COMP 170 Discrete Mathematical Tools for CS
2007 Fall Semester – Written Assignment # 9
Distributed: Nov 22, 2007 – Due: Nov 29, 2007

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.

<u>Some Notes:</u>

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions can either be submitted at the end of your Thursday lecture section or, before 5PM, in the collection bin in front of Room 4213A.

Problem 1: The eight kings and queens are removed from a deck of cards, and then two of these cards are selected (from the eight). What is the probability that the king or queen of spades is among the cards selected?

Problem 2: Calculate

$$\sum_{\substack{i_1,i_2,i_3:\\ 1\le i_1<i_2<i_3\le 5}} i_1 \cdot i_2 \cdot i_3$$

Problem 3 In this problem, a *black card* is a spade or a club.
Remove one card from an ordinary deck of cards. What is the probability that it is an ace, a diamond, or black? Use the inclusion-exclusion formula to solve this problem.

Problem 4: In this exercise you will solve the following problem:
If you roll eight dice, what is the probability that each of the numbers 1 through 6 appears on top at least once?

For $1 \le i \le 6$, let E_i be the event that number i doesn't show up on any of the dice.

(a) Write a formula for $P(E_i)$.

(b) Let $k \le 6$ and $1 \le i_1 < i_2 < \cdots < i_k \le 6$.
Write a formula for $P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k})$.

(c) Now use the inclusion-exclusion formula to write a formula for $P(E_1 \cup E_2 \cup \cdots \cup E_6)$.
This is the probability that *some* number doesn't appear when you roll eight die.
Your formula should use the summation sign, powers and binomial coeficients.

(d) Using the solution to (c), write down the probability that each of the numbers 1 through 6 appears on top at least once (a solution in the form of a sum is fine; it is not necessary to actually calculate the value of the sum).

Problem 5: In this exercise you will solve the following problem:
If you are hashing n keys into a hash table with k locations (buckets), what is the probability that every location gets at least one key? This probability can be expressed as a formula using the summation ($\sum$) symbol.

Hint: To solve this problem let E_i be the event that bucket E_i is empty. Then $E_1 \cup E_2 \cup \cdots \cup E_k$ is the event that at least one bucket is empty.

Let X be the event that every bucket gets at least one key*. Then X is the complement of $E_1 \cup E_2 \cup \cdots \cup E_k$ and the problem is asking you to find*

$$P(X) = 1 - P(E_1 \cup E_2 \cup \cdots \cup E_k).$$

You can now use the inclusion-exclusion formula to find $P(E_1 \cup E_2 \cup \cdots \cup E_k)$.

Challenge Problem A line of 100 people are waiting to enter an airplane. They each hold a ticket for one of the 100 seats on the flight. (For convenience, assume that the nth passenger in line has a ticket for seat number n, e.g., the first person has a ticket for seat number 1, etc.).

Unfortunately, the first person in line is crazy, will ignore the seat number on his ticket, and will pick a random seat out of the 100 to sit in. All of the other passengers are quite normal, though, and will go to their proper seat unless someone is already sitting in it. If someone is already sitting in their seat, they will choose an unoccupied seat at random and sit in it.

What is the probability that the last (100^{th}) person will sit in his own (the 100^{th}) seat?