ASSIGNMENT 3: COMP2711H

FALL 2015

Q1 The Inclusion-Exclusion Principle covered in the lecture on Sets is the following:
Given a finite number of finite sets, $A_1, A_2, \ldots A_n$, we have

$$|\cup_{i=1}^{n} A_i| = \sum_{1\le i\le n} |A_i| - \sum_{1\le i<j\le n} |A_i \cap A_j| + \sum_{1\le i<j<k\le n} |A_i \cap A_j \cap A_k| - \ldots + (-1)^{n+1} |\cap_{i=1}^{n} A_i| .$$

Give an induction proof for the Inclusion-Exclusion Principle stated above. (15 marks)

Q2 Prove by induction on n that the product $1 \times 3 \times 5 \times \cdots \times (2n-1)$ of the first n odd natural numbers is equal to $\frac{(2n)!}{2^n n!}$. (10 marks)

Q3 Use strong mathematical induction to prove that every integer greater than 1 is either a prime number or the product of prime numbers. (10 marks)

Q4 Let A and B be two sets with m and n elements, respectively, where $m \le n$. What is the total number of one-to-one functions from A to B? Justify your answer. (10 marks)

Q5 Let A and B be two sets with m and n elements, respectively, where $m \ge n$. Use an inclusion-exclusion argument to find the total number of onto functions from A to B. (15 marks)

Q6 Suppose that a_1, a_2, a_3 are three odd integers. Show that at least one of the differences $a_i - a_j$ (for $i \ne j$) is divisible by 4. (10 marks)

Q7 Consider sequences of length n over the set $\{a, b, c, d\}$. How many such sequences contain at least one pair of adjacent characters that are the same? (10 marks)

Q8 Let $c_{(m,n)}$ be the number of onto functions from a set A of m elements to a set B of n elements, where $m \ge n \ge 1$. Find a formula relating $c_{(m,n)}$ to $c_{(m-1,n)}$ and $c_{(m-1,n-1)}$. (10 marks)

Q9 A store sells 30 kinds of balloons. How many different combinations of 24 balloons can be chosen? (10 marks)

Date: Handed out on September 30, due on October 7 at the *beginning* of the lecture.