

# The Birthday Paradox

Suppose 25 people are in a room. What is the probability that at least two of them share a birthday?

Less than  $1/2$ ?

Actually it's greater than  $1/2$ .

We will see the analysis of the problem of calculating the probability of event:

$A_n$ : There are  $n$  people in a room and at least two of them share a birthday.

(We will assume that a year has 365 days and there are no twins in the room.)

This will be very similar to the analysis of  
hashing  $n$  keys into a table of size 365.

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Sample space  $S$ : All  $n$ -tuples  $(l_1, l_2, \dots, l_n)$  where the  $l_i$  are birthdays, i.e. numbers in  $[1, 365]$ .

$$|S| = 365^n$$

Now let  $B_n$  be the event that no two people share a birthday.

The number of such outcomes is

$$365 \times 364 \times \dots \times (365 - (n - 1)) = 365^n$$

So, by Theorem 5.12 seen in class,

$$P(B_n) = \frac{365^n}{365^n}$$

Note: Easy to calculate using  $P(B_1) = 1$ ,

$$P(B_{n+1}) = P(B_n) \times \frac{365-n}{365}$$

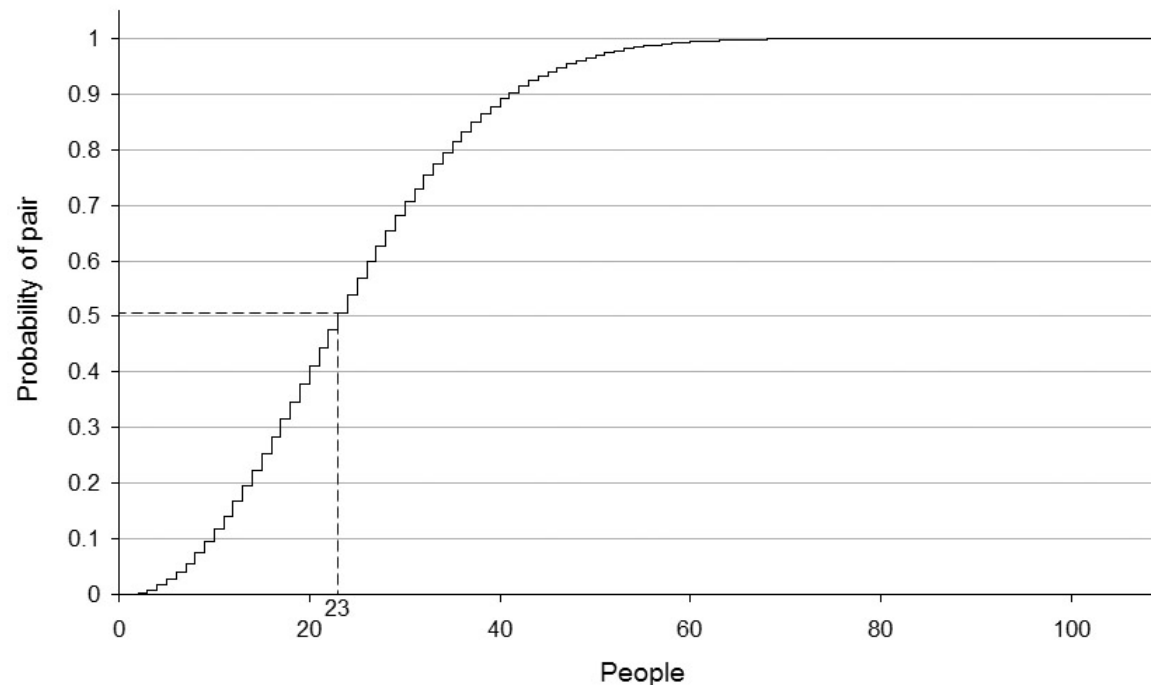
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$A_n$ : At least two of the  $n$  people share a birthday.

$B_n$ : No two of the  $n$  people share a birthday.

$A_n$  and  $B_n$  are complementary so

$$P(A_n) = 1 - P(B_n) = 1 - \frac{365^n}{365^n}$$



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$n$	$A_n$	$B_n$	$n$	$A_n$	$B_n$
1	0.00000000	1.00000000	16	0.28360400	0.71639599
2	0.00273972	0.99726027	17	0.31500766	0.68499233
3	0.00820416	0.99179583	18	0.34691141	0.65308858
4	0.01635591	0.98364408	19	0.37911852	0.62088147
5	0.02713557	0.97286442	20	0.41143838	0.58856161
6	0.04046248	0.95953751	21	0.44368833	0.55631166
7	0.05623570	0.94376429	22	0.47569530	0.52430469
8	0.07433529	0.92566470	23	0.50729723	0.49270276
9	0.09462383	0.90537616	24	0.53834425	0.46165574
10	0.11694817	0.88305182	25	0.56869970	0.43130029
11	0.14114137	0.85885862	26	0.59824082	0.40175917
12	0.16702478	0.83297521	27	0.62685928	0.37314071
13	0.19441027	0.80558972	28	0.65446147	0.34553852
14	0.22310251	0.77689748	29	0.68096853	0.31903146
15	0.25290131	0.74709868	30	0.70631624	0.29368375