Bottom- Up – Approach :   (2-3 tree)   (P.T.O)

```
          Q
        /   \
      F      T
     / \    / \
    C  GL  S  VW
```

⇓ insert M

```
          Q
        /   \
      F      T
     / \    / \
    C  GLM S  VW
```

⇓

```
          Q
        /   \
      G      T
     / \    / \
   CF  LM  S  VW
```

⇓ insert N

```
          Q
        /   \
      G      T
     / \    / \
   CF  LMN S  VW
```

⇓

```
          Q
        /     \
      G M      T
     /|  \    / \
   CF L  N  S   VW
```

Insert A

```
                    ⇑
                    Q
                  /   \
              B F M    T
             /|  | \  / \
            A C GL N S  VW
```
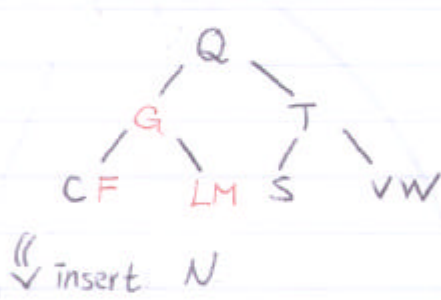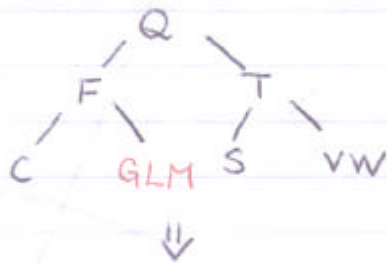
⇑

```
                    Q
                  /   \
               F M     T
              /|  \   / \
           ABC GL  N S  VW
```

⇑ insert B

```
                    Q
                  /   \
               F M     T
              /|  \   / \
            AC GL  N S  VW
```

⇑

```
                    Q
                  /   \
               G M     T
              /|  \   / \
          ACF L  N S  VW
```

```
            M
          /   \
       B F      Q T̄
      /|\      /|\
     A C GL N S  VW
```
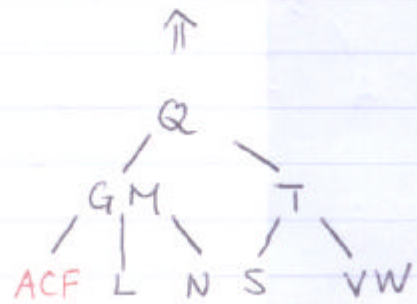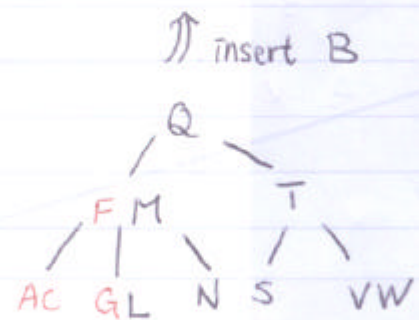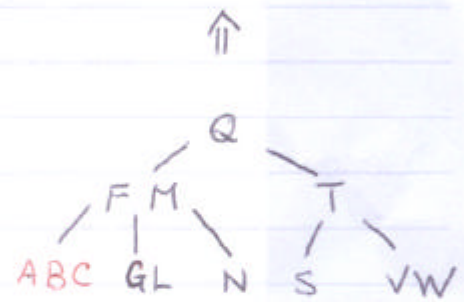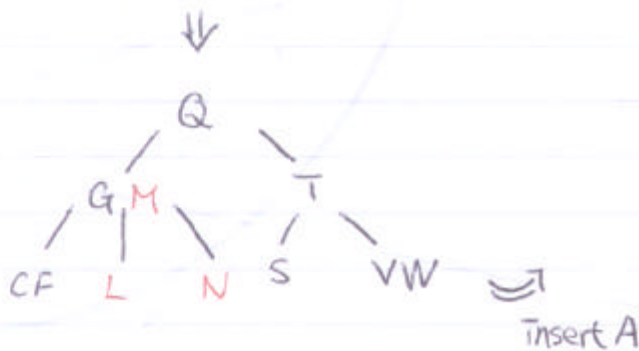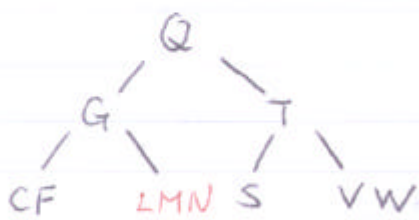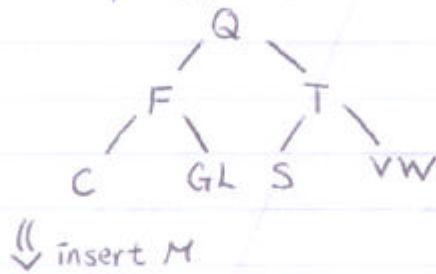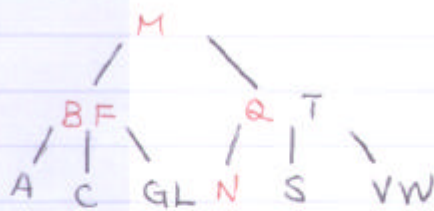
Bottom-Up-Insert:

① Find the location to insert, and insert it
② If # of keys is not exceed the limit, done!
③ If # of keys > limit
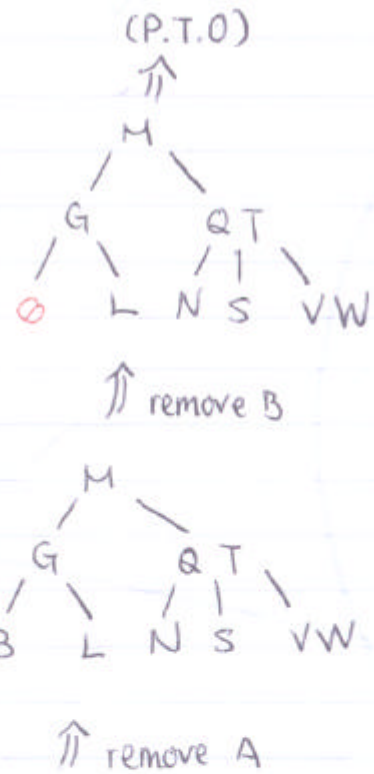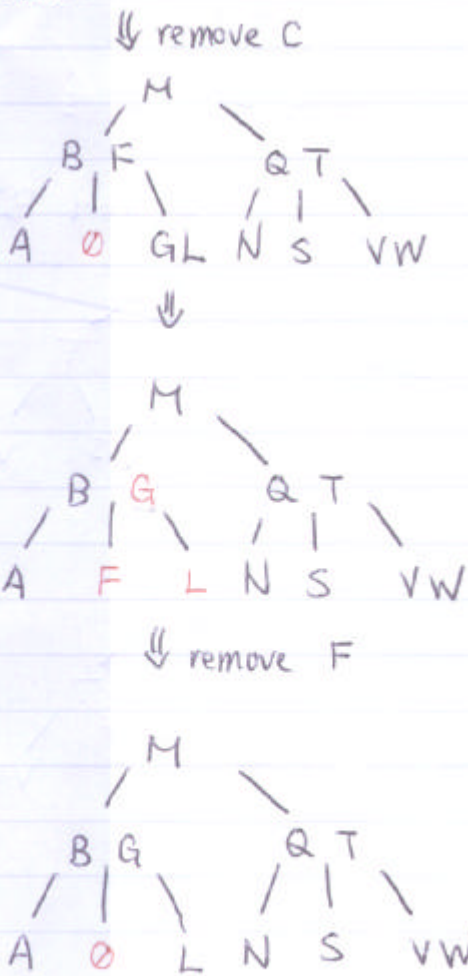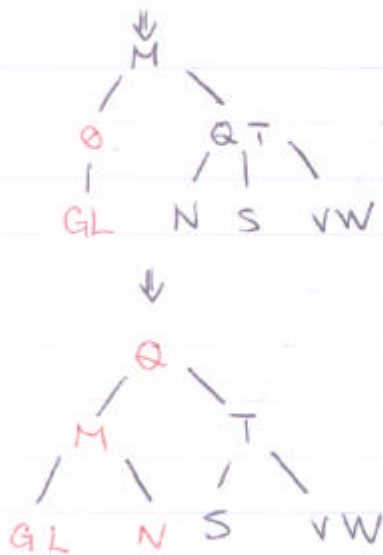 - consider rotation with slibling (Left slibling, then right slibling) if slibling have space left.
  - if both slibling are full then do a split and carry one the fix-up start at the original parent.

Deletion :

⇓ remove C

```
           M
         /   \
      B F      Q T
     /|\      /|\
    A Ø GL N S  VW
         ⇓
```

```
           M
         /   \
      B G      Q T
     /|\      /|\
    A F L N S  VW
       ⇓ remove F
```

```
            M
          /   \
       B G      Q T
      /|\\     /|\
     A Ø  L N S  VW
```
⇁

(P.T.O)
⇑

```
           M
         /   \
      G        Q T
     / \      /|\
    Ø    L  N S  VW
       ⇑ remove B
```

```
           M
         /   \
      G        Q T
     / \      /|\
    B    L  N S  VW
       ⇑ remove A
```

```
            M
          /   \
       G        Q T
      / \      /|\
    AB    L  N S  VW
```

```
      ↓↓
      M
     /   \
   Ø     Q T
   |    / | \
  GL   N S  VW

        ↓

       Q
     /   \
   M      T
  / \    / \
 GL  N  S   VW
```

Bottom-Up-Delete :

① Find the location to delete, and delete it,

② if the location is a leaf :

    ②-1 if # key ⩾ min keys , done !

    ②-2 if # key < min key

        – consider rotation with slibling (left then right).
          if slibling have > min keys

        – if both slibling have only min. number of key,
          then do a merge with the slibling (left then
          right), then carry on fix-up start at the
          original parent.

③ if the location is not a leaf:

        – find predecessor k' and replace k and
          delete k'

        – find successor k'' and replace k and
          delete k''