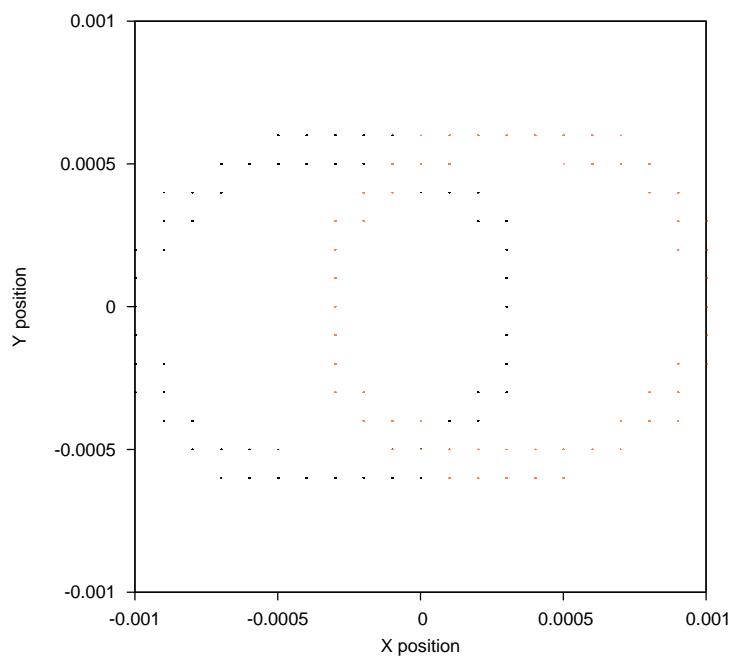


Figure 6: Configuration 2 - Inner Bar

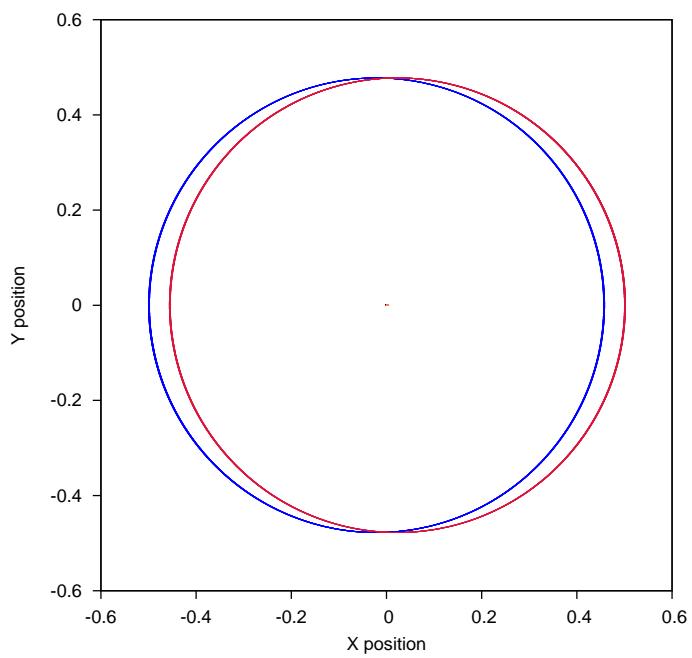


Figure 7: Configuration 3

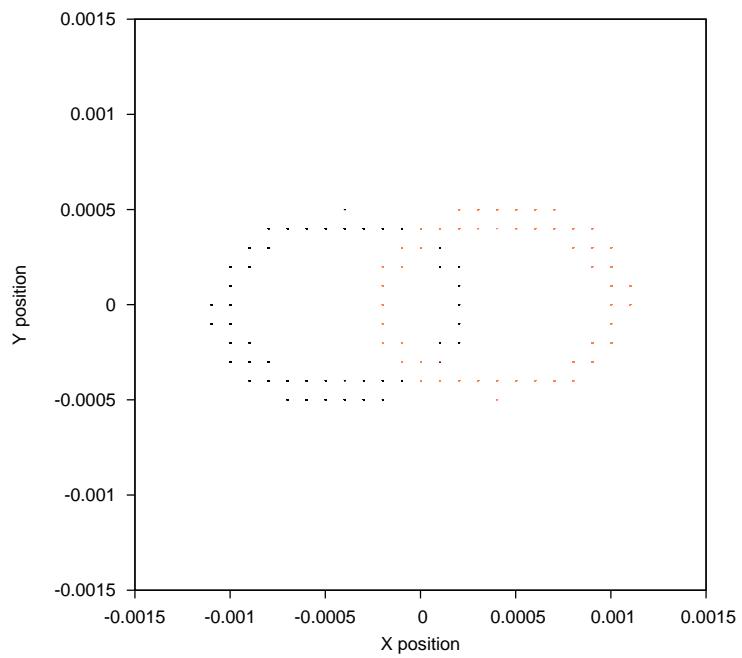


Figure 8: Configuration 3 - Inner Bar

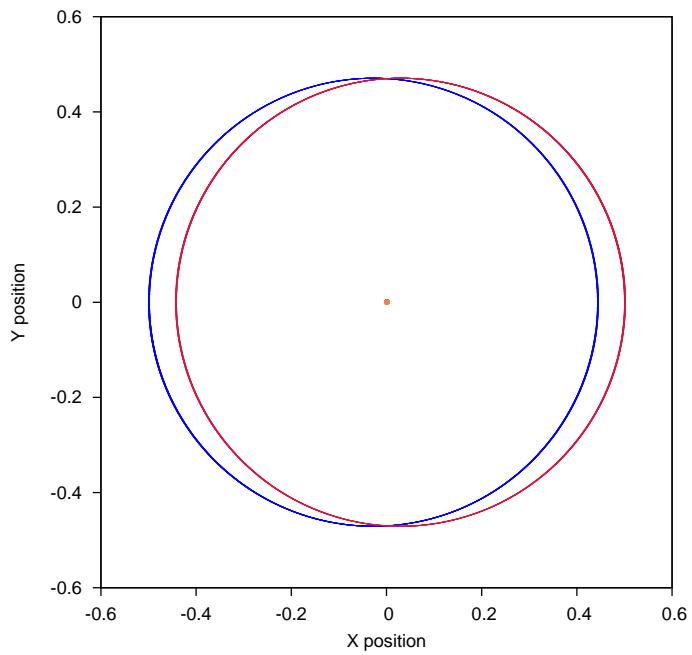


Figure 9: Configuration 4

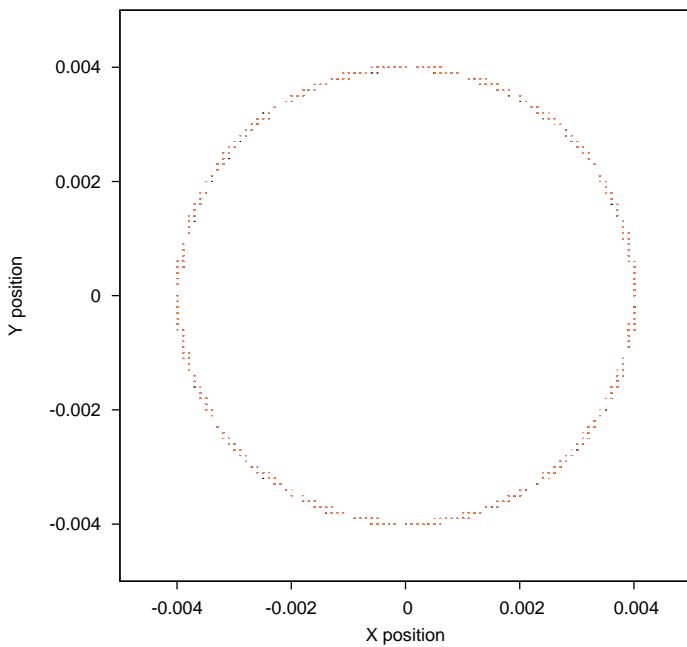


Figure 10: Configuration 4 - Inner Bar

1

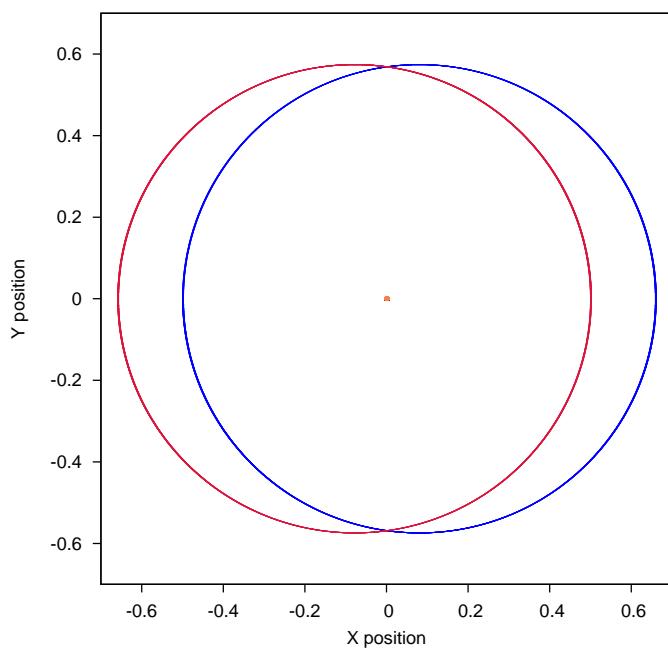


Figure 11: Configuration 5

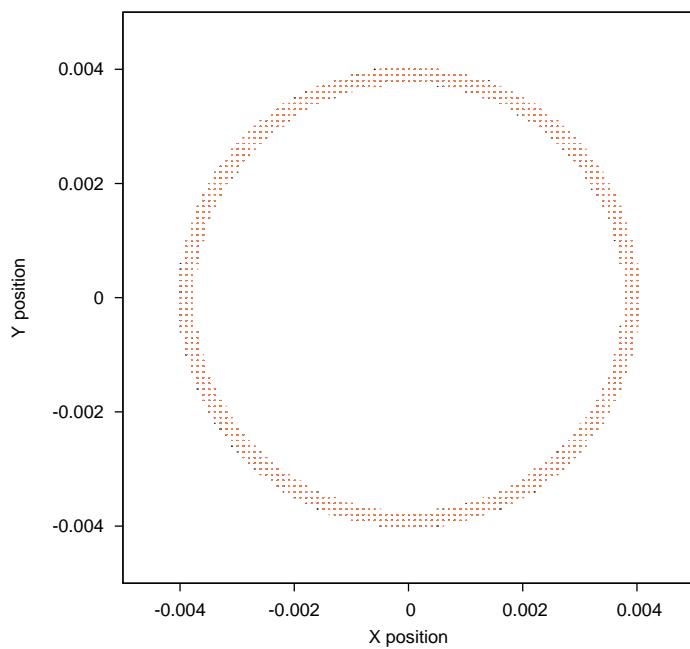


Figure 12: Configuration 5 - Inner Bar

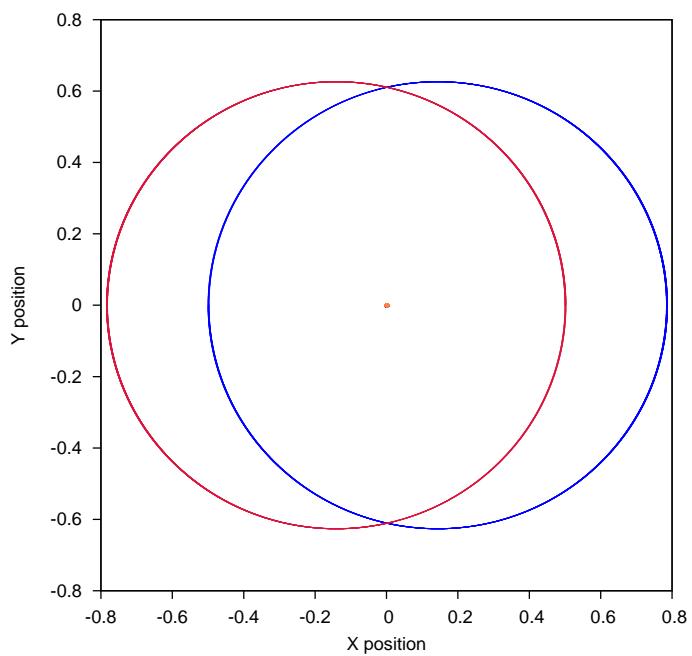


Figure 13: Configuration 6

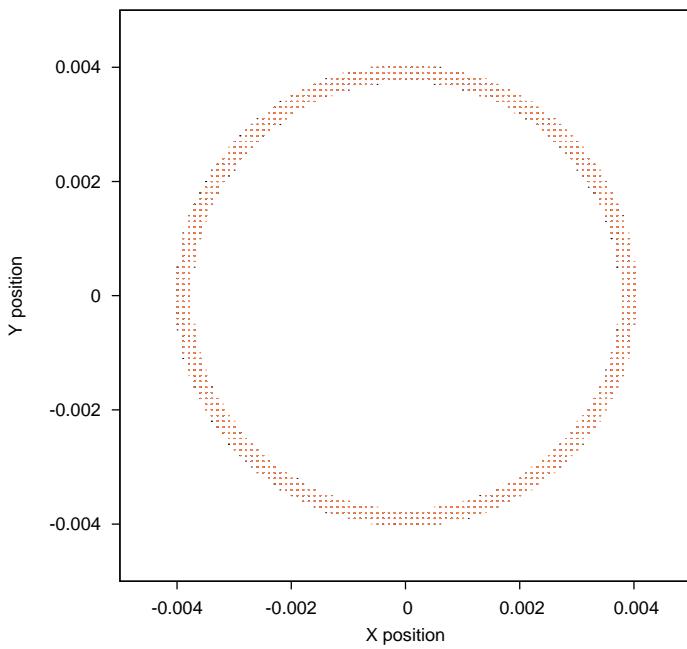


Figure 14: Configuration 6 - Inner Bar

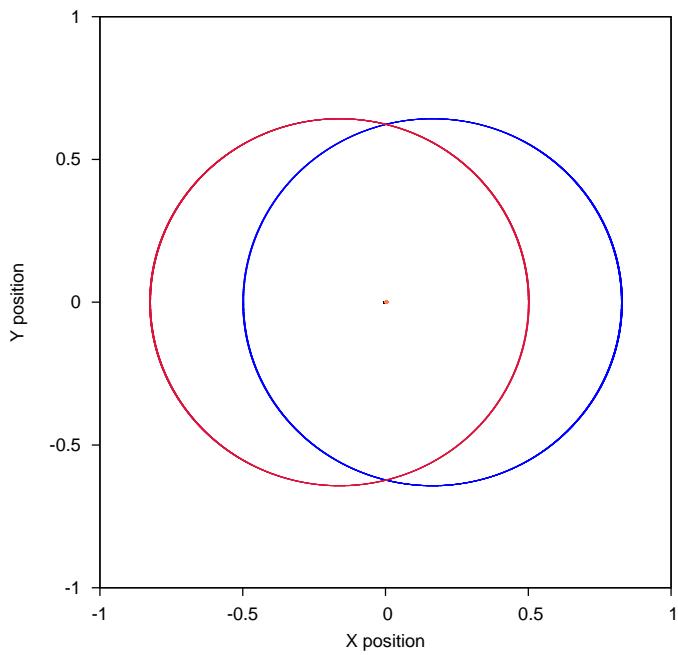


Figure 15: Configuration 7

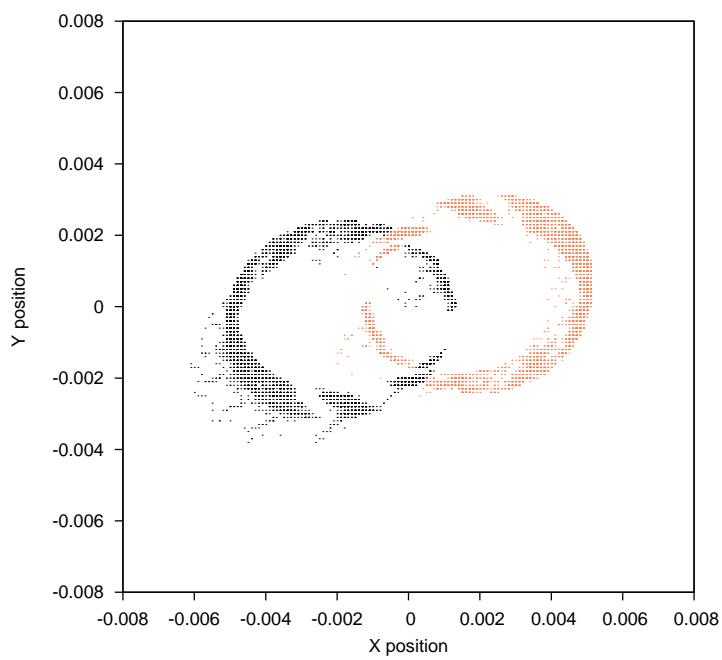


Figure 16: Configuration 7 - Inner Bar

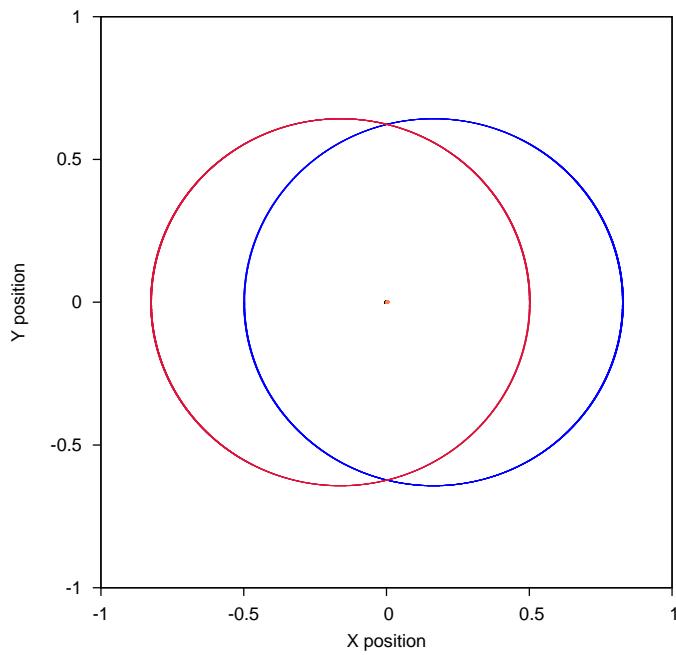


Figure 17: Configuration 8

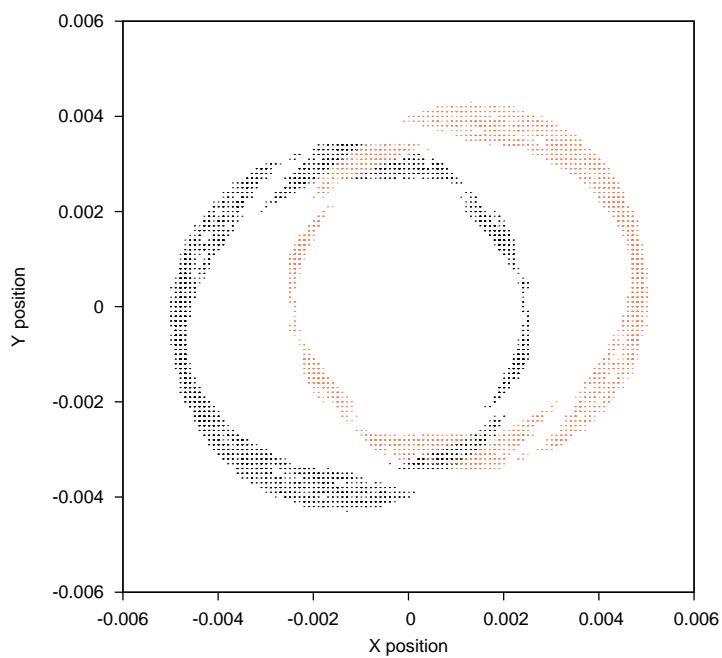


Figure 18: Configuration 8 - Inner Bar

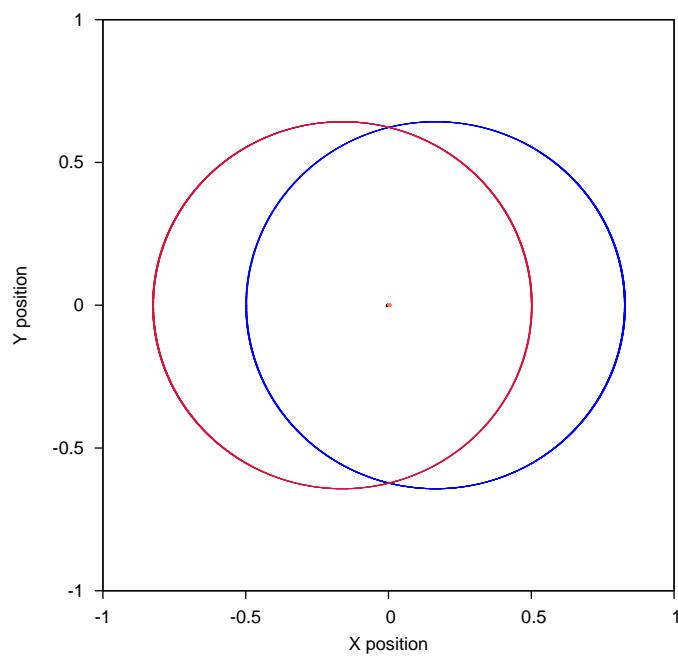


Figure 19: Configuration 9

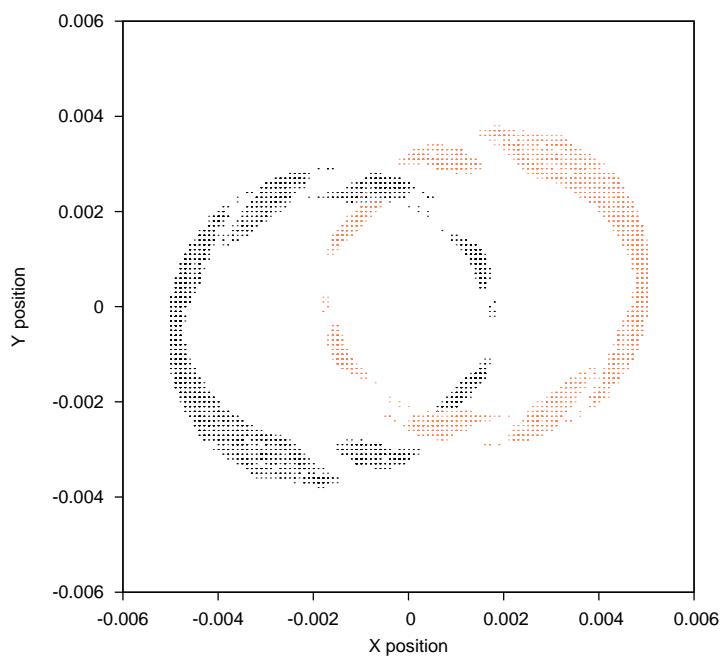


Figure 20: Configuration 9 - Inner Bar

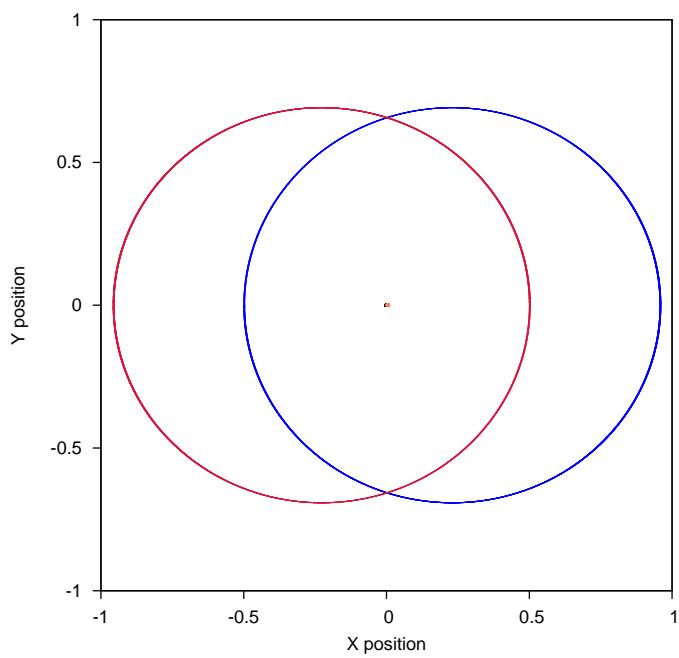


Figure 21: Configuration 10

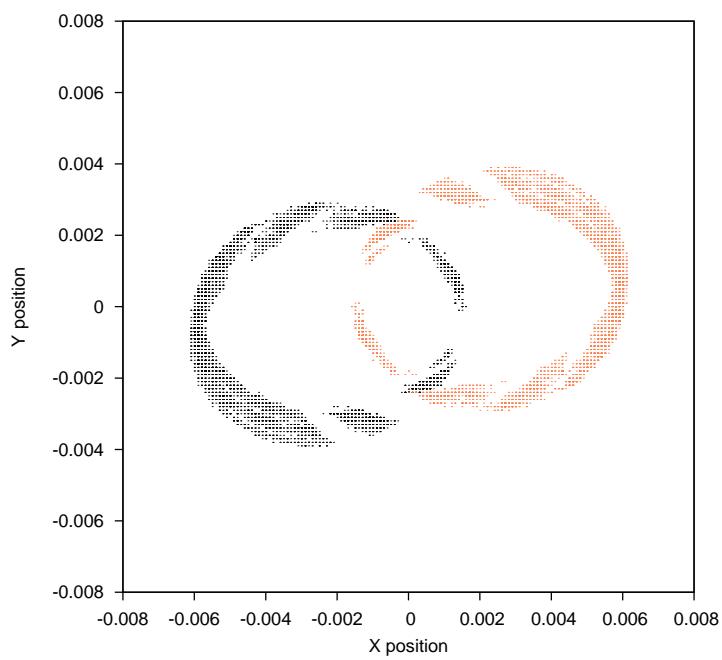


Figure 22: Configuration 10 - Inner Bar

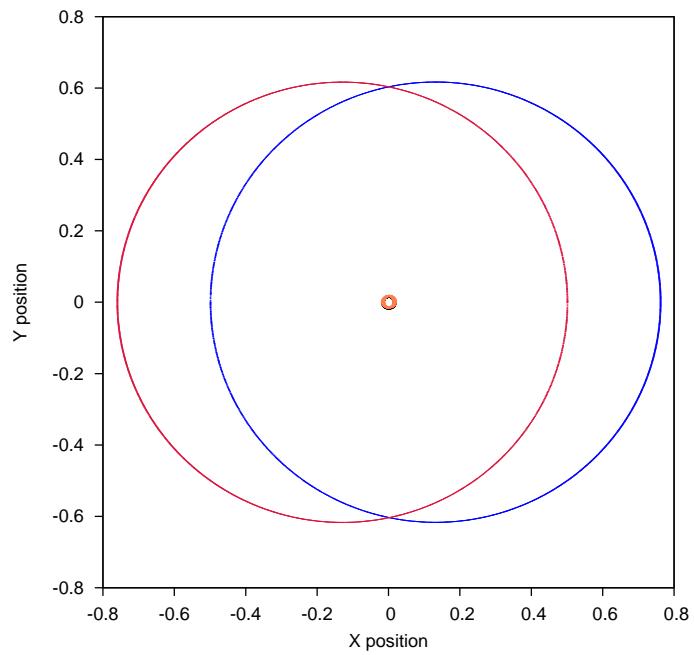


Figure 23: Configuration 11

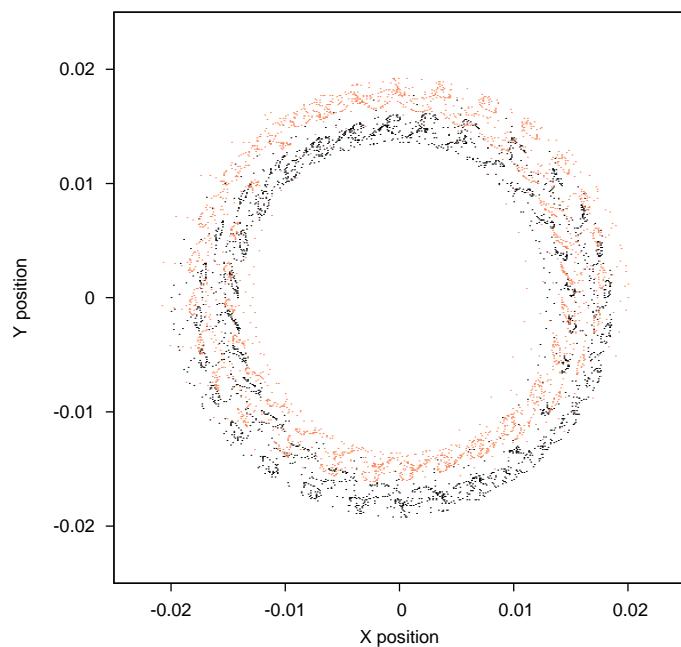


Figure 24: Configuration 11 - Inner Bar

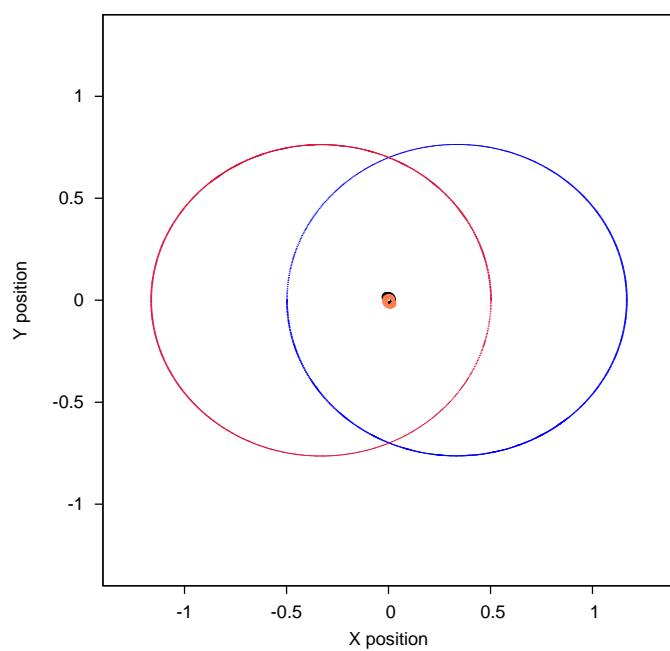


Figure 25: Configuration 12

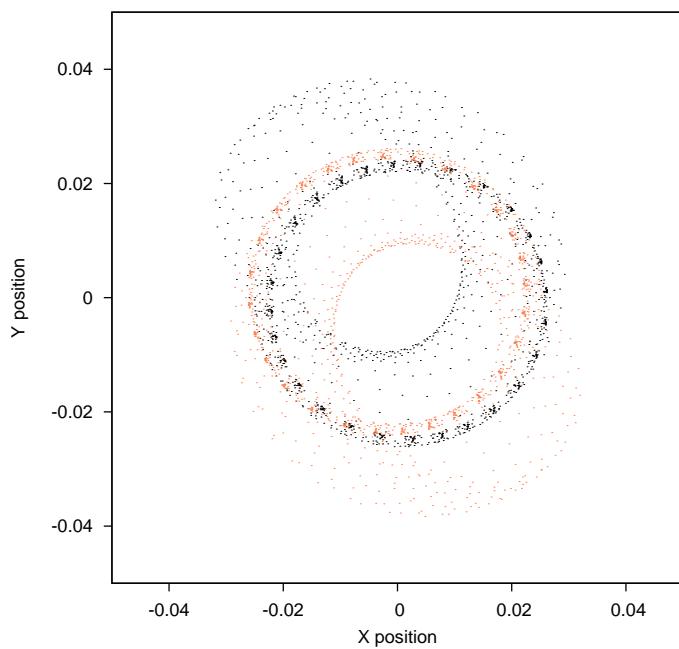


Figure 26: Configuration 12 - Inner Bar

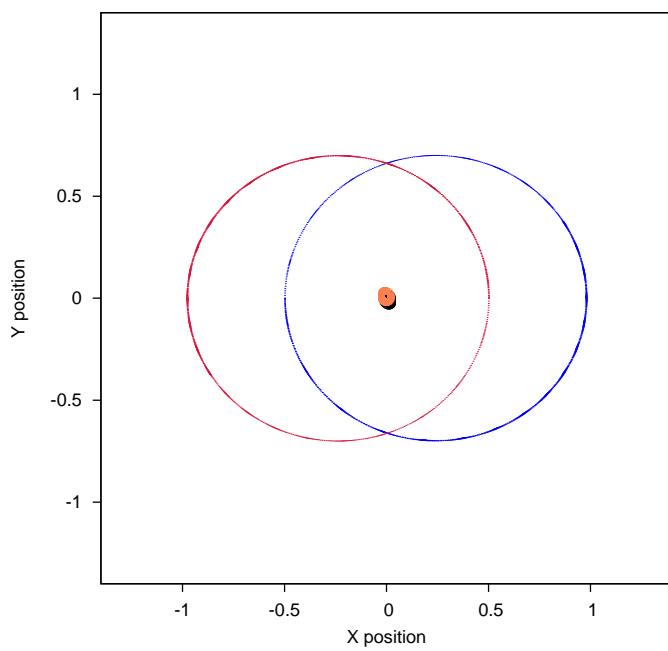


Figure 27: Configuration 13

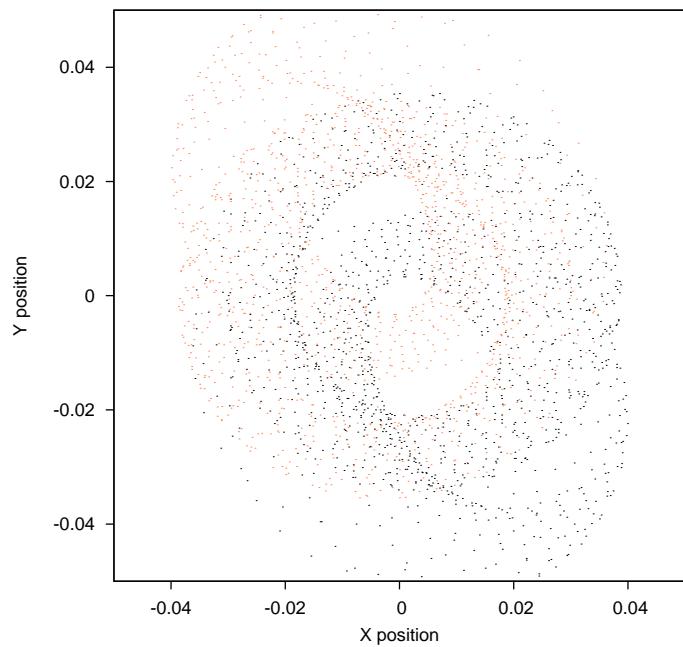


Figure 28: Configuration 13 - Inner Bar

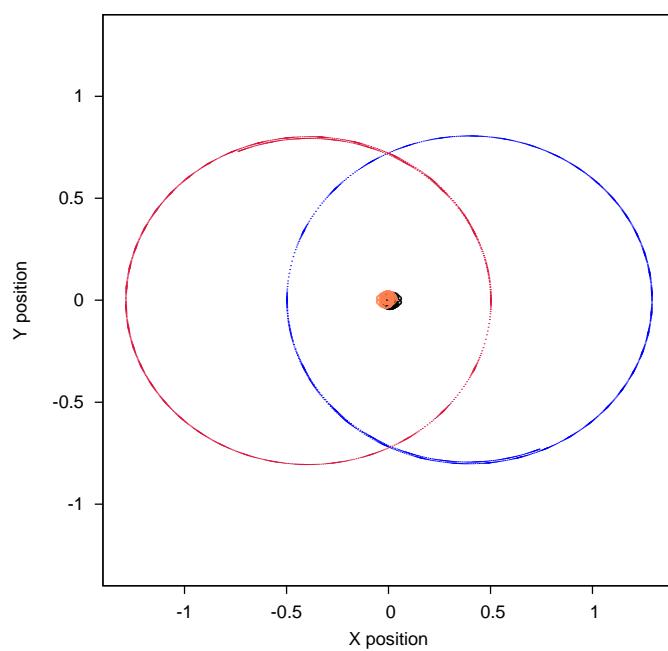


Figure 29: Configuration 14

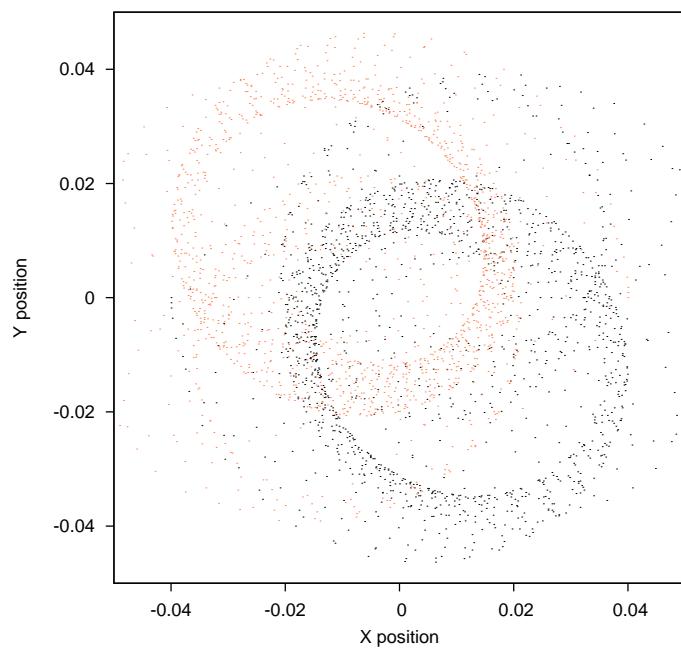


Figure 30: Configuration 14 - Inner Bar

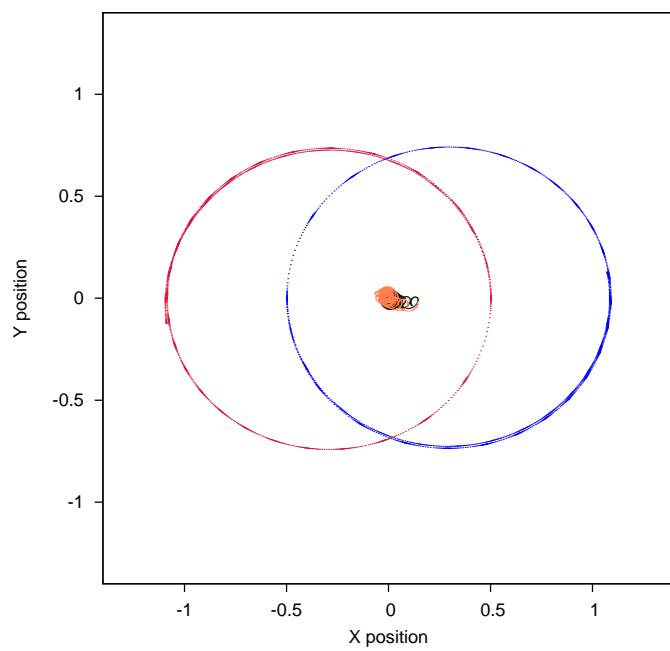


Figure 31: Configuration 15

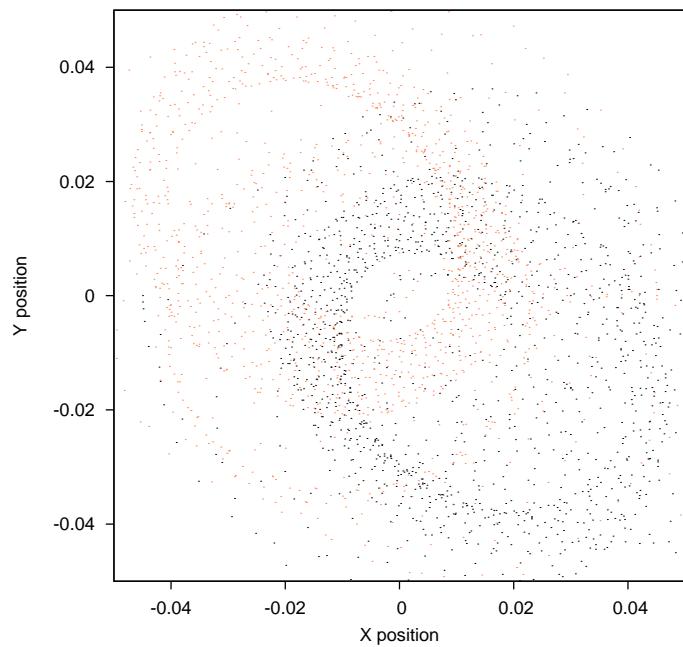


Figure 32: Configuration 15 - Inner Bar

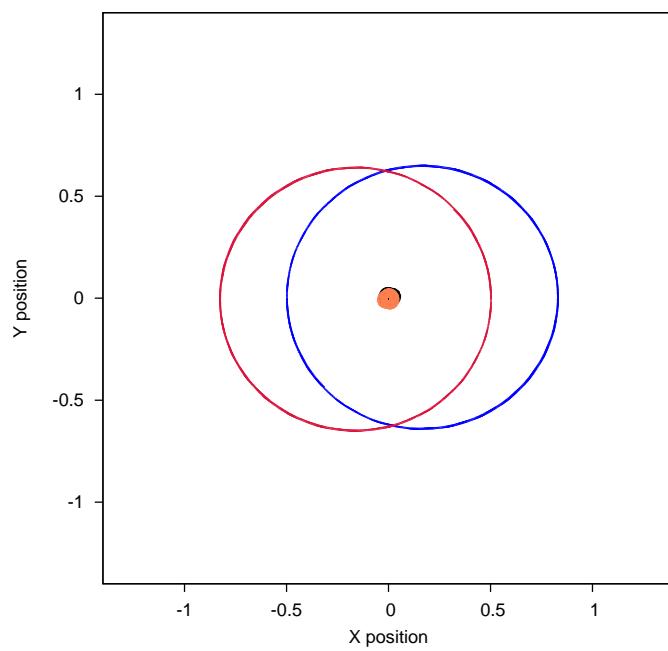


Figure 33: Configuration 16

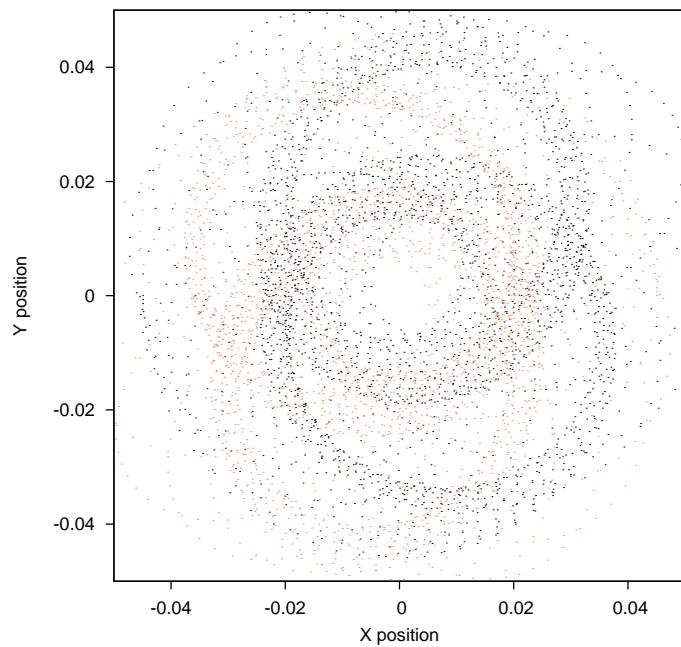


Figure 34: Configuration 16 - Inner Bar

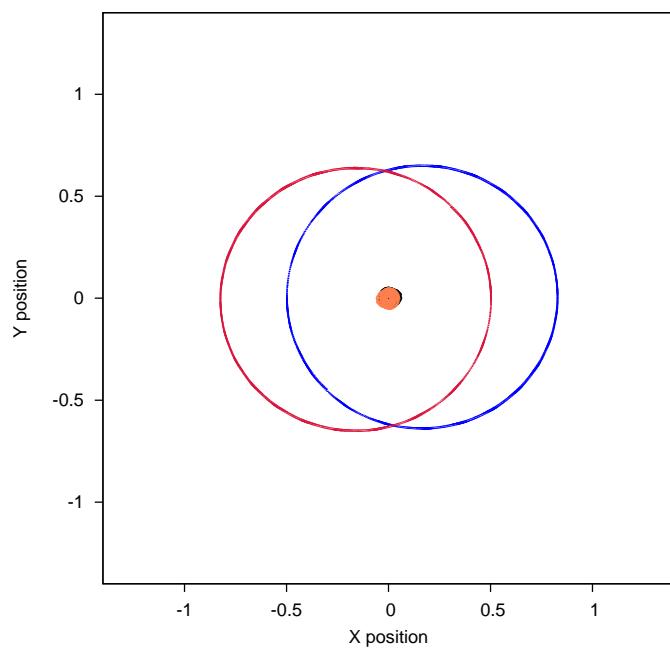


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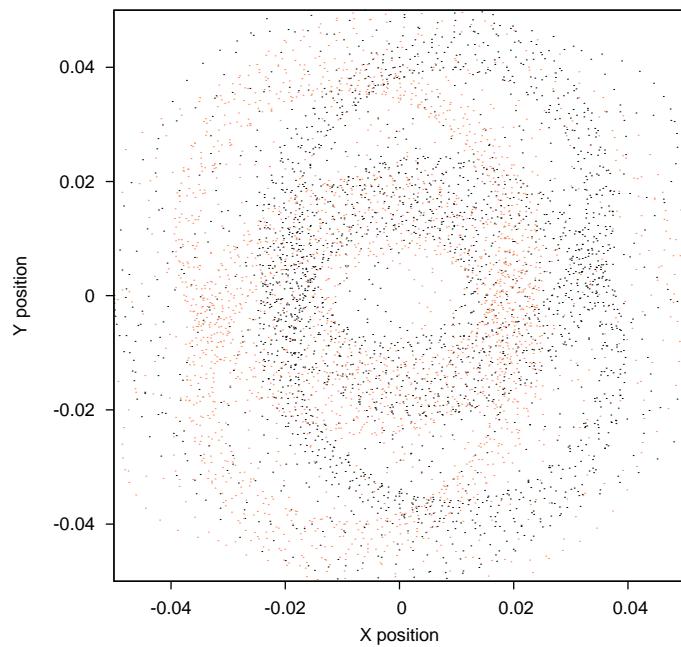


Figure 36: Configuration 17 - Inner Bar

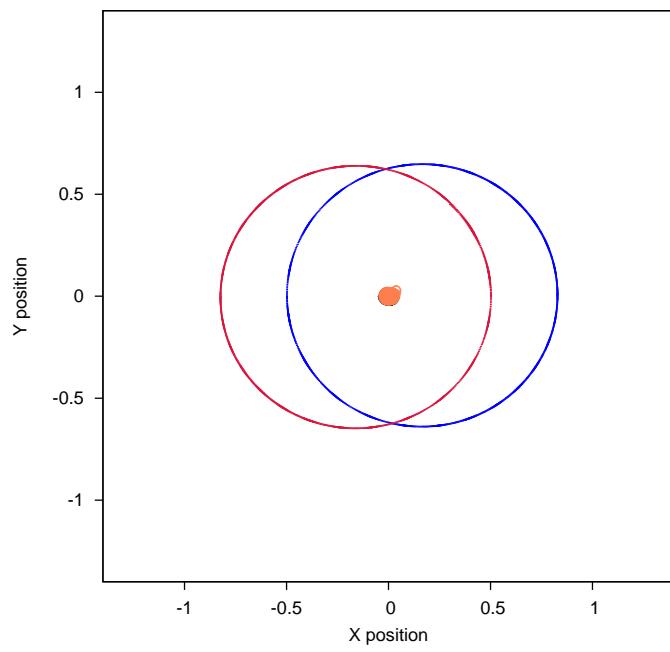


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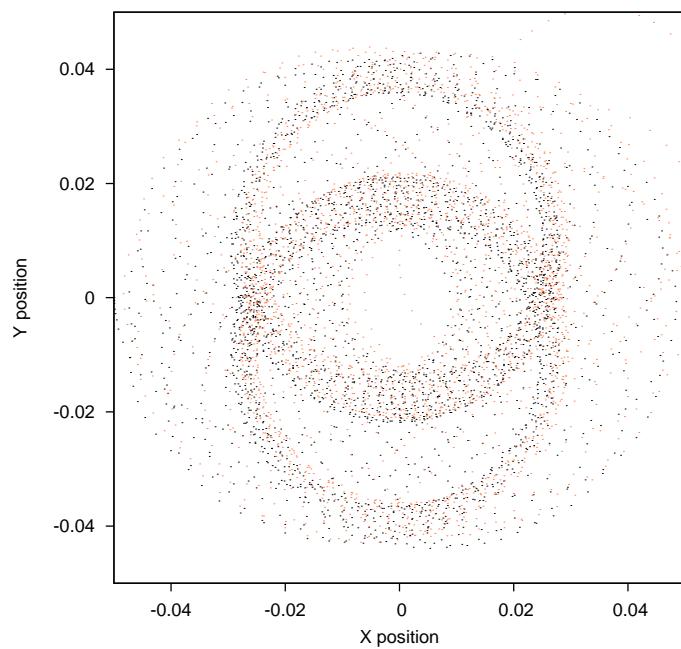


Figure 38: Configuration 18 - Inner Bar

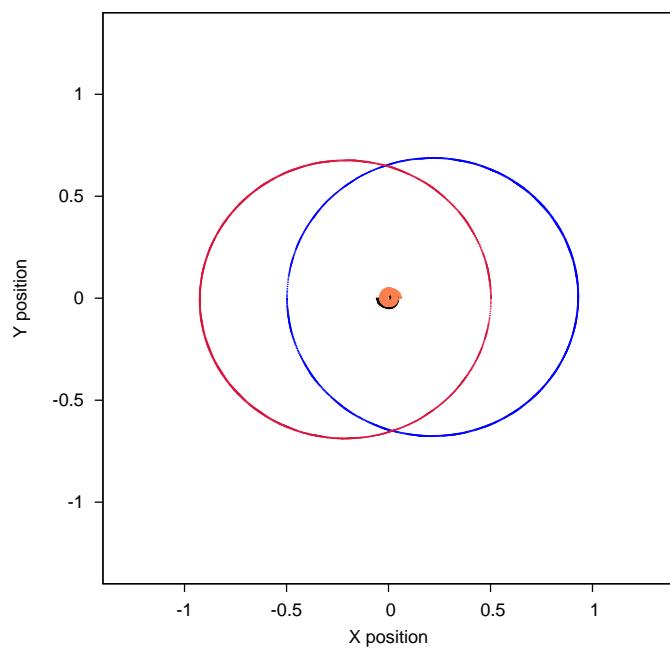


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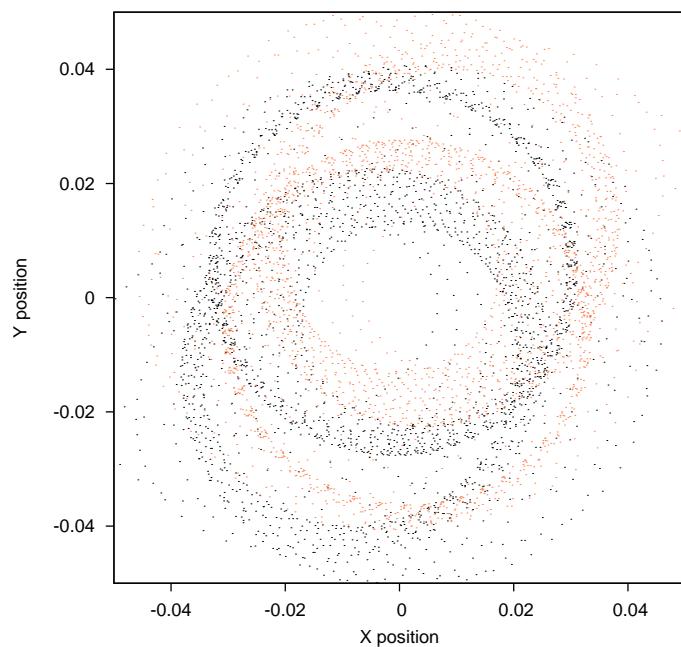


Figure 40: Configuration 19 - Inner Bar

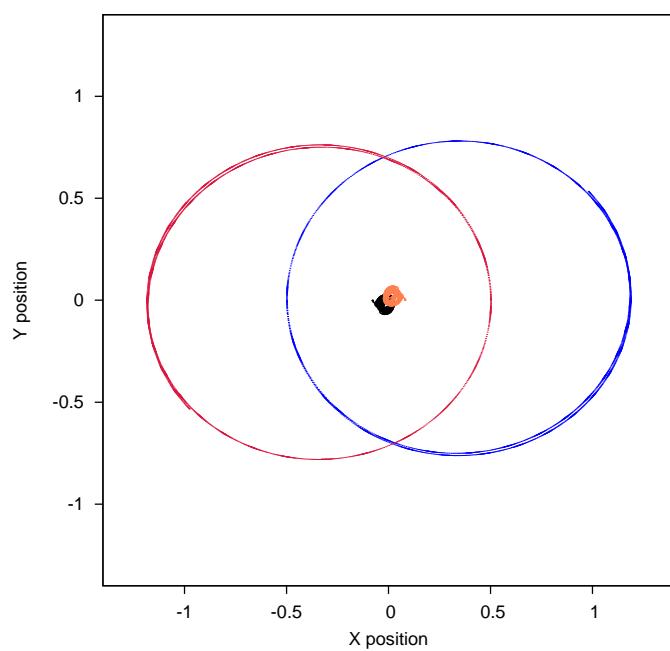


Figure 41: Configuration 20

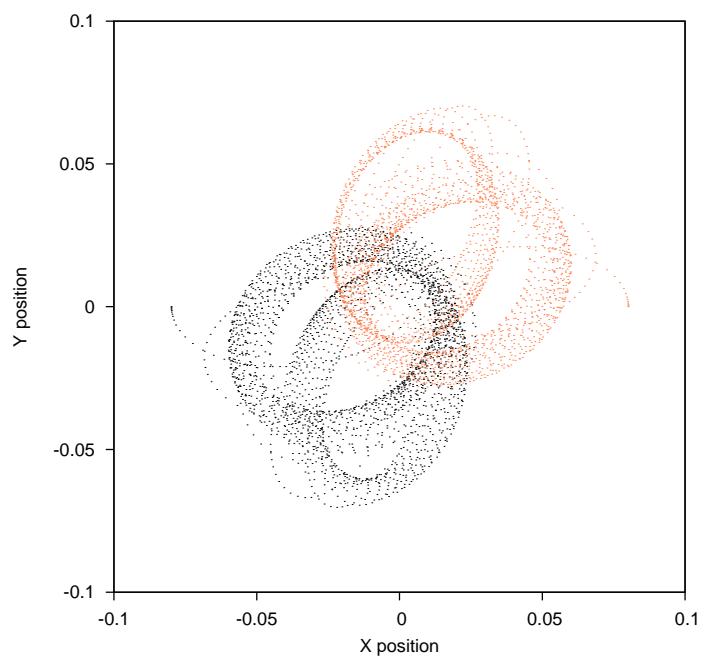


Figure 42: Configuration 20 - Inner Bar

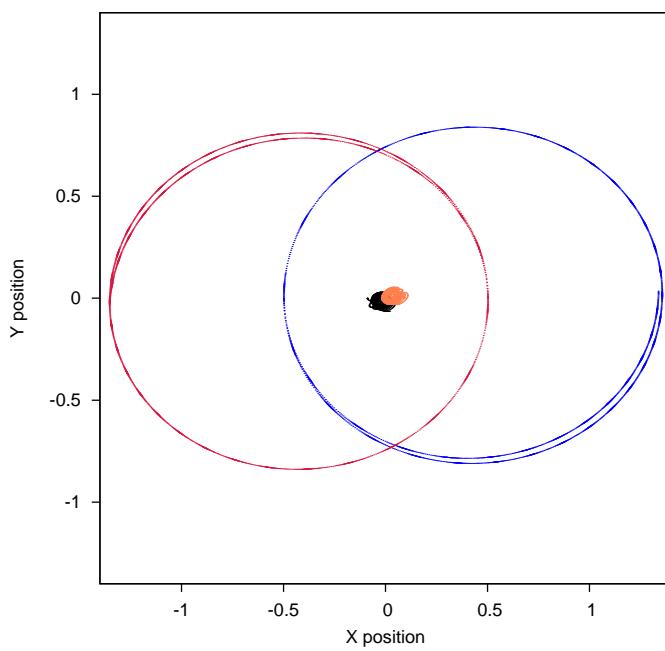


Figure 43: Configuration 21

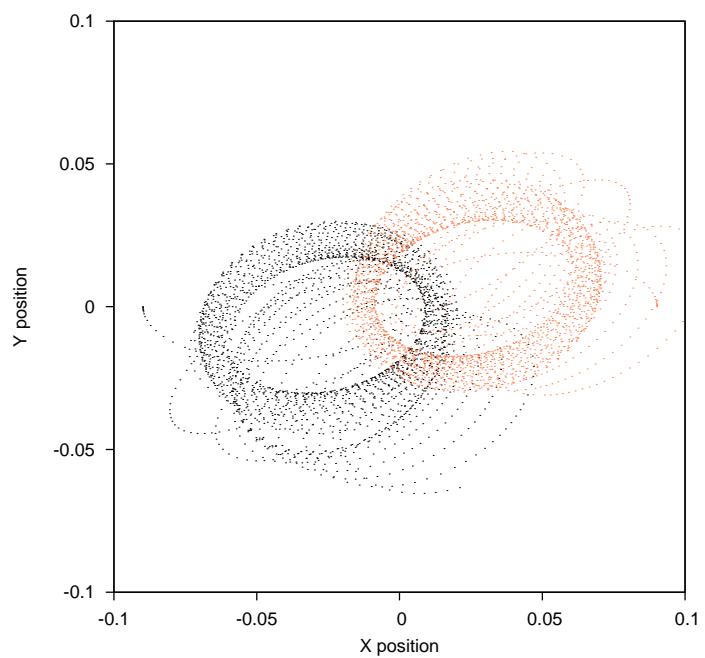


Figure 44: Configuration 21 - Inner Bar

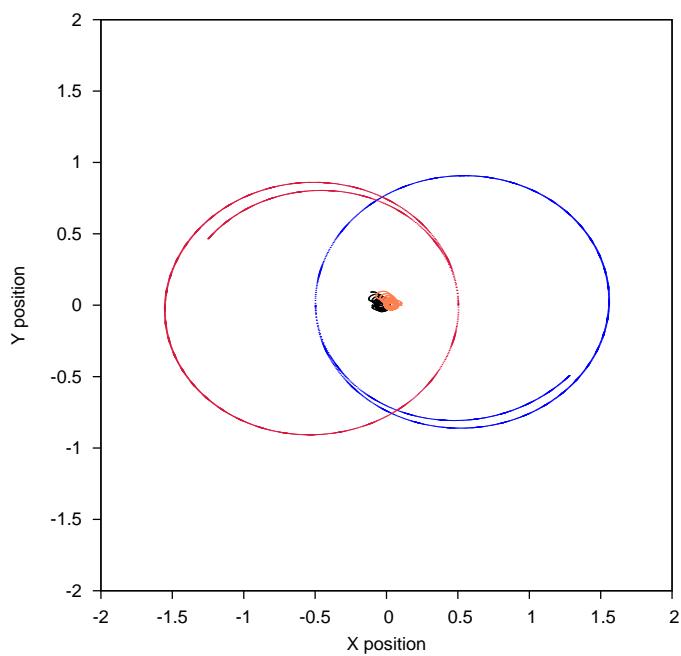


Figure 45: Configuration 22

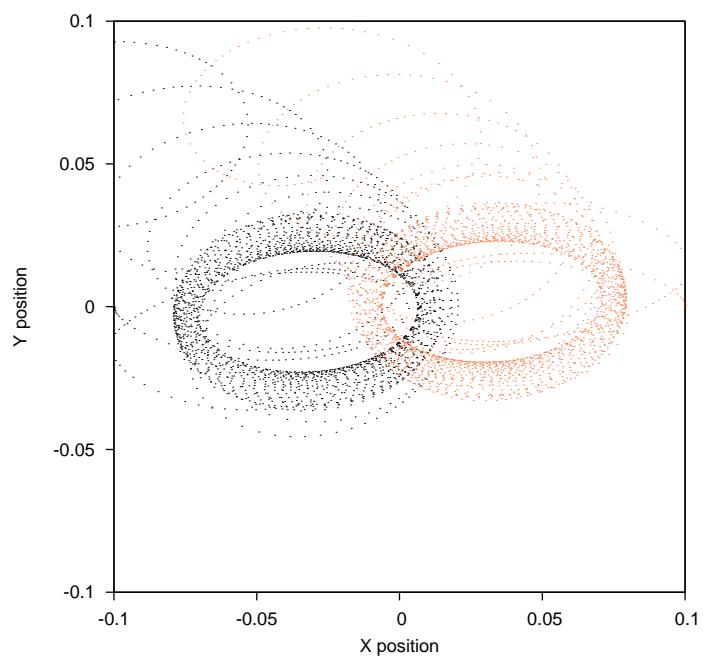


Figure 46: Configuration 22 - Inner Bar

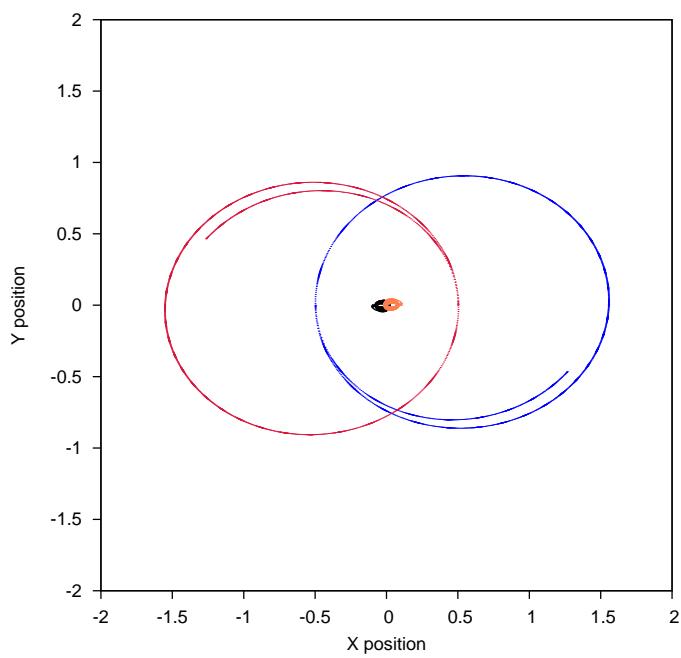


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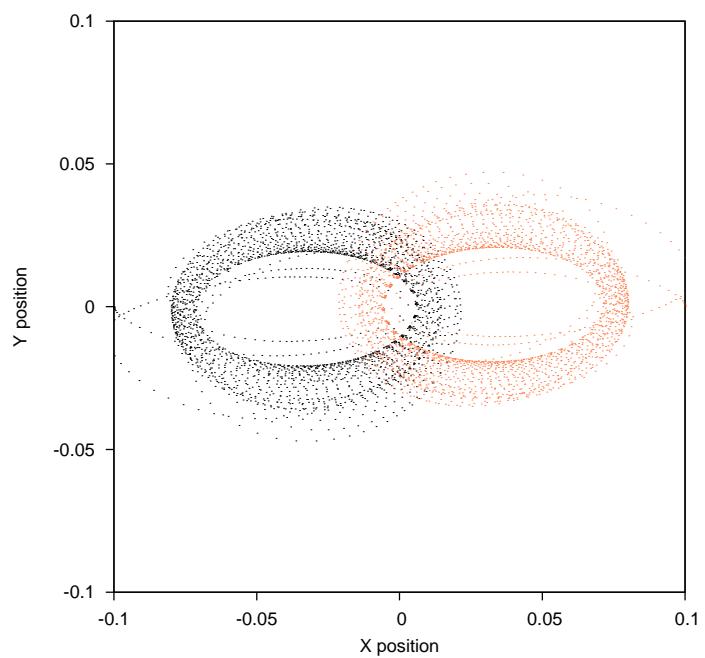


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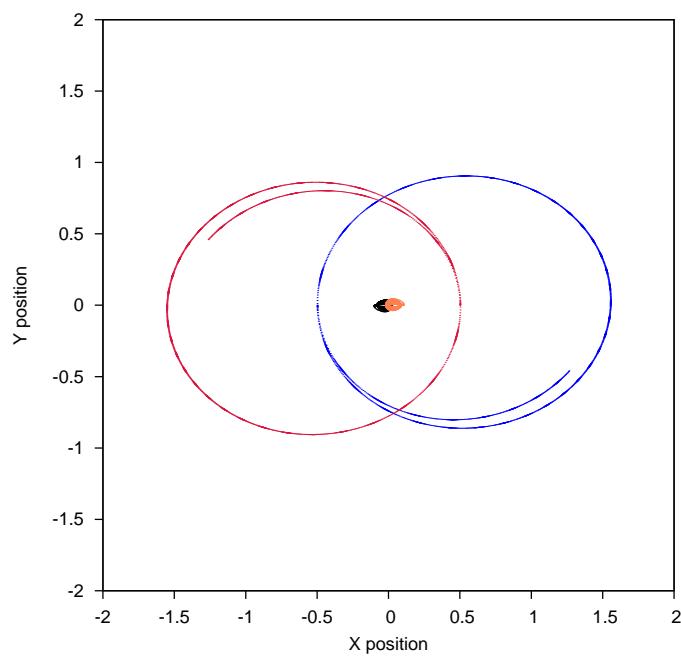


Figure 49: Configuration 24

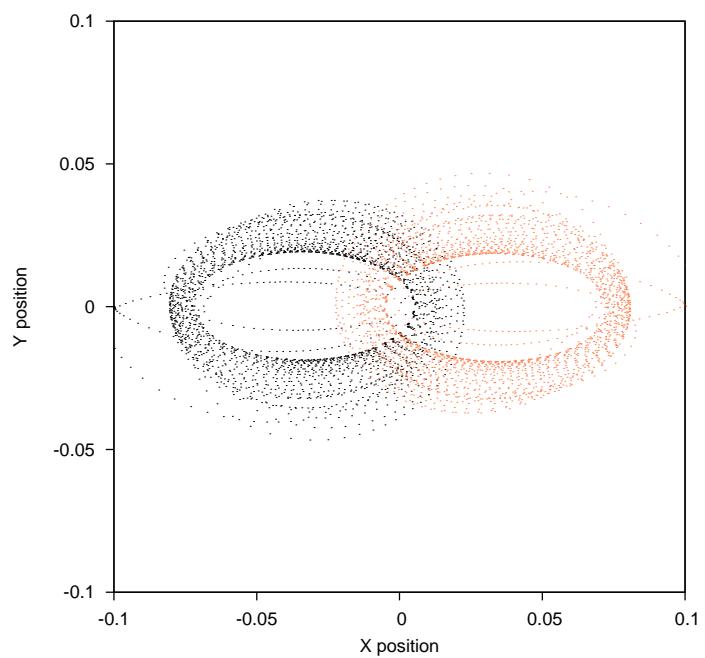


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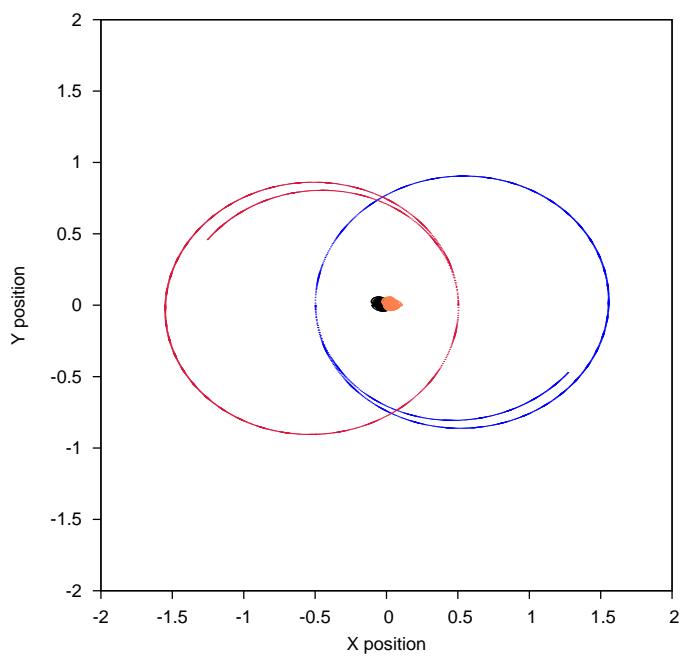


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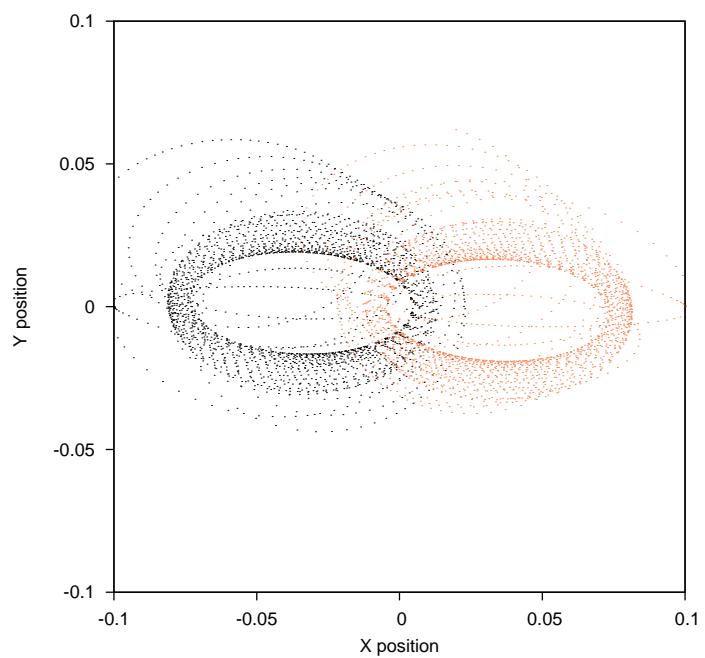


Figure 52: Configuration 25 - Inner Bar

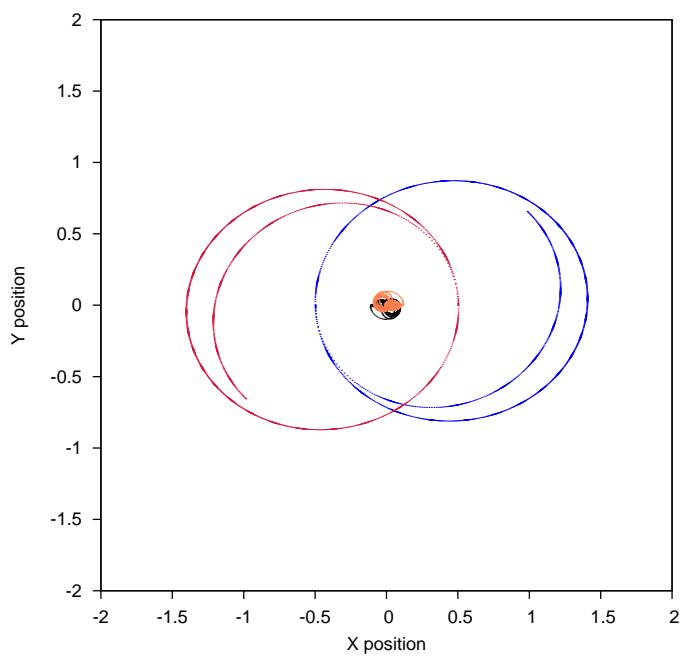


Figure 53: Configuration 26

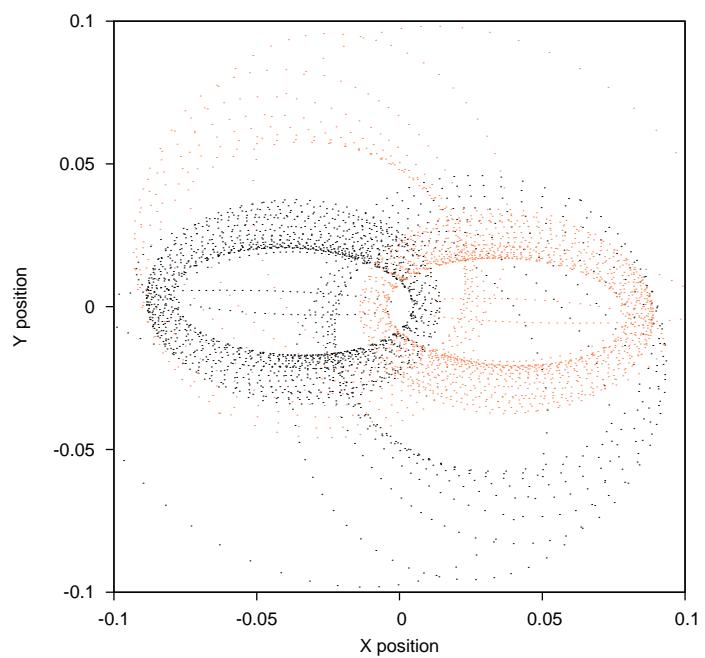


Figure 54: Configuration 26 - Inner Bar

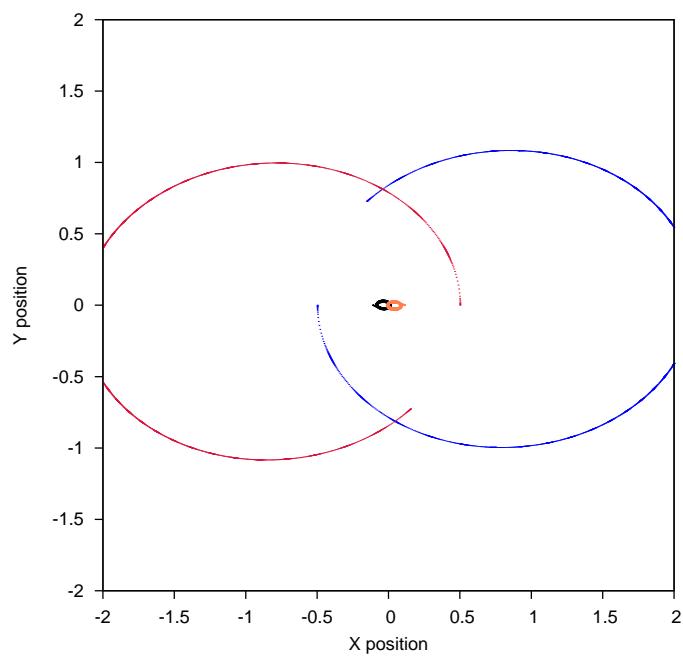


Figure 55: Configuration 27

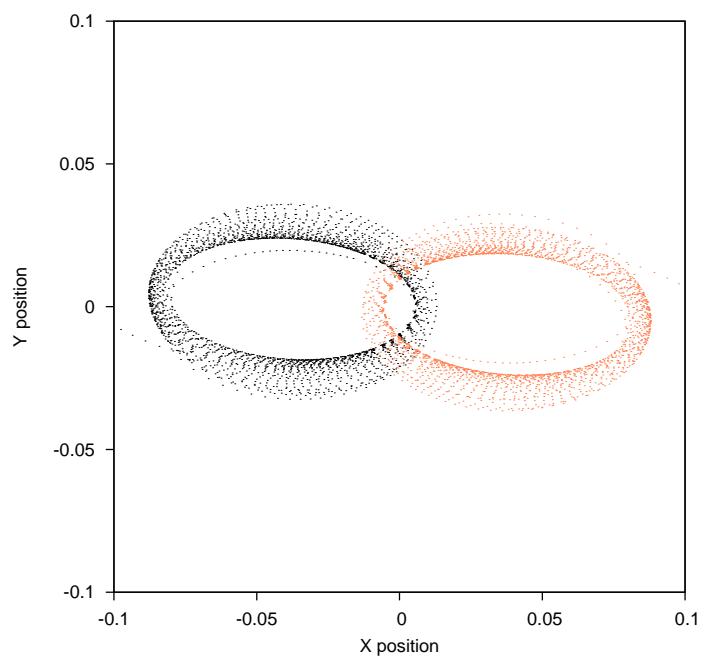


Figure 56: Configuration 27 - Inner Bar

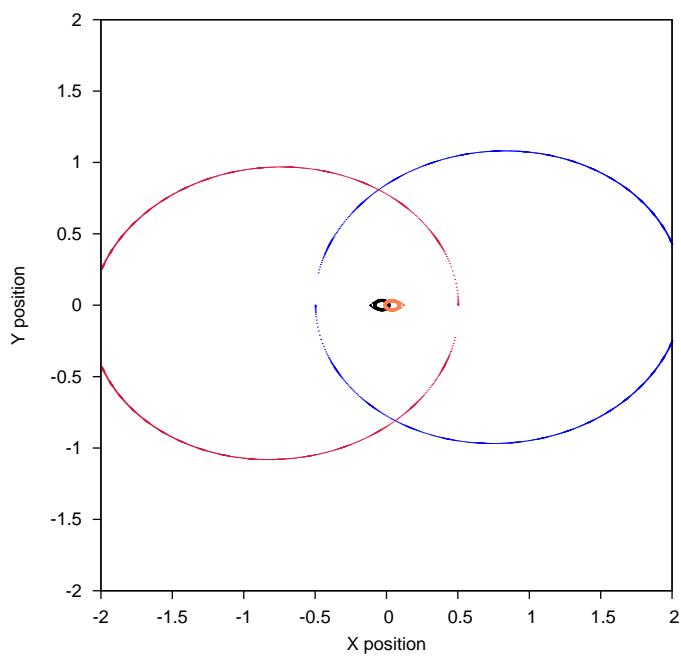


Figure 57: Configuration 28

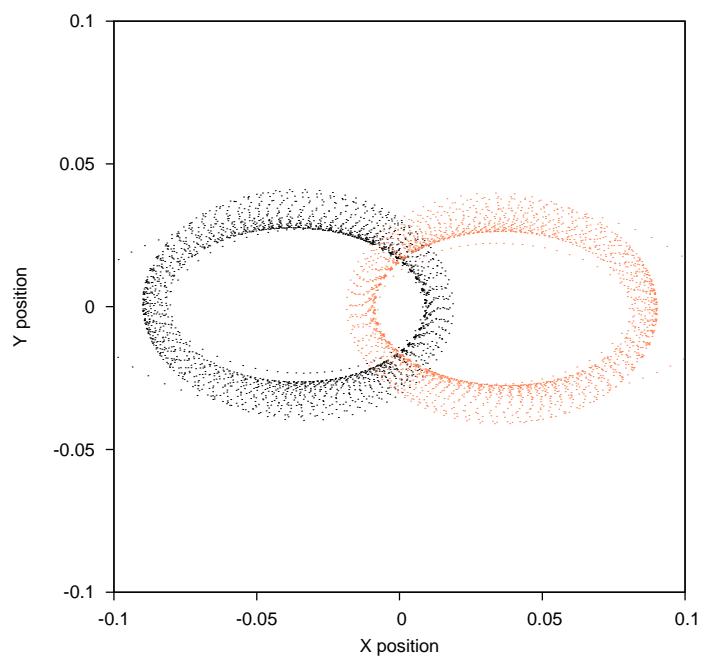


Figure 58: Configuration 28 - Inner Bar

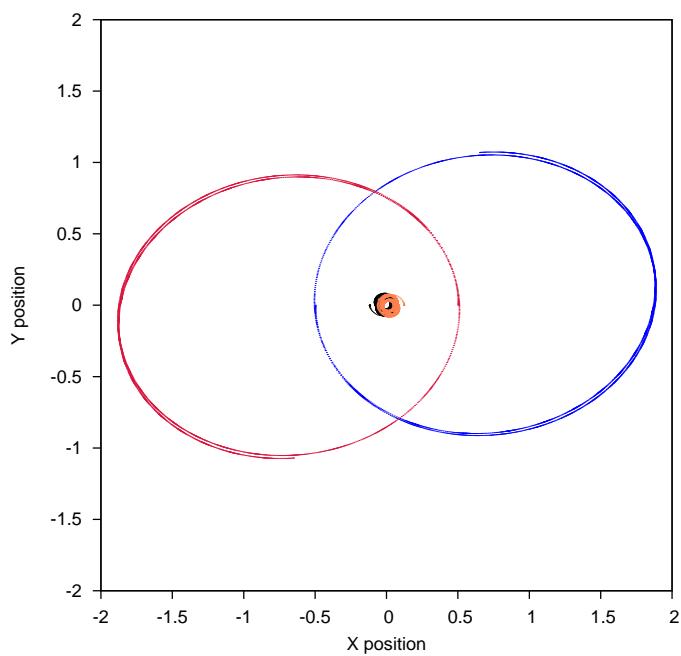


Figure 59: Configuration 29

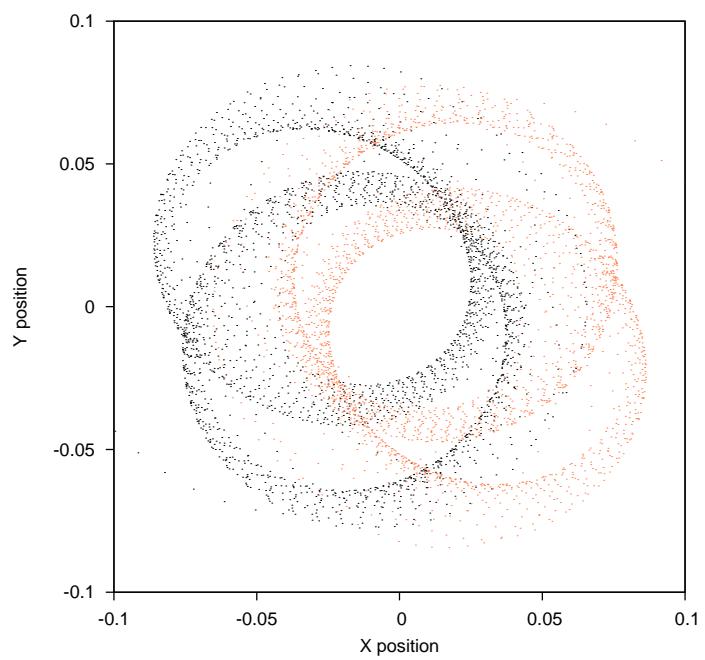


Figure 60: Configuration 29 - Inner Bar

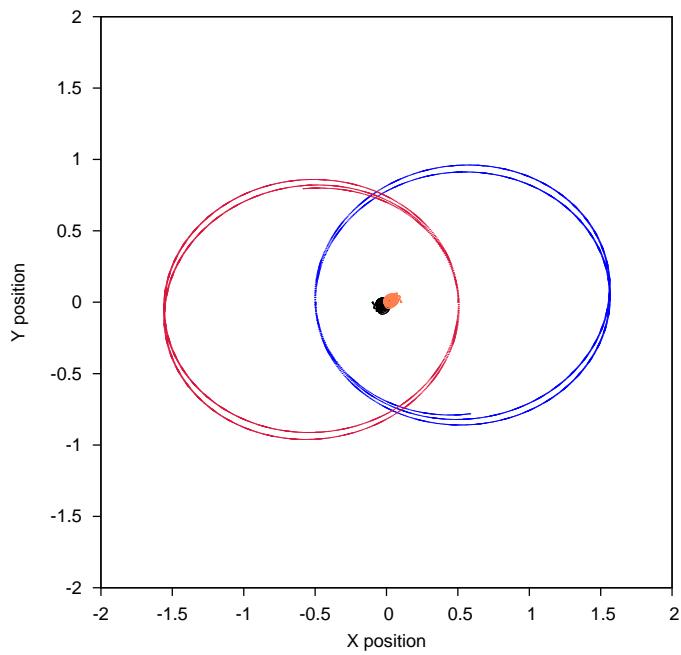


Figure 61: Configuration 30

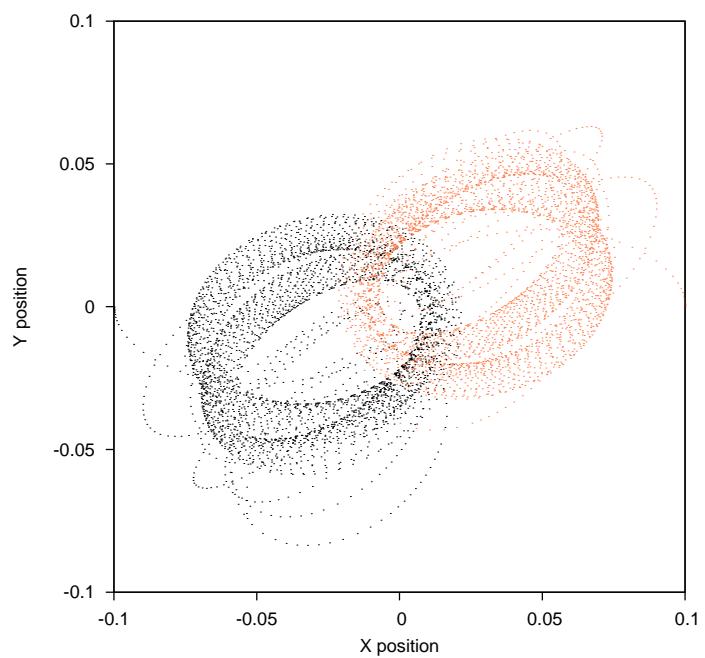


Figure 62: Configuration 30 - Inner Bar

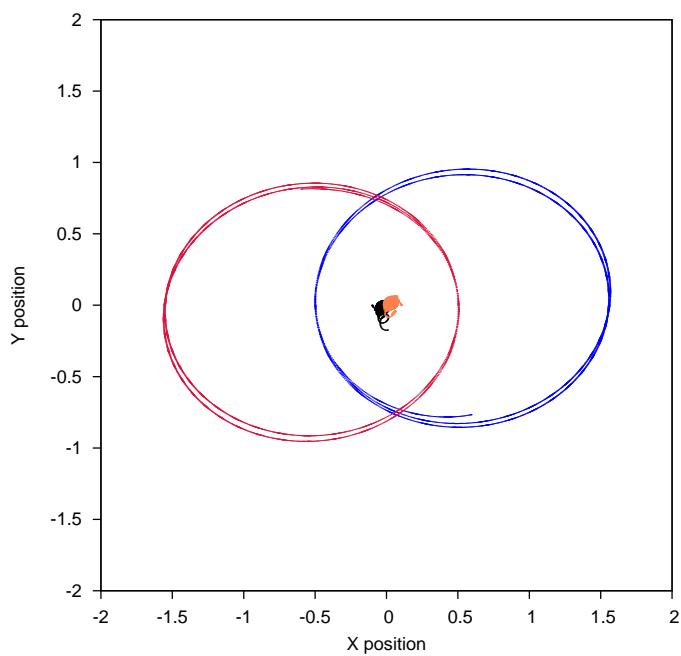


Figure 63: Configuration 31

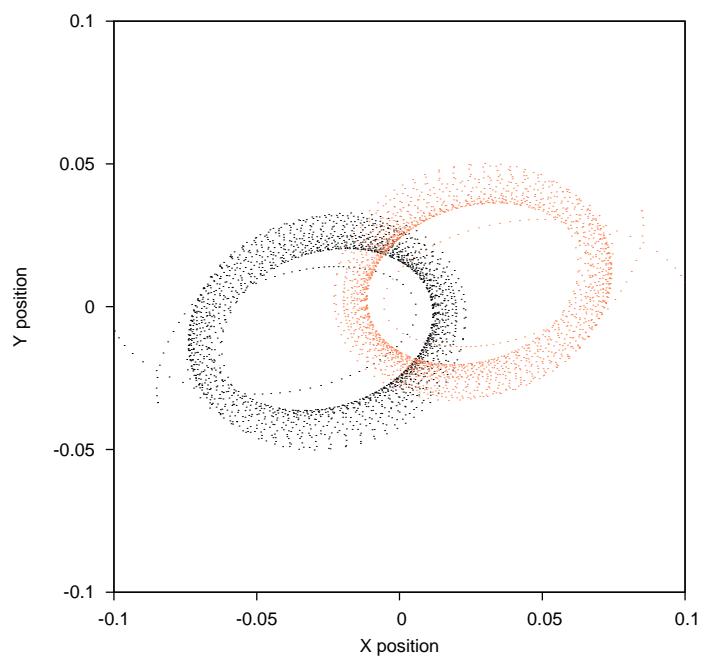


Figure 64: Configuration 31 - Inner Bar

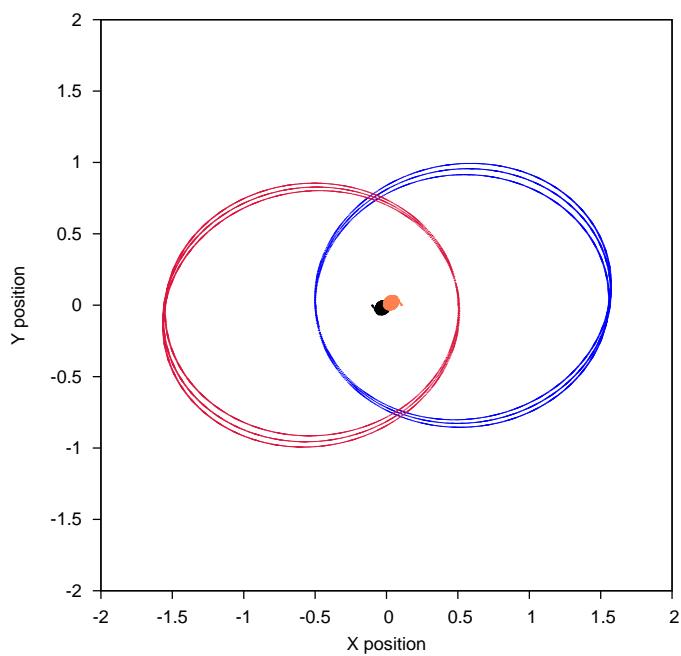


Figure 65: Configuration 32

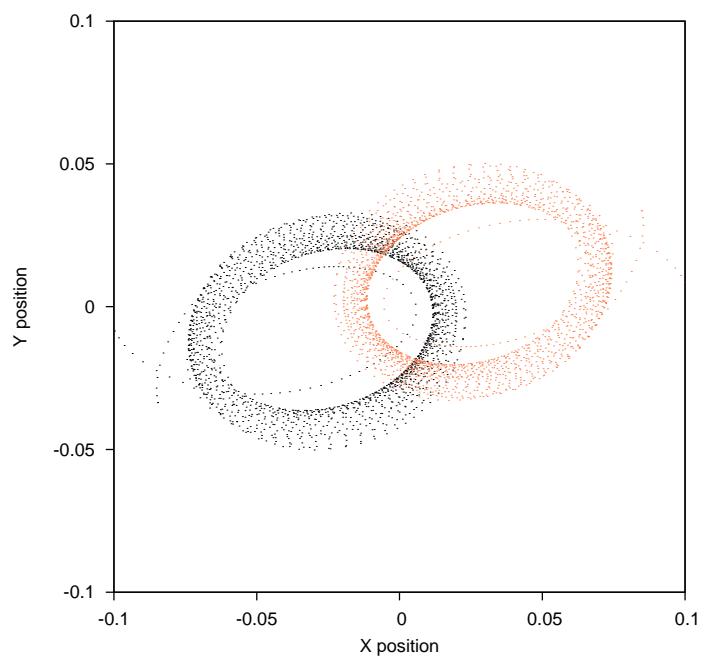


Figure 66: Configuration 32 - Inner Bar

## 4 Discussion

The final three runs (configurations 30, 31, and 32) were by far the most stable. The orbital periods vary slightly with time so it is difficult to give accurate figures, so I am using the periods of the first orbit. The following table shows the periods of the two binaries together with the ratios of the corresponding periods. The data shows that as the periodic ratios tends

Table 2: Periods

Config.	Inner	Outer	Ratio	Length of simulation
30	.51	11.56	1:22.7	25
31	.54	11.5	1:21.3	25
32	.55	11.5	1:20.9	35

towards 1:21 the system considerably stabilises. Run 32 was by far the most stable run and the orbits persisted until the integrator errored with a 'Step size underflow'. 1:21 could possibly be an orbital resonance. Fourier analysis? Rewrite in Python, automate, introduce machine learning algorithm

(Garzon, 2014), (Erwin and Sparke, 2002), (Maciejewski, 2002), (Maciejewski, 2003), (Maciejewski, 2008), (Maciejewski and Athanassoula, 2008), (Maciejewski and Small, 2010) (Maciejewski and Sparke, 1998), (Maciejewski and Sparke, 1999), (Manos and Athanassoula, 2011), (Shen and Debattista, 2008), (Shen and Debattista, 2009), (Debattista and Shen, 2007),

## References

- Aarseth, S., Gravitational N-Body Simulations, 2003, Cambridge University Press
- AlvarezRamirez, M., Medina, M., A Review of the Planar Caledonian Four-Body Problem, 2014, TODO
- Binney, J., Tremaine, S., Galactic Dynamics, 2008, Princeton University Press
- Collins, G., The Foundations Of Celestial Mechanics, 2004, Pachart Publishing House TODO
- Debattista, V., Shen, J., Long-Lived Double-Barred Galaxies from Psuedobulges, 2007, The Astrophysical Journal, 654: L127L130, 2007 January 10
- Du, M., Shen, J., Debattista, V., Forming Double-Barred Galaxies from Dynamically Cool Inner Disks, 2015, The Astrophysical Journal, 804:139 (10pp), 2015 May 10
- Du, M., Debattista, V., Shen, J., Kinematic Properties of Double-Barred Galaxies: Simulations vs. Integral-Field Observations, 2016, arXiv:1607.00585v1
- Erwin, P., Double-Barred Galaxies I. A Catalog of Barred Galaxies with Stellar Secondary Bars and Inner Disks, 2003, arXiv:astro-ph/0310806v2
- Erwin, P., Double-Barred Galaxies, 2011, Mem. S.A.It. Suppl. Vol. 18, 145
- Erwin, P., Sparke, L., Double Bars, Inner Disks, and Nuclear Rings in Early-Type Disk Galaxies, 2002, The Astronomical Journal, 124:6577, 2002 July
- Garzon, F., Lopez-Corredoira, M., Dynamical evolution of two associated galactic bars, 2014, arXiv:1409.1916.v1
- Heggie D., Hut, P., The Gravitational Million-Body Problem, 2002, TODO
- de Lorenzo-Caceres, A., Vazdekis, A., Aguerri, K., A., L., Corsini, E., M., Debattista, V., P., Constraining the formation of inner bars. Photometry, kinematics and stellar populations in NGC357, 2002, arXiv:1111.1718v1
- de Lorenzo-Caceres, A., Falcon-Barroso, J., Vazdekis, A., Distinct stellar populations in the inner bars of double-barred galaxies, 2013, arXiv:1302.5701v1

- Maciejewski, W., Constraints on nested bars implications for gas inflow, 2002, arXiv:astro-ph/0202110v1
- Maciejewski, W., Chaos or Order in Double Barred Galaxies?, 2003, arXiv:astro-ph/0304432v1
- Maciejewski, W., Orbits in corotating and counterrotating double bars, 2008, arXiv:0801.1471v1
- Maciejewski, W., Athanassoula, A., Regular motions in double bars. II. Survey of trajectories and 23 models, 2008, arXiv:0805.3967v1
- Maciejewski, W., Small, E., Orbital Support of Fast and Slow Inner Bars in Double Barred Galaxies, 2010, arXiv:1006.4574v1
- Maciejewski, W., Sparke, L., Bars within Bars in Galaxies, 1998, arXiv:astro-ph/9812228v1
- Maciejewski, W., Sparke, L., Orbits Supporting Bars within Bars, 1999, arXiv:astro-ph/9911281v1
- Malhotra, R., Orbital Resonances and Chaos in the Solar System, 1998, Solar system Formation and Evolution ASP Conference Series, Vol. 149, 1998
- Manos, T., Athanassoula, E., Regular and chaotic orbits in barred galaxies - I. Applying the SALI/GALI method to explore their distribution in several models, 2011, arXiv:1102.1157v2
- Moiseev, A., V., Velocity dispersion of stars and gas motion in double-barred galaxies, 2001, arXiv:astro-ph/0111220v1
- Moulton, F., An Introduction to Celestial Mechanics, 1914, Palmyrin Library  
TODO
- Saha, K., Maciejewski, W., Spontaneous formation of double bars in dark matter dominated galaxies, 2013, arXiv:1304.7108v1
- Sellwood, J., Wilkinson, A., Dynamics of Barred Galaxies, 2006, arXiv:astro-ph/0608665v1
- Shen, J., Debattista, V., Long-lived double-barred galaxies in N-body simulations, 2008, Mem. S.A.It. Vol. 00, 169
- Shen, J., Debattista, V., Observable Properties of Double-Barred Galaxies in N-Body Simulations, 2009, The Astrophysical Journal, 690:758772, 2009 January 1

Steves, B., Roy, A., Some special restricted four-body problems-I. Modelling the Caledonian problem, 1998, Phrt. Spucx, Si., Vol. 46. No. I I /) 12, pp. 1465-1474, 1998

Szell, A., Erdi, B., Sandor, Zs., Steves, B., Chaotic and stable behaviour in the Caledonian Symmetric Four-Body Problem, 2004, Mon. Not. R. Astron. Soc. 347, 380388 (2004)

Trenti, M., Hut, P., Gravitational N-body Simulations, 2008, arXiv:0806.3950v1

Wozniak, H., How can double-barred galaxies be long-lived?, 2015, A and A 575, A7 (2015) Numerical Recipes