COMP 170 Discrete Mathematical Tools for CS
2007 Fall Semester – Written Assignment # 8
Distributed: Nov 15, 2007 – Due: Nov 22, 2007

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.

Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions can either be submitted at the end of your Thursday lecture section or, before 5PM, in the collection bin in front of Room 4213A.

Problem 1: Show by induction that any solution to a recurrence of the form

$$T(n) \leq 2T\left(\frac{n}{3}\right) + c \log_3 n$$

is $O(n \log_3 n)$. You may assume that n is a power of 3 and $c > 0$.
What happens if you replace 2 with 3? Explain why.
Would it make a difference if you used a different base for the logarithm (only an intuitive explanation is needed here)?

Problem 2: Show by induction that

$$T(n) = \begin{cases} 8T(n/2) + n \log n & n > 1 \\ d & n = 1 \end{cases}$$

has $T(n) = O(n^3)$ for any solution $T(n)$. You may assume that n is a power of 2 and $d > 0$.

Problem 3: A student taking a ten-question, true-false diagnostic test knows none of the answers and must guess at each one. Compute the probability that the student gets a score of 80 or higher. What is the probability that the grade is 70 or lower?

Problem 4: A die is made of a cube with a square painted on one side, a circle on two sides, and a triangle on three sides. If the die is rolled twice, what is the probability that the two shapes you see on top are the same?

Problem 5: For each of the two problems answer the following:
Which is more likely, or are both equally likely?
(The *suits* in a deck of cards are diamonds, spades, hearts and clubs.)

(a) Drawing an ace and a king when you draw 2 cards from among the 13 spades, or drawing an ace and a king when you draw 2 cards from an ordinary deck of 52 playing cards?

(b) Drawing an ace and a king of the same suit when you draw 2 cards from a deck, or drawing an ace and a king when you draw 2 cards from among the 13 spades?

Challenge Problem: Suppose there are $n \leq 365$ people in a room. What is the probability that at least *three* of them share a birthday? Express the probability in terms of n. Show how you derived your answer.
(You may assume that there are 365 days in a year and there are no twins in the room. Your answer may be expressed using $\binom{m}{k}$ and the summation ($\sum$) and product ($\prod$) symbols.)