COMP 3711H – Fall 2014
Tutorial 8

1. Let G be a connected undirected graph with weights on the edges. Assume that all the edge weights are distinct. Prove that the edge with the smallest weight must be included in any minimum spanning tree of G. *You have to prove this from first principles, i.e., you are not allowed to use the Lemmas proven in class or assume the correctness of Kuskal's or Prim's algorithm.*

2. Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 32 \rangle$.

3. Consider the following algorithm for the chain matrix multiplication problem:

   Suppose $n > 2$ and p_i is the smallest of $p_0, \cdots, p_n$. Break the product after A_i, and recursively apply this procedure to the product $A_1..A_i$ and $A_{i+1}..A_n$.

   Does this algorithm work (i.e, does it minimize the number of multiplications needed)? If yes, prove your conclusion. If not, give a counter-example.

4. Let $G = (V, E)$ be a connected undirected graph in which all edges have weight either 1 or 2. Give an $O(|V| + |E|)$ algorithm to compute a minimum spanning tree of G. Justify the running time of your algorithm. (*Note:* You may either present a new algorithm or just show how to modify an algorithm taught in class.)

5. Give an $O(n^2)$ time dynamic programming algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers (i.e, each successive number in the subsequence is greater than or equal to its predecessor). For example, if the input sequence is $\langle 5, 24, 8, 17, 12, 45 \rangle$, the output should be either $\langle 5, 8, 12, 45 \rangle$ or $\langle 5, 8, 17, 45 \rangle$.

   Hint: Let $d[i]$ be the length of the longest increasing subsequnce whose last item is item i.