

## ASSIGNMENT 2: COMP2711H

FALL 2015

- Q1 Let  $Q(n)$  be the predicate “ $n^2 \leq 30$  with domain being the set  $\mathbb{Z}^+$  of all positive integers.”
- What is  $Q(2)$ ? (5 marks)
  - Find the truth set of  $Q(n)$ . (5 marks)
- Q2 Find a counterexample to show the following statement is false.  
“ $\forall$  positive integers  $m$  and  $n$ ,  $mn \geq m + n$ .” (6 marks)
- Q3 Write down a negation for the following statement:  
“ $\forall x \in \mathbb{R}$ , if  $x(x + 1) > 0$  then  $x > 0$  or  $x < -1$ .” (10 marks)
- Q4 Write down a negation for the following statement:  
“ $\exists$  a book  $b$  such that  $\forall$  people  $p$ ,  $p$  has read  $b$ .” (10 marks)
- Q5 Give a direct proof of the theorem “The sum of two even numbers  $x$  and  $y$  is even.” (10 marks)
- Q6 Give a proof by contradiction of the theorem “If  $x^3$  is odd, so is  $x$ .” (10 marks)
- Q7 Prove the theorem “Let  $n \in \mathbb{N}$  be such that  $n$  is not divisible by 2 and 3. Then  $n^2 - 1$  is divisible by 24”. (12 marks)
- Q8 Define a relation  $Q$  on  $\mathbb{R}$  as follows: For all  $x$  and  $y$  in  $\mathbb{R}$ ,  $xQy$  if and only if  $x - y$  is rational. (20 marks)
- (a) Is  $q$  reflexive?
  - (b) Is  $q$  symmetric?
  - (c) Is  $q$  antisymmetric?
  - (d) Is  $q$  transitive?
- Q9 Let  $D$  be the relation on  $\mathbb{Z}$  defined as follows: For all  $m$  and  $n$  in  $\mathbb{Z}$ ,  $mDn$  if and only if  $3|(m^2 - n^2)$ .
- (a) Prove that  $D$  is an equivalence relation. (11 marks)
  - (b) Describe the equivalence classes of  $D$ . (11 marks)