COMP 170 Discrete Mathematical Tools for CS
2010 Spring Semester – Written Assignment # 7
Distributed: April 8, 2010 – Due: April 15 2010

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.
Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) fromthe textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions should be submitted before 5PM of the due date, in the collection bin in front of Room 4213A (This is near the TA labs).

Problem 1: Prove that
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
for all integers $n \geq 1$.

Problem 2: For what values of $n \geq 1$ is $n! \geq 5 \cdot 2^n$? Use mathematical induction to show that your answer is correct.

Problem 3: Prove that every integer greater than 7 is a sum of a nonnegative integer multiple of 3 and a nonnegative integer multiple of 5.
(Hint: first prove the three base cases *of $n = 8, 9, 10$ and then prove the inductive step assuming that $n > 10$.)*

Problem 4: Find the error in the following "proof" that all positive integers n are equal:
Let $p(n)$ be the statement that all numbers in an n-element set of positive integers are equal.
Then p(1) is true.
Now assume $p(n-1)$ is true, and let N be the set of the first n integers.
Let N' be the set of the first $n-1$ integers, and let N'' be the set of the last

$n-1$ integers.
By $p(n-1)$, all members of N' are equal, and all members of N'' are equal. Thus, the first $n-1$ elements of N are equal and the last $n-1$ elements of N are equal, and so all elements of N are equal. Therefore, all positive integers are equal.

Problem 5: Consider the recurrence $M(n) = 2M(n-1)+2$, with base case of $M(1) = 1$.
(a) State the solution to this recurrence
(you may use Theorem 4.1 in the book).
(b) Use induction to prove that this solution is correct.