HKUST – Department of Computer Science and Engineering
COMP170: Discrete Math Tools for CS – Spring 2010

#	LS

Final Examination

Date: Monday May 31, 2010 Time: 16:30- 19:00

Name: _____	Student ID: _____
Email: _____	Lecture and Tutorial: _____

Instructions

- This is a closed book examination. It consists of ?? pages and ?? questions.
- Please write your name, student ID, email, lecture and tutorial sections on this page.
- For each subsequent page, please write your student ID at the top of the page in the space provided.
- Please sign the honor code statement on page 2.
- Answer all the questions within the space provided on the examination paper. You may use the back of the pages for your rough work. The last three pages are scrap paper and may also be used for rough work. Each question is on a separate page (and sometimes has an extra page for you to do work on). This is for clarity and is not meant to imply that each question requires a full page answer. Many can be answered using only a few lines.
- Only use notation given in class. Do not use notation that you have learnt outside of this class that is nonstandard.
- Calculators may be used for the examination.

Student ID: _____

As part of HKUST's introduction of an honor code, the HKUST Senate has recommended that all students be asked to sign a brief declaration printed on examination answer books that their answers are their own work, and that they are aware of the regulations relating to academic integrity. Following this, please read and sign the declaration below.

I declare that the answers submitted for this examination are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University regulations governing academic integrity.

Student's Name: _____

Student's Signature: _____

Student ID: _____

Definitions and Formulas: This page contains some definitions used in this exam and a list of formulas (theorems) that you may use in the exam (without having to provide a proof). Note that you might not need all of these formulas on this exam.

Definitions:

1. $N = \{0, 1, 2, 3, \ldots\}$, the set of non-negative integers.
2. $Z^+ = \{1, 2, 3, \ldots\}$, the set of positive integers.
3. R is the set of *real numbers*.
4. R^+ is the set of positive *real numbers*.

Formulas:

1. $\binom{n}{i} = \frac{n!}{i!\,(n-i)!}$
2. If $0 < i < n$ then $\binom{n}{i} = \binom{n-1}{i-1} + \binom{n-1}{i}$.
3. $\neg(p \wedge q)$ is equivalent to $\neg p \vee \neg q$.
4. $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$.
5. $\sum_{i=1}^{n-1} i = n(n-1)/2$
6. $\sum_{i=1}^{n-1} i^2 = \frac{2n^3-3n^2+n}{6}$
7. If $r \neq 1$ then $\sum_{i=0}^{n-1} r^i = \frac{1-r^n}{1-r}$.
8. If $r \neq 1$ then $\sum_{i=0}^{n} ir^i = \frac{nr^{n+2}-(n+1)r^{n+1}+r}{(1-r)^2}$.
9. The inclusion-exclusion theorem:

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{k=1}^{n}(-1)^{k+1} \sum_{\substack{i_1,i_2,\ldots,i_k: \\ 1\le i_1<i_2<\ldots<i_k\le n}} P(E_{i_1} \cap E_{i_2} \cap \ldots \cap E_{i_k})$$

10. If X is a random variable, then $E(X)$ denotes the *Expectation of* X and $V(X) = E\left((X - E(X))^2\right)$ denotes the *Variance of* X.
11. $f(n) = O(g(n))$ if there exist some $N > 0$ and positive constant c such that $\forall n > N$, $f(n) \le c \cdot g(n)$.
12. $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$ and $g(n) = O(f(n))$.