Binary, Octal, and Hex Numbers and Complements

In **positional notation**, a decimal number $d_n d_{n-1} \ldots d_1 d_0 . d_{-1} d_{-2} \ldots d_{-q}$ is equal to

$$d_n \times 10^n + d_{n-1} \times 10^{n-1} + \ldots + d_0 \times 10^0 + d_{-1} \times 10^{-1} + \ldots + d_{-q} \times 10^{-q}$$

Example: $2349_{10} = 2 \times 1000 + 3 \times 100 + 4 \times 10 + 9 \times 1$

In the binary system, the positional notation also applies, with the only exception that the base is 2 instead of 10. Examples:

$$\begin{aligned} 100110_2 &= 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ 110.001_2 &= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} \end{aligned}$$

Octal is a base-8 number system (digits: $0, 1, 2, \ldots, 7$) and hexadecimal is a base-16 number system (digits: $0, 1, \ldots, 9, A, B, C, D, E, F$). Examples:

$$\begin{aligned} 60.3_8 &= 6 \times 8^1 + 0 \times 8^0 + 3 \times 8^{-1} \\ &= 48.375_{10} \\ 3C.FD_{16} &= 3 \times 16^1 + 12 \times 16^0 + 15 \times 16^{-1} + 13 \times 16^{-2} \\ &= 60.988_{10} \quad \text{(correct to 3 decimal places)} \end{aligned}$$

Decimal-to-binary Conversion. Examples: (i) Convert 125_{10} into binary. (ii) Convert 125.4375_{10} into binary.

Decimal-to-octal and **decimal-to-hexadecimal** conversions are similar.

Given a binary number, we can readily convert it into its octal (by grouping together 3 bits) and hexadecimal (by grouping together 4 bits) equivalences. Examples: $110110101_2 = 665_8 = 1B5_{16}$, $10101.11_2 = 25.6_8 = 15.C_{16}$.

To represent negative numbers in binary, we have the **sign-and-magnitude**, **1's-complement** and **2's-complement**. Octal and hex have similar methods.

Sign-and-magnitude. The most significant (or leftmost) bit is used as the sign bit. Examples: $11100 = -12$, $01100 = 12$.

1's complement. To negate a number, invert all the bits, including the sign bit. Thus, it has "double zero", namely $+0$ and -0. Assume a m-bit number. The range of representable numbers is $-(2^{m-1} - 1)$ to $2^{m-1} - 1$.

2's complement. A number is negated by 1's complement and then add 1. It thus eliminates the problem of "double zero". Range of representable numbers is -2^{m-1} to $2^{m-1} - 1$.

Examples of negating 6 in *1's complement* and *2's complement* notation. Assume $m = 8$.

$$
\begin{array}{ll}
00000110 & (+6) \\
11111001 & (-6 \text{ in 1's complement}) \\
11111010 & (-6 \text{ in 2's complement})
\end{array}
$$

Exercise

1. Convert 375_{10} into its equivalence in
   (a) binary ____________
   (b) hexadecimal ____________
   (c) octal ____________

2. Convert the following numbers into its decimal equivalence:
   (a) 1011010_2 ____________
   (b) $5A_{16}$ ____________
   (c) 132_8 ____________

3. What is the binary equivalence of the following numbers in (i) sign-and-magnitude; (ii) 1's complement and (iii) 2's complement? Assume a 8-bit number.
   (a) -24 (i) ____________ (ii) ____________ (iii) ____________
   (b) 24 (i) ____________ (ii) ____________ (iii) ____________
   (c) -128 (i) ____________ (ii) ____________ (iii) ____________