

Proof by contradiction ϵ

Variations

Rule 12: If from $p(x), \neg q(x)$, we
can derive $r(x)$ and $\neg r(x)$
conclude $p(x) \Rightarrow q(x)$

Variant 1: $r(x) = p(x)$

If from $p(x), \neg q(x)$ we
can derive $\neg p(x)$

conclude $p(x) \Rightarrow q(x)$

Variant 2: $r = \text{a known fact}$

If from $p(x), \neg q(x)$ we can
derive $\neg r$

conclude $p(x) \Rightarrow q(x)$

Variant 3: r is something arises
in proof

Variant 4:

{ If from ~~$\neg q(x)$~~ $\neg q(x)$ we
can derive $\neg p(x)$
Conclude $p(x) \Rightarrow q(x)$

$\rightarrow \neg q(x) \Rightarrow \neg p(x)$

If $p(x)$ is always true (e.g. $1+1=2$)

Rule 12 becomes:

If from $\neg q(x)$, we can
derive r , $\neg r$
Conclude q

Used when proving " $\sqrt{2}$ not rational".