COMP 170 Discrete Mathematical Tools for CS
2005 Fall Semester – Written Assignment # 7
Distributed: Oct 27, 2005 – Due: Nov 3, 2005 *at end of class*
REVISED & CORRECTED October 28, 2053

The top of your submission should contain (i) your name, (ii) your student ID #, (ii) your email address and (iv) your tutorial section.

Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. A solution that consists of just a number will be counted as wrong.

2nd Note: Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.

3rd Note: Most of these problems are taken (some modified) from sections 3.3, 4.1 and 4.2 of the textbook

Problem 1: (a) Construct a contrapositive proof that for all real numbers x, if $x^2 - 2x \neq -1$ then $x \neq 1$.
(b) Construct a proof by contradiction that for all real numbers x, if $x^2 - 2x \neq -1$ then $x \neq 1$.

Problem 2 Use induction to prove that, $\forall n \geq 1$, n an integer,

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3.$$

Problem 3: Prove that every number greater than 7 is a sum of a nonnegative integer multiple of 3 and a nonnegative integer multiple of 5.
(Hint: First prove the *three* base cases of $n = 8, 9, 10$ and then prove the inductive step assuming that $n > 10$.)

Problem 4: Find the error in the following proof that all positive integers n are equal:
Let $p(n)$ be the statement that all numbers in an n-element set of positive integers are equal. Then p(1) is true. Now assume $p(n-1)$ is true, and let N be the set of the first n integers. Let N' be the set of the first $n-1$ integers, and let N'' be the set of the last $n-1$ integers. By $p(n-1)$, all members of N' are equal, and all members of N'' are equal. Thus, the first $n-1$ elements of N are equal and the last $n-1$ elements of N are equal, and so all elements of N are equal. Therefore, all positive integers are equal.

Problem 5: For what values of $n \geq 1$ is $n! \geq 5 \cdot 2^n$? Prove by induction.

Problem 6: (*Note: A typo in this problem was corrected. The two n's became $(n+1)$'s.*)

The Fibonacci numbers are defined by $F(0) = F(1) = 1$ and $\forall n \geq 2$, $F(n) = F(n-1) + F(n-2)$. The closed form solution to the Fibonacci numbers is

$$F(n) = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^{n+1} - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^{n+1}.$$

Use strong induction to prove that this formula is correct.

Problem 7: Consider the recurrence $M(n) = 2M(n-1)+2$, with base case of $M(1) = 1$.
(a) State the solution to this recurrence
(you may use Theorem 4.1 in the book).
(b) Use induction to prove that this solution is correct.