# Computer Security

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**COMP4631**

# Lecture 06: One-Key Block Ciphers

## Outline of this Lecture

- One-key block ciphers and their security
- Simple substitution ciphers and their security

# Ciphers and Their Classifications

**Definition:** A cipher is an encryption-and-decryption system for hiding information. The Julius Caesar cipher covered in Lecture 5 is an example.

**Classifications:**

- Ciphers are classified into two types:
  **"one-key ciphers"** and **"two-key ciphers"**.
- One-key ciphers are further classified into another two types:
  **"one-key block ciphers"** and **"one-key stream ciphers"**.

We will introduce and study them later.

# One-Key Block Ciphers

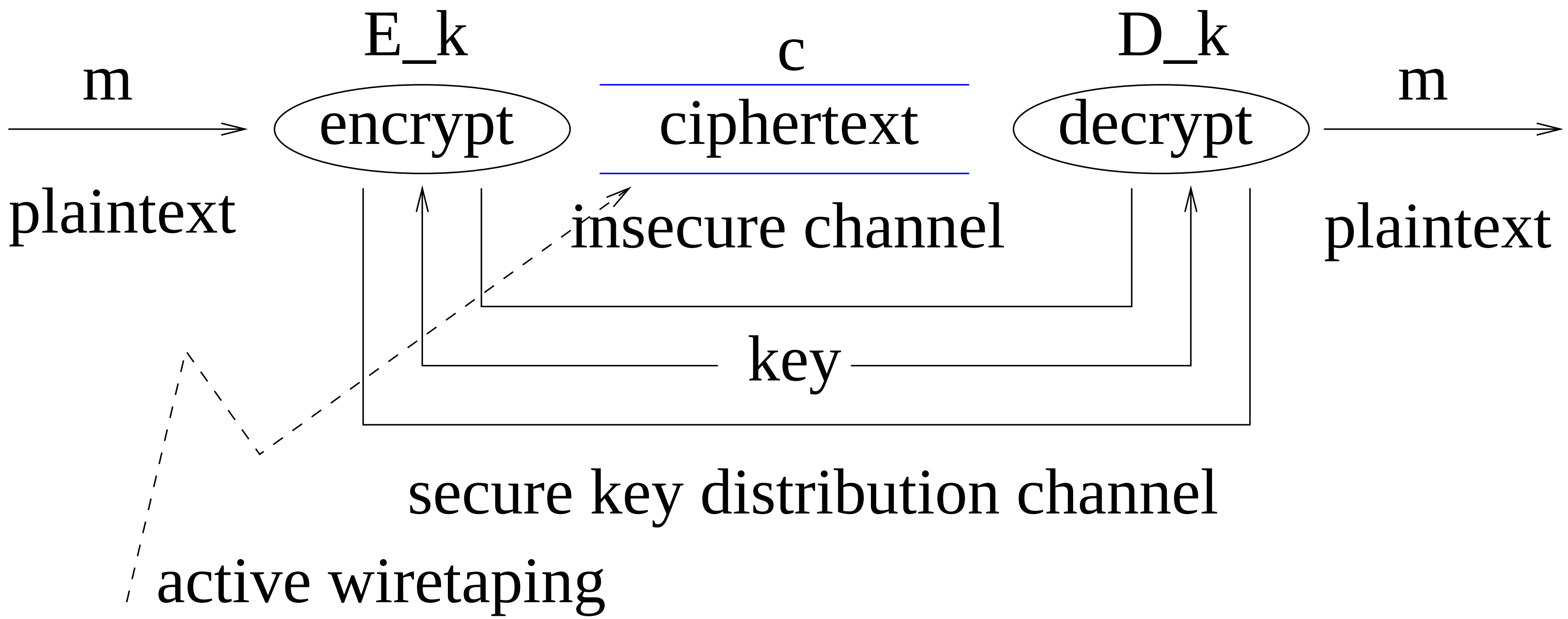
# One-key Block Ciphers

A 5-tuple $(\mathcal{M}, \mathcal{C}, \mathcal{K}, E_k, D_k)$, where

- $\mathcal{M}$, $\mathcal{C}$, $\mathcal{K}$ are respectively the plaintext space, ciphertext space, and key space;
- Any $k \in \mathcal{K}$ could be the encryption and decryption key;
- $E_k$ and $D_k$ are encryption and decryption transformations with $D_k(E_k(m)) = m$ for each $m \in \mathcal{M}$.
- Encryption: $c = E_k(m)$, where $E_k$ is usually applied to blocks or characters of the plaintext $m$.
- Decryption: $m = D_k(c)$, where $D_k$ is usually applied to blocks or characters of the ciphertext $c$.

The ciphertext $c = E_k(m)$ depends only on $k$ and $m$, is time-independent.

# Classical Information Channel



# Attacks on One-Key Block Ciphers

**Ciphertext-only attack:** A cryptanalyst determines the decryption transformation $D_k$ or key $k$, or the plaintext from intercepted ciphertext $c$.

**Known-plaintext attack:** A cryptanalyst determines the decryption transformation $D_k$ or key $k$, from a ciphertext-plaintext pair $(c, m)$.

# Security Requirements for One-key Block Ciphers

- The security should depend on the confidentiality of the key, so it is usually assumed that the algorithms $E_k$ and $D_k$ are known to a cryptanalyst.
- It should be computationally infeasible for a cryptanalyst to determine the plaintext $m$, given a ciphertext $c$.
- It should be computationally infeasible for a cryptanalyst to systematically determine the decryption transformation $D_k$ or key $k$ from intercepted ciphertext $c$, even if the corresponding plaintext $m$ is known.

**Question:** How do you design a one-key block cipher meeting these requirements?

# Simple Substitution Ciphers

A Special Type of One-Key Block Ciphers

# Description of Simple Substitution Ciphers

Let $f$ be a 1-to-1 mapping from alphabet $A$ to alphabet $B$. It is a 5-tuple $(\mathcal{M}, \mathcal{C}, \mathcal{K}, E_k, D_k)$, where

- $\mathcal{M} = A^*$ and $\mathcal{C} = B^*$, i.e., all finite strings of characters.
- $\mathcal{K}$ is the set of all possible $f$.
- $k = f \in \mathcal{K}$ is the encryption and decryption key;
- For a message $m = m_0 m_1 m_2 \cdots$,

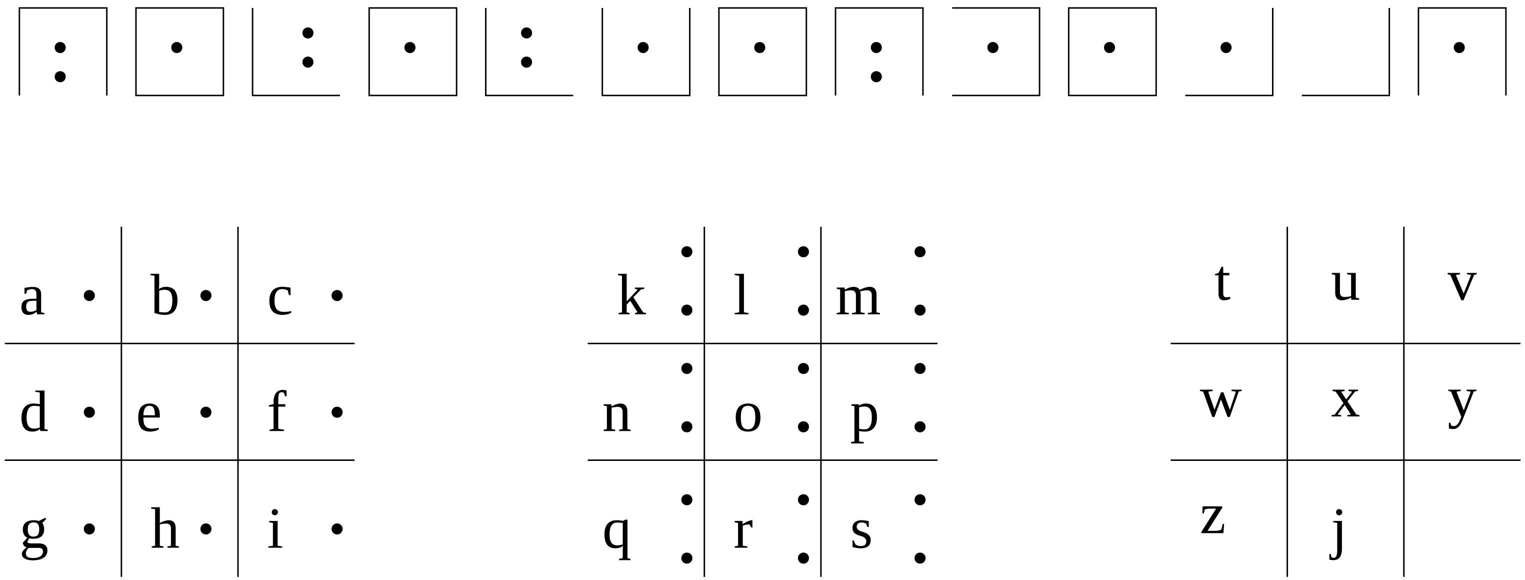
$$E_k(m) = f(m_0) f(m_1) f(m_2) \cdots$$

- For a ciphertext $c = c_0 c_1 c_2 \cdots$,

$$D_k(c) = f^{-1}(c_0) f^{-1}(c_1) f^{-1}(c_2) \cdots$$

# First Example of Simple Substitution Ciphers

**Example:** Let $A$ be the English alphabet and $B$ the set of the 26 characters given in the following figure. The following mapping $f$ defines a simple substitution cipher, i.e., the churchyard cipher:



# Second Example of Simple Substitution Ciphers

Let $A = B$ be the English alphabet. We identify letters with digits:

$$\begin{array}{cccccc} a & b & c & \cdots & y & z \\ 0 & 1 & 2 & \cdots & 24 & 25 \end{array}$$

Take any $(k_0, k_1)$ with $\gcd(k_0, 26) = 1$ and $0 \le k_0 \le 25$, define the 1-to-1 mapping $f$ by

$$f(a) = (ak_0 + k_1) \bmod 26.$$

It is called the **affine cipher**, where the key $k = (k_0, k_1)$ or $k = f$.

If $(k_0, k_1) = (1, 3)$, it is the **Caesar cipher**. RENAISSANCE is encrypted as UHQDLVVDQFH.

**Question:** Why should $\gcd(k_0, 26) = 1$?

# The Security of Simple Substitution Ciphers

**Claim 1:** A simple substitution cipher is **not** secure with respect to known-plaintext attacks.

**Claim 2:** A simple substitution cipher is insecure with respect to ciphertext-only attacks!

**Question: Why** a simple substitution cipher is **insecure** with respect to ciphertext-only attacks?

# Frequency Distribution of Single English Letters

| $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ | $I$ | $J$ | $K$ | $L$ | $M$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8.0 | 1.5 | 3.0 | 4.0 | 13.0 | 2.0 | 1.5 | 6.0 | 6.5 | 0.5 | 0.5 | 3.5 | 3.0 |

| $N$ | $O$ | $P$ | $Q$ | $R$ | $S$ | $T$ | $U$ | $V$ | $W$ | $X$ | $Y$ | $Z$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7.0 | 8.0 | 2.0 | 0.2 | 6.5 | 6.0 | 9.0 | 3.0 | 1.0 | 1.5 | 0.5 | 2.0 | 0.2 |

**Remark:** In the table, 8.0 means 8.0%. E appears the most, and Z the least. The uneven distribution of letters makes it easy to break simple substitution ciphers.

# Frequency Distribution of Digraphs & Trigraphs

**Definition:** A digraph (also called bigram) is a sequence of two English letter, e.g., **th**
A trigraph is a sequence of three English letters, e.g., **the**

**The most frequent digraphs:** th, he, in, er, an, re, on, at, en, nd, ed, or, es, ti, te, it, is, st, to, ar, of, ng, ha, al

**The most frequent trigraphs:** the, and, tha, hat, ent, ion, for, tio, has, edt, tis, ers, res, ter, con, ing, men, tho

**Remark:** Some digraphs and trigraphs do not appear at all.

**Question:** What do the uneven distributions (of single letters, digraphs and trigraphs) mean to the security of simple substitution ciphers?

# Redundancy in Human Languages

**Language redundancy:** E.g., in "hwever", "hoever" and "howeer", you can easily determine the missing letters.

**Comment:** Shannon information theory can be used to give a rigorous measure of redundancy in a human language.
See, Denning, Cryptography and Data Security, 1982.

**Why redundancy in human languages?**

**Comment:** The uneven distributions of single English letters and digraphs are due to the redundancy in a human language.

**Comment:** The amount of redundancy in a human language affects the security of a one-key block cipher.

**Remark:** Chinese has less redundancy than English!

# Security of Simple Substitution Ciphers

**Claim:** Simple substitution ciphers are not secure with respect to ciphertext-only attacks. **Why?**

**Claim:** For English, about 28 letters in a piece of ciphertext are needed to "break" a simple substitution cipher.

See, Denning, Cryptography and Data Security, 1982.

# Breaking Simple Substitution Ciphers

**Ciphertext-only attack:** Given a piece of ciphertext $c$ encrypted with a simple substitution cipher, we want to determine the key $k = f$ that is a 1-to-1 mapping from the English alphabet $A$ to another set $B$ of characters.

**Cryptanalysis:** For the given piece of ciphertext $c$, we compute the frequency distributions of letters and digraphs in $B$, and then compare them with those of the English letters, and try to match them. If the number of characters in $c$ is long enough (in theory, 28 characters should work), the key is uniquely determined.

**Exercise:** On the course web page there are ten pieces of ciphertext.

# Summary

- We defined one-key block ciphers and talked about their security issues in general.
- We discussed simple substitution ciphers, and realized that a cipher may be insecure if it is not well designed.
- In Appendix A, a "secure" cipher (the one-time pad) is introduced, but is not "practical".

**Question:** Is there any **secure** and **practical** cipher?

This is a very hard question!
We will spend three more lectures on one-key ciphers.

# Appendix A: The One-Time Pad

# The One-Time Pad

**Question:** Is there any unbreakable cipher?

**One-time pad:**

- Each message is encoded into a binary string (e.g., using the ASCII code).
- Generate a secret key, which is a **"random binary string"** with the same length as the message.
- The ciphertext is the bitwise exclusive-or of the message with the secret key (the encryption process).
- The message is the bitwise exclusive-or of the ciphertext with the secret key (the decryption process).
- A secret key is used only for one message and is then discarded (a special key usage policy).

# Questions Regarding the One-Time Pad

**Question:** How do you prove that it is unbreakable?

Let $m = m_\ell m_{\ell-1} \cdots m_2 m_1$ be the message, and let $k = k_\ell k_{\ell-1} \cdots k_2 k_1$ be the secret key. Then the ciphertext

$$c = m \oplus k = (m_\ell \oplus k_\ell)(m_{\ell-1} \oplus m_{\ell-1}) \cdots (m_2 \oplus k_2)(m_1 \oplus k_1).$$

**Intuitive Proof:** Let $x$ be a random bit and let $y$ be a message bit. Define $z = x \oplus y$. Knowing $z$ does not give you any information about $y$, as $y = 0$ and $y = 1$ are equally likely (due to the fact $x$ is a random bit).

**Question:** Is this a practical cipher?

- How to generate a long random binary string?
- How to distribute the secret key to the other party?

# Appendix B: Transposition Ciphers

# Permutations of $\mathbf{Z}_d$ for Transposition Ciphers

Let $\mathbf{Z}_d$ denote the set of integers 0 through $d-1$. A **permutation** $f$ of $\mathbf{Z}_d$ is a one-to-one function from $\mathbf{Z}_d$ to itself.

**Question:** What is the total number of permutations on $\mathbf{Z}_d$?

**Example:** Let $d = 4$ and define $f$ by

$$\begin{array}{rcccc} i: & 0 & 1 & 2 & 3 \\ f(i): & 2 & 0 & 3 & 1 \end{array}$$

Then $f$ is a permutation of $\mathbf{Z}_4$.

**Question:** What is the inverse permutation $f^{-1}$?

# Description of Transposition Ciphers

Let $f$ be a permutation of $\mathbf{Z}_d$. It is a 5-tuple $(\mathcal{M}, \mathcal{C}, \mathcal{K}, E_k, D_k)$, where

- $\mathcal{M} = \mathcal{C} =$ set of all finite strings of English letters.
- $\mathcal{K}$ is the set of all possible pairs $(d, f)$.
- $k = (d, f) \in \mathcal{K}$ is the secret key; and
- A message is divided into blocks of length $d$. For each message block $m = m_0 \cdots m_{d-1}$,

$$E_k(m) = m_{f(0)} \cdots m_{f(d-1)}$$

- For each ciphertext block $c = c_0 \cdots c_{d-1}$,

$$D_k(c) = c_{f^{-1}(0)} \cdots c_{f^{-1}(d-1)}$$

# An Example of Transposition Ciphers

**Example:** Let $d = 4$ and define $f$ by

$$\begin{array}{rcccc} i: & 0 & 1 & 2 & 3 \\ f(i): & 2 & 0 & 3 & 1 \end{array}$$

The message RENAISSANCES is broken into groups of 4 letters and encrypted into

$$\begin{array}{lclll} \text{position} & & 0123 & 0123 & 0123 \\ m & = & \text{RENA} & \text{ISSA} & \text{NCES} \\ E_k(m) & = & \text{NRAE} & \text{SIAS} & \text{ENSC.} \end{array}$$

**Exercise:** Decrypt the ciphertext NRAESIASENSC.

# The Security of Transposition Ciphers

**Question:** How do you detect a cipher as a transposition cipher?

**Question:** Is a transposition cipher secure with respect to known-plaintext attacks?

**Question:** Is a transposition cipher secure with respect to ciphertext-only attacks? If yes, justify your conclusion. If no, demonstrate how to **break** it.

**Remark:** These are left to students as exercises.