COMP2611 Spring 2022 Homework #1

Note:

- **The deadline of this homework is at 11:55pm on Monday, 14 March 2022 (Hong Kong Time, UTC+8). NO late submissions will be accepted!**
- **Work out the answers of the questions either directly on the hardcopy of this document or on your own paper sheets. Then scan all the answer pages into a single pdf file "homework1_<studID>.pdf", or take photos and zip them into a single zip file "homework1_<stdID>.zip". Make sure every detail of the answers is clearly visible in your submission (verify this on the scanned pages before submitting), otherwise marks may be deducted.**
- **We only accept e-submissions at the Canvas. To submit, first find the Canvas page of COMP2611, homework 1, and then upload the file. You can upload for multiple times, only the last one before the deadline will be marked.**
- **Make sure you keep the original copy of your homework until the homework score is finalized.**

Name :

Student ID :

Email :

Question	Marks
1. Boolean Algebra	/10
2. K-map	/10
3. Combinational Logic	/12
4. Sequential Logic	/8
5. Sequential Logic Circuit Timing Chart	/10
total	/50

Question 1: Boolean Algebra (10 points)

Prove the logic equivalence as stated in the following equation with Boolean Algebra.

$$ACE + \bar{A}BE + \bar{B}\bar{C}\bar{D} + B\bar{C}E + \bar{C}DE + \bar{A}E = E + \bar{B}\bar{C}\bar{D}$$

Answer:

$ACE + \bar{A}BE + \bar{B}\bar{C}\bar{D} + B\bar{C}E + \bar{C}DE + \bar{A}E$

$= E(AC + \bar{A}B + B\bar{C} + \bar{C}D + \bar{A}) + \bar{B}\bar{C}\bar{D}$

$= E(AC + \bar{A}B + \bar{A} + B\bar{C} + \bar{C}D) + \bar{B}\bar{C}\bar{D}$

$= E(AC + \bar{A} + B\bar{C} + \bar{C}D) + \bar{B}\bar{C}\bar{D}$

……

$= E(AC + \bar{A}C + \bar{A} + B\bar{C} + \bar{C}D) + \bar{B}\bar{C}\bar{D}$

……

$= E(AC + \bar{A}C + \bar{A} + B\bar{C} + \bar{C}D) + \bar{B}\bar{C}\bar{D}$

$= E(AC + \bar{A} + B\bar{C} + \bar{C}D) + \bar{B}\bar{C}\bar{D}$

……

$= E(AC + \bar{A} + B\bar{C} + \bar{C}D) + \bar{B}\bar{C}\bar{D}$

$= E(\bar{A} + C + B\bar{C} + \bar{C}D) + \bar{B}\bar{C}\bar{D}$

……

$= E(\bar{A} + C + B\bar{C} + BC + \bar{C}D + CD) + \bar{B}\bar{C}\bar{D}$

……

$= E(\bar{A} + C + B\bar{C} + BC + \bar{C}D + CD) + \bar{B}\bar{C}\bar{D}$

$= E(\bar{A} + C + B + D) + \bar{B}\bar{C}\bar{D}$

……

$= E(\bar{A} + C + B + D) + \bar{B}\bar{C}\bar{D}$

$= E(\bar{A} + \overline{\bar{B}\bar{C}\bar{D}}) + \bar{B}\bar{C}\bar{D}$

……

$= \bar{A}E + \overline{\bar{B}\bar{C}\bar{D}}E + \bar{B}\bar{C}\bar{D}$

$= \bar{A}E + E + \bar{B}\bar{C}\bar{D}$

……

$= \bar{A}E + E + \bar{B}\bar{C}\bar{D}$

$= E + \bar{B}\bar{C}\bar{D}$

if exactly the simplest answer then 10 points. For any extra (correct) term, we will subtract 2 points if the complete expression is still correct. For example, if his/her answer has 3 terms, and the expression is correct we will have 10-2*1=8 points.

In the case he/she have all the 6 terms, if he/she attempts to do some simplification(s) with some correct steps then he/she will get 10-2*4=2 points for attempting the question

If just copy blindly either the left-hand-side or the right-hand-side of the equality sign, or if no correct step is shown then 0 point.

If the final answer is wrong search for the last correct expression and apply the above to give points.

Question 2: K-map (10 points)

The Boolean expression for the output ***F*** in the inputs ***A, B, C*** and ***D*** is given in **Sum-of-Product (SoP):**

$$F = m_0 + m_1 + m_2 + m_5 + m_8 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15}$$

Assume input ***D*** corresponds to the least significant digit and ***A*** corresponds to the most significant digit. Use the K-map approach to simplify the logic function and write its simplest form. (Note: You should circle the 1's group(s) clearly in your K-map.)

Answer:

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	1	1	1	1
10	1	0	1	1

$$F = AC + AB + \bar{A}\bar{C}D + \bar{B}\bar{D}$$

Each correct group on the K-map, 2 points in total 4*2=8 points, final correct expression 2 points. Every extra but correct group -0.5 point (deduct for once only), every wrong group -1 point. For example, if the correct expression is correct but there is an extra group in the K-map and an extra term in the expression, the student can get 4*2+2-0.5 = 9.5 points. If the correct expression is incorrect, there are two correct group as well as one wrong group, the student can get 2*2-1-0.5 = 2.5 points.

Question 3: Combinational Logic (12 points)

We want to design a circuit that verifies whether a 2-input AND gate works correctly. The combinational logic circuit denoted by the square in the diagram below takes the inputs of a 2-input AND gate (i.e. inputs $\mathbf{A_0}$, $\mathbf{A_1}$) and determines whether the output of the AND gate (i.e $\mathbf{A_2}$) is correct (truth table of an AND gate is provided below for quick reference). If the AND gate is outputting correctly, the circuit will output 1 in **F,** otherwise it will output 0 in **F**.

Inputs		Correct AND gate output
A_1	A_0	A_2
0	0	0
0	1	0
1	0	0
1	1	1

a) Construct the truth table for the output ***F*** (be careful about the positions of $\mathbf{A_2}$,$\mathbf{A_1}$,$\mathbf{A_0}$ in the table) (4 points)

Answer:

Input			**Output**
A_2	A_1	A_0	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Each correct output 0.5 points, in total 8*0.5=4 points

b) Derive the Boolean expression for the output ***F*** in the inputs A_2, A_1, and A_0 (i.e. A_2, A_1, A_0) in both **Sum-of-Product (SoP)** and **Product-of-Sum (PoS)** formats with minterms and maxterms respectively. (4 points)

Answers:

Sum-of-Products:

$$F = m_0 + m_1 + m_2 + m_7$$

or

$$F = \overline{A_2}\,\overline{A_1}\,\overline{A_0} + \overline{A_2}\,\overline{A_1}\,A_0 + \overline{A_2}\,A_1\overline{A_0} + A_2A_1A_0$$

Each correctly written minterm, 0.5 points, in total 4*0.5=2points

Product-of-Sums:

$$F = M_3 M_4 M_5 M_6$$

or

$$F = (A_2 + \overline{A_1} + \overline{A_0})(\overline{A_2} + A_1 + A_0)$$
$$(\overline{A_2} + A_1 + \overline{A_0})(\overline{A_2} + \overline{A_1} + A_0)$$

Each correctly written maxterm, 0.5 points, in total 4*0.5=2points

c) Use K-map to below simplify the **SoP** representation for the circuit. Write the expression in its simplest form. (4 points)

Answer:

A_2 \ A_1A_0	00	01	11	10
0	1	1	0	1
1	0	0	1	0

$$F = \overline{A_1}\ \overline{A_2} + \overline{A_0}\ \overline{A_2}\ + A_0 A_1 A_2$$

Each correct group in the k-map 1 point

Final expression if consistent with the above groups, then 1 point

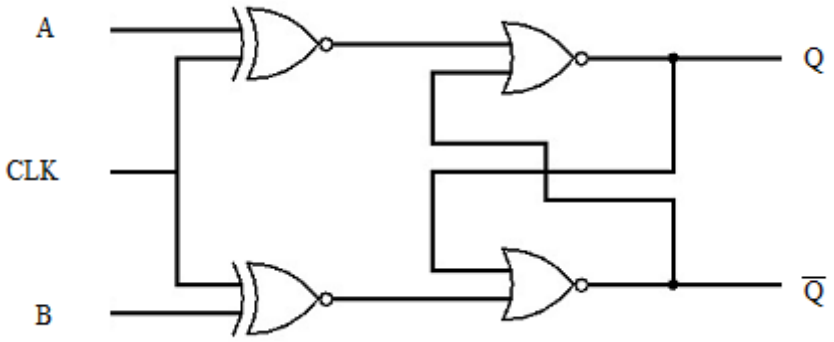
Question 4: Sequential Logic (8 points)

Complete the truth table of the sequential logic circuit given below. Assume $\boldsymbol{Q_{t+1}}$ is the value of the output $\boldsymbol{Q}$ after the corresponding logic gates have been given enough time to produce the output according to the inputs.

If the older value $\boldsymbol{Q_t}$ will be preserved ($\boldsymbol{Q_{t+1} = Q_t}$) by the input combination, put "**Latch**" in the table for the output $\boldsymbol{Q_{t+1}}$.

If the input combination(s) is forbidden, put "**Forbidden**" in the table.

If the new $\boldsymbol{Q_{t+1}}$ value will be stable and there's no relationship between the new value $\boldsymbol{Q_{t+1}}$ and the old value $\boldsymbol{Q_t}$, put the specific new value ("0" or "1") in the table. (7 points)



The two gates at the left in the circuit above are XNOR (Exclusive NOR) gates, the truth table is as follows:

Inputs		Output
A	B	A XNOR B
0	0	1
0	1	0
1	0	0
1	1	1

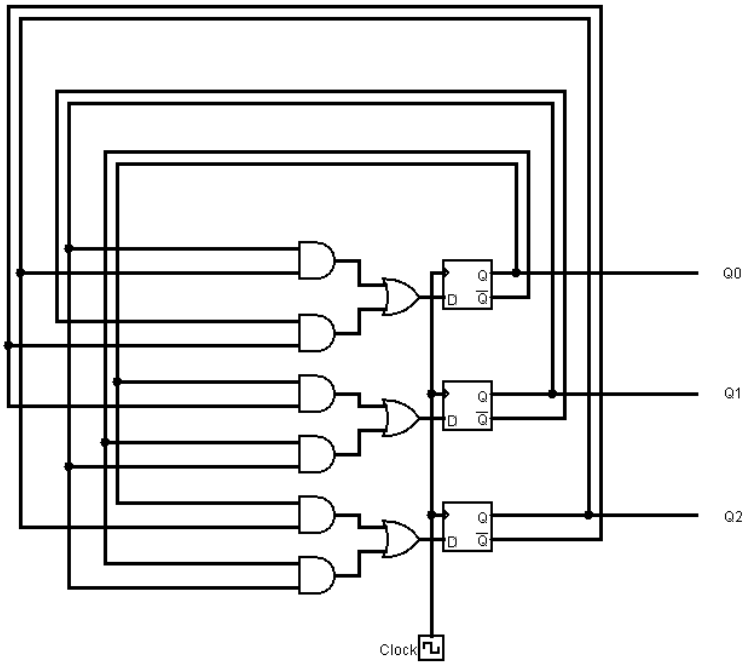
Answer:

Input			Output
C	A	B	Q_{t+1}
0	0	0	**Forbidden**
0	0	1	**0**
0	1	0	**1**
0	1	1	**Latch**
1	0	0	**Latch**
1	0	1	**1**
1	1	0	**0**
1	1	1	**Forbidden**

Each correct output entry 1 point

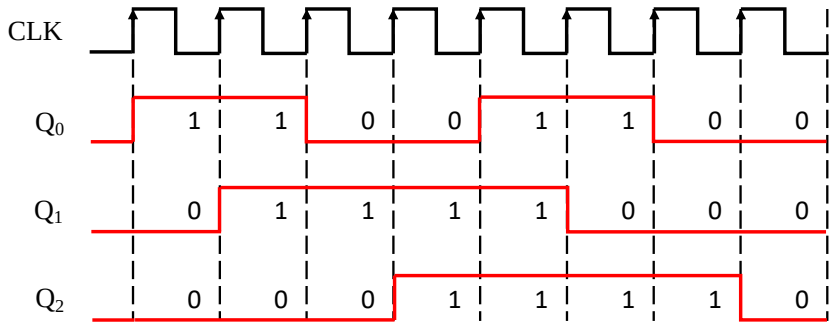
Question 5: Sequential Logic Circuit Timing Chart (10 points)

A circuit composed of 3 **rising edge-triggered** D flip-flops is shown below. Ignore propagation delays. Assume the $\mathbf{Q_2}$, $\mathbf{Q_1}$ and $\mathbf{Q_0}$ of the D flip-flops are all **0** initially.



a) Complete the timing diagram below for the outputs of this circuit (8 points).

Answer:



Each rising edge all Q_0,Q_1,Q_2 correct then 1 point. Total 8 rising edges

If Q_0,Q_1,Q_2 for all the rising edges are correct, but there are mistakes overall, the mistakes in each clock cycle -0.5points.

b) State what the circuit does by referring to the timing diagram (hint: consider **$Q_2Q_1Q_0$** to be a binary value). (2 points)

It generates 3-bit Gray code in $Q_2Q_1Q_0$, the values are generated as the follows during each rising edge of the CLK

000 → 001 → 011 → 010 → 110 → 111 → 101 → 100 → 000 → 001 → …

2 points for correct answer.

If you answer "Grey code", you will get 2 points; if you list sequence 000 → 001 → 011 → 010 → 110 → 111 → 101 → 100 → 000 → 001 → …, either in binary or decimal you will get 2 points; if there is no loop or "repeat" or "cycle" like word, you will get 1 point.