$n = 5$

Increasing Triples		3 element subsets
$(1, 2, 3)$	$\xrightarrow{f}$	$\{1, 2, 3\}$
$(1, 2, 4)$		$\{1, 2, 4\}$
$(1, 2, 5)$		$\{1, 2, 5\}$
$(1, 3, 4)$		$\{1, 3, 4\}$
$(1, 3, 5)$		$\{1, 3, 5\}$
$(1, 4, 5)$		$\{1, 4, 5\}$
$(2, 3, 4)$		$\{2, 3, 4\}$
$(2, 3, 5)$		$\{2, 3, 5\}$
$(2, 4, 5)$		$\{2, 4, 5\}$
$(3, 4, 5)$		$\{3, 4, 5\}$

$\{3, 1, 2\}$ or $\{2, 1, 3\}$

same as ~~$\{3, 1, 2\}$ or $\{2, 1, 3\}$~~ etc.

ex

$$f(?) = \{4, 1, 3\}$$

$$f((1, 3, 4)) = \{4, 1, 3\}$$

Ex: - a 5 element permutation of $n = 7$

$\boxed{5\ 2\ 7\ 6\ 1}$

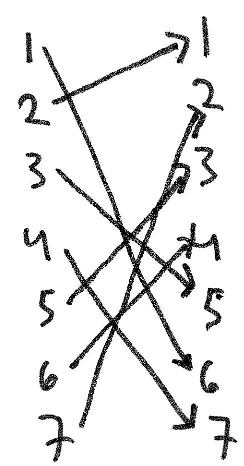
- another 5 element permutation

$\boxed{2\ 5\ 7\ 6\ 1}$

- a 7 element permutation

$\boxed{6\ 1\ 5\ 7\ 3\ 4\ 2}$

can interpret this list as a function
$L_1 L_2 \ldots L_7$ where $f(i) = L_i$



This is 1-1 function
from set to itself
which is what we
previously called
a **permutation**

n=4

3 element subsets	3 element perms
{1, 2, 3}	(1,2,3) (1,3,2) (2,1,3) (2,3,1) (3,1,2) (3,2,1)
{1, 2, 4}	(1,2,4) (1,4,2) (2,1,4) (2,4,1) (4,1,2) (4,2,1)
{1, 3, 4}	(1,3,4) (1,4,3) (3,1,4) (3,4,1) (4,1,3) (4,3,1)
{2, 3, 4}	(2,3,4) (2,4,3) (3,2,4) (3,4,2) (4,2,3) (4,3,2)

$3!$

$\updownarrow$

$6 \cdot$ (# 3 elem subsets)

= # 3 elem perms

$$n^{\underline{k}} = n(n-1)(n-2) \cdots (n-k+1)$$

$$= \frac{n(n-1)(n-2) \quad (n-k+1)\left[(n-k)(n-k-1) \cdots 3 \cdot 2 \cdot 1\right]}{\left[(n-k)(n-k-1) \cdots 3 \cdot 2 \cdot 1\right]}$$

$$= \frac{n!}{(n-k)!}$$

Let S be a set

A is a <u>subset</u> of S
if every item in A is in S

$$\boxed{A \subseteq S}$$

The complement of A (relative to S) is all items in S not in A.

$$\bar{A} = \{ x \in S \text{ but } x \overset{\text{not in}}{\notin} A \}$$

example: $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 4\}$

$\bar{A} = \{2, 3, 5, 6\}$

Note: $|\bar{A}| = |S| - |A|$

$S = \{1, 2, \dots, n\}$

X = all subsets of size $\underline{k}$ in S

X' = all subsets of size $\underline{n-k}$ in S

Note: $|X| = \binom{n}{k}$ $\quad |X'| = \binom{n}{n-k}$

Let $A \subseteq S$. Define $f(A) = \overline{A}$

Claim: $f : X \to X'$ is a bijection

Then $\binom{n}{k} = |X| = |X'| = \binom{n}{n-k}$

Example: $S = \{1, 2, 3, 4, 5\}$

$k = 2$

<u>sets of size 2</u>		<u>sets of size n-2</u>
$\{1,2\}$	$\xrightarrow{f}$	$\{3,4,5\}$
$\{1,3\}$	$\xrightarrow{f}$	$\{2,4,5\}$
$\{1,4\}$	$\xrightarrow{f}$	$\{2,3,5\}$
$\{1,5\}$	$\xrightarrow{f}$	$\{2,3,4\}$
$\{2,3\}$	$\xrightarrow{f}$	$\{1,4,5\}$
$\{2,4\}$	$\longrightarrow$	$\{1,3,5\}$
$\{2,5\}$	$\xrightarrow{f}$	$\{1,3,4\}$
$\{3,4\}$	$\xrightarrow{f}$	$\{1,2,5\}$
$\{3,5\}$	$\xrightarrow{f}$	$\{1,2,4\}$
$\{4,5\}$	$\xrightarrow{f}$	$\{1,2,3\}$