

**COMP 170 Discrete Mathematical Tools for CS**  
**2007 Fall Semester – Written Assignment # 2**  
**Distributed: Sept 13, 2007 – Due: Sept 20, 2007**

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.

Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- These problems are taken (some modified) from section 1.2 of the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions can either be submitted at the end of your Thursday lecture section or, before 5PM, in the collection bin in front of Room 4213A.

**Problem 1:** There are more functions from the real numbers to the real numbers than most of us can imagine. In discrete mathematics, however, we often work with functions from a finite set  $S$  with  $s$  elements to a finite set  $T$  with  $t$  elements. Thus, there are only a finite number of functions from  $S$  to  $T$ . How many functions are there from  $S$  to  $T$  in this case?

**Problem 2:** Consider the sets  $S_4 = \{a, b, c, d\}$  and  $S_5 = \{1, 2, 3, 4, 5\}$ .

- (a) How many functions are there from the set  $S_4$  to  $S_5$ ?
- (b) How many *injections* (*one-to-one* functions) are there from the set  $S_4$  to  $S_5$ ?
- (c) How many *surjections* (*onto* functions) are there from the set  $S_4$  to  $S_5$ ?
- (d) How many *bijections* are there from the set  $S_4$  to  $S_5$ ?
- (e) How many functions are there from the set  $S_5$  to  $S_4$ ?
- (f) How many *injections* are there from the set  $S_5$  to  $S_4$ ?
- (g) How many *surjections* are there from the set  $S_5$  to  $S_4$ ?
- (h) How many *bijections* are there from the set  $S_5$  to  $S_4$ ?
- (i) How many functions are there from the set  $S_5$  to  $S_5$ ?
- (j) How many *injections* are there from the set  $S_5$  to  $S_5$ ?
- (k) How many *surjections* are there from the set  $S_5$  to  $S_5$ ?
- (l) How many *permutations* are there from the set  $S_5$  to  $S_5$ ?

**Problem 3:** Suppose you are choosing participants for a panel discussion on allowing karaoke on campus. You must choose 5 administrators from a group of 10 and 4 students from a group of 15. In how many ways can this be done?

**Problem 4:** Suppose you are organizing a panel discussion on allowing karaoke on campus. Participants will sit behind a long table in the order in which you list them. You must choose 5 administrators from a group of 10 and 4 students from a group of 15.

(a) If the administrators must sit together in a group and the students must sit together in a group, in how many ways can you choose and list the 9 people?

(b) If you must alternate students and administrators, in how many ways can you choose and list them?

**Problem 5:** The Executive Committee (Exco) of a student society consists of four Year 1 students, five Year 2 students, and three Year 3 students.

(a) How many different subcommittees of six members can be formed from the members of the Exco?

(b) Suppose we want to create a Treasury Subcommittee and an Events Subcommittee, each containing three members, with the restriction that *no student serves on both the Treasury Subcommittee and the Events Subcommittee*. How many different ways are there to choose students to serve on the two different committees?

(c) Consider problem (b) with the added restriction that the *Treasury Subcommittee contains exactly one student from each year*. How many different ways are there to choose students to serve on the two different committees?

(d) How many different subcommittees of six members can be formed from the members of the Exco if we require that a subcommittee must contain at least one student from each year?

**Challenge Problem:** *This question does not have to be done. However, if you can do it properly, then you can be sure that you really understand this section. The best submitted solutions to this problem will be given extra-credit.*

Let  $N_{m,n}$  be the number of surjections or onto functions from a set  $S_m$  of  $m$  elements to a set  $S_n$  of  $n$  elements, where  $m \geq n \geq 1$ . Find a formula relating  $N_{m,n}$  to  $N_{m-1,n}$  and  $N_{m-1,n-1}$ .