COMP 3711H – Fall 2014
Tutorial 11 – Revised Nov 28, 2014

1. **Open Addressing**
   Let table size be $m = 15$ (with items indexed from $0 \dots 14$).
   Use the hash function $h(x) = (x \bmod 15)$ and linear hashing to hash the items $19, 6, 18, 34, 25, 34$ in that order.
   Draw the resulting table.

2. **Universal Hashing**
   Recall the universal hash function family defined by

$$h_{a,b}(x) = \Big((ax + b) \bmod p\Big) \bmod m$$

   where $a \in Z_p^*$, $b \in Z_p$ and p is a prime with $p \geq U$. Let $p = 17$, $m = 5$. For all $x = 0, 1, \dots, 16$ write the values for $h_{1,0}(x)$. Now write all the values for $h_{2,2}(x)$. e

3. **Divide and Conquer for closest pair**
Let $P = \{p_1, p_2, \ldots, p_n\}$ be n two-dimensional points and define

$$\delta(P) = \min_{p,p' \in P : p \neq p'} d(p, p')$$

to be the closest pair distance of P.
Let X be a real value and split P on the line $x = X$ so that

$$P_L = \{p \in P \,:\, p.x \leq X\}, \quad P_R = \{p \in P \,:\, p.x > X\}.$$

Suppose you are given the closest pair distance of the two sets:

$$\delta_L = \delta(P_L) \quad \text{and} \quad \delta_R = \delta(P_R).$$

Set $\delta' = \min(\delta_L, \delta_R)$ and define the points contained by the δ' strips to the left and right of the line $x = X$ by

$$S_L = \{p \in P_L \,:\, X - p.x \leq \delta'\}, \quad S_R = \{p \in P_R \,:\, p.x - X \leq \delta'\}$$

(a) Prove that
$$\delta(P) = \min(\, \delta_L,\, \delta_R,\, d(S_L, S_R)\,)$$
where $d(P_1, P_2) = \min\{d(p_i, p_2), \,:\, p_1 \in P_1, p_2 \in P_2\}$.

(b) Suppose that you are given the values δ_L and δ_R and each of the sets P_L and P_R sorted by y-coordinate. Show how to calculate $\delta(P) = \min(\, \delta_L,\, \delta_R,\, d(S_L, S_R)\,)$ in $O(n)$ time.

Hint. In $O(n)$ time first find S_L and P_L, each sorted by y coordinate. Then show how, in $O(|S_L| + |S_R|)$ time, you can find $d(S_L, S_R)$ by using the ideas from the gridding lemma.

(c) Now construct a divide and conquer algorithm for finding $\delta(P)$ that works by
(i) Finding the median by x-coordinate of P. Set this x coordinate to be X.
(ii) Split P on X into P_L and P_R.
(iii) Recusively find $\delta(P_L)$ and $\delta(P_R)$
(iv) Use the ideas above to find $\delta(P)$ using $O(n)$ extra time

Note that the recursion will terminate when $P = \{p\}$ or $P = \{p, p'\}$. In those cases $delta(P) = \infty$ or $delta(P) = d(p, p')$ can be found in $O(1)$ time.
The correctness of the algorithm follows from (a) and (b).

Show how to implement the algorithm in $O(n \log^2 n)$ time.

(d) Can you improve this to $O(n \log n)$ time?