

COMP 170 Discrete Mathematical Tools for CS
2006 Fall Semester – Written Assignment # 8
Distributed: Nov 21, 2006 – Due: Nov 28, 2006 *at end of class*

The top of your submission should contain (i) your name, (ii) your student ID #, (iii) your email address and (iv) your tutorial section.

Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. A solution that consists of just a number will be counted as wrong.

2nd Note: Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.

3rd Note: Most of these problems are taken (some modified) from sections 5.1 and 5.2 of the textbook.

4th Note: Your assignment can either be submitted at the end of your Tuesday lecture session or before 5PM in the collection bin in front of room 4213A.

Problem 1: A die is made of a cube with a square painted on one side, a circle on two sides, and a triangle on three sides. If the die is rolled twice, what is the probability that the two shapes you see on top are different?

Problem 2: For each of the two problems answer the following:
Which is more likely, or are both equally likely?
(In this and later problems, the *suits* in a deck of cards are diamonds, spades, hearts and clubs.)

- (a) Drawing an ace and a king when you draw 2 cards from among the 13 spades, or drawing an ace and a king when you draw 2 cards from an ordinary deck of 52 playing cards?
- (b) Drawing an ace and a king of the same suit when you draw 2 cards from a deck, or drawing an ace and a king when you draw 2 cards from among the 13 spades?

Problem 3: The eight kings and queens are removed from a deck of cards, and then two of these cards are selected (from the eight). What is the probability that the king or queen of spades is among the cards selected?

Problem 4: Calculate

$$\sum_{\substack{i_1, i_2, i_3: \\ 1 \leq i_1 < i_2 < i_3 \leq 5}} i_1 \cdot i_2 \cdot i_3$$

Problem 5: In this problem, a *black card* is a spade or a club.
Remove one card from an ordinary deck of cards. What is the probability that it is an ace, a diamond, or black? Use the inclusion-exclusion formula to solve this problem.

Problem 6: In this problem you will solve the following problem:

If you roll eight dice, what is the probability that each of the numbers 1 through 6 appears on top at least once?

For $1 \leq i \leq 6$, let E_i be the event that number i doesn't show up on any of the dice.

- (a) Write a formula for $P(E_i)$.
- (b) Let $k \leq 6$ and $1 \leq i_1 < i_2 < \cdots < i_k \leq 6$.
Write a formula for $P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k})$.
- (c) Now use the inclusion-exclusion formula to write a formula for $P(E_1 \cup E_2 \cup \cdots \cup E_6)$.
This is the probability that *some* number doesn't appear when you roll eight die.
Your formula should use the summation sign, powers and binomial coefficients.
- (d) Using the solution to (c), write down the probability that each of the numbers 1 through 6 appears on top at least once (a solution in the form of a sum is fine; it is not necessary to actually calculate the value of the sum).

Challenge Problem: A group of n married couples sits around a circular table for a discussion of marital problems. The counselor assigns each person to a seat at random.

- (a) What is the probability that the husband and wife of one given couple, say couple i , sit next to each other?
- (b) What is the probability that for each of the couples in a given set of k ($1 \leq k \leq n$) couples, the husband and wife sit next to each other?
- (c) What is the probability that no husband and wife sit next to each other?