COMP 170 Discrete Mathematical Tools for CS
2007 Fall Semester – Written Assignment # 6
Distributed: Oct 25, 2007 – Due: Nov 1, 2007

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.

Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions can either be submitted at the end of your Thursday lecture section or, before 5PM, in the collection bin in front of Room 4213A.

Problem 1: Construct a proof for each of the following:
(a) For all integers m and n, if m is even and n is odd, then $m+n$ is odd.
(b) For all integers m and n, if m and n are both odd, then mn is odd.

Problem 2: (a) Construct a contrapositive proof that for all real numbers x, if $x^2 - 2x \neq -1$ then $x \neq 1$.
(b) Construct a proof by contradiction that for all real numbers x, if $x^2 - 2x \neq -1$ then $x \neq 1$.

Problem 3: Prove that

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

for all integers $n \geq 1$.

Problem 4: For what values of $n \geq 1$ is $n! \geq 5 \cdot 2^n$? Use mathematical induction to show that your answer is correct.

Problem 5: Prove that every integer greater than 7 is a sum of a nonnegative integer multiple of 3 and a nonnegative integer multiple of 5.
(Hint: first prove the three base cases *of* $n = 8, 9, 10$ *and then prove the inductive step assuming that* $n > 10$*.)*

Problem 6: Find the error in the following "proof" that all positive integers n are equal:
Let $p(n)$ be the statement that all numbers in an n-element set of positive integers are equal. Then p(1) is true. Now assume $p(n-1)$ is true, and let N be the set of the first n integers. Let N' be the set of the first $n-1$ integers, and let N'' be the set of the last $n-1$ integers. By $p(n-1)$, all members of N' are equal, and all members of N'' are equal. Thus, the first $n-1$ elements of N are equal and the last $n-1$ elements of N are equal, and so all elements of N are equal. Therefore, all positive integers are equal.

Challenge Problem: A triangle on the Cartesian plane is represented by the x- and y-coordinates of each of its three vertices. Show that there exists no equilateral triangle whose coordinates are all integers. (Recall that a triangle is equilateral if all of its sides have the same length.)