ASSIGNMENT 1: COMP2711H

FALL 2015

Q1. If $|A| = 5$, what is the value of $|P(A)|$? (11 marks)

Q2. What could you say about two nonempty sets A and B if $A \times B = B \times A$? (11 marks)

Q3. Is it true that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ for any sets A, B and C? If yes, please give a proof. Otherwise, please give a counterexample. (11 marks)

Q4. Evaluate (write down the true table of) the compound compositional form $\sim (p \wedge q) \vee (p \vee q)$. (11 marks)

Q5. Let $Q(x, y)$ be the predicate "If $x < y$ then $x^2 < y^2$" with domain for both x and y being the set $\mathbb{R}$ of real numbers.

(a) What is $Q(-2, 1)$? (6 marks)

(b) Find out the truth set of $Q(x, y)$. (6 marks)

Q6. Rewrite the following statement informally without using variables or quantifiers:

"$\exists$ a set A such that A has 16 subsets." (8 marks)

Q7. Write a negation for the following statements:

(a) "$\exists$ a movie such that m is over 6 hours long." (6 marks)

(b) "$\forall$ real numbers x, if $x^2 \geq 1$ then $x > 0$." (6 marks)

Q8. A **tautology** is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a tautology is a **tautological statement**.

A **contradication** is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a **contradication** is a contradictory statement.

Use true tables to establish which of the following statement forms are tautologies and which are contradictions.

(a) $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$. (6 marks)

(b) $(p \wedge \sim q) \wedge (\sim p \vee q)$. (6 marks)

Q9. Two propositional forms on the same variables are **(logically) equivalent** if they have the same result column in their truth tables. We use the notation $F \equiv G$, and sometimes the notation $F = G$.

Hence, a tautology is equivalent to the boolean constant T, and a contradiction is equivalent to the boolean constant F.

(a) Prove that $\sim (\sim p) \equiv p$. (6 marks)

(b) Prove that $\sim p \vee q \equiv p \rightarrow q$. (6 marks)

Date: Handed out on 9 September, due on Sept. 16 at the *beginning* of the lecture.