3 successes in 5 Bernoulli Trials

C_1 : S S S F F $\qquad p^3(1-p)^2$

C_2 : S S F S F $\qquad p^3(1-p)^2$

C_3 : S S F F S $\qquad p^3(1-p)^2$

$\vdots$

F F S S S $\qquad p^3(1-p)^2$

# of cases: $m = \binom{5}{3}$

P(3 successes)

$= P(C_1) + P(C_2) + \cdots + P(C_m)$

$= m\, p^3(1-p)^2$

$= \binom{5}{3} p^3(1-p)^2$

k successes in n Bernoulli Trials

- prob of sequence w/ k successes
  $n-k$ failures

$$= p^k (1-p)^{n-k}$$

- # of such sequences:

$$\binom{n}{k}$$

- $P(k \text{ successes})$

$$= \binom{n}{k} p^k (1-p)^{n-k}$$

Theorem 5.8

X: # of successes in n

Bernolli trials

$$P(X=k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & 0 \le k \le n \\ 0 & \text{if not} \end{cases}$$

X: <u>Binomial Random Var.</u>

<u>Binomial distribution</u>

Expected Value of RV

$\approx$ Average over many experiments

- X: result of rolling a die
- $E(x)$?
- First consider rolling the die 100 times

2, 4, 1, 2, 3, 5, 6, ..., 4, 6

Average

$$= \frac{2+4+1+2+3+ \cdots + 4+6}{100}$$

$$= 1 \cdot \frac{\#\text{ of } 1\text{'s}}{100} + 2 \cdot \frac{\#\text{ of } 2\text{'s}}{100} + \cdots$$

$$\approx 1 \cdot P(X=1) + 2 \cdot P(X=2) + \cdots$$

$$= E(x).$$

Today: 27-11-2008

- Flip coin n times,

  # of heads to <u>expect</u> ?

OR: Repeat many times

$\boxed{\text{flip coin } n \text{ times}}$

# of heads on average ?

- Flip coin until head,

  # of flips to <u>expect</u> ?

OR: Repeat many times

$\boxed{\text{flip coin until head}}$

Average # of flips

Lemma 5.9

- Biased Coin: $p(T) = \frac{2}{3}$
- Toss it 3 times, X: # of tails

	s_1	s_2	s_3	s_4	...	s_8
S:	TTT	TTH	THT	HTT	....	HHH
X:	3	2	2	2		0

$$E(X) = 3 \cdot p(X=3) + 2 \cdot p(X=2) + 1 \cdot p(X=1) + 0 \cdot p(X=0)$$

$$= X(s_1)\,p(s_1) + X(s_2)\,p(s_2) + X(s_3)\,p(s_3) + X(s_4)\,p(x_4) \cdots + X(s_8)\,p(s_8)$$

$$= \sum_{s \in S} X(s)\,p(s)$$

Proof of Theorem 5.11

$$E(cX) = \sum_{s \in S} c X(s) p(s)$$

$$= c \sum_{s \in S} X(s) p(s)$$

$$= c E(X).$$

P42

$$\sum_{i=0}^{\infty} (1-p)^i = \lim_{n\to\infty} \sum_{i=0}^{n} (1-p)^i$$

$$= \lim_{n\to\infty} \frac{1-(1-p)^{n+1}}{1-(1-p)}$$

$$= \frac{1}{1-(1-p)} = \frac{1}{p}$$

Known fact: $r \neq 1$

$$\sum_{i=0}^{n} i\, r^i = \frac{n r^{n+2} - (n+1) r^{n+1} + r}{(1-r)^2}$$

$0 < r < 1$

$$\sum_{i=0}^{\infty} i\, r^i = \frac{r}{(1-r)^2}$$

Expected number of trials

$$= 1 \cdot p + 2 \cdot (1-p) p + 3 \cdot (1-p)^2 p$$

$$+ \cdots + i \cdot (1-p)^{i-1} p + \cdots$$

$$= \sum_{i=1}^{\infty} i (1-p)^{i-1} p$$

$$= \frac{p}{1-p} \sum_{i=1}^{\infty} (1-p)^i i \qquad \Rightarrow L17\text{-}8$$

$$= \frac{p}{1-p} \frac{1-p}{(1-(1-p))^2}$$

$$= \frac{p}{p^2} = \frac{1}{p}$$

Theorem 5.13