Example: k = 184, j = 69

$$i k[i] = j[i]q[i] + r[i] k[i] j[i] r[i] q[i] y[i] x[i]$$
 $0 184 = 69(2) + 46 184 69 46 2$
 $1 69 = 46(1) + 23 69 46 23 1$
 $2 46 = 23(2) + 0 46 23 0 2$

1) First run the regular GCD algorithm: get gcd(184,69) = 23

Recall that (**)
$$y[i-1] = x[i]$$
 and (*) $x[i-1] = y[i] - q[i-1]x[i]$ and we want $j[i]x[i] + k[i]y[i] = gcd(k[i], j[i])$

$$i k[i] = j[i]q[i] + r[i] k[i] j[i] r[i] q[i] y[i] x[i]$$
 $0 184 = 69(2) + 46 184 69 46 2$
 $1 69 = 46(1) + 23 69 46 23 1$
 $2 46 = 23(2) + 0 46 23 0 2$

- 1) First run the regular GCD algorithm: get gcd(184,69) = 23
- 2) Then calculate y[2] = 0, x[2] = 1

Recall that (**)
$$y[i-1] = x[i]$$
 and (*) $x[i-1] = y[i] - q[i-1]x[i]$ and we want $j[i]x[i] + k[i]y[i] = gcd(k[i], j[i])$

$$i k[i] = j[i]q[i] + r[i] k[i] j[i] r[i] q[i] y[i] x[i]$$
 $0 184 = 69(2) + 46 184 69 46 2$
 $1 69 = 46(1) + 23 69 46 23 1$
 $2 46 = 23(2) + 0 46 23 0 2$

- 1) First run the regular GCD algorithm: get gcd(184,69) = 23
- 2) Then calculate y[2] = 0, x[2] = 1
- 3) Continue bottom-up, calculating the x[i], y[i] from (*) and (**)

Recall that (**)
$$y[i-1] = x[i]$$
 and (*) $x[i-1] = y[i] - q[i-1]x[i]$ and we want $j[i]x[i] + k[i]y[i] = gcd(k[i], j[i])$

$$i k[i] = j[i]q[i] + r[i] k[i] j[i] r[i] q[i] y[i] x[i]$$
 $0 184 = 69(2) + 46 184 69 46 2$
 $1 69 = 46(1) + 23 69 46 23 1 1 -1$
 $2 46 = 23(2) + 0 46 23 0 2$

- 1) First run the regular GCD algorithm: get gcd(184,69) = 23
- 2) Then calculate y[2] = 0, x[2] = 1
- 3) Continue bottom-up, calculating the x[i], y[i] from (*) and (**)

$$i k[i] = j[i]q[i] + r[i] k[i] j[i] r[i] q[i] y[i] x[i]$$
 $0 184 = 69(2) + 46 184 69 46 2 -1 3$
 $1 69 = 46(1) + 23 69 46 23 1 1 -1$
 $2 46 = 23(2) + 0 46 23 0 2 0 1$

- 1) First run the regular GCD algorithm: get gcd(184,69) = 23
- 2) Then calculate y[2] = 0, x[2] = 1
- 3) Continue bottom-up, calculating the x[i], y[i] from (*) and (**)

Recall that (**)
$$y[i-1] = x[i]$$
 and (*) $x[i-1] = y[i] - q[i-1]x[i]$ and we want $j[i]x[i] + k[i]y[i] = gcd(k[i], j[i])$

$$i k[i] = j[i]q[i] + r[i] k[i] j[i] r[i] q[i] y[i] x[i]$$
 $0 184 = 69(2) + 46 184 69 46 2 -1 3$
 $1 69 = 46(1) + 23 69 46 23 1 1 -1$
 $2 46 = 23(2) + 0 46 23 0 1$

- 1) First run the regular GCD algorithm: get gcd(184,69) = 23
- 2) Then calculate y[2] = 0, x[2] = 1
- 3) Continue bottom-up, calculating the x[i], y[i] from (*) and (**)
- 4) We are done! Note that 184(-1) + 69(3) = 23 = gcd(184, 69).

Example: k = 99, j = 63

$$i \quad k[i] = j[i]q[i] + r[i] \quad k[i] \quad j[i] \quad r[i] \quad q[i] \quad y[i] \quad x[i]$$
 $0 \quad 99 = 63(1) + 36 \quad 99 \quad 63 \quad 36 \quad 1$
 $1 \quad 63 = 36(1) + 27 \quad 63 \quad 36 \quad 27 \quad 1$
 $2 \quad 36 = 27(1) + 9 \quad 36 \quad 27 \quad 9 \quad 1$
 $3 \quad 27 = 9(3) + 0 \quad 27 \quad 9 \quad 0 \quad 3$

1) First run the regular GCD algorithm: get gcd(99,63) = 9

$$i k[i] = j[i]q[i] + r[i] k[i] j[i] r[i] q[i] y[i] x[i]$$
 $0 99 = 63(1) + 36 99 63 36 1$
 $1 63 = 36(1) + 27 63 36 27 1$
 $2 36 = 27(1) + 9 36 27 9 1$
 $3 27 = 9(3) + 0 27 9 0 3 0 1$

- 1) First run the regular GCD algorithm: get gcd(99,63) = 9
- 2) Then calculate y[3] = 0, x[3] = 1

Recall that (**)
$$y[i-1] = x[i]$$
 and (*) $x[i-1] = y[i] - q[i-1]x[i]$ and we want $j[i]x[i] + k[i]y[i] = gcd(k[i], j[i])$

$$i k[i] = j[i]q[i] + r[i] k[i] j[i] r[i] q[i] y[i] x[i]$$
 $0 99 = 63(1) + 36 99 63 36 1$
 $1 63 = 36(1) + 27 63 36 27 1$
 $2 36 = 27(1) + 9 36 27 9 1 1 -1$
 $3 27 = 9(3) + 0 27 9 0 3 0 1$

- 1) First run the regular GCD algorithm: get gcd(99,63) = 9
- 2) Then calculate y[3] = 0, x[3] = 1
- 3) Continue bottom-up, calculating the x[i], y[i] from (*) and (**)

Recall that (**)
$$y[i-1] = x[i]$$
 and (*) $x[i-1] = y[i] - q[i-1]x[i]$ and we want $j[i]x[i] + k[i]y[i] = gcd(k[i], j[i])$

$$i \quad k[i] = j[i]q[i] + r[i] \quad k[i] \quad j[i] \quad r[i] \quad q[i] \quad y[i] \quad x[i]$$
 $0 \quad 99 = 63(1) + 36 \quad 99 \quad 63 \quad 36 \quad 1$
 $1 \quad 63 = 36(1) + 27 \quad 63 \quad 36 \quad 27 \quad 1 \quad -1 \quad 2$
 $2 \quad 36 = 27(1) + 9 \quad 36 \quad 27 \quad 9 \quad 1 \quad 1 \quad -1$
 $3 \quad 27 = 9(3) + 0 \quad 27 \quad 9 \quad 0 \quad 3 \quad 0 \quad 1$

- 1) First run the regular GCD algorithm: get gcd(99,63) = 9
- 2) Then calculate y[3] = 0, x[3] = 1
- 3) Continue bottom-up, calculating the x[i], y[i] from (*) and (**)

$$i \quad k[i] = j[i]q[i] + r[i] \quad k[i] \quad j[i] \quad r[i] \quad q[i] \quad y[i] \quad x[i]$$
 $0 \quad 99 = 63(1) + 36 \quad 99 \quad 63 \quad 36 \quad 1 \quad 2 \quad -3$
 $1 \quad 63 = 36(1) + 27 \quad 63 \quad 36 \quad 27 \quad 1 \quad -1 \quad 2$
 $2 \quad 36 = 27(1) + 9 \quad 36 \quad 27 \quad 9 \quad 1 \quad 1 \quad -1$
 $3 \quad 27 = 9(3) + 0 \quad 27 \quad 9 \quad 0 \quad 3 \quad 0 \quad 1$

- 1) First run the regular GCD algorithm: get gcd(99,63) = 9
- 2) Then calculate y[3] = 0, x[3] = 1
- 3) Continue bottom-up, calculating the x[i], y[i] from (*) and (**)

Recall that (**)
$$y[i-1] = x[i]$$
 and (*) $x[i-1] = y[i] - q[i-1]x[i]$ and we want $j[i]x[i] + k[i]y[i] = gcd(k[i], j[i])$

$$i \quad k[i] = j[i]q[i] + r[i] \quad k[i] \quad j[i] \quad r[i] \quad q[i] \quad y[i] \quad x[i]$$
 $0 \quad 99 = 63(1) + 36 \quad 99 \quad 63 \quad 36 \quad 1 \quad 2 \quad -3$
 $1 \quad 63 = 36(1) + 27 \quad 63 \quad 36 \quad 27 \quad 1 \quad -1 \quad 2$
 $2 \quad 36 = 27(1) + 9 \quad 36 \quad 27 \quad 9 \quad 1 \quad 1 \quad -1$
 $3 \quad 27 = 9(3) + 0 \quad 27 \quad 9 \quad 0 \quad 3 \quad 0 \quad 1$

- 1) First run the regular GCD algorithm: get gcd(99,63) = 9
- 2) Then calculate y[3] = 0, x[3] = 1
- 3) Continue bottom-up, calculating the x[i], y[i] from (*) and (**)
- 4) We are done! Note that 99(2) + 63(-3) = 9 = gcd(99, 63).