L15

p6

Sum	Outcomes	#
12	6, 6	1
11	5, 6 6, 5	2
10	5, 5 4, 6 6, 4	3
9	3, 6 4, 5 5, 4 6, 3	4
8	2, 6 3, 5 4, 4 5, 3 6, 2	5

$$P(E \cup F \cup G \cup H)$$

$$= P(E) + P(F) + P(G) + P(H)$$

$$- P(E \cap F) - P(E \cap G) - P(E \cap H)$$

$$- P(F \cap G) - P(F \cap H) - P(G \cap H)$$

$$+ P(E \cap F \cap G) + P(E \cap F \cap H)$$

$$+ P(E \cap G \cap H) + P(F \cap G \cap H)$$

$$- P(E \cap F \cap G \cap H)$$

Theorem 5·3

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{\substack{i_1, i_2, \cdots, i_k \\ 1 \le i_1 < i_2 < \cdots < i_k \le n}} P(E_{i_1} \cap \cdots \cap E_{i_k})$$

$= \sum P($individual events$)$

$- \sum P($pairs$)$

$+ \sum P($Triples$)$

$- \sum P($Quadruples$)$

$\cdots$

Theorem 5.3 Another perspective

To compute $P(\bigcup_{i=1}^{n} E_i)$:

* Consider all subsequences of events

* For each subsequence, get the prob of the intersection of the events

* Add the prob val to result if length of sequence is odd

* Subtract the prob val from result if length of subsequence is Even

Last step in the proof of Theorem 5.3

$$\sum_{k=1}^{n-1} (-1)^{k+1} \sum_{\substack{i_1, i_2, \cdots, i_k \\ 1 \le i_1 < i_2 \cdots < i_k \le n-1}} P(E_{i_1} \cap \cdots \cap E_{i_k})$$

$$+ P(E_n) + \sum_{k=1}^{n-1} (-1)^{k+2} \sum_{\substack{i_1, i_2, \cdots, i_k \\ 1 \le i_1 < i_2 \cdots < i_k \le n-1}} P(E_{i_1} \cap \cdots \cap E_{i_k} \cap E_n)$$

$$\stackrel{?}{=} \sum_{k=1}^{n} (-1)^{k+1} \sum_{\substack{i_1, i_2 \cdots i_k \\ 1 \le i_1 < i_2 < \cdots < i_k \le n}} P(E_{i_1} \cap \cdots \cap E_{i_k})$$

LHS: $\left\{\begin{array}{l}\text{1st term: All subsequences w/o } E_n \\ \text{2nd + 3 term: All subsequences w } E_n\end{array}\right.$

$\to$ * All subsequences of $E_1, E_2, \cdots E_n$

* + : length of subsequence is <u>odd</u>

* − : length of subsequence is <u>even</u>

= RHS

Back pack Example

Students: $1, 2, \cdots, n$

Back packs: $1, 2, \cdots, n$

$f(x)$: the backpack student x gets back

$f: [1, \cdots, n] \longrightarrow [1, \cdots, n]$

1-to-1: Different students get back different backpacks

Onto: Every backpack is taken

f is a permutation of $[1 \cdots n]$

The process: students getting back backpacks

Sample space: S_n — set of all permutations of $[1..n]$

$|S_n| = n!$

$\boxed{\text{Fist Event}}$

k given students $x_1, x_2, \ldots x_k$ getting back their own backpack

$f(x_i) = x_i \qquad i = 1, 2, \ldots k$

\# of outcomes: $(n-k)!$

$$P(\text{1st Event}) = \frac{(n-k)!}{n!}$$

$$= D_{n,k} \quad \text{(notation)}$$