# AE227 - Numerical Fluid Flow: Assignment 1

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Deadline: $24^{th}$ February 2024 (Monday)

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**Instructions**

1. The assignment is due on Monday, $24^{th}$ February 2024, by 23:59 hrs.
2. The assignment carries 10 points.
3. Matrices are represented by uppercase boldface alphabets, for example '$\mathbf{A}$'.
4. Vectors are represented by lowercase boldface alphabets, for example '$\mathbf{u}$' and '$\mathbf{b}$'.
5. Superscripts in parenthesis always imply iteration index. For example $\mathbf{u}^{(k)}$ implies vector $\mathbf{u}$ at $k^{th}$ iteration.

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## Problem: Solving System of Linear Equations

Any system of linear equations can be represented as a Matrix-Vector equation. Consider such system of linear equations expressed in the standard notation as given below.

$$\mathbf{A}\mathbf{u} = \mathbf{b} \tag{1}$$

where $\mathbf{A}$ is a full rank square matrix, $\mathbf{b}$ is the known RHS vector and $\mathbf{u}$ is the solution vector. In this assignment, you are required to find $\mathbf{u}$ using different methods as described in class for given $\mathbf{A}$ and $\mathbf{b}$ using **Jacobi**, **Gauss-Seidel** (GS), **Steepest Gradient Descent** (SGD) and **Conjugate Gradient** (CG) iterative methods. Solve the above-mentioned linear system using SGD and CG with the same initial guess of $\mathbf{u}^{(0)} = [1, 1, 1, ..., 1]^T$.
Please note that the iterations should be continued until the relative change in the solution of system of linear equations ($\mathbf{A}\mathbf{u} = \mathbf{b}$) from one iteration to another is less than $10^{-13}$. More precisely, stop the iterations when

$$\frac{||\mathbf{u}^{(k+1)} - \mathbf{u}^{(k)}||_2}{||\mathbf{u}^{(k)}||_2} \leq 10^{-13} \tag{2}$$

The aforementioned cases are available for download at folder *Assignment_1* at `https://github.com/hkishnani/Numerical_Fluid_Flow_AE227_TA.git`.
Submit a short report, which contains the following deliverables for different cases of $\mathbf{A}$ and $\mathbf{b}$ to find $\mathbf{u}$:

1. Perform the iterations for Jacobi, GS, SGD and CG methods until the convergence criteria (given above) is met. Plot the relative change in the solution (LHS of eq. 2) versus the iteration number (k) for each method on a single plot. Repeat the procedure for all the six systems (six different plots). In the plot, the relative change in the solution (y-axis) should be in base-10 logarithmic scale (For example, see the command "semilogy" in matlab) but iteration index (x-axis) must be on linear scale .

2. Report the optimal method for which the number of iterations required to reach the convergence criteria (Eq. 2) is minimum in each linear system.

3. In a table, report the number of iterations required corresponding to Jacobi, GS, SGD and CG methods for all systems to reach the desired convergence criteria along with your chosen initial guess.

4. Also confirm that these values match with the ones obtained using solvers as provided in inbuilt libraries of your preferred programming language.

5. Fill in the values in given table, for all linear systems. For finding the condition number ($\kappa(\mathbf{A})$) and spectral radius ($\rho(\mathbf{A})$) of the given matrix $\mathbf{A}$, use of functions provided by external libraries is allowed.

6. Based on your understanding explain the effect of condition number ($\kappa(\mathbf{A})$) and spectral radius ($\rho(\mathbf{A})$) on the convergence behavior of different methods. (Hint: Plotting might provide some insight)

**Note:**

1. Give appropriate captions and labels to all plots.

2. In the report, name the sections according to the appropriate deliverables.

3. Report and Code must be submitted in a zip or tar.gz folder named as following :
**NAME_SRNO_ASSIGNMENT1.zip**

| **Matrix Shape** | **20x20** | **40x40** | **80x80** | **160x160** | **320x320 a** | **320x320 b** |
|---|---|---|---|---|---|---|
| $\rho(A)$ **(Value)** | | | | | | |
| $\kappa(A)$ **(value)** | | | | | | |
| **SPD (True / False)** | | | | | | |
| **Sparsity (0 - 1)** | | | | | | |
| **Block diagonal (True/False)** | | | | | | |
| **Rank (integer)** | | | | | | |