

Recapitulation from Previous Lectures

Recap: Grammars

Name the productions of a language:

$$\textit{start} \rightarrow \textit{letter}(\textit{letter} \mid \textit{digit})^*$$
$$\textit{letter} \rightarrow \textit{a} \mid \textit{b} \mid \textit{c} \mid \dots \mid \textit{z}$$
$$\textit{digit} \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

“*start*”, “*letter*”, “*digit*” are part of a distinct set of identifiers called *non-terminals*, often denoted abstractly by letters P, Q, R, S, \dots with S usually the Starting symbol

Regular grammars: *no recursion* (only use Kleene star)

Context-free grammars: *recursion is allowed*

Context-Free Grammars (CFG) and Derivations

Example CFG:

$$S \rightarrow \varepsilon \mid aSb$$

Semantics of CFGs given by *rewriting derivations*

Example derivation:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaa\varepsilon bbb = aaabbb$$

Recap: Parse Trees And Abstract Syntax Trees

Parse trees uniquely specify how an input was recognized by the grammar.

Parse trees *contain all information* needed to reconstruct the input.

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Parse trees uniquely specify how an input was recognized by the grammar.

Parse trees *contain all information* needed to reconstruct the input.

Abstract syntax trees (ASTs) omit *syntactic details* not relevant to program semantics

AST can be pretty-printed,

but the result may not correspond to the specific input (loss of information)

Example: Expressions

An expression grammar:

$$expr \rightarrow \text{intLiteral} \mid \text{ident} \mid expr \ op \ expr \mid '(' \ expr \ ')'$$
$$op \rightarrow + \mid *$$

A possible AST for it:

```
enum Expr:
  case IntLit(n: Int)
  case Var(name: String)
  case Add(e1: Expr, e2: Expr)
  case Mult(e1: Expr, e2: Expr)
```

Notice: no parenthesis case; no “op”

Recap: Ambiguities

Some grammars are ambiguous.

$$\begin{aligned} \text{expr} &\rightarrow \text{intLiteral} \mid \text{ident} \mid \text{expr op expr} \mid '(' \text{expr} ')' \\ \text{op} &\rightarrow + \mid * \end{aligned}$$

How to parse these?

- ▶ “ $x * 42 + y$ ”
- ▶ “ $x + 42 + y$ ”

Removing ambiguities requires transforming the grammar.

Generalities on Grammars

Chomskys Classification of Grammars (1959)

- ▶ Type 0, *unrestricted*: arbitrary string rewrite rules

Equivalent to Turing machines!

$$eXb \rightarrow eXeX \rightarrow Y$$

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$O(n)$ -space Turing machines

$$aXb \rightarrow acXb$$

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Type 3 \subset Type 2 \subset Type 1 \subset Type 0

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Decidable even for type 1 grammars (by eliminating epsilons – Chomsky, 1959)

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Simple algorithm for CFGs: “CYK” (CockeYoungerKasami), takes $O(n^3)$ time

Better complexity possible (L. G. Valiant, 1975) – reduce to matrix multiplication
 n^k for k between 2 and 3

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However, parsing general grammars has *little interest*

for *computer* language processing, and in particular for *compiler design*.

Most *reasonable* programming languages have *reasonable* grammars!

Recursive Descent LL(1) Parsing

- useful parsing technique
- to make it work, we might need to transform the grammar

A Rule of While Language Syntax

// Where things work very nicely for recursive descent!

statmt ::=

println (stringConst , ident)

| ident = expr

| if (expr) statmt (else statmt)?

| while (expr) statmt

| { statmt }*

Parser for the statmt (rule -> code)

```
def skip(t : Token) = if (lexer.token == t) lexer.next
  else error("Expected"+ t)
def statmt = {
  if (lexer.token == Println) { lexer.next;
    skip(openParen); skip(stringConst); skip(comma);
    skip(identifier); skip(closedParen)
  } else if (lexer.token == Ident) { lexer.next;
    skip(equality); expr
  } else if (lexer.token == ifKeyword) { lexer.next;
    skip(openParen); expr; skip(closedParen); statmt;
    if (lexer.token == elseKeyword) { lexer.next; statmt }
  }
  // | while ( expr ) statmt
```

Continuing Parser for the Rule

```
// | while ( expr ) statmt
```

```
} else if (lexer.token == whileKeyword) { lexer.next;  
    skip(openParen); expr; skip(closedParen); statmt
```

```
// | { statmt* }
```

```
} else if (lexer.token == openBrace) { lexer.next;  
    while (isFirstOfStatmt) { statmt }  
    skip(closedBrace)
```

```
} else { error("Unknown statement, found token " +  
    lexer.token) }
```

How to construct if conditions?

```
statmt ::= println ( stringConst , ident )  
        | if ( expr ) statmt (else statmt)?  
        | while ( expr ) statmt
```

- Look what each alternative starts with to decide what to parse
- Here: we have terminals at the beginning of each alternative
- More generally, we have 'first' computation, as for regular expressions
- Consider a grammar G and non-terminal N

$L_G(N) = \{ \text{set of strings that N can derive} \}$

e.g. $L(\text{statmt})$ – all statements of while language

$\text{first}(N) = \{ a \mid aw \text{ in } L_G(N), a - \text{terminal}, w - \text{string of terminals} \}$

$\text{first}(\text{statmt}) = \{ \text{println}, \text{ident}, \text{if}, \text{while}, \{ \} \}$

$\text{first}(\text{while (expr) statmt}) = \{ \text{while} \}$

- we will give an algorithm

Formalizing and Automating Recursive Descent: LL(1) Parsers

Task: Rewrite Grammar to make it suitable for recursive descent parser

- Assume the priorities of operators as in Java

```
expr ::= expr (+|-|*|/) expr  
       | name | '(' expr ')'  
name ::= ident
```

Grammar vs Recursive Descent Parser

```
expr ::= term termList
termList ::= + term termList
           | - term termList
           | ε
term ::= factor factorList
factorList ::= * factor factorList
            | / factor factorList
            | ε
factor ::= name | ( expr )
name ::= ident
```

Note that the abstract trees we would create in this example do not strictly follow parse trees.

```
def expr = { term; termList }
def termList =
  if (token==PLUS) {
    skip(PLUS); term; termList
  } else if (token==MINUS)
    skip(MINUS); term; termList
  }
def term = { factor; factorList }
...
def factor =
  if (token==IDENT) name
  else if (token==OPAR) {
    skip(OPAR); expr; skip(CPAR)
  } else error("expected ident or ")
```


Rough General Idea

$$\begin{array}{l} A ::= B_1 \dots B_p \\ \quad | C_1 \dots C_q \\ \quad | D_1 \dots D_r \end{array}$$



```
def A =  
  if (token  $\in$  T1) {  
    B1 ... Bp  
  } else if (token  $\in$  T2) {  
    C1 ... Cq  
  } else if (token  $\in$  T3) {  
    D1 ... Dr  
  } else error("expected T1,T2,T3")
```

where:

T1 = **first**(B₁ ... B_p)

T2 = **first**(C₁ ... C_q)

T3 = **first**(D₁ ... D_r)

first(B₁ ... B_p) = {a $\in \Sigma$ | B₁...B_p \Rightarrow ... \Rightarrow aw }

T1, T2, T3 should be **disjoint** sets of tokens.

Computing **first** in the example

```
expr ::= term termList
termList ::= + term termList
           | - term termList
           | ε
term ::= factor factorList
factorList ::= * factor factorList
            | / factor factorList
            | ε
factor ::= name | ( expr )
name ::= ident
```

$\text{first}(\text{name}) = \{\mathbf{ident}\}$

$\text{first}((\text{expr})) = \{ (\}$

$\text{first}(\text{factor}) = \text{first}(\text{name})$
 $\quad \cup \text{first}((\text{expr}))$
 $= \{\mathbf{ident}\} \cup \{ (\}$
 $= \{\mathbf{ident}, (\}$

$\text{first}(* \text{factor factorList}) = \{ * \}$

$\text{first}(/ \text{factor factorList}) = \{ / \}$

$\text{first}(\text{factorList}) = \{ *, / \}$

$\text{first}(\text{term}) = \text{first}(\text{factor}) = \{\mathbf{ident}, (\}$

$\text{first}(\text{termList}) = \{ +, - \}$

$\text{first}(\text{expr}) = \text{first}(\text{term}) = \{\mathbf{ident}, (\}$

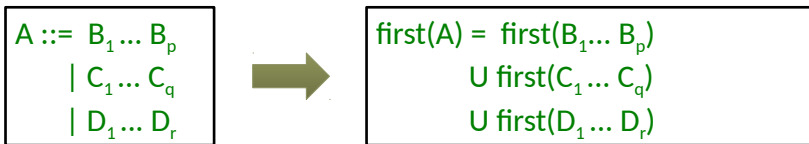
Algorithm for **first**: Goal

Given an arbitrary context-free grammar with a set of rules of the form $X ::= Y_1 \dots Y_n$ compute first for each right-hand side and for each symbol.

How to handle

- alternatives for one non-terminal
- sequences of symbols
- nullable non-terminals
- recursion

Rules with Multiple Alternatives



Sequences

$\text{first}(B_1 \dots B_p) = \text{first}(B_1)$ if not nullable(B_1)

$\text{first}(B_1 \dots B_p) = \text{first}(B_1) \cup \dots \cup \text{first}(B_k)$

if nullable(B_1), ..., nullable(B_{k-1}) and
not nullable(B_k) or $k=p$

Abstracting into Constraints

recursive grammar: constraints over finite sets: expr' is $\text{first}(\text{expr})$

```
expr ::= term termList
termList ::= + term termList
           | - term termList
           | ε
term ::= factor factorList
factorList ::= * factor factorList
            | / factor factorList
            | ε
factor ::= name | ( expr )
name ::= ident
```

nullable: termList , factorList

```
expr' = term'
termList' = {+}
           ∪ {-}

term' = factor'
factorList' = {*}
            ∪ { / }

factor' = name' ∪ { ( }
name' = { ident }
```

For this nice grammar, there is no recursion in constraints.
Solve by substitution.

Example to Generate Constraints

$S ::= X \mid Y$
 $X ::= \mathbf{b} \mid SY$
 $Y ::= ZX\mathbf{b} \mid Y\mathbf{b}$
 $Z ::= \varepsilon \mid \mathbf{a}$



$S' = X' \cup Y'$
 $X' =$

terminals: \mathbf{a}, \mathbf{b}

non-terminals: S, X, Y, Z

reachable (from S):

productive:

nullable:

First sets of terminals:

$S', X', Y', Z' \subseteq \{\mathbf{a}, \mathbf{b}\}$

Example to Generate Constraints

$$\begin{aligned} S &::= X \mid Y \\ X &::= \mathbf{b} \mid S Y \\ Y &::= Z X \mathbf{b} \mid Y \mathbf{b} \\ Z &::= \varepsilon \mid \mathbf{a} \end{aligned}$$

$$\begin{aligned} S' &= X' \cup Y' \\ X' &= \{\mathbf{b}\} \cup S' \\ Y' &= Z' \cup X' \cup Y' \\ Z' &= \{\mathbf{a}\} \end{aligned}$$

terminals: **a, b**

non-terminals: **S, X, Y, Z**

reachable (from S): S, X, Y, Z

productive: X, Z, S, Y

nullable: Z

These constraints are recursive.
How to solve them?

$$S', X', Y', Z' \subseteq \{\mathbf{a}, \mathbf{b}\}$$

How many candidate solutions

- in this case?
- for k tokens, n nonterminals?

Iterative Solution of **first** Constraints

	S'	X'	Y'	Z'
1.	$\{\}$	$\{\}$	$\{\}$	$\{\}$
2.	$\{\}$	$\{b\}$	$\{b\}$	$\{a\}$
3.	$\{b\}$	$\{b\}$	$\{a,b\}$	$\{a\}$
4.	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a\}$
5.	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a\}$

$$S' = X' \cup Y'$$

$$X' = \{b\} \cup S'$$

$$Y' = Z' \cup X' \cup Y'$$

$$Z' = \{a\}$$

- Start from all sets empty.
- Evaluate right-hand side and assign it to left-hand side.
- Repeat until it stabilizes.

Sets grow in each step

- initially they are empty, so they can only grow
- if sets grow, the RHS grows (\cup is monotonic), and so does LHS
- they cannot grow forever: in the worst case contain all tokens

Constraints for Computing Nullable

- Non-terminal is nullable if it can derive ϵ

$S ::= X \mid Y$
 $X ::= \mathbf{b} \mid SY$
 $Y ::= ZX\mathbf{b} \mid Y\mathbf{b}$
 $Z ::= \epsilon \mid \mathbf{a}$



$S' = X' \mid Y'$
 $X' = 0 \mid (S' \& Y')$
 $Y' = (Z' \& X' \& 0) \mid (Y' \& 0)$
 $Z' = 1 \mid 0$

$S', X', Y', Z' \in \{0,1\}$

0 - not nullable

1 - nullable

| - disjunction

& - conjunction

	S'	X'	Y'	Z'
1.	0	0	0	0
2.	0	0	0	1
3.	0	0	0	1

again monotonically growing

Computing first and nullable

- Given any grammar we can compute
 - for each non-terminal X whether nullable(X)
 - using this, the set first(X) for each non-terminal X
- General approach:
 - generate constraints over finite domains, following the structure of each rule
 - solve the constraints iteratively
 - start from least elements
 - keep evaluating RHS and re-assigning the value to LHS
 - stop when there is no more change

Summary: Algorithm for nullable

```
nullable = {}  
changed = true  
while (changed) {  
  changed = false  
  for each non-terminal X  
    if ((X is not nullable) and  
      (grammar contains rule  $X ::= \epsilon \mid \dots$ )  
      or (grammar contains rule  $X ::= Y_1 \dots Y_n \mid \dots$   
        where  $\{Y_1, \dots, Y_n\} \subseteq \text{nullable}$ )  
    then {  
      nullable = nullable  $\cup$  {X}  
      changed = true  
    }  
}
```

Summary: Algorithm for **first**

for each nonterminal X : $\text{first}(X) = \{\}$

for each terminal t : $\text{first}(t) = \{t\}$

repeat

for each grammar rule $X ::= Y(1) \dots Y(k)$

for $i = 1$ to k

if $i=1$ or $\{Y(1), \dots, Y(i-1)\} \subseteq \text{nullable}$ **then**

$\text{first}(X) = \text{first}(X) \cup \text{first}(Y(i))$

until none of $\text{first}(\dots)$ changed in last iteration

Follow sets. LL(1) Parsing Table

Exercise Introducing Follow Sets

Compute nullable, first for this grammar:

$\text{stmtList} ::= \epsilon \mid \text{stmt stmtList}$

$\text{stmt} ::= \text{assign} \mid \text{block}$

$\text{assign} ::= \text{ID} = \text{ID} ;$

$\text{block} ::= \text{beginof ID stmtList ID ends}$

Describe a parser for this grammar and explain how it behaves on this input:

beginof myPrettyCode

x = u;

y = v;

myPrettyCode **ends**

How does a recursive descent parser look like?

```
def stmtList =  
  if (???) {}           what should the condition be?  
  else { stmt; stmtList }  
  
def stmt =  
  if (lex.token == ID) assign  
  else if (lex.token == beginof) block  
  else error("Syntax error: expected ID or beginonf")  
...  
  
def block =  
  { skip(beginof); skip(ID); stmtList; skip(ID); skip(ends) }
```

Problem Identified

$\text{stmtList} ::= \epsilon \mid \text{stmt stmtList}$

$\text{stmt} ::= \text{assign} \mid \text{block}$

$\text{assign} ::= \text{ID} = \text{ID} ;$

$\text{block} ::= \text{beginof ID stmtList ID ends}$

Problem parsing stmtList :

- **ID** could start alternative stmt stmtList
- **ID** could **follow** stmt , so we may wish to parse ϵ that is, do nothing and return
- For nullable non-terminals, we must also compute what **follows** them

LL(1) Grammar - good for building recursive descent parsers

- Grammar is LL(1) if for each nonterminal X
 - first sets of different alternatives of X are disjoint
 - if $\text{nullable}(X)$, $\text{first}(X)$ must be disjoint from $\text{follow}(X)$ and only one alternative of X may be nullable
- For each LL(1) grammar we can build recursive-descent parser
- Each LL(1) grammar is unambiguous
- If a grammar is not LL(1), we can sometimes transform it into equivalent LL(1) grammar

Computing if a token can **follow**

$$\mathbf{first}(B_1 \dots B_p) = \{a \in \Sigma \mid B_1 \dots B_p \Rightarrow \dots \Rightarrow aw\}$$

$$\mathbf{follow}(X) = \{a \in \Sigma \mid S \Rightarrow \dots \Rightarrow \dots Xa \dots\}$$

There exists a derivation from the start symbol that produces a sequence of terminals and nonterminals of the form $\dots Xa \dots$
(the token a follows the non-terminal X)

Rule for Computing Follow

Given $X ::= YZ$ (for reachable X)

then $\text{first}(Z) \subseteq \text{follow}(Y)$

and $\text{follow}(X) \subseteq \text{follow}(Z)$

now take care of nullable ones as well:

For each rule $X ::= Y_1 \dots Y_p \dots Y_q \dots Y_r$

$\text{follow}(Y_p)$ should contain:

- $\text{first}(Y_{p+1} Y_{p+2} \dots Y_r)$
- also $\text{follow}(X)$ if $\text{nullable}(Y_{p+1} Y_{p+2} \dots Y_r)$

Compute nullable, first, follow

$\text{stmtList} ::= \epsilon \mid \text{stmt stmtList}$

$\text{stmt} ::= \text{assign} \mid \text{block}$

$\text{assign} ::= \text{ID} = \text{ID} ;$

$\text{block} ::= \text{beginof ID stmtList ID ends}$

Is this grammar LL(1)?

Conclusion of the Solution

The grammar is not LL(1) because we have

- nullable(stmtList)
- $\text{first}(\text{stmt}) \cap \text{follow}(\text{stmtList}) = \{\mathbf{ID}\}$
- If a recursive-descent parser sees **ID**, it does not know if it should
 - finish parsing stmtList or
 - parse another stmt

Table for LL(1) Parser: Example

$S ::= B \text{ EOF}$

(1)

$B ::= \epsilon \mid B (B)$

(1)

(2)

nullable: B

$\text{first}(S) = \{ (, \text{EOF} \}$

$\text{follow}(S) = \{ \}$

$\text{first}(B) = \{ (\}$

$\text{follow}(B) = \{), (, \text{EOF} \}$

empty entry:
when parsing S,
if we see),
report error

Parsing table:

	EOF	()
S	{1}	{1}	{ }
B	{1}	{1,2}	{1}

parse conflict - choice ambiguity:
grammar not LL(1)

1 is in entry because (is in follow(B)

2 is in entry because (is in first(B(B))

Table for LL(1) Parsing

Tells which alternative to take, given current token:

choice : Nonterminal x Token \rightarrow Set[Int]

$A ::=$	(1) $B_1 \dots B_p$
	(2) $C_1 \dots C_q$
	(3) $D_1 \dots D_r$

if $t \in \text{first}(C_1 \dots C_q)$ add 2 to choice(A,t)
if $t \in \text{follow}(A)$ add K to choice(A,t) where K is nullable

For example, when parsing A and seeing token t

choice(A,t) = {2} means: parse alternative 2 ($C_1 \dots C_q$)

choice(A,t) = {3} means: parse alternative 3 ($D_1 \dots D_r$)

choice(A,t) = {} means: report syntax error

choice(A,t) = {2,3} : not LL(1) grammar

General Idea when parsing nullable(A)

$$\begin{array}{l} A ::= B_1 \dots B_p \\ \quad | C_1 \dots C_q \\ \quad | D_1 \dots D_r \end{array}$$



```
def A =  
  if (token ∈ T1) {  
    B1 ... Bp  
  } else if (token ∈ (T2 ∪ TF)) {  
    C1 ... Cq  
  } else if (token ∈ T3) {  
    D1 ... Dr  
  } // no else error, just return
```

where:

$T_1 = \text{first}(B_1 \dots B_p)$

$T_2 = \text{first}(C_1 \dots C_q)$

$T_3 = \text{first}(D_1 \dots D_r)$

$T_F = \text{follow}(A)$

Only one of the alternatives can be nullable (here: 2nd)
T₁, T₂, T₃, T_F should be pairwise **disjoint** sets of tokens.

Concrete Parser Implementation

Concrete Parser Implementation

In practice, also want to ***produce*** *abstract syntax trees*,
not just recognize languages!

⇒ Make our recursive-descent methods return AST instances

How to concisely deal with parser state?

Example Language

Consider the following definitions:

enum Token:

- case** Ident(name: String)
- case** OpenParen
- case** CloseParen
- case** Plus
- case** Times

// "A + B * C" \Rightarrow Ident("A"),Plus,Ident("B"),Times,Ident("C")

enum Expr:

- case** Var(name: String)
- case** Add(lhs: Expr, rhs: Expr)
- case** Mult(lhs: Expr, rhs: Expr)

// ... \Rightarrow Add(Var("A") , Mult(Var("B"), Var("C"))))

Mutable Parser Architecture

```
class Parser(ite: Iterator[Token]):  
  
    // Parser state manipulation:  
    var cur: Option[Token] = ite.nextOption  
    def consume: Unit =  
        cur = ite.nextOption  
  
    // define parser here:  
    def expr = ...  
  
object Parser:  
    def parse(ts: Iterable[Token]): Expr =  
        val p = Parser(ts.iterator)  
        val res = p.expr // entry point  
        if p.cur.nonEmpty then fail("input not fully consumed")  
        res
```

Parsing Atomic Expressions

// Helper method:

```
def skip(tk: Token): Unit =  
  if cur != Some(tk)  
    then fail("expected " + tk + ", found " + cur)  
  consume
```

// Unambiguous "atomic" expressions:

```
def atom: Expr = cur match  
  case Some(Ident(nme)) =>  
    consume  
    Var(nme)  
  case OpenParen =>  
    consume  
    val e = expr  
    skip(CloseParen)  
    e  
  case _ => fail("expected atomic expression, found " + cur)
```

Implementing Precedence and Associativity Right

Idea: make operator-parsing methods return *lists*,
then *reassociate* these lists correctly.

```
def atom = ... // as before

def multipliedAtoms: List[Expr] = ??? // parse: * atom * atom * ...

def product: Expr = ??? // parse: atom * atom * ...

def addedProducts: List[Expr] = ??? // parse: + prod + prod + ...

def expr: Expr = ??? // parse: prod + prod + ...
```

Implementing Precedence and Associativity Right

```
def expr: Expr =  
  val p = product; val ps = addedProducts  
  ps.foldLeft(p)((l, r) ⇒ Add(l, r))  
  
def addedProducts: List[Expr] = cur match  
  case Some(Plus) ⇒  
    consume; product :: addedProducts  
  case _ ⇒ Nil  
  
def product: Expr =  
  val a = atom; val as = multipliedAtoms  
  as.foldLeft(a)((l, r) ⇒ Mult(l, r))  
  
def multipliedAtoms: List[Expr] = cur match  
  case Some(Times) ⇒  
    consume; atom :: multipliedAtoms  
  case _ ⇒ Nil
```

Note: Debugging Parsers in Scala

Very simple yet effective ways of debugging Scala implementations:

Note: Debugging Parsers in Scala

Very simple yet effective ways of debugging Scala implementations:

- ▶ Use the pprint library to display readable trees.

```
pprint.log(ExprParser.parse(ts))
```

```
Test2.scala:49 ExprParser.parse(ts): Infix(  
  lhs = Infix(  
    lhs = Infix(lhs = Var(name = "A"), op = "*", rhs = Var(name = "B")),  
    op = "_",  
    rhs = Var(name = "C")  
  ),  
  op = "/",  
  rhs = Var(name = "D")  
)
```

Note: Debugging Parsers in Scala

Very simple yet effective ways of debugging Scala implementations:

- ▶ Use the pprint library to display readable trees.

```
pprint.log(ExprParser.parse(ts))
```

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)
```

- ▶ Use the sourcecode library to display line numbers or definition names.

Note: Debugging Parsers in Scala

- Use the sourcecode library to display line numbers or definition names.

Instrumenting helper methods:

```
class TypeParser(ite: Iterator[Token], debug: Boolean):  
  
  var _cur = ite.nextOption  
  
  def cur(using n: sourcecode.Name) =  
    if debug then println(s"⇒ ${n.value}\t inspects ${_cur}")  
    _cur  
  
  def consume(using n: sourcecode.Name) =  
    if debug then println(s"⇒ ${n.value}\t consumes ${_cur}")  
    _cur = ite.nextOption  
  
  ...
```

Note: Debugging Parsers in Scala

- Use the sourcecode library to display line numbers or definition names.

Creates a *trace* of the parser execution without any modification to parser definitions!

```
⇒ expr      inspects Some(OpenParen)
⇒ expr      consumes Some(OpenParen)
⇒ expr      inspects Some(OpenParen)
⇒ expr      consumes Some(OpenParen)
⇒ expr      inspects Some(Ident(A))
⇒ expr      consumes Some(Ident(A))
⇒ exprCont  inspects Some(Oper(*))
⇒ exprCont  consumes Some(Oper(*))
⇒ expr      inspects Some(Ident(B))
⇒ expr      consumes Some(Ident(B))
⇒ exprCont  inspects Some(Oper(-))
⇒ exprCont  inspects Some(Oper(-))
⇒ exprCont  consumes Some(Oper(-))
...
```

Note: Removing Mutation

Mutation used in these slides for conciseness.

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Refer to Tutorial 4 solutions for a *purely functional* parser (working on *lists* of tokens)

```
def ty(ts: List[Token]): (Type, List[Token]) =  
  val (us, rest) = unions(ts)  
  rest match  
    case Arrow :: rest  $\Rightarrow$   
      val (t, rest2) = ty(rest); (Infix(us, Fun, t), rest2)  
    case _  $\Rightarrow$  (us, rest)
```

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To make such parser *streaming*, simply use a Scala *LazyList* instead of a *List*

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  rest match  
    case Arrow :: rest =>  
      val (t, rest2) = ty(rest); (Infix(us, Fun, t), rest2)  
    case _ => (us, rest)
```

To make such parser *streaming*, simply use a Scala *LazyList* instead of a *List*

Lookahead simpler to implement:

```
case Ident(nme) :: OpenBracket :: rest => ...
```