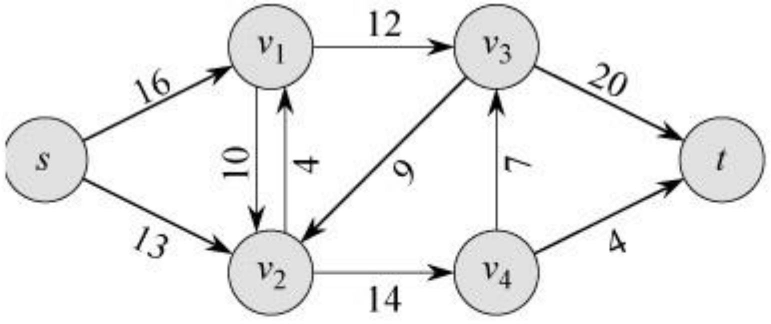
Two-Way Edges or Antiparallel edges

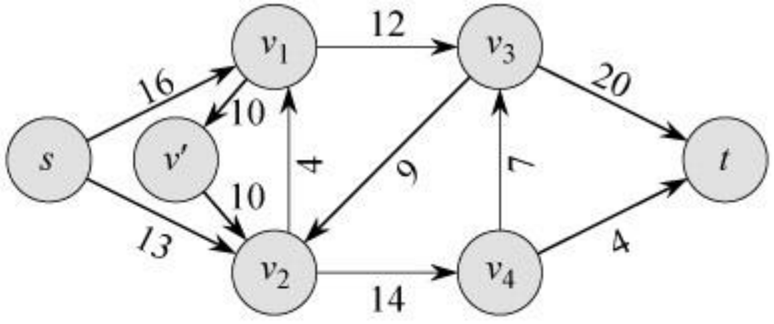
- Vis-A-Vis the solution to Question 2 part (2) in Homework 4 some students have asked whether an input to the Max-Flow problem can have capacity constraints in BOTH directions, i.e., the graph will, for some pair u,v, contain both $(u, v), (v, u) \in E$ with capacity constraints in both directions, e.g., between v_1, v_2 in (a) below
- We call these *Two-Way* or *Antiparallel* edges and they model two-way traffic.
- The answer is YES, the input to the Max-Flow problem IS allowed to have Two-Way edges.
- If you work through the definitions of residual capacities you'll note that they do, formally allow this and are well-defined so

you may assume that the Ford-Fulkerson algorithm can handle two-way edges

- We emphasize that it is the INPUT graph, with capacities, that has two way edges.
- In the actual flow solution, positive flow will only go in one direction



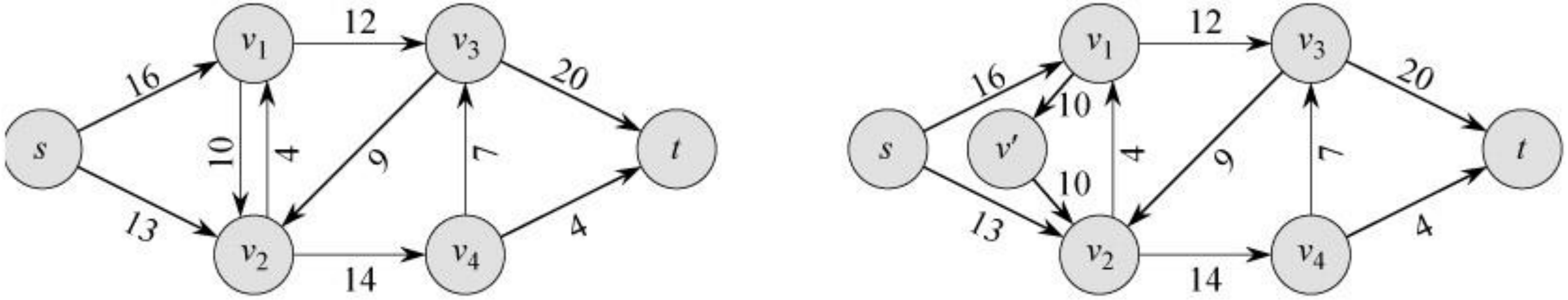
(a)



(b)

How to modify Ford-Fulkerson so that all edges are one-way

- Although the Ford-Fulkerson algorithm does work with two-way edges you might find them intuitively ``uncomfortable''. Below, we sketch a technique that allows you to modify problems with two way edges by an equivalent problem with only one way edges.
- The idea is, for each two way pair, to replace the edge in one of the directions by creating an auxiliary vertex and a path going through that vertex (see (b) below).
- This transforms the problem into one in which all edges go in only one direction.
- The total number of vertices and edges in the transformed graph are less than twice that of the original graph and a max-flow in the transformed graph corresponds to a max-flow in the original graph and vice-versa. This means that the problem for the original graph can be solved (in asymptotically the same time) by solving the problem in the simplified transformed graph which only has edges in one direction.



(a) (b)