

# Teoria da Computação

## Tese de Church Turing

Leonardo Takuno  
{leonardo.takuno@gmail.com}

Centro Universitário Senac

# Sumário

- 1 Máquina de Turing
- 2 Linguagem Turing-Reconhecível
- 3 Linguagem Turing-Decidível
- 4 Tese de Church-Turing
- 5 Exercícios

# Sumário

- 1 Máquina de Turing
- 2 Linguagem Turing-Reconhecível
- 3 Linguagem Turing-Decidível
- 4 Tese de Church-Turing
- 5 Exercícios

## Definição Formal

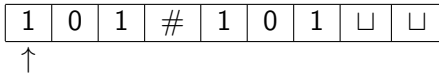
Uma **máquina de Turing** é uma 7-upla  $(Q, \Sigma, \Gamma, \delta, q_0, q_{aceita}, q_{rejeita})$ , onde  $Q, \Sigma, \Gamma$ , são todos conjuntos finitos e

- 1  $Q$  é o conjunto de estados
- 2  $\Sigma$  é o alfabeto de entrada não contém o simbolo em branco  $\sqcup$
- 3  $\Gamma$  é o alfabeto da fita, onde  $\sqcup \in \Gamma$ , e  $\Sigma \subseteq \Gamma$
- 4  $\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$  é a função de transição
- 5  $q_0 \in Q$  é o estado inicial.
- 6  $q_{aceita} \in Q$  é o estado de aceitação, e
- 7  $q_{rejeita} \in Q$  é o estado de rejeição, onde  $q_{rejeita} \neq q_{aceita}$

# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

Exemplo  $x = 101\#101$



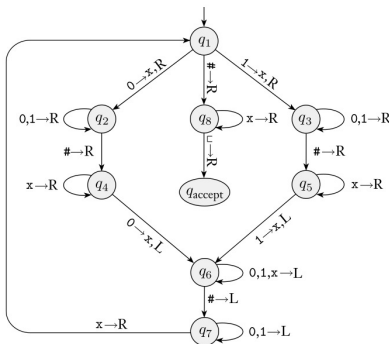
# Máquina de Turing

$M_1$  = “sobre a cadeia de entrada  $x$

- 1 Faça um zigue-zague ao longo da fita checando posições correspondentes de ambos os lados do símbolo  $\#$  para verificar se elas contêm o mesmo símbolo. Se elas não contêm, ou se nenhum  $\#$  for encontrado, *rejeite*. Marque os símbolos à medida que eles são verificados para manter registro de quais símbolos têm correspondência.
- 2 Quando todos os símbolos à esquerda do  $\#$  tiverem sido marcados, verifique a existência de algum símbolo remanescente à direita do  $\#$ . Se resta algum símbolo, *rejeite*; caso contrário, *aceite*.”

# Máquina de Turing

$$L(M1) = \{w\#w \mid w \in \{0, 1\}^*\}$$

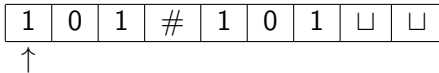


Nota: Transições para o estado  $q_{reject}$  são implícitos sobre o símbolo que não aparecem nos estados.

# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

Exemplo  $x = 101\#101$

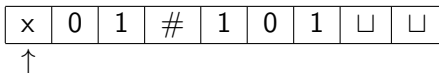




# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

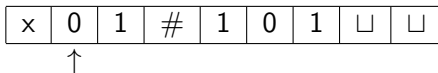
Exemplo  $x = 101\#101$



# Máquina de Turing

$B = \{w\#w \mid w \in \{0,1\}^*\}.$

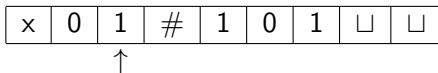
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

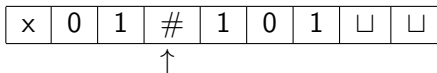
Exemplo  $x = 101\#101$



# Máquina de Turing

$B = \{w\#w \mid w \in \{0,1\}^*\}.$

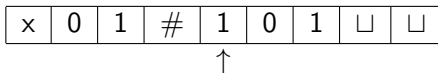
Exemplo  $x = 101\#101$



# Máquina de Turing

$B = \{w\#w \mid w \in \{0,1\}^*\}.$

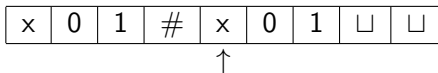
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

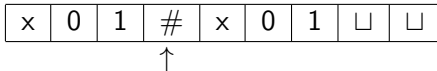
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

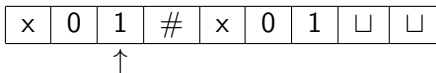
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

Exemplo  $x = 101\#101$

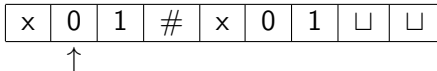




# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

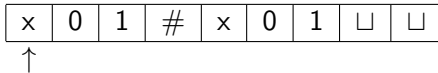
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

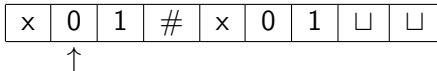
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

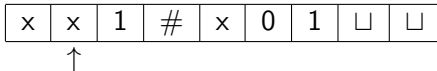
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

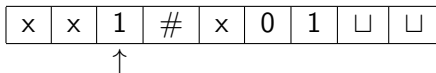
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

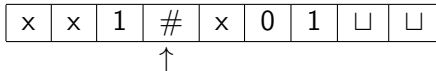
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

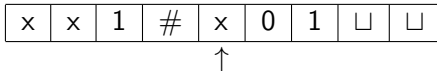
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

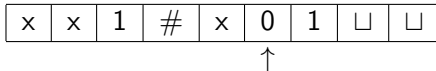
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

Exemplo  $x = 101\#101$

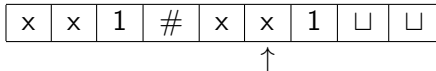




# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

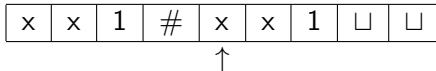
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

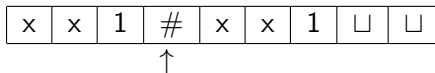
Exemplo  $x = 101\#101$



# Máquina de Turing

$B = \{w\#w \mid w \in \{0,1\}^*\}.$

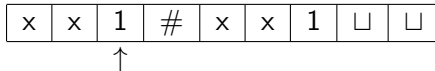
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

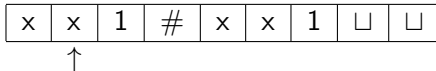
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

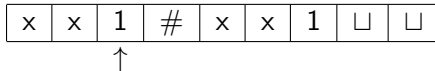
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

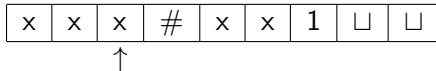
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

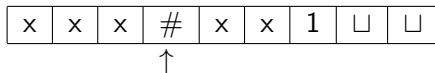
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

Exemplo  $x = 101\#101$

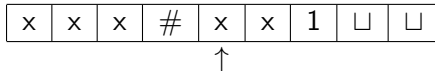




# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

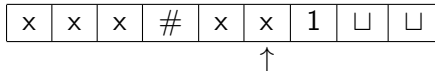
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

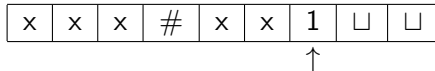
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

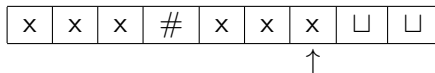
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

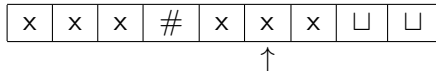
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

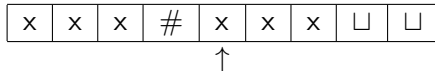
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

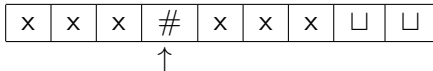
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

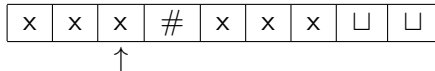
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

Exemplo  $x = 101\#101$

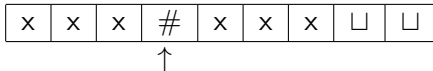




# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

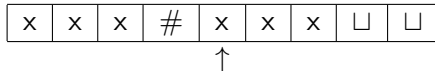
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

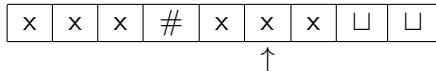
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

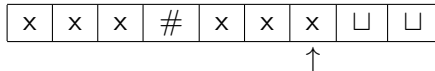
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

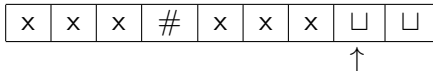
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

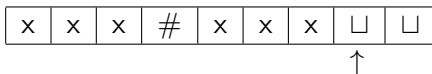
Exemplo  $x = 101\#101$



# Máquina de Turing

$$B = \{w\#w \mid w \in \{0,1\}^*\}.$$

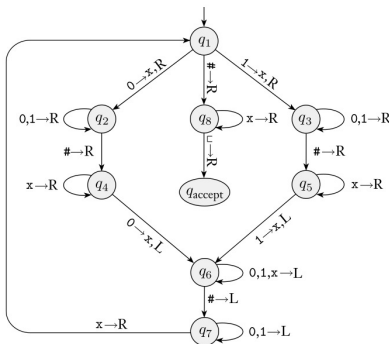
Exemplo  $x = 101\#101$



M1 aceita  $x$ . Logo  $x \in B$ .

## Exercício 1

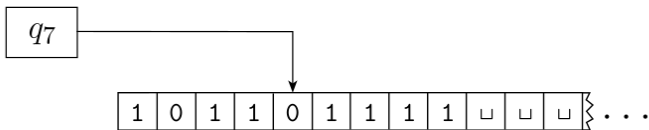
$$L(M1) = \{w\#w \mid w \in \{0, 1\}^*\}$$



Identifique as componentes da definição da Máquina de Turing no grafo orientado acima.

## Configurações

- O estado corrente, o conteúdo corrente da fita, e a localização da cabeça de leitura é chamado de **configuração**.



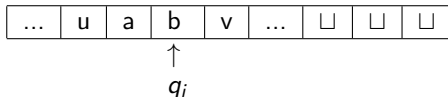
configuração: 1011 $q_7$ 01111



## Configurações

- Sejam  $u, v \in \Gamma^*$ ;  $a, b, c \in \Gamma$ ;  $q_i, q_j \in Q$  e  $M$  uma máquina de Turing com função de transição  $\delta$ .
  - Dizemos que a configuração “ $uaq_i bv$ ” produz a configuração “ $uacq_j v$ ” se:

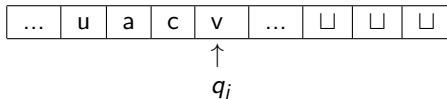
$$\delta(q_i, b) = (q_j, c, R)$$



## Configurações

- Sejam  $u, v \in \Gamma^*$ ;  $a, b, c \in \Gamma$ ;  $q_i, q_j \in Q$  e  $M$  uma máquina de Turing com função de transição  $\delta$ .
  - Dizemos que a configuração “ $uaq_i bv$ ” produz a configuração “ $uacq_j v$ ” se:

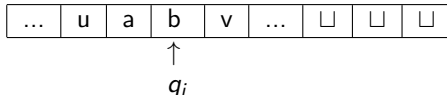
$$\delta(q_i, b) = (q_j, c, R)$$



## Configurações

- Sejam  $u, v \in \Gamma^*$ ;  $a, b, c \in \Gamma$ ;  $q_i, q_j \in Q$  e  $M$  uma máquina de Turing com função de transição  $\delta$ .
  - Similarmente, “ $uaq_i bv$ ” produz a configuração “ $uq_j acv$ ” se:

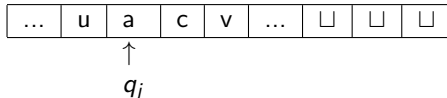
$$\delta(q_i, b) = (q_j, c, L)$$



## Configurações

- Sejam  $u, v \in \Gamma^*$ ;  $a, b, c \in \Gamma$ ;  $q_i, q_j \in Q$  e  $M$  uma máquina de Turing com função de transição  $\delta$ .
  - Similarmente, “ $uaq_i bv$ ” produz a configuração “ $uq_j acv$ ” se:

$$\delta(q_i, b) = (q_j, c, L)$$



## Configurações

- **configuração inicial** : A MT está no estado  $q_0$  com a cabeça de leitura situado na posição mais a direita da cadeia  $w$ :  
“ $q_0 w$ ”
- **configuração de parada** : um estado na qual a máquina está ou em estado de aceitação (**configuração de aceitação**) ou no estado de rejeição (**configuração de rejeição**):
  - configuração de aceitação: “ $uq_{accept}v$ ”
  - configuração de rejeição: “ $uq_{reject}v$ ”

## Configurações

- Uma máquina de Turing  $M$  aceita a entrada  $w \in \Sigma^*$ , se, e somente se, há uma seqüência finita de configurações  $C_1, C_2, \dots, C_k$  com
  - $C_1$  a configuração inicial: " $q_0 w$ ";
  - para  $i = 1, \dots, k - 1$   $C_i$  produz  $C_{i+1}$ ;
  - $C_k$  é uma configuração de aceitação " $uq_{accept} v$ ".
- A linguagem que consiste de todas as entradas que são aceitas por  $M$  é denotada por  $L(M)$ .

# Sumário

- 1 Máquina de Turing
- 2 Linguagem Turing-Reconhecível
- 3 Linguagem Turing-Decidível
- 4 Tese de Church-Turing
- 5 Exercícios

## MT e Linguagem

**Definição 3.5:** Chame uma linguagem de **Turing-reconhecível**, se alguma máquina de Turing a reconhece.

- Um linguagem Turing-reconhecível muitas vezes é chamada de **linguagem recursivamente enumerável**.
- Se  $w \notin L$ , a máquina pode parar em um estado de rejeição, ou pode entrar em loop indefinidamente.
- Como distinguir entre um tempo de computação muito longa e um processamento que nunca pára?



# Sumário

- 1 Máquina de Turing
- 2 Linguagem Turing-Reconhecível
- 3 Linguagem Turing-Decidível**
- 4 Tese de Church-Turing
- 5 Exercícios

## MT e Linguagem

O problema com MT que reconhecem uma linguagem é que elas podem entrar em loop para alguma entrada. Queremos uma máquina que sempre pára. Chamamos tais máquinas de decisores.

**Definição 3.6:** Chame uma linguagem de **Turing-decidível** ou simplesmente decidível se alguma máquina de Turing a decide.

Decisores responderão se uma string pertence a linguagem ou não à linguagem.

NOTE: Toda linguagem decidível é Turing-reconhecível.

Um linguagem decidível é muitas vezes chamada de **linguagem recursiva**.

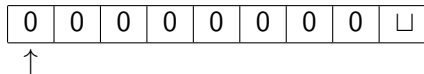
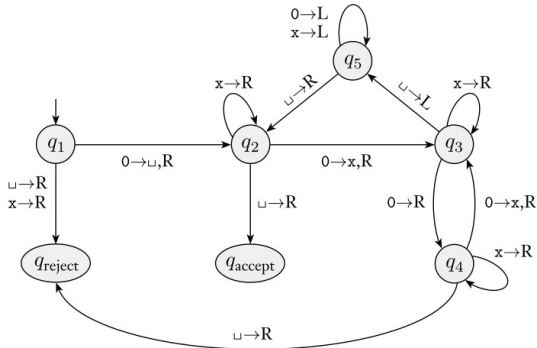
$$A = \{0^{2^n} \mid n \geq 0\}$$

A linguagem consistindo em todas as cadeia de 0s cujo comprimento é uma potência de 2.

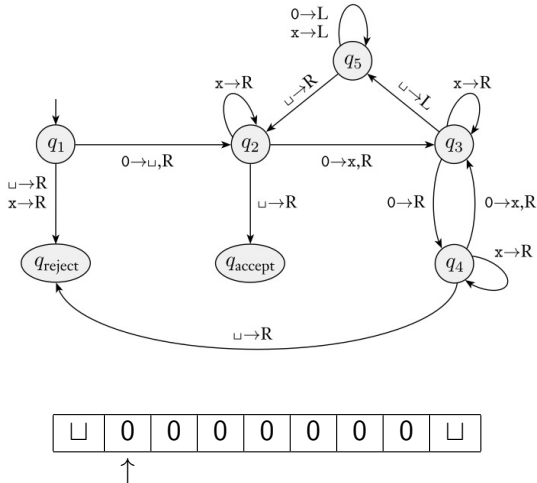
$M_2 =$  “Sobre a cadeia de entrada  $w$ :

- 1 Faça uma varredura da esquerda para a direita na fita, marcando um 0 não, e outro, sim.
- 2 Se no estágio 1, a fita continha um único 0, aceite.
- 3 Se no estágio 1, a fita continha mais que um único 0 e o número de 0s era ímpar, rejeite
- 4 Retorne a cabeça para a extremidade esquerda da fita.
- 5 Vá para o estágio 1.

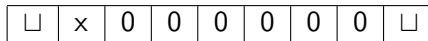
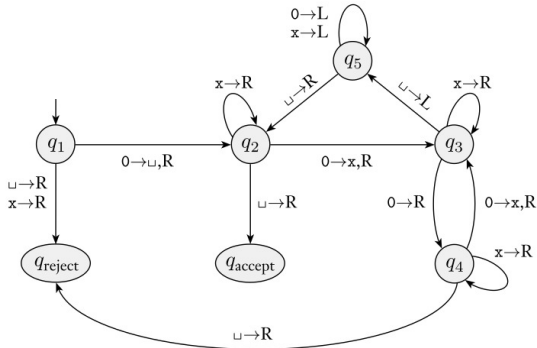
$$A = \{0^{2^n} \mid n \geq 0\}$$



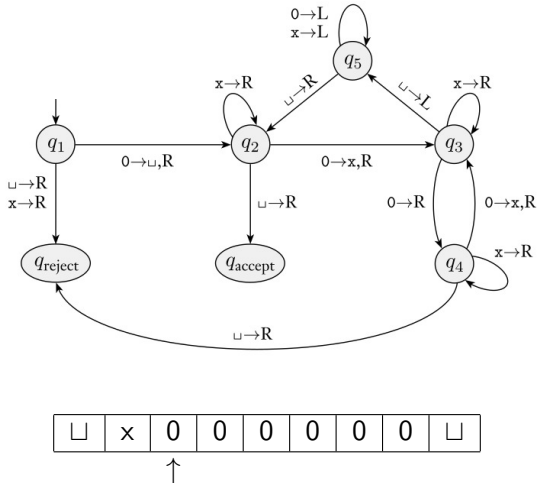
$$A = \{0^{2^n} \mid n \geq 0\}$$



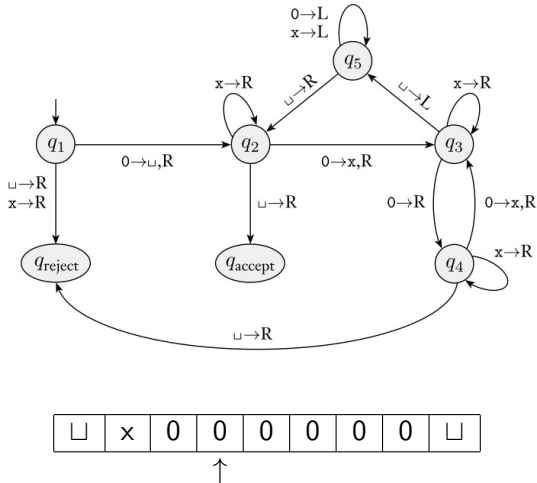
$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$

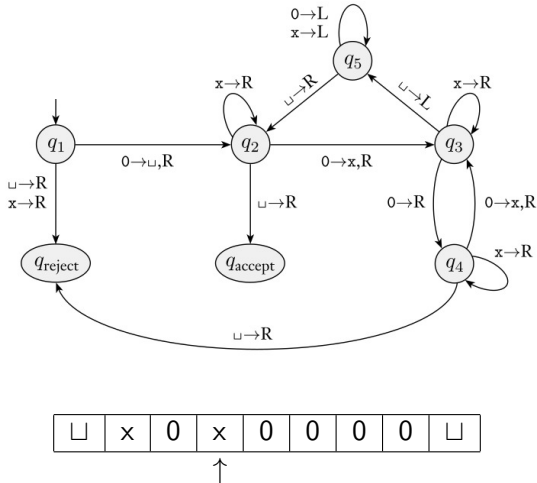


$$A = \{0^{2^n} \mid n \geq 0\}$$

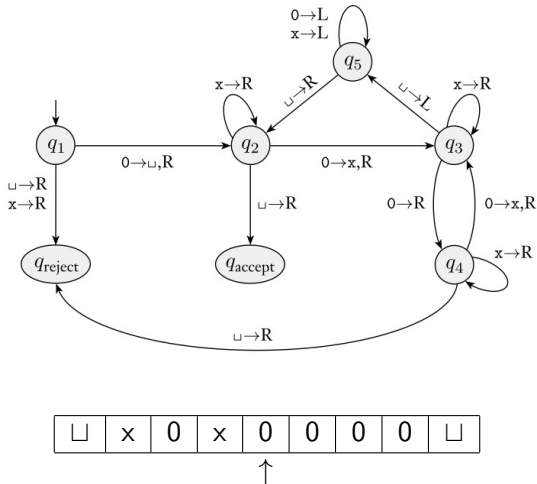




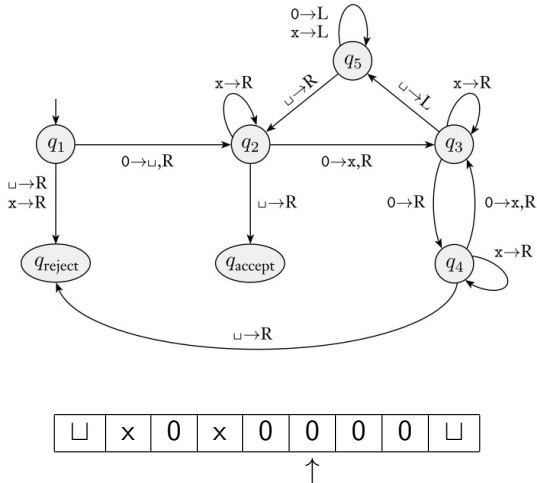
$$A = \{0^{2^n} \mid n \geq 0\}$$



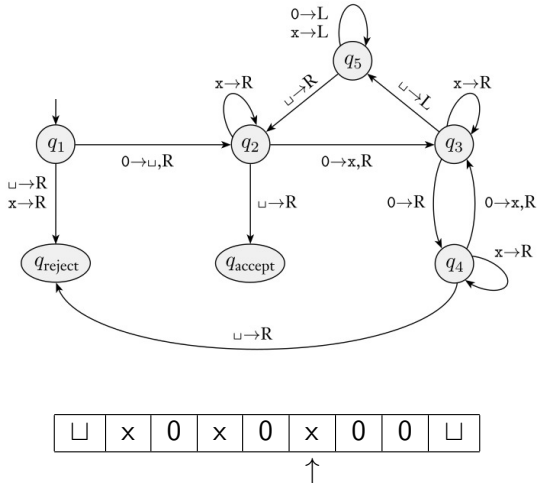
$$A = \{0^{2^n} \mid n \geq 0\}$$



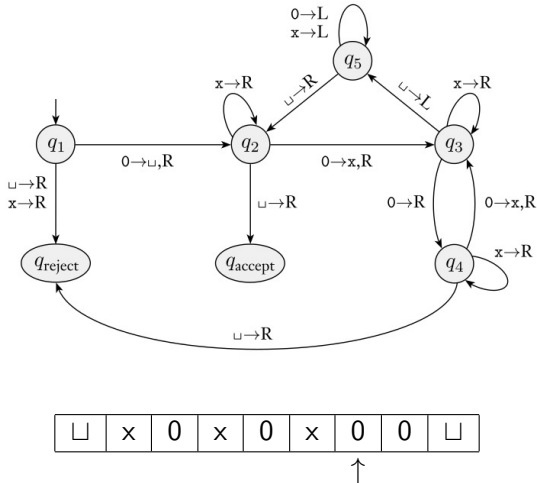
$$A = \{0^{2^n} \mid n \geq 0\}$$



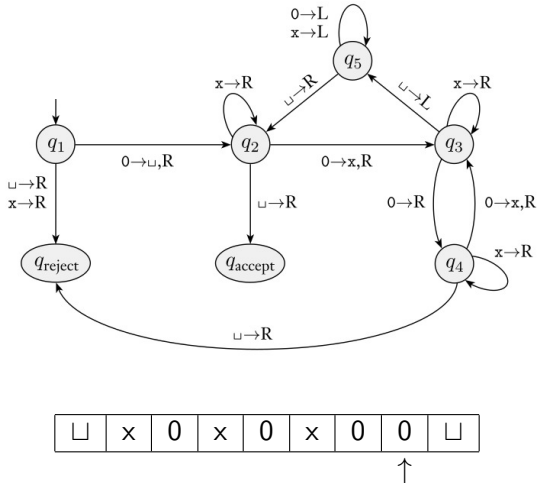
$$A = \{0^{2^n} \mid n \geq 0\}$$



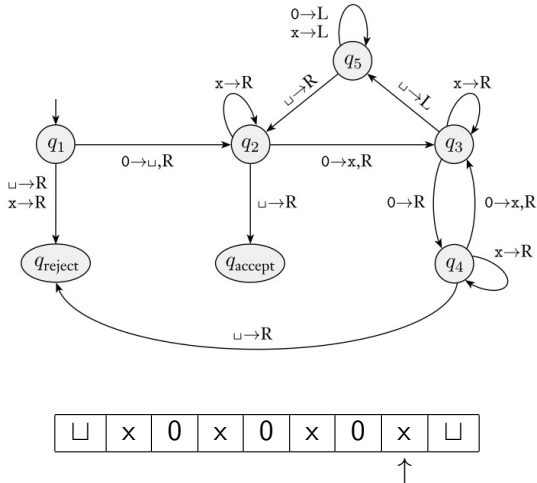
$$A = \{0^{2^n} \mid n \geq 0\}$$



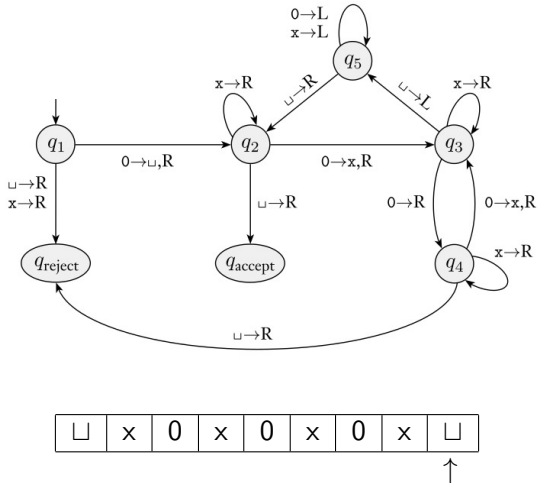
$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$

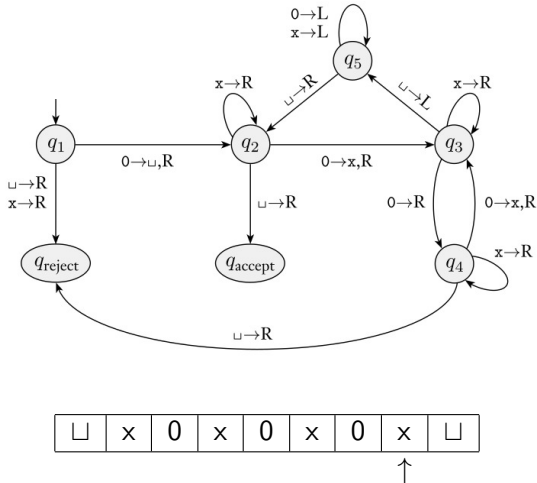


$$A = \{0^{2^n} \mid n \geq 0\}$$

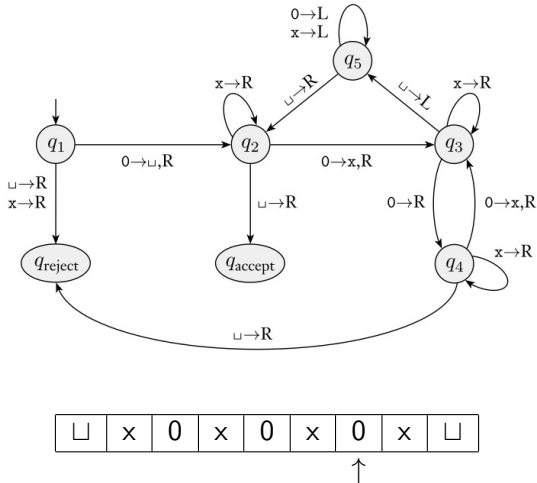




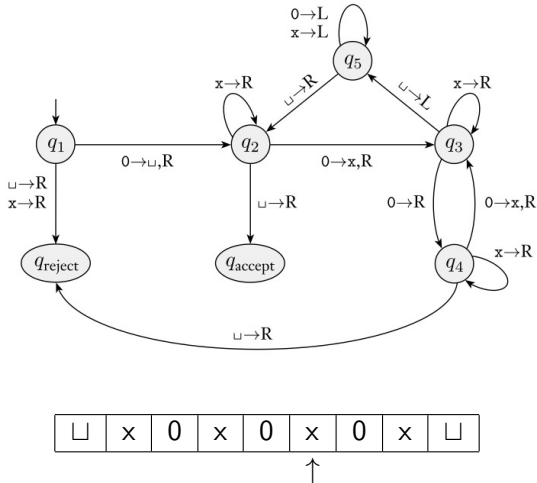
$$A = \{0^{2^n} \mid n \geq 0\}$$



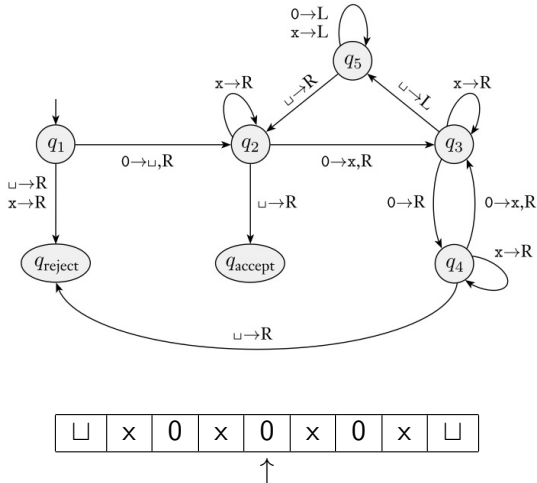
$$A = \{0^{2^n} \mid n \geq 0\}$$



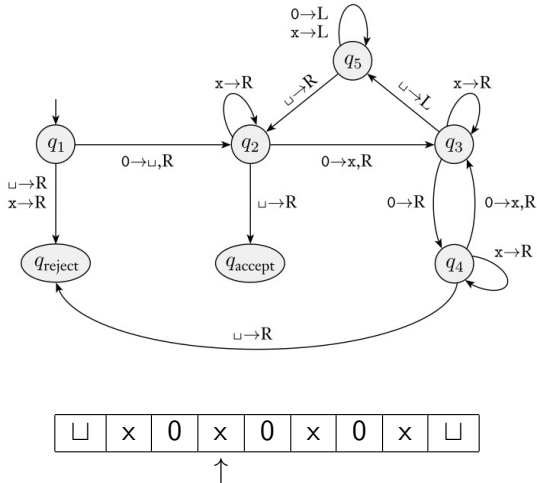
$$A = \{0^{2^n} \mid n \geq 0\}$$



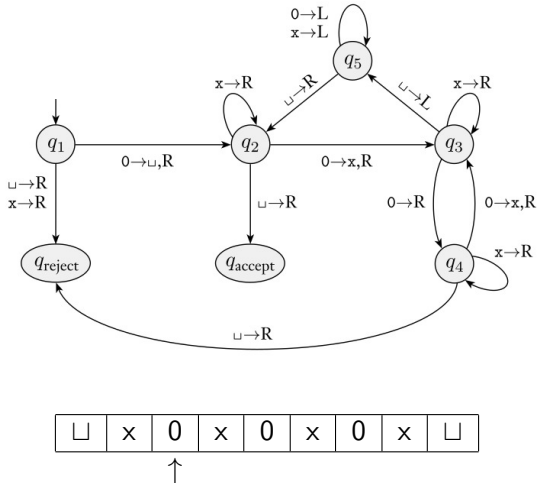
$$A = \{0^{2^n} \mid n \geq 0\}$$



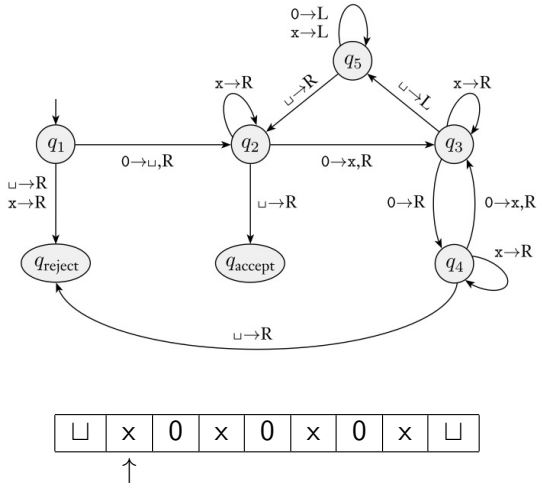
$$A = \{0^{2^n} \mid n \geq 0\}$$



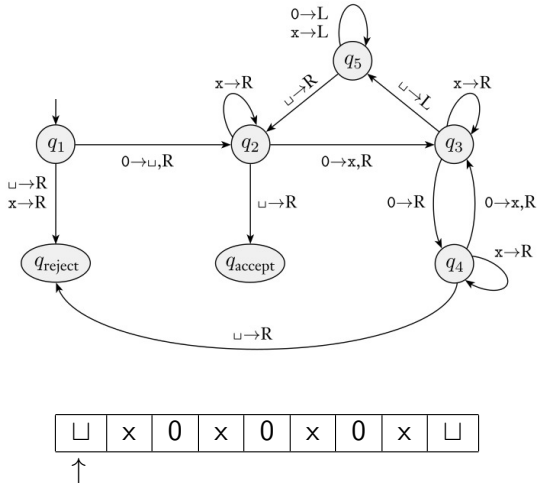
$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$

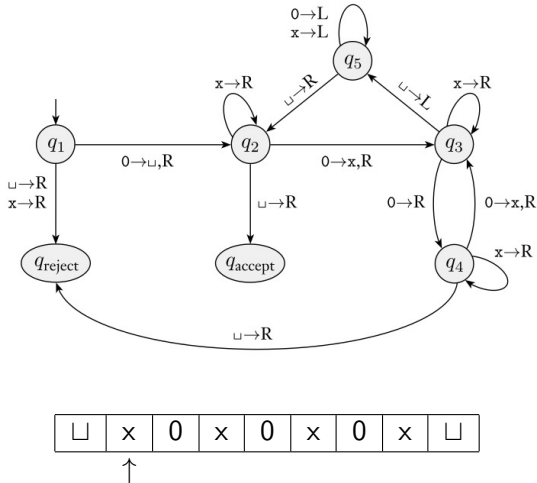


$$A = \{0^{2^n} \mid n \geq 0\}$$

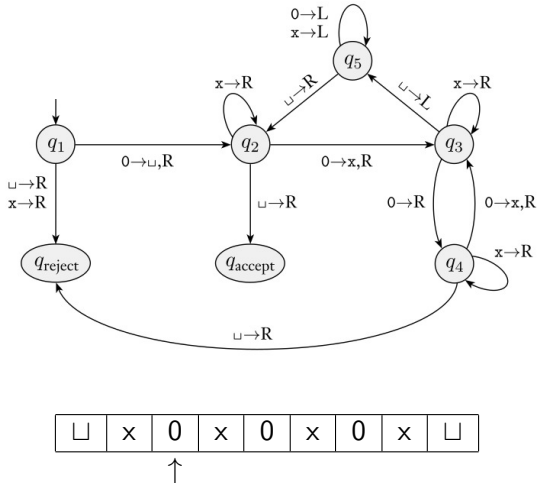




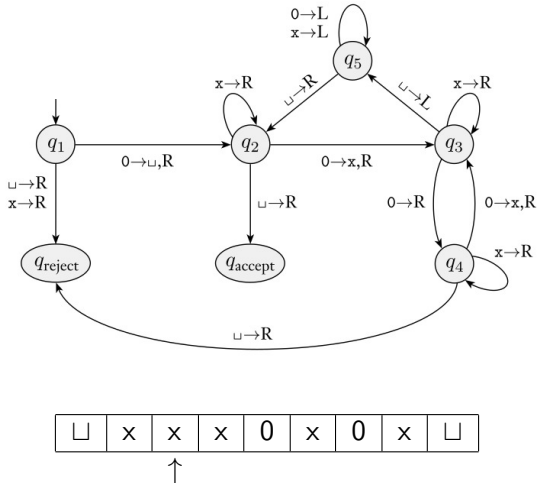
$$A = \{0^{2^n} \mid n \geq 0\}$$



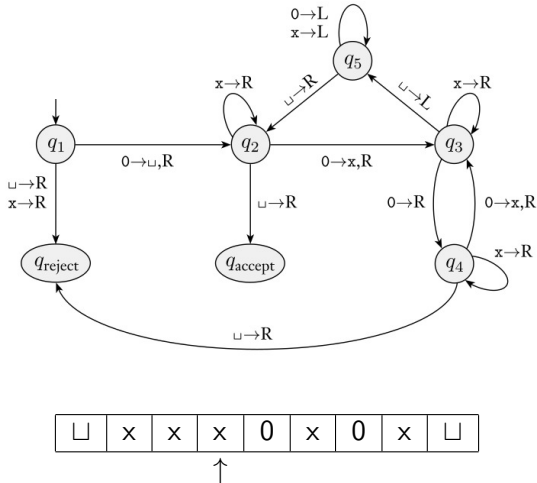
$$A = \{0^{2^n} \mid n \geq 0\}$$



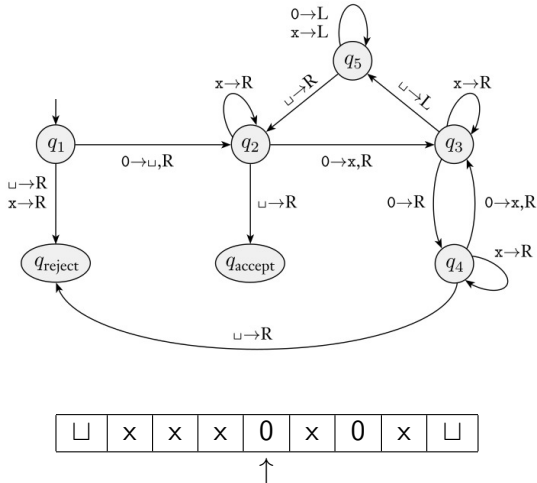
$$A = \{0^{2^n} \mid n \geq 0\}$$



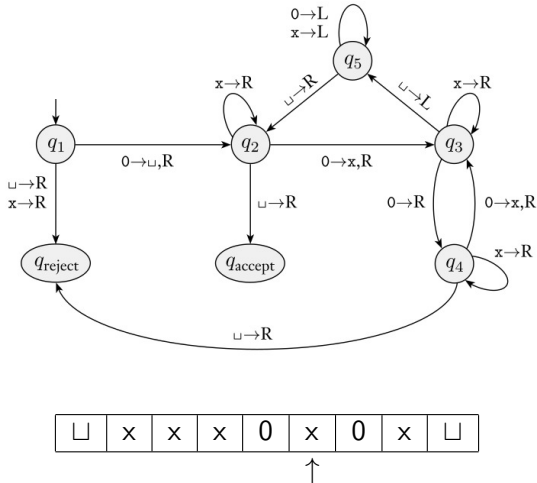
$$A = \{0^{2^n} \mid n \geq 0\}$$



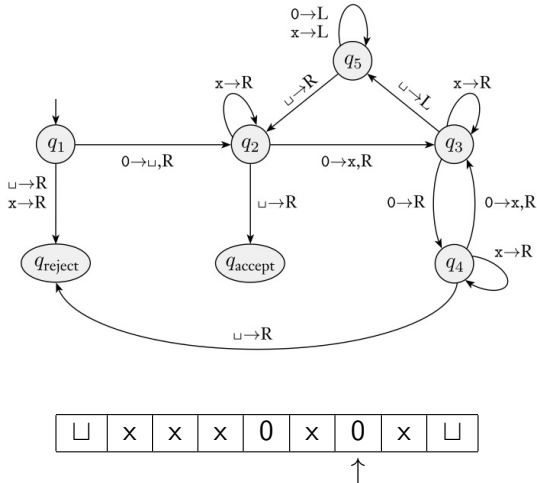
$$A = \{0^{2^n} \mid n \geq 0\}$$



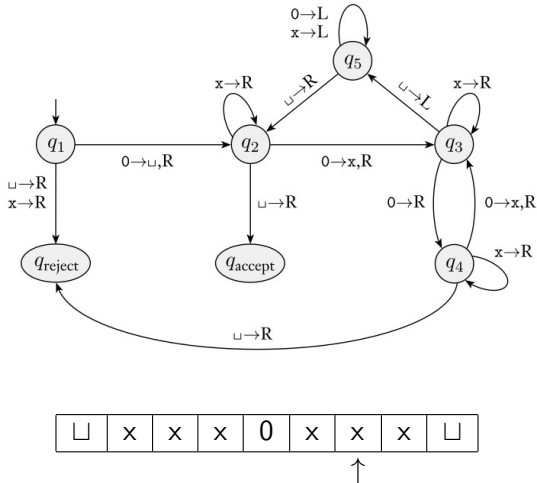
$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$

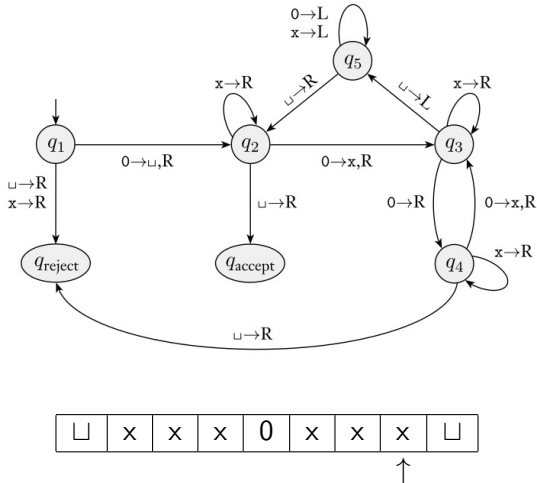


$$A = \{0^{2^n} \mid n \geq 0\}$$

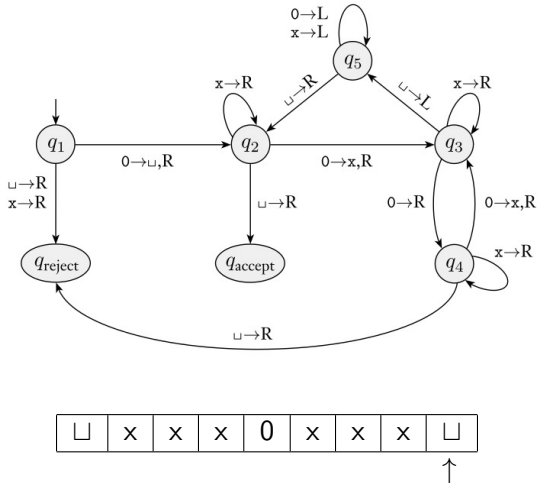




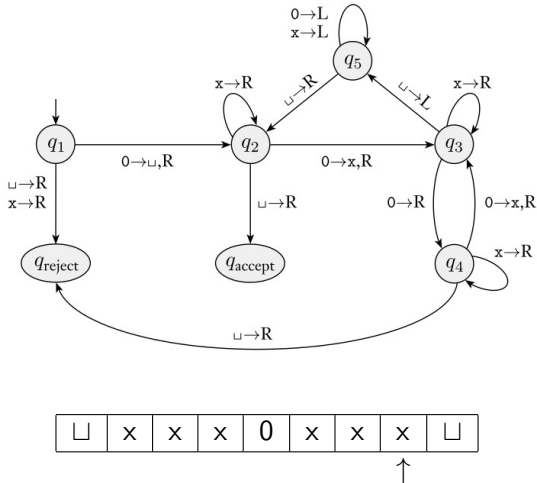
$$A = \{0^{2^n} \mid n \geq 0\}$$



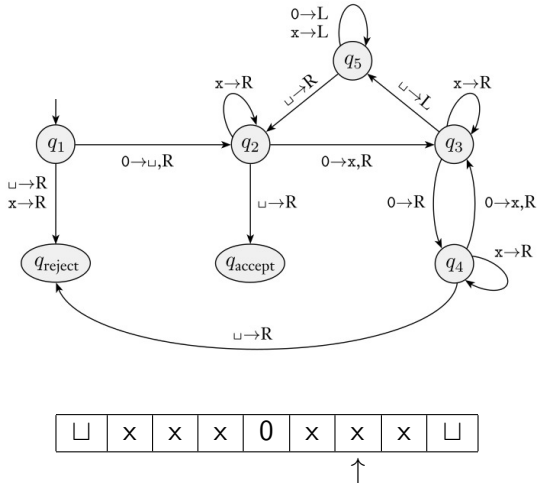
$$A = \{0^{2^n} \mid n \geq 0\}$$



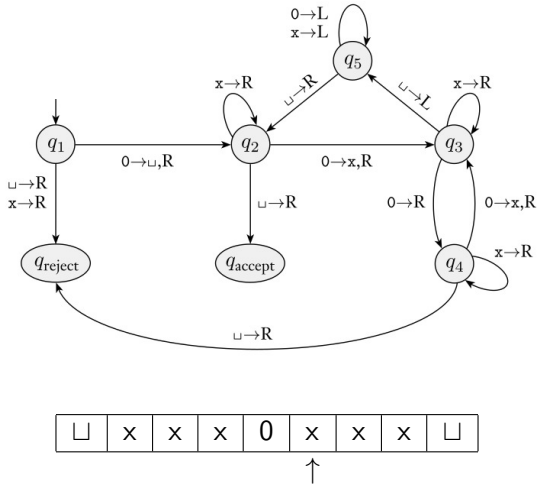
$$A = \{0^{2^n} \mid n \geq 0\}$$



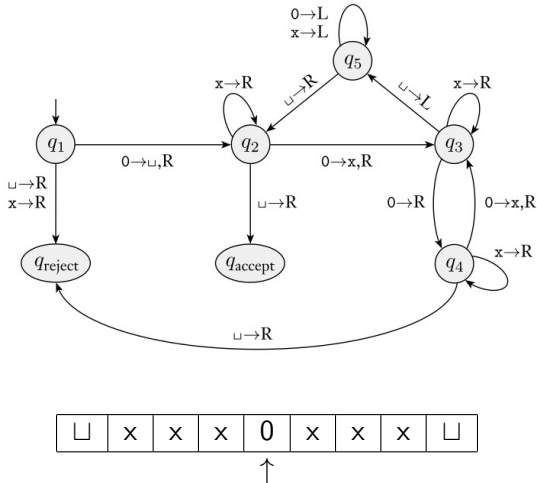
$$A = \{0^{2^n} \mid n \geq 0\}$$



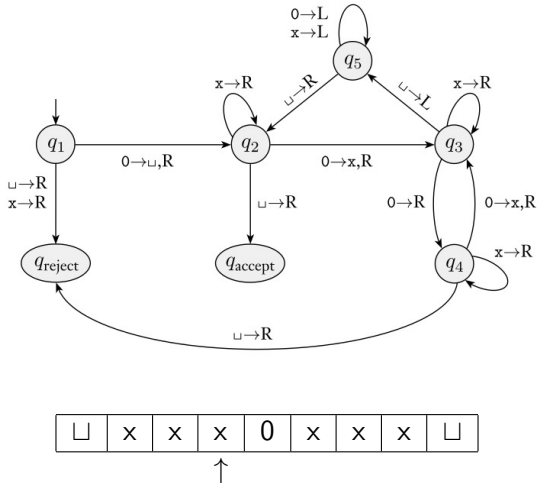
$$A = \{0^{2^n} \mid n \geq 0\}$$



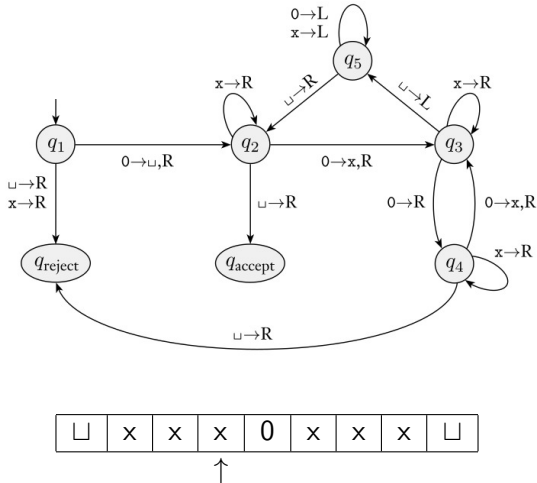
$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$

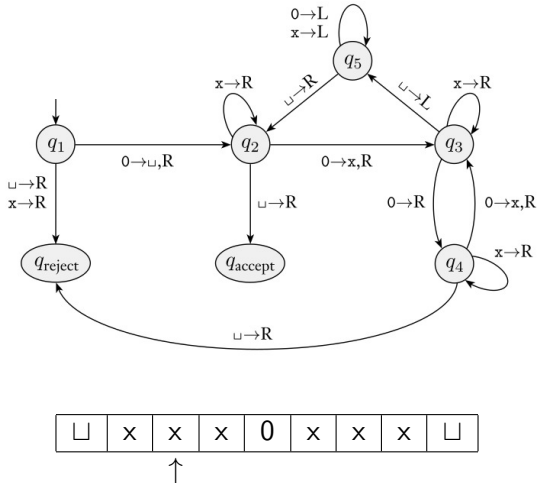


$$A = \{0^{2^n} \mid n \geq 0\}$$

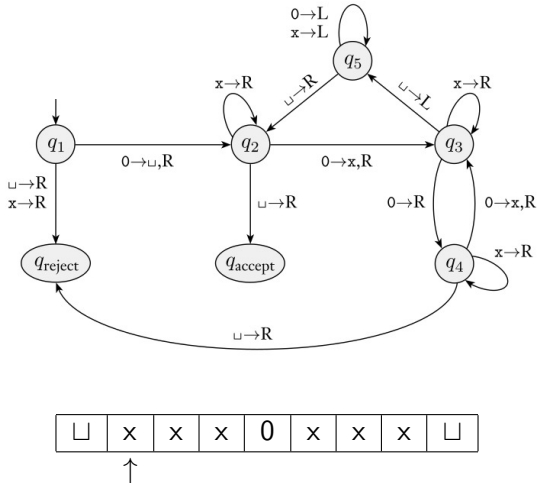




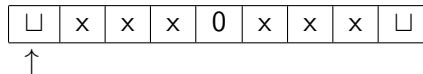
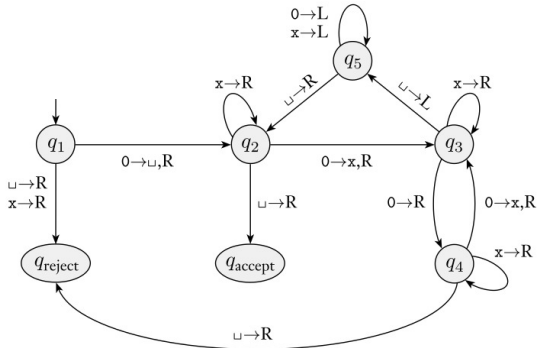
$$A = \{0^{2^n} \mid n \geq 0\}$$



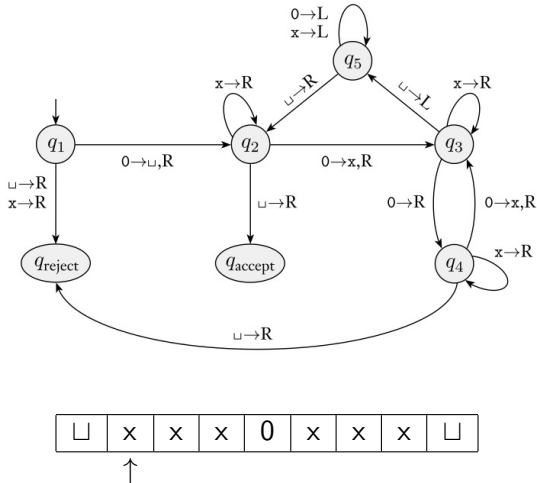
$$A = \{0^{2^n} \mid n \geq 0\}$$



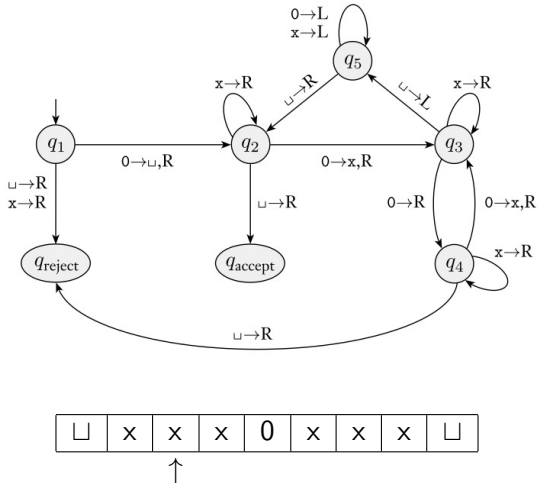
$$A = \{0^{2^n} \mid n \geq 0\}$$



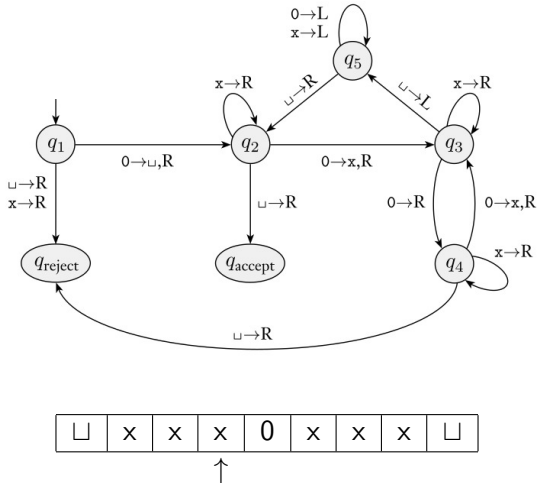
$$A = \{0^{2^n} \mid n \geq 0\}$$



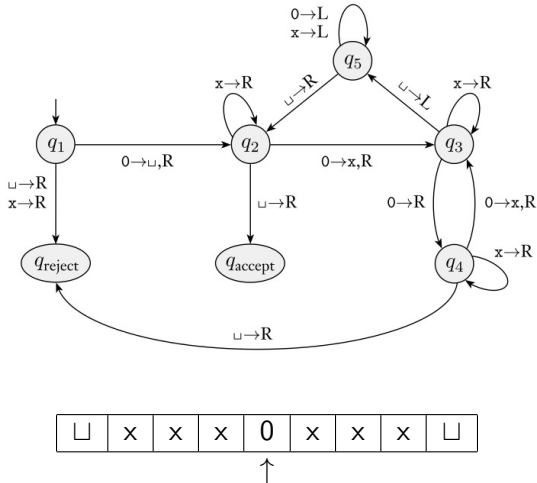
$$A = \{0^{2^n} \mid n \geq 0\}$$



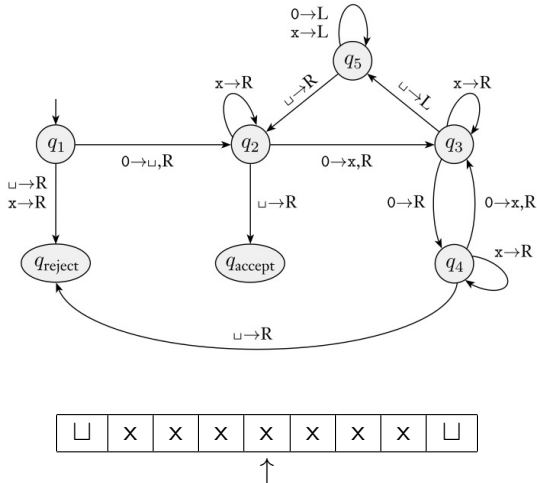
$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$

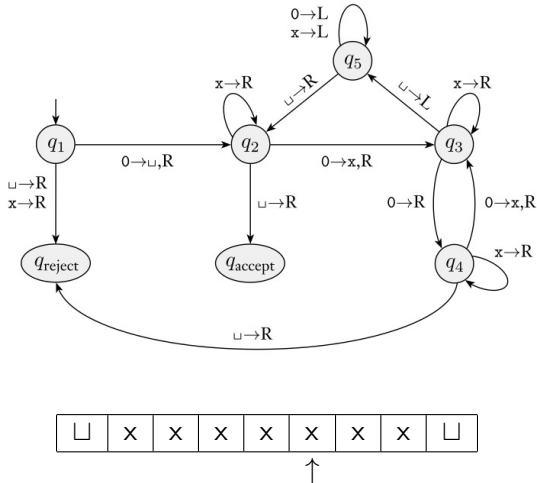


$$A = \{0^{2^n} \mid n \geq 0\}$$

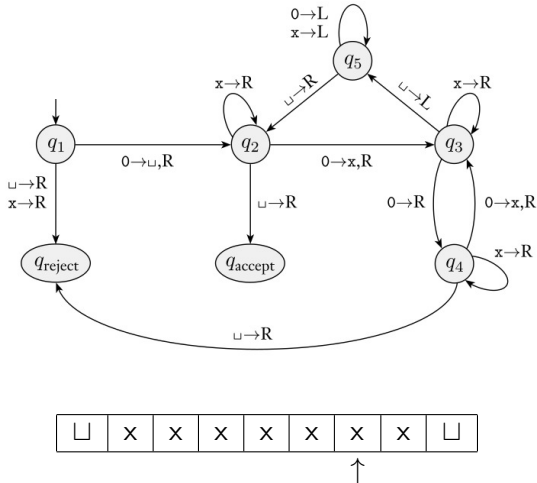




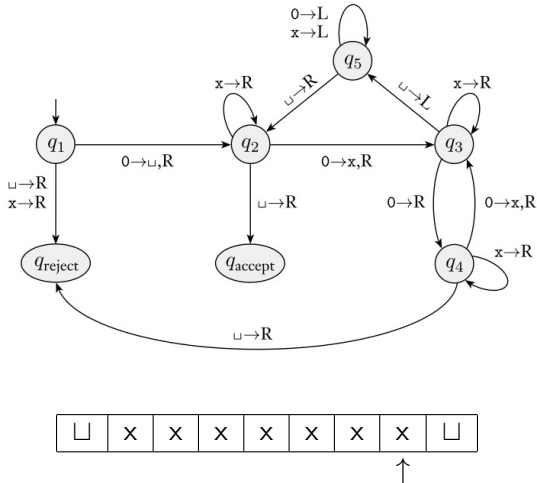
$$A = \{0^{2^n} \mid n \geq 0\}$$



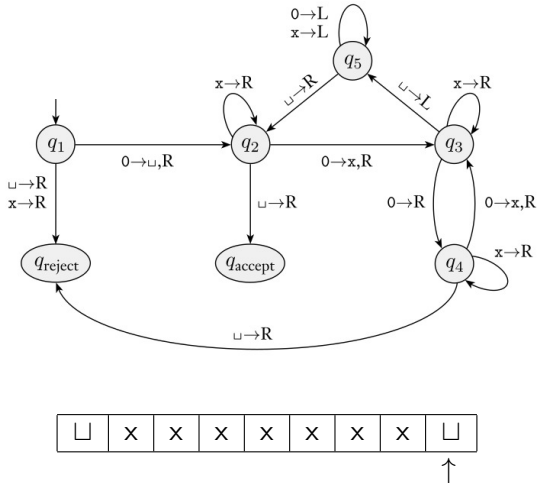
$$A = \{0^{2^n} \mid n \geq 0\}$$



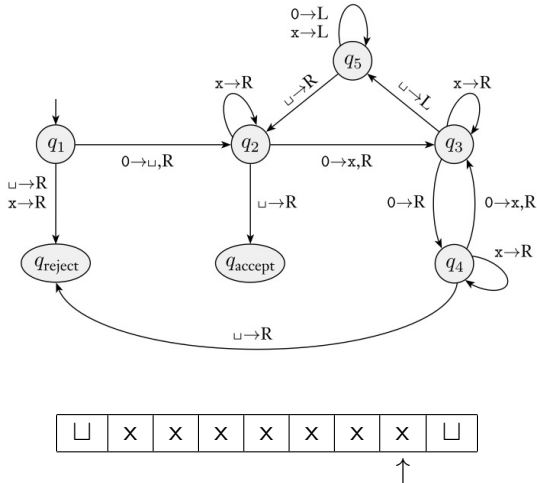
$$A = \{0^{2^n} \mid n \geq 0\}$$



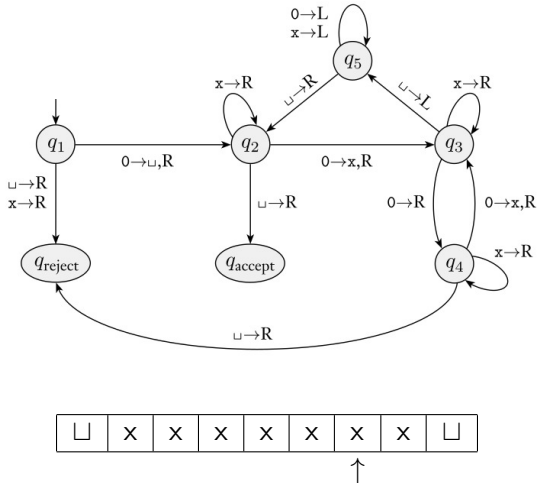
$$A = \{0^{2^n} \mid n \geq 0\}$$



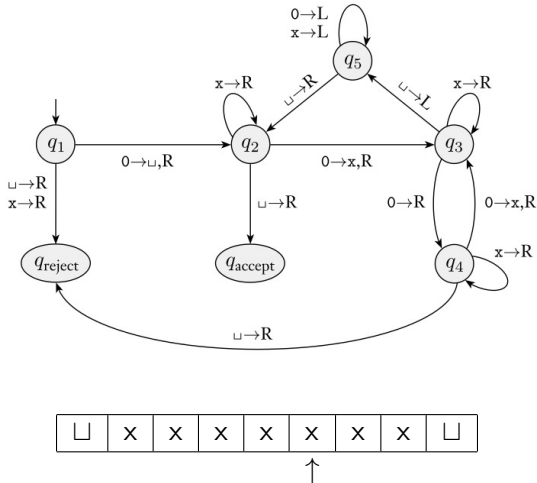
$$A = \{0^{2^n} \mid n \geq 0\}$$



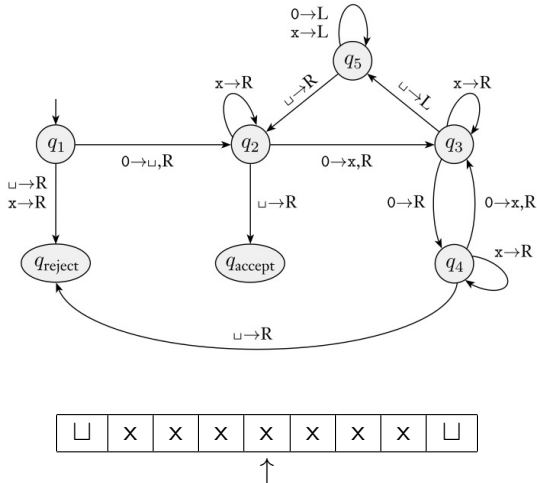
$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$

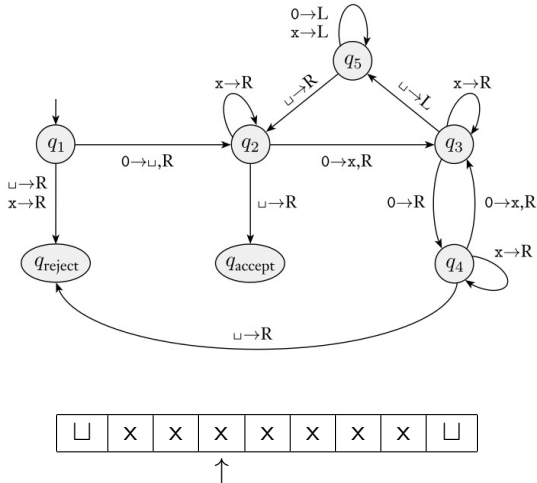


$$A = \{0^{2^n} \mid n \geq 0\}$$

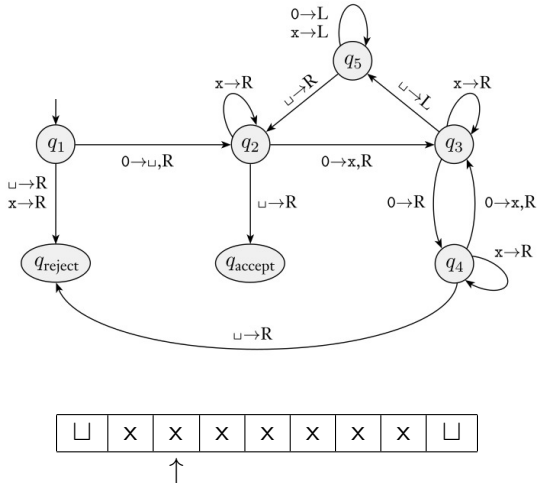




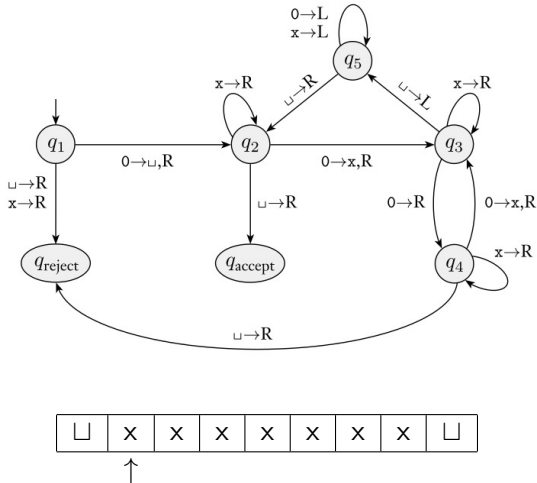
$$A = \{0^{2^n} \mid n \geq 0\}$$



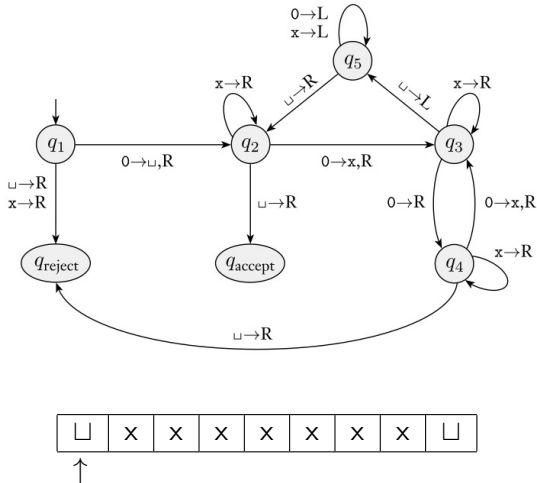
$$A = \{0^{2^n} \mid n \geq 0\}$$



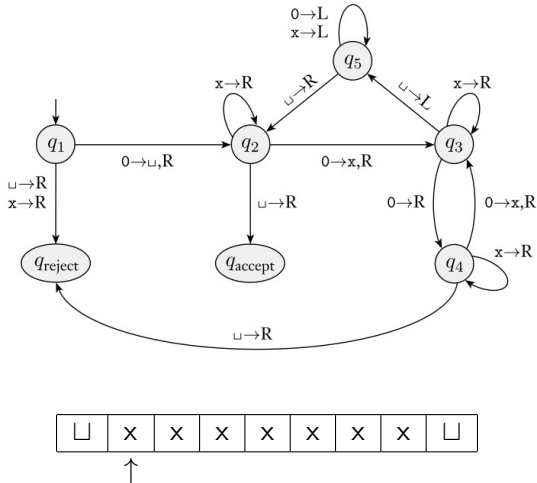
$$A = \{0^{2^n} \mid n \geq 0\}$$



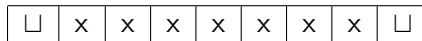
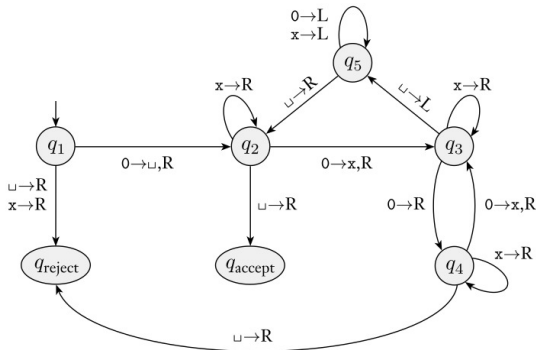
$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$



$$A = \{0^{2^n} \mid n \geq 0\}$$



Aceita

## configurações

$$A = \{0^{2^n} \mid n \geq 0\}$$

$q_1 0000$	$\sqcup q_5 x 0 x \sqcup$	$\sqcup x q_5 x x \sqcup$
$\sqcup q_2 000$	$q_5 \sqcup x 0 x \sqcup$	$\sqcup q_5 x x x \sqcup$
$\sqcup x q_3 00$	$\sqcup q_2 x 0 x \sqcup$	$q_5 \sqcup x x x \sqcup$
$\sqcup x 0 q_4 0$	$\sqcup x q_2 0 x \sqcup$	$\sqcup q_2 x x x \sqcup$
$\sqcup x 0 x q_3 \sqcup$	$\sqcup x x q_3 x \sqcup$	$\sqcup x q_2 x x \sqcup$
$\sqcup x 0 q_5 x \sqcup$	$\sqcup x x x q_3 \sqcup$	$\sqcup x x q_2 x \sqcup$
$\sqcup x q_5 0 x \sqcup$	$\sqcup x x q_5 x \sqcup$	$\sqcup x x x q_2 \sqcup$
		$\sqcup x x x \sqcup q_{aceita}$

## Exercício 2

- a) Obtenha as componentes da definição da Máquina de Turing  $M_2$ .
- b) Dê a sequência de configurações de  $M_2$  sobre a cadeia 000.



# Sumário

- 1 Máquina de Turing
- 2 Linguagem Turing-Reconhecível
- 3 Linguagem Turing-Decidível
- 4 Tese de Church-Turing**
- 5 Exercícios

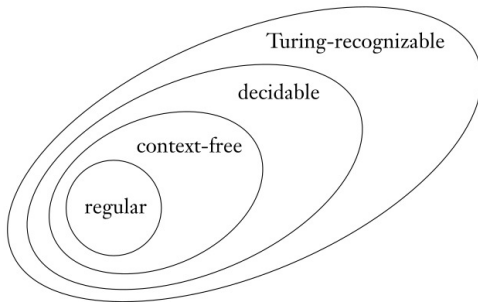
# Tese de Church-Turing

Por que estudar máquinas de Turing?

Noção intuitiva de algoritmos igual Algoritmos de máquinas de Turing
--

Esta equivalência não pode ser provada, até agora nenhum algoritmo foi encontrado que não pudesse ser implementado em uma máquina de Turing.

# Hierarquia de Linguagem



# Sumário

- 1 Máquina de Turing
- 2 Linguagem Turing-Reconhecível
- 3 Linguagem Turing-Decidível
- 4 Tese de Church-Turing
- 5 Exercícios

## Exercícios

- 3) Construa a Máquina de Turing para a linguagem  $L = 01^*0$ .  
Apresente a descrição em algoritmo, o diagrama de máquina de Turing e a descrição formal.
- 4) Construa a Máquina de Turing para a linguagem  $L = 0^n1^n$ .  
Apresente a descrição em algoritmo, o diagrama de máquina de Turing e a descrição formal.