Simula SSCP 2021 Project

Flow Instabilities in Aneurysms

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Background

Aneurysms

An aneurysm is the enlargement of an artery caused by weakness in the arterial wall. The term derives from the greek word *aneurysma* meaning "dilation" and is so named as aneurysm often resembles an outward bulging balloon; see e.g. (B) in Figure 1. Intracranial aneurysms are found in 0.4-3% of the population and the incidence of rupture is approximately 12 per 100,000 individuals. Thus roughly 600 people suffer ruptured intracranial aneurysms in Norway every year.

Aneurysms are usually detected by chance when a patient has their brain x-rayed, for example to check for trauma after an injury. On detection, it is unclear how to proceed. It is estimated that roughly 25% of aneurysms are dangerous and should be operated upon. It is, however, not clear what constitutes a dangerous aneurysm. Nor is it clear how to quantify the risk of aneurysm rupture.

Flow Instabilities

In fluid dynamics, hydrodynamic stability is the field which analyses the stability and the onset on instability of fluid flows. In [3, 1], kinetic energy instability is studied using the methods of Reynolds and Orr. Given a base flow $\mathbf{u}$ in some domain Ω, this method can find the most unstable perturbation $\mathbf{v}_\lambda$ as the solution of a generalized eigenvalue problem

$$A\mathbf{v}_\lambda = B\lambda\mathbf{v}_\lambda,$$

where $\mathbf{v}_\lambda$ corresponds to the *growth rate of the kinetic energy* of the instability.

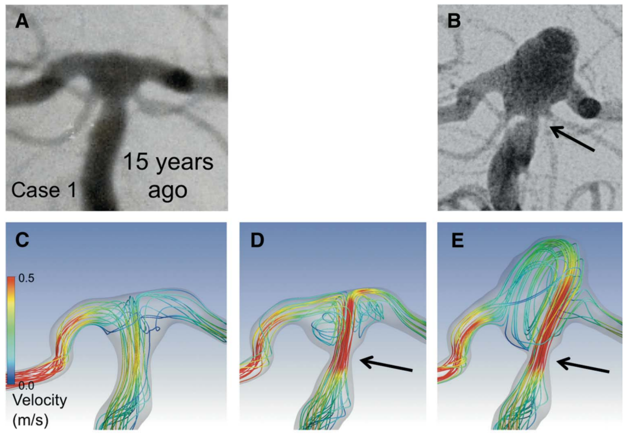

Figure 1: Results from [2] showing a ruptured basilar terminus aneurysm. Angiograms taken 15 years before (A) and then after rupture . (C)-(E) computational fluid dynamics simulations on the original geometry (C), an intermediate geometry (D) and the final geometry (E). Colored lines indicate streamlines colored according to velocity magnitude at peak systole.

Project proposal

Main objective

The main objective is to explore if the Reynolds-Orr method for determining instabilities can be used to quantify the risk of an aneurysm rupture. More specifically, the goal is to find the most unstable flow perturbation $\mathbf{v}_\lambda$, and associated perturbation energy growth rate λ, for the geometries in Figure 1(A) and 1(B).

Suggested workflow

1. Flow instability in idealized geometry (tools: fenics, slepc, mshr)
   (a) Mesh a cylinder (using e.g. mshr)
   (b) Write down flow equations for steady state NS flow in this cylinder with parameters
      i. Constant viscosity ν
      ii. Pressure drop Δp across domain

   iii. No slip on lateral boundary
   iv. Cylinder radius
   (c) Solve for baseflow and compare with Poiseuille flow solution
   (d) Write down eigenvalue problem for flow instability
   (e) Compute eigenvalues and eigenmodes
   (f) Q: Can this be reduced to 2D?

2. Flow instability in patient geometry
   (a) Write down flow equations for steady state blood flow
      i. Value for pressure drop Δp
      ii. Value for ν
   (b) Mesh the two geometries in Figure 1(A) and 1(B) using the Aneurysm workflow software.
   (c) Compute base flow for each geometry using Oasis.
   (d) Compute most unstable flow perturbation for each geometry using SLEPc.
   (e) Compare flow perturbations and energy growth rate for the two geometries.
   (f) If time permits: Add perturbation to base flow and see what happens.

Deliverables

Over the course of the project, please deliver

- Code for finding flow instabilities in idealized geometry (dependencies: fenics, mshr, slepc)
- Code for finding flow instabilities in realistic geometry
- Journal club-style 10 minute presentation of their project (17 August, 17:00 CET)
- 8-10 page paper to be submitted as a student report in a Simula SpringerBrief (due 1 September)

We should also keep in mind the possibility of writing a full paper!

Starting point

I will share my code for [1]. This includes:

- solve.py, which contains a solver for ns with no-slip boundary conditions (using fenics)
- get_eigenvalue_matrices.py, which uses fenics to set up the petsc arrays for A and B in the eigenvalue problem
- setup_slepc_solver.py, which sets up slepc to solve the eigenvalue problem
- find_instability.py, which solves for the baseflow and runs the eigenvalue solver

My code uses Nitsche's method to implement slip boundary conditions on an inner cylinder. You can substitute this with no-slip boundary conditions on the arterial walls and pressure boundary conditions on the top and bottom domains. NB: This is going to change the variational formulations.

Collaboration tools

Suggestions:

- Git for code
- Conda environment or docker for running code
- Overleaf for writing together

Tip: Try to write down as much as possible. You should get on paper sketches of the problem geometry, the flow equations, the eigenvalue problem and flow parameters.

References

[1] I. G. Gjerde and L. R. Scott. Kinetic-energy instability of flows with slip boundary conditions, 2021.

[2] K. Kono, O. Masuo, N. Nakao, and H. Meng. De novo cerebral aneurysm formation associated with proximal stenosis. *Neurosurgery*, 73(6):E1080–E1090, Dec. 2013.

[3] L. R. Scott. Kinetic energy flow instability with application to couette flow, Aug. 2020.