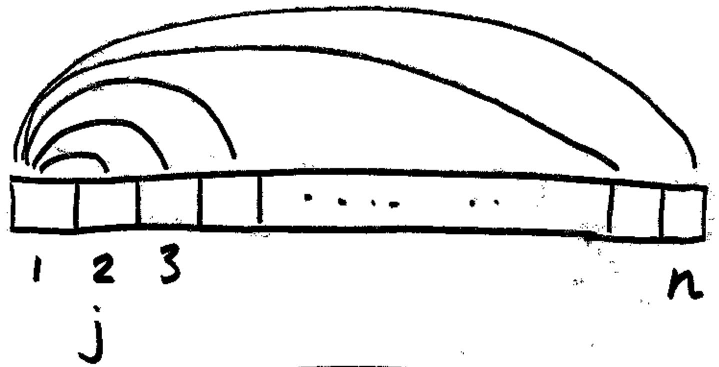
S

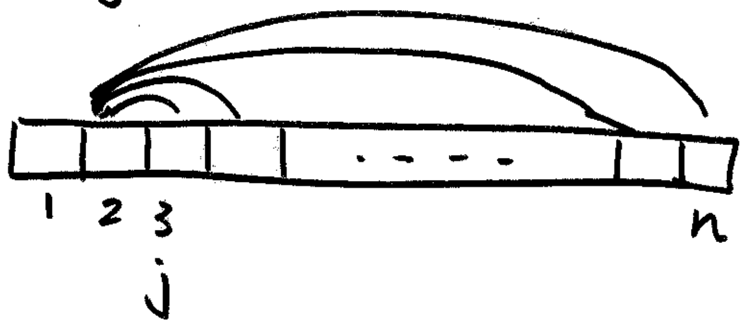
\# of comparisons

$i=1$:



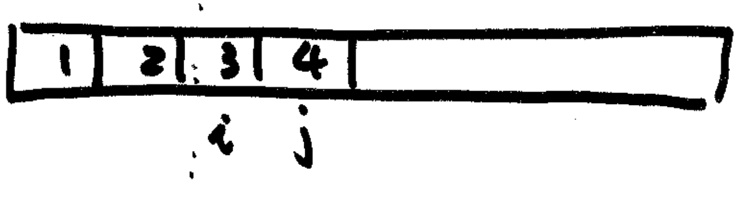
$n-1 = |S_1|$

$i=2$:

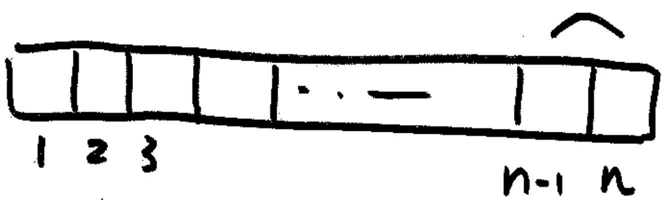


$n-2 = |S_2|$

$i=t$



$i=n-1$:



$1 = |S_{n-1}|$

Total # of comparisons $= (n-1) + (n-2) + \cdots + 1$

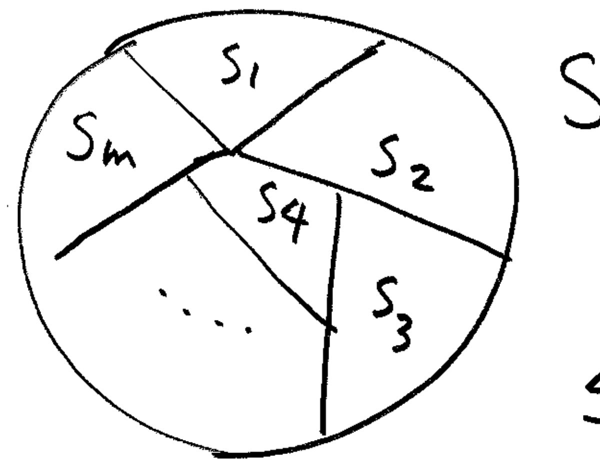
$S = S_1 \cup S_2 \cup \cdots \cup S_{n-1}$

HS1

Partition

$S_1, S_2, \cdots S_m$ is

a partition of S if



S_i: blocks
subsets

(1): $S = S_1 \cup S_2 \cup \cdots \cup S_m$

(2): $S_1, S_2, \cdots S_m$ are mutually disjoint.

The sum principle

$$|S| = |S_1| + |S_2| + \cdots + |S_m| = \sum_{i=1}^{m} |S_i|$$

Matrix Multiplication

* $A : r \times n$, $B : n \times m$

* $C = A \cdot B : \quad r \times m$

$$C[i, j] = \sum_{k=1}^{n} A[i, k] \cdot B[k, j]$$

$k=0 \quad S = 0$

$1 \quad S = A[i,1]\,B[1,j]$

$2 \quad S = A[i,1]\,B[1,j] + A[i,2]\,B[2,j]$

$\vdots \quad S = \quad \vdots$

$n \quad S = A[i,1]\,B[1,j] + \cdots + A[i,n]\,B[n,j]$

Entry Code

$$S = S_0 \cup S_1 \cup \cdots \cup S_9$$

S	S_0	S_1	S_9
All codes	Those start w/ 0	Those start w/ 1	Those start w/ 9

$|S_i|$ = # of codes with 3 digits

$$|S| = 10 \times |S_i| = 10 \times 10^3 = 10^4$$

Repeat argument on S_i :

$$|S_i| = 10^3$$

Function

$f : S \longrightarrow T$

S		T
1	$\rightarrow$	Sam
2	$\rightarrow$	Mary
3	$\rightarrow$	Sarah

All Functions from $\{1,2\} \rightarrow \{a, b\}$

$f(1)$	$f(2)$
a	a
a	b
b	a
b	b

2-element list from $\{a, b\}$

$\#: 2 \times 2 = 4$

L2-2

Counting Functions

of functions 2-element set $\longrightarrow$ 3-element set

= # of 2-element lists from 3-element set

$= 3 \times 3$ $\boxed{f(1)\ f(2)}$

# of functions 3-element set $\longrightarrow$ 2-element set

= # of 3-element lists from 2-element set

$= 2 \times 2 \times 2 = 8$ $\boxed{f(1)\ f(2)\ f(3)}$

Bijection

$f: S \to T \implies |S| = |T|$

<u>Claim</u>: # of (i, j, k) s.t. $1 \le i < j < k \le n$

$=$ # of 3-element subsets from $\{1, 2, \dots, n\}$

<u>Proof</u>: X — set of those triples

Y — Set of those 3-elememt subsets

Define $f : X \longrightarrow Y$: $f((i, j, k)) = \{i, j, k\}$

(1). f is bijective :

$$(i, j, k) \neq (i', j', k') \Rightarrow \{i, j, k\} \neq \{i', j', k'\}$$
$$\Longleftrightarrow f((i, j, k)) \neq f((i', j', k'))$$

$$(1, 2, 3) \neq (1, 3, 4) \Rightarrow \{1, 2, 3\} \neq \{1, 3, 4\}$$

(2) • f is surjective:

- any 3-element subset can be written as

  $r = \{i, j, k\}$ s.t. $i < j < k$

- Example: $\{3, 1, 2\}$ is the same as $\{1, 2, 3\}$

$$\boxed{\begin{array}{c} (1,2,3) \in X \\ \downarrow f \\ \{1,2,3\} = \{3,1,2\} \end{array}}$$

- $(i, j, k) \in X$

  $f((i, j, k)) = \{i, j, k\} = r$

- So, f is onto.

$(1) + (2) \Rightarrow f$ is bijection.

$\Rightarrow |X| = |Y|$. Claim proved.

3-element subsets	3-element permutations
$\{1,2,3\}$	$\left.\begin{matrix}123, & 132, & 213\\ 231, & 312, & 321\end{matrix}\right\} 6$
$\{1\ 2\ 4\}$	$\left.\begin{matrix}124, & 142, & 214\\ 241, & 412, & 421\end{matrix}\right\} 6$
⋯	⋯

$$\#\text{ of 3-element subsets} = \frac{\#\text{ of 3-element permutations}}{6}$$

$$n! = n(n-1)\cdots(n-k+1)(n-k)\cdots 1$$

$$(n-k)! = (n-k)\cdots 1$$

$$n^{\underline{k}} = n(n-1)\cdots(n-k+1)$$

$$n^{\underline{k}} = \frac{n!}{(n-k)!}$$

L2-7

<u>k-element subsets</u>	<u>k-element permutations</u>
$\{1, 2, \ldots, k\}$	$\left.\begin{array}{l} 12\cdots k \quad 21\cdots k \quad \cdots \\ \cdots \end{array}\right\} k!$
$\cdots$	

$$\# \text{ of } k\text{-element subsets}$$

$$= \frac{\# \text{ of } k\text{-element permutations}}{k!}$$

$$= \frac{n^{\underline{k}}}{k!} = \frac{n!}{k!\,(n-k)!}$$

<u>Theorem 1.2</u>