COMP 170 Discrete Mathematical Tools for CS
2010 Spring Semester – Challenge Problem # 2
Distributed: March 4, 2010 – Due: March 11, 2010

Your solutions should contain (i) your name, (ii) your student ID #, (ii) your email address and (iv) your tutorial section.

Your solutions should be submitted before 5PM on March 11, 2010, in the collection bin in front of Room 4213A (this is near the TA labs).
Note that there is no assignment due this week.

Analyzing the HKID card check digit scheme.

Hong Kong ID cards are in the format $LD_1D_2D_3D_4D_5D_6(C)$ where

- L is a letter (sometimes two letters but we will assume one letter)
- D_i is a digit from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and
- C is a *check digit* from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A\}$, A denoting the value 10, calculated from $LD_1D_2D_3D_4D_5D_6$.

C is calculated as follows.

First assign every letter L a number L' by setting $A = 1$, $B = 2$, ... $Z = 26$. For an ID card $\mathbf{LD_1D_2D_3D_4D_5D_6}(\mathbf{C})$ to be valid it must satisfy

$$(8L' + 7D_1 + 6D_2 + 5D_3 + 4D_4 + 3D_5 + 2D_6 + C) \bmod 11 = 0. \qquad (1)$$

Equivalently,

$$C = 11 - \Big((8L' + 7D_1 + 6D_2 + 5D_3 + 4D_4 + 3D_5 + 2D_6) \bmod 11\Big). \qquad (2)$$

As an example **D246824**(**8**) is a legal Hong Kong ID number, since D corresponds to 4 and

$$\begin{aligned} 8*4+7*2+6*4+5*6+4*8+3*2+2*4+1*8 &= 154 \\ &= 14*11 \equiv 0 \pmod{11} \end{aligned}$$

Similarly **D246823**(**A**) is legal because the check digit A corresponds to 10 and

$$8*4+7*2+6*4+5*6+4*8+3*2+2*3+1*10 = 154 = 0 \bmod 11$$

as well. **D246824**(**8**), **D246834**(**5**) and **Z246834**(**5**) are also legal Hong Kong ID numbers.

On the other hand **D246834(8)** is *not* a valid HKID number because it fails the check digit test;

$$\begin{aligned} 8*4+7*2+6*4+5*6+4*8+3*3+2*4+1*8 &= 157 \\ &\equiv 3 \pmod{11} \neq 0. \end{aligned}$$

The two most common errors when transcribing ID numbers are

- **Single Digit Errors**, i.e., making a mistake in one digit from $D_1D_2D_3D_4D_5D_6$. An example would be writing $D2468\mathbf{3}4(8)$ instead of $D2468\mathbf{2}4(8)$
- **Two Digit Transpositions**, i.e., swapping two different digits from $D_1D_2D_3D_4D_5D_6$. An example would be writing $D24\mathbf{2}8\mathbf{6}4(8)$ instead of $D24\mathbf{6}8\mathbf{2}4(8)$

<u>**Problem 1:**</u> Prove that the HKID check digit scheme will catch all Single Digit errors and all Two Digit Transpositions. That is, a valid HKID number written with either a Single Digit Error or a Two Digit Transposition will always be invalid.

<u>**Problem 2:**</u> Using base 11 in Equations (1) and (2) is very awkward since it sometimes requires using a *letter A* as a check *digit.* Explain why the designers of the HKID check digit scheme did not use base 10 instead. That is, why didn't they replace Equations (1) and (2) with

$$(8L' + 7D_1 + 6D_2 + 5D_3 + 4D_4 + 3D_5 + 2D_6 + C) \bmod 10 = 0 \tag{3}$$

and

$$C = 10 - \Big((8L' + 7D_1 + 6D_2 + 5D_3 + 4D_4 + 3D_5 + 2D_6) \bmod 10\Big)? \tag{4}$$

<u>**Problem 3:**</u> In the last problem you showed that a specific base 10 scheme would not have worked properly.

Would it have been possible for the designers to have created a modified base 10 scheme that would work? More specifically, is it possible to design a good check digit scheme by choosing α_i, $i = 0 \dots 7$, such that, for all i, $\alpha_i \in \{0,1,2,3,4,5,6,7,8,9\}$ and C is defined by

$$(\alpha_0 L' + \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 + \alpha_4 D_4 + \alpha_5 D_5 + \alpha_6 D_6 + \alpha_7 C) \bmod 10 = 0. \tag{5}$$

A base 10 check digit scheme is *good* if it can detect all Single Digit Errors and Two Digit Transpositions. To solve this problem you either have to list the values α_i, $i = 0 \dots 7$, and prove that they define a good base 10 check digit scheme or prove that no good base 10 check digit scheme exists.