A strong-induction example

Let $T(n)$ be defined by

$$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 + \sum_{i=0}^{n-1} T(i) & \text{if } n > 0 \end{cases}$$

Then $T(n) = 2^n$.

A strong-induction example

Let $T(n)$ be defined by

$$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 + \sum_{i=0}^{n-1} T(i) & \text{if } n > 0 \end{cases}$$

Then $T(n) = 2^n$.

Proof:

(i) Base Case: $T(0) = 1 = 2^0$.

A strong-induction example

Let $T(n)$ be defined by

$$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 + \sum_{i=0}^{n-1} T(i) & \text{if } n > 0 \end{cases}$$

Then $T(n) = 2^n$.

Proof:

(i) Base Case: $T(0) = 1 = 2^0$.

(ii) Induction hypothesis: Assume that $T(i) = 2^i$ for all $0 \leq i < n$.

A strong-induction example

Let $T(n)$ be defined by

$$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 + \sum_{i=0}^{n-1} T(i) & \text{if } n > 0 \end{cases}$$

Then $T(n) = 2^n$.

Proof:

(i) Base Case: $T(0) = 1 = 2^0$.

(ii) Induction hypothesis: Assume that $T(i) = 2^i$ for all $0 \leq i < n$.

Then $T(n) = 1 + \sum_{i=0}^{n-1} T(i)$ Definition

A strong-induction example

Let $T(n)$ be defined by

$$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 + \sum_{i=0}^{n-1} T(i) & \text{if } n > 0 \end{cases}$$

Then $T(n) = 2^n$.

Proof:

(i) Base Case: $T(0) = 1 = 2^0$.

(ii) Induction hypothesis: Assume that $T(i) = 2^i$ for all $0 \leq i < n$.

$$\begin{aligned} \text{Then } T(n) &= 1 + \sum_{i=0}^{n-1} T(i) && \text{Definition} \\ &= 1 + \sum_{i=0}^{n-i} 2^i && \text{Strong induction hypothesis} \end{aligned}$$

A strong-induction example

Let $T(n)$ be defined by

$$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 + \sum_{i=0}^{n-1} T(i) & \text{if } n > 0 \end{cases}$$

Then $T(n) = 2^n$.

Proof:

(i) Base Case: $T(0) = 1 = 2^0$.

(ii) Induction hypothesis: Assume that $T(i) = 2^i$ for all $0 \le i < n$.

$$\text{Then } T(n) = 1 + \sum_{i=0}^{n-1} T(i) \qquad \text{Definition}$$

$$= 1 + \sum_{i=0}^{n-i} 2^i \qquad \text{Strong induction hypothesis}$$

$$= 1 + (2^n - 1) = 2^n$$