

Definitions and Formulas: This page contains some definitions used in this exam and a list of formulas (theorems) that you may use in the exam (without having to provide a proof). Note that you might not need all of these formulas on this exam.

Definitions

1. $N = \{0, 1, 2, 3, \dots\}$, the set of non-negative integers.
2. $Z^+ = \{1, 2, 3, \dots\}$, the set of positive integers.
3. R is the set of *real numbers*.
4. R^+ is the set of positive *real numbers*.

Formulas:

1. $\binom{n}{i} = \frac{n!}{i!(n-i)!}$
2. If $0 < i < n$ then $\binom{n}{i} = \binom{n-1}{i-1} + \binom{n-1}{i}$.
3. $\neg(p \wedge q)$ is equivalent to $\neg p \vee \neg q$.
4. $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$.
5. $\sum_{i=1}^{n-1} i = n(n-1)/2$.
6. $\sum_{i=1}^{n-1} i^2 = \frac{2n^3 - 3n^2 + n}{6}$.
7. If $r \neq 1$ then $\sum_{i=0}^{n-1} r^i = \frac{1-r^n}{1-r}$
8. If $r \neq 1$ then $\sum_{i=0}^n i r^i = \frac{nr^{n+2} - (n+1)r^{n+1} + r}{(1-r)^2}$
9. The inclusion-exclusion theorem:

$$P\left(\bigcup_{i=1}^n E_i\right) = \sum_{k=1}^n (-1)^{k+1} \sum_{\substack{i_1, i_2, \dots, i_k: \\ 1 \leq i_1 < i_2 < \dots < i_k \leq n}} P(E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_k})$$

10. If X is a random variable, then $E(X)$ denotes the *Expectation of X* and $V(X) = E((X - E(X))^2)$ denotes the *Variance of X* .