Machine Learning

Lecture 13: Value-Based Deep Reinforcement Learning

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This set of notes is based on the references listed at the end and internet resources.

Outline

Motivation

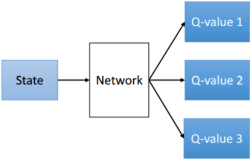
Q-learning:

- **Repeat**
  - Choose a for the state s (ϵ-greedy with $\arg\max_a Q(s,a)$)
  - Take action a, observe r and s'
  - Update:

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$
$$s \leftarrow s'$$

- Difficult to represent $Q(s,a)$ as a lookup table when the number of states is large.
  - Atari games: A state is an image with 210×160 pixels and each pixel has 128 possible color values.
  - The total number of states is: $(210 \times 160)^{128}$.
- Cannot determine actions for states never visited (such states are many).

Deep Q-Networks

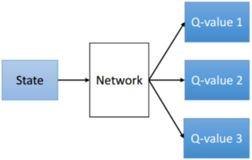


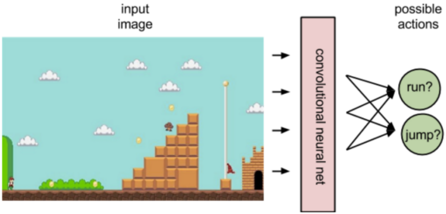
- In DQN, we aim to approximate the optimal state-action value function $Q^*(s, a)$ using a neural network with parameters θ:

$$Q(s, a; \theta) \approx Q^*(s, a)$$

- A function is defined over all possible states (images) without enumerating them.
- There is information for picking actions at any states.

Deep Q-Networks





- In DQN, we aim to approximate the optimal state-action value function $Q^*(s, a)$ using a neural network with parameters θ:

$$Q(s, a; \theta) = Q^*(s, a)$$

- **Key question**: How to learn θ from experiences with the environment?

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \ldots$$

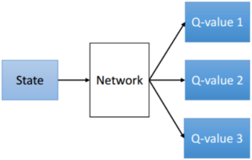
Outline

Learning DQN

Q-learning:

- **Repeat**
  - Choose a for the state s (ϵ-greedy with $\arg\max_a Q(s,a)$)
  - Take action a, observe r and s'
  - Update:

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$
$$s \leftarrow s'$$

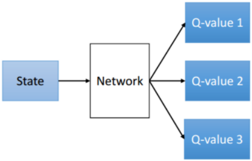


- We will learn θ iteratively, just as $Q(s,a)$ is learned iteratively in Q-learning.
- As the first step toward a DQN learning algorithm, we can change the the Q-value update step into a θ update step:

$$\theta \leftarrow \text{update}(\theta; s, a, s', r)$$

- To do so, we need come up with a objective function and update θ using its gradient.

Learning from experience tuple $e = (s, a, s', r)$



- We want to change θ so that $Q(s, a; \theta)$ gets closer to $Q^*(s, a)$.
- The problem is that we do not know the true target $Q^*(s, a)$.
- So, we use the **TD target**:

$$y = r(s, a) + \gamma \max_{a'} Q(s', a'; \theta)$$

  - Let Q' be the results if we run one value iteration on $Q(s, a; \theta)$. It is closer to Q^* than Q.
  - As explained in the previous lecture, y is an unbiased estimation of Q' based on the experience tuple.

Learning from experience tuple $e = (s, a, s', r)$

- Our target is now to push $Q(s, a; \theta)$ toward the TD target. So, we use the following loss function:

$$L(\theta) = (y - Q(s, a; \theta))^2 = ([r(s, a) + \gamma \max_{a'} Q(s', a'; \theta)] - Q(s, a; \theta))^2 \quad (1)$$

- The update rule for θ is:

$$\theta \leftarrow \theta - \alpha \nabla_\theta ([r(s, a) + \gamma \max_{a'} Q(s', a'; \theta)] - Q(s, a; \theta))^2 \quad (2)$$

where α is the learning rate.

Moving Target

- The TD target $y = r(s, a) + \gamma \max_{a'} Q(s', a'; \theta)$ depends on the parameters θ.
- Every time the parameters are updated, the target is also changed.
- So, learning θ with update rule (2) is like chasing a moving target.
- This makes learning unstable.

$$\theta_0$$

$$\theta_1 \leftarrow \theta_0 - \alpha \nabla_\theta (\underline{y_{\theta_0}} - Q(s_1, a_1; \theta))^2$$

$$\theta_2 \leftarrow \theta_1 - \alpha \nabla_\theta (\underline{y_{\theta_1}} - Q(s_2, a_2; \theta))^2$$

......

Target Network

- To deal with the moving target issue, create a **target network** that has the same structure as the DQN. Let θ^- be its parameters.
- Use the target network to compute the TD target y
- Set $\theta^- = \theta$ once in a while (every C steps).

$$\theta \leftarrow \theta - \alpha \nabla_\theta ([r(s,a) + \gamma \max_{a'} Q(s', a'; \theta^-)] - Q(s,a;\theta))^2 \tag{3}$$

$C=10$

t	θ	θ^-
1	θ_0	θ_0
2	θ_1	θ_0
:		:
10	θ_9	θ_0
11	θ_{10}	θ_{10}
:		θ_{10}

Deep Q-Learning with Target Network

Repeat:

- Take action a in current state s, observe r and s'
- Update the parameters:

$$\begin{aligned} \theta &\leftarrow \theta - \alpha \nabla_\theta([r(s,a) + \gamma \max_{a'} Q(s', a'; \theta^-)] - Q(s, a; \theta))^2 \\ s &\leftarrow s' \end{aligned}$$

- $\theta^- \leftarrow \theta$ in every C steps.

Outline

Experience Replay

- In Deep Q-Learning with target network
  - Only the latest experience tuple is used for parameter update.
- The idea of **experience replay** is:
  - Store experience tuples in a buffer
  - Use a random minibatch experience tuples for parameter update.

Deep Q-Learning with Target Network and Experience Replay

Repeat:

- Take action a in current state s, observe r and s'; add experience tuple (s, a, s', r) to a buffer D; $s \leftarrow s'$
- Sample a minibatch $B = \{s_j, a_j, s'_j, r_j\}$ from D.
- Update the parameters

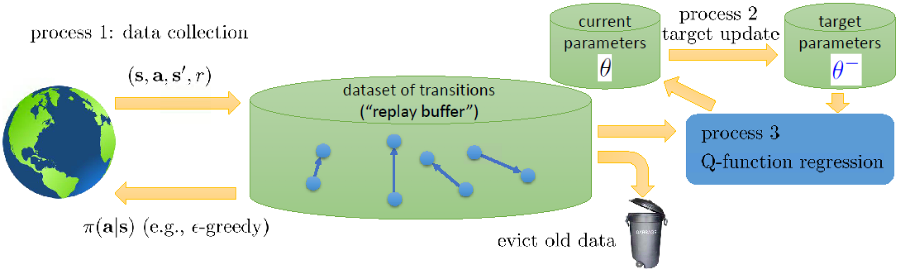
$$\theta \quad \leftarrow \quad \theta - \alpha \nabla_\theta \sum_j ([r(s_j, a_j) + \gamma \max_{a'_j} Q(s'_j, a'_j; \theta^-)] - Q(s_j, a_j; \theta))^2$$

- $\theta^- \leftarrow \theta$ in every C steps.

Advantages of Experience Replay

- With experience replay, each experience tuple is potentially used multiple times. Hence better data efficiency.
- With experience replay, each update improves $Q(s, a; \theta)$ using signals from multiple points (experience tuples). Hence faster convergence.
- Without experience replay, experiences tuples used in consecutive parameter updates are strongly correlated. Experience replay breaks the correlations and hence reduces variance of the updates.
- Experience replay avoids oscillations or divergence in the parameters.

A More General View of DQN

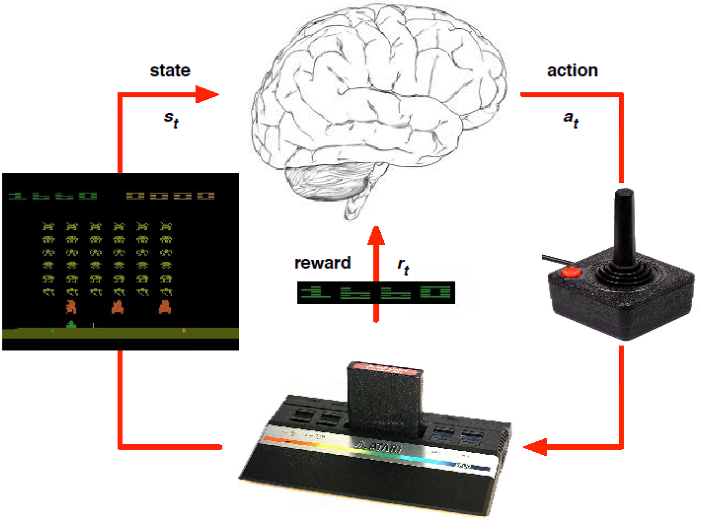


- Processes 1 and 3 run at the same pace, while Process 2 is slower.
- Old experiences are discarded when buffer is full.

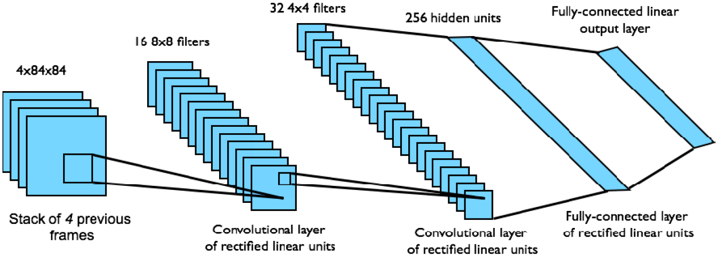
Outline

Atari Games

Space Invaders

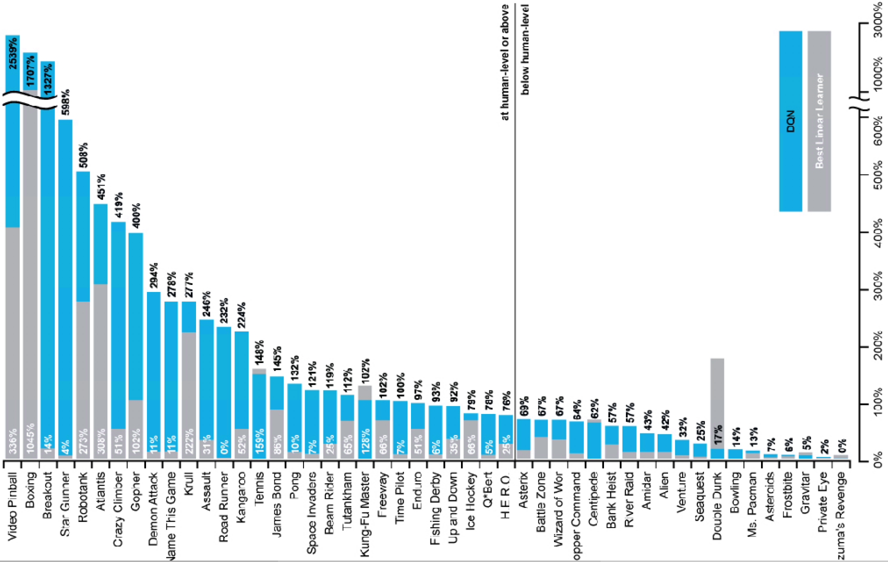


DAN in Atari



- End-to-end learning of values $Q(s; a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q(s; a)$ for 18 joystick/button positions
- Reward is change in score for that step
- Network architecture and hyperparameters fixed across all games

Comparable with or superior to human at majority of games



Results on Breakout

https://www.youtube.com/watch?v=V1eYniJ0Rnk

Outline

Overestimate of Value function

- In (3), we estimate TD Target as follows:

$$y_Q = r(s,a) + \gamma \max_{a'} Q(s', a'; \theta^-)$$

The subscript Q indicates that y_Q is the estimate used in Q-learning (DQN).

- The Q-values used are not accurate, and are influenced by random factors denoted as ω^-. So, we can write:

$$y_Q = r(s,a) + \gamma \max_{a'} Q(s', a'; \theta^-, \omega^-)$$

- If we run the process many times and take average of the y values, we get

$$E_{\omega^-}[y_Q] = r(s,a) + \gamma E_{\omega^-}[\max_{a'} Q(s', a'; \theta^-, \omega^-)]$$

Overestimate of Value function

- Ideally, we would like to first get a robust estimate of Q by take average over many run, i.e.,

$$E_{\omega^-}[Q(s', a'; \theta^-, \omega^-)]$$

and then use it to estimate the TD target:

$$\bar{y} = r(s, a) + \gamma \max_{a'} E_{\omega^-}[Q(s', a'; \theta^-, \omega^-)]$$

- On average, the y_Q overestimates the ideal value $\bar{y}$ because

$$\begin{aligned} E_{\omega^-}[y_Q] &= r(s, a) + \gamma E_{\omega^-}[\max_{a'} Q(s', a'; \theta^-, \omega^-)] \\ &\geq r(s, a) + \gamma \max_{a'} E_{\omega^-}[Q(s', a'; \theta^-, \omega^-)] \\ &= \bar{y} \end{aligned}$$

Note that. for any two random variables X_1 and X_2, $E[\max(X_1, X_2)] \geq \max(E[X_1], E[X_2])$

Double DQN

- $a^* = \arg\max_{a'} Q(s', a'; \theta, \omega)$

$$y_{DoubleQ} = r(s,a) + \gamma Q(s', a^*; \theta^-, \omega^-)$$

- Taking average, we get

$$E_{\omega^-}[y_{DoubleQ}] = r(s,a) + \gamma E_{\omega^-}[Q(s', a^*; \theta^-, \omega^-)]$$

This is closer to

$$\bar{y} = r(s,a) + \gamma \max_{a'} E_{\omega^-}[Q(s', a'; \theta^-, \omega^-)]$$

than $$E_{\omega^-}[y_Q] = r(s,a) + \gamma E_{\omega^-}[\max_{a'} Q(s', a'; \theta^-, \omega^-)]$$

An Analogy

- TD Target in DQN:

$$y_Q = r(s,a) + \gamma \max_{a'} Q(s', a'; \theta^-)$$

The second term estimates future optimal value of s' in two steps:

- Picks next action a', and
- Evaluate Q(s', a')

In both steps, we use θ^-

An Analogy

- Task: Estimate highest exam score of a class given current situation s'.
- Method 1: Ask one teacher θ^- to
  - Estimate the score of each student a': $Q(s', a'; \theta^-)$.
  - Pick best student: $a^* = \arg\max_{a'} Q(s', a'; \theta^-)$
  - Report $Q(s', a^*; \theta^-)$.

  Problem: Best student picked by teach θ^- might not the best after all. Score for a^* is overestimated, and hence max score of class is overestimated.
- Method 2:
  - Ask one teacher θ to pick best student: $a^* = \arg\max_{a'} Q(s', a'; \theta)$
  - Ask one teacher θ^- to estimate her score: $Q(s', a^*; \theta^-)$

Double DQN

- Update rule for DQN:

$$\theta \leftarrow \theta - \alpha \nabla_\theta([r(s,a) + \gamma \max_{a'} Q(s', a'; \theta^-)] - Q(s,a;\theta))^2$$

Or, equivalently:

$$\theta \leftarrow \theta - \alpha \nabla_\theta([r(s,a) + \gamma Q(s', a^*; \theta^-)] - Q(s,a;\theta))^2$$

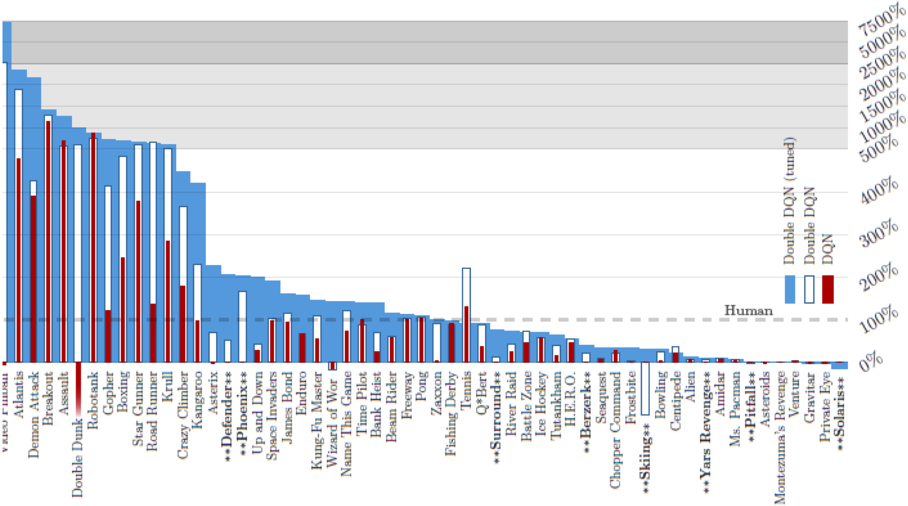
where $a^* = \arg\max_{a'} Q(s', a'; \theta^-)$.

- Update rule for Double DQN:

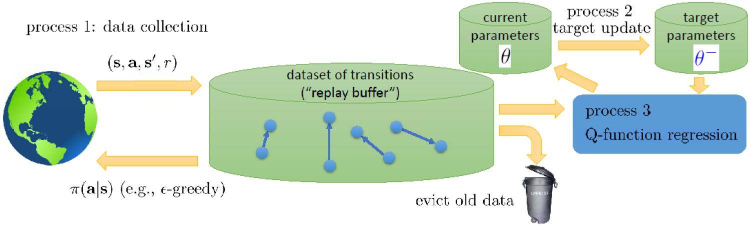
$$\theta \leftarrow \theta - \alpha \nabla_\theta([r(s,a) + \gamma Q(s', a^*; \theta^-)] - Q(s,a;\theta))^2 \tag{4}$$

where $a^* = \arg\max_{a'} Q(s', a'; \theta)$.

DQN versus Double DQN



Why Prioritized Experience Replay?



- In DQN and Double DQN, experience transitions are uniformly sampled from a replay memory for the purpose of Q-function regression. (Process 3)
- This approach does not take the significance of the experience transitions into consideration.

Prioritized Experience Replay

- The significance of a experience tuple (s, a, s', r) is measured using the **TD error**:

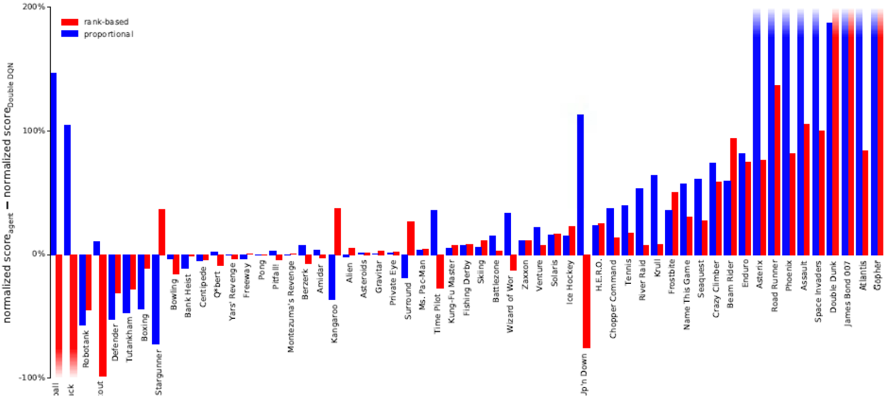
$$\delta = [r(s, a) + \gamma \max_{a'} Q(s', a'; \theta)] - Q(s, a; \theta)$$

- Let $\{s_i, a_i, s'_i, r_i)\}$ be the collection of experience tuples in the replay memory.
- For each update of θ, experience tuples are sampled using the following distribution:

$$P(i) = \frac{p_i^\alpha}{\sum_k p_k^\alpha}$$

- p_i is a function of δ_i: $p_i = |\delta_i| + \epsilon$ or $p_i = \frac{1}{rank(i)}$
- α determines how much prioritization is used.$\alpha = 0$ amounts to uniform sampling.

Prioritized Experience Replay Improves Double DQN in Most Cases

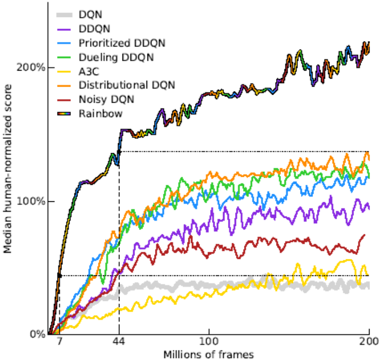


Code Example

github.com/vmayoral/basic_reinforcement_learning/blob/master/tutorial6/README.md

RAINBOW on Atari Games [1]

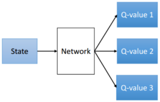
RAINBOW = Double DQN + Prioritized Replay + Dueling Networks + Multi-step Learning + Distributional RL:



[1] Hessel *et al.* 2017: Rainbow: Combining Improvements in Deep Reinforcement Learning

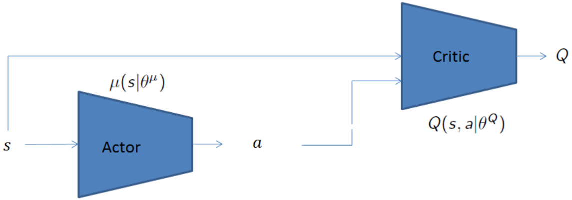
Deep Deterministic Policy Gradient (DDPG) [2]

- DQN using a neural network to represent $Q(s, a)$



$$\pi(s) = \arg\max_a Q(s, a)$$

- Cannot deal with continuous actions.
- DDPG: One network for $Q(s, a)$: **Critic** $Q(s, a|\theta^Q)$
  another network for $\pi(s) = \arg\max_a Q(s, a)$: **Actor** $\mu(s|\theta^\mu)$



[2] Lillicrap *et al.* 2016, CONTINUOUS CONTROL WITH DEEP REINFORCEMENT LEARNING

Deep Deterministic Policy Gradient (DDPG)

for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $\boxed{y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})}$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\boxed{\nabla_{\theta^\mu} J \approx \frac{1}{N}\sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}}$$

- DQN: $y = r(s,a) + \gamma \max_{a'} Q(s', a'; \theta)$
- Actor update: $\max_{\theta^\mu} J = Q(s, \mu(s|\theta^\mu)|\theta^Q)$

Deep Deterministic Policy Gradient (DDPG)

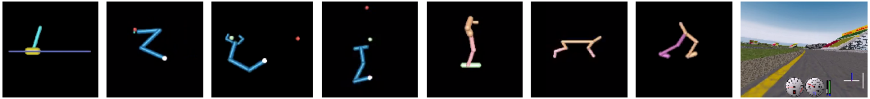


Figure 1: Example screenshots of a sample of environments we attempt to solve with DDPG. In order from the left: the cartpole swing-up task, a reaching task, a gasp and move task, a puck-hitting task, a monoped balancing task, two locomotion tasks and Torcs (driving simulator). We tackle all tasks using both low-dimensional feature vector and high-dimensional pixel inputs. Detailed descriptions of the environments are provided in the supplementary. Movies of some of the learned policies are available at https://goo.gl/J4PIAz.

- **Twin Delayed DDPG (TD3)**: Substantially improved version of DDPG https://spinningup.openai.com/en/latest/algorithms/td3.html

References

- Sergey Levine. Deep Reinforcement Learning, http://rail.eecs.berkeley.edu/deeprlcourse/, 2018.
- David Silver. Tutorial: Deep Reinforcement Learning, http://hunch.net/∼beygel/deep_rl_tutorial.pdf, 2016.
- Tambet Matiisen. Demystifying Deep Reinforcement Learning. https://ai.intel.com/demystifying-deep-reinforcement-learning/, 2015.
- Watkins. (1989). Learning from delayed rewards: introduces Q-learning
- Riedmiller. (2005). Neural fitted Q-iteration: batch-mode Q-learning with neural networks
- Mnih et al. (2013). Human-level control through deep reinforcement learning: Q-learning with convolutional networks for playing Atari.
- Van Hasselt, Guez, Silver. (2015). Deep reinforcement learning with double Q-learning: a very effective trick to improve performance of deep Q-learning.
- Schaul, T., Quan, J., Antonoglou, I., and Silver, D. (2016). Prioritized experience replay. In ICLR, 2016.
- Wang, Schaul, Hessel, van Hasselt, Lanctot, de Freitas (2016). Dueling network architectures for deep reinforcement learning: separates value and advantage estimation in Q-function.