COMP 170 Discrete Mathematical Tools for CS
2006 Fall Semester – Challenge Problems in Probability
Distributed: Dec 1, 2006 – Due: by Dec 7, 2006 *at end of class*

The top of your submission should contain (i) your name, (ii) your student ID #, (iii) your email address and (iv) your tutorial section.

Note that these are challenge problems and do not have to be handed in unless you want to.

2nd Note: Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.

As mentioned in class, sometimes probability theory gives us unexpected answers. The two problems on this sheet are famous probability "paradoxes".

Problem 1: I have two boxes in front of me and choose some value x. Then I flip a fair coin. If the coin comes up heads, I put x dollars in the left box and $2x$ dollars in the right one. If the coin comes up tails, I put x dollars in the right box and $2x$ dollars in the left one.
(Note: To simplify this problem you should assume that x is any integral power of 2, i.e., $x = 2^i$ where $i \in \{0, -1, 1, -2, 2, -3, 3, \ldots\}$. You should also assume that there are coins in every possible denomination, $x = 2^i$, no matter how small or large.)

We now play the following game: You open the left box. You can either keep the money you see, or decide to open the right box and keep the amount there. Which should you do?

Let's perform the calculation. Let z be the amount of money that you see in the left box and X the random variable denoting the amount of money in the right box. Then, with probability $\frac{1}{2}$, $X = \frac{z}{2}$ and with probability $\frac{1}{2}$, $X = 2z$. So

$$E(X) = \frac{1}{2} \cdot \frac{z}{2} + \frac{1}{2} \cdot (2z) = \frac{5}{4} z > z.$$

This means that *it is always better for you to switch and take the money in the right box!*
This is obviously **impossible**, since the left and right box have equal probability of containing $2x$ dollars so switching should not help you.

What went wrong here?

[*Note: If after thinking about this for a while you don't see the problem, please come speak with the instructor to get hints.*]

Problem 2: You are a contestant on a game show. In this game show, there are three curtains. Behind one of the curtains is a new car, and behind the other two are cans of Spam. You get to pick one of the curtains. After you pick one of the curtains, the host, who we assume knows where the car is, reveals what is behind one of the curtains that you did not pick, showing you some cans of Spam. He then asks you if you would like to switch your choice of curtain. Should you switch? Why or why not? To get credit for this answer you neeed to *fully* mathematically work out the probability that switching gets you the car.

[*Note: This is a well-known tricky problem. If you do find a solution somewhere for this, you should (i) say where you found the solution and (ii) write out the proof in your own words/symbols.]*