

Heterogeneous Parallel Programming

COMP4901D

Prefix Scan

Overview

- Prefix Scan the Primitive
- Sequential implementation
- Work-Inefficient parallel implementation
- Work-efficient parallel implementation

Prefix Scan

- Frequently used for parallel work assignment and resource allocation
- A key primitive in many parallel algorithms to convert serial computation into parallel computation
- A foundational parallel computation pattern

Definition of Prefix Scan

Definition: *The all-prefix-sums operation takes a binary associative operator \oplus , and an array of n elements*

$$[x_0, x_1, \dots, x_{n-1}],$$

and returns the array

$$[x_0, (x_0 \oplus x_1), \dots, (x_0 \oplus x_1 \oplus \dots \oplus x_{n-1})].$$

Example: If \oplus is addition, then the all-prefix-sums operation on the array
would return

$$\begin{array}{l} [3 \ 1 \ 7 \ 0 \ 4 \ 1 \ 6 \ 3], \\ [3 \ 4 \ 11 \ 11 \ 15 \ 16 \ 22 \ 25]. \end{array}$$

Example of Prefix Scan

- Assume that we have a 100-inch sausage to feed 10 people
 - We know how much each person wants in inches:
[3 5 2 7 28 4 3 0 8 1]
 - How do we cut the sausage quickly?
 - How much will be left?
-
- Method 1: cut the sections sequentially: 3 inches first, 5 inches second, 2 inches third, etc.
 - Method 2: calculate prefix sum and cut in parallel:
[3, 8, 10, 17, 45, 49, 52, 52, 60, 61]
(39 inches left)

Building Block for Parallel Algorithms

- Scan is a simple and useful parallel building block
- Convert recurrences from sequential
 `for(j=1;j<n;j++) out[j] = out[j-1] + f(j);`
- into parallel:
 `forall(j) { temp[j] = f(j) }; scan(out, temp);`
- Useful for many parallel algorithms:
 - Radix sort, Quicksort, String comparison, Lexical analysis
 - Stream compaction, Polynomial evaluation
 - Solving recurrences, Tree operations
 - Histograms,

Other Applications

- Assigning camp slots
- Assigning farmer market space
- Allocating memory to parallel threads
- Allocating memory buffer for communication channels

Inclusive Sequential Addition

- Given a sequence $[x_0, x_1, x_2, \dots]$,
- Calculate output $[y_0, y_1, y_2, \dots]$
- Such that
 - $y_0 = x_0$
 - $y_1 = x_0 + x_1$
 - $y_2 = x_0 + x_1 + x_2$
 - ...
- Recursive definition: $y_i = y_{i-1} + x_i$

A Work-Efficient C Implementation

```
y[0] = x[0];
```

```
for (i = 1; i < Max_i; i++) y[i] = y [i-1] + x[i];
```

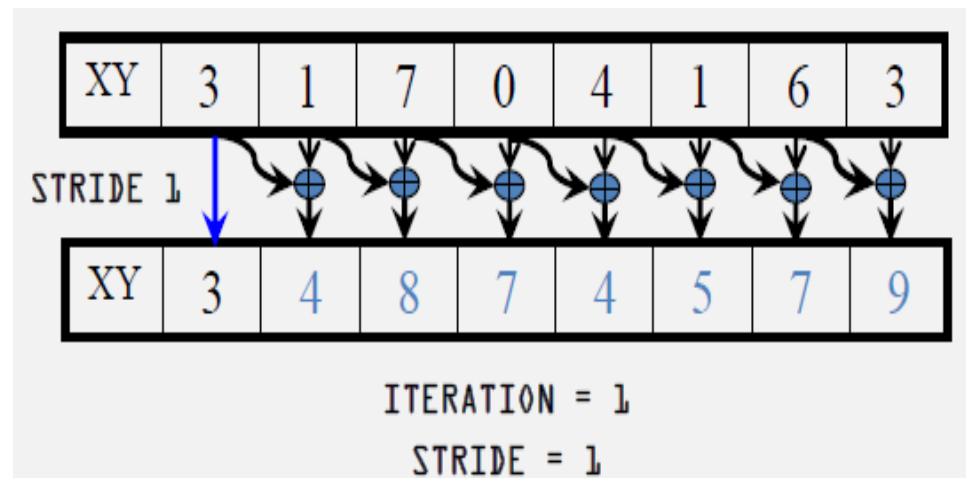
- Computationally efficient:
- N additions needed for N elements - $O(N)$!

A Simple Parallel Algorithm

- Assign one thread to calculate each y element
- Have every thread to add up all x elements needed for the y element
 - $y_0 = x_0$
 - $y_1 = x_0 + x_1$
 - $y_2 = x_0 + x_1 + x_2$
 - ...

A Better Parallel Algorithm

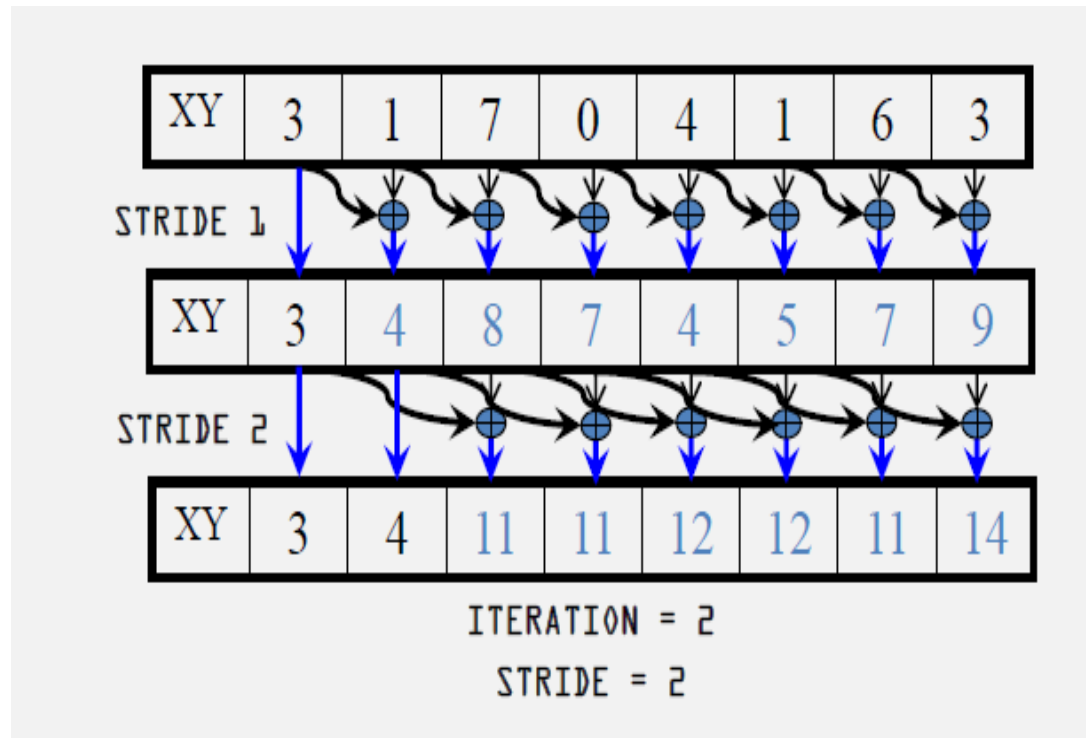
1. Read input from device global memory to shared memory
2. Iterate $\log(n)$ times; stride from 1 to $n-1$:
double stride each iteration



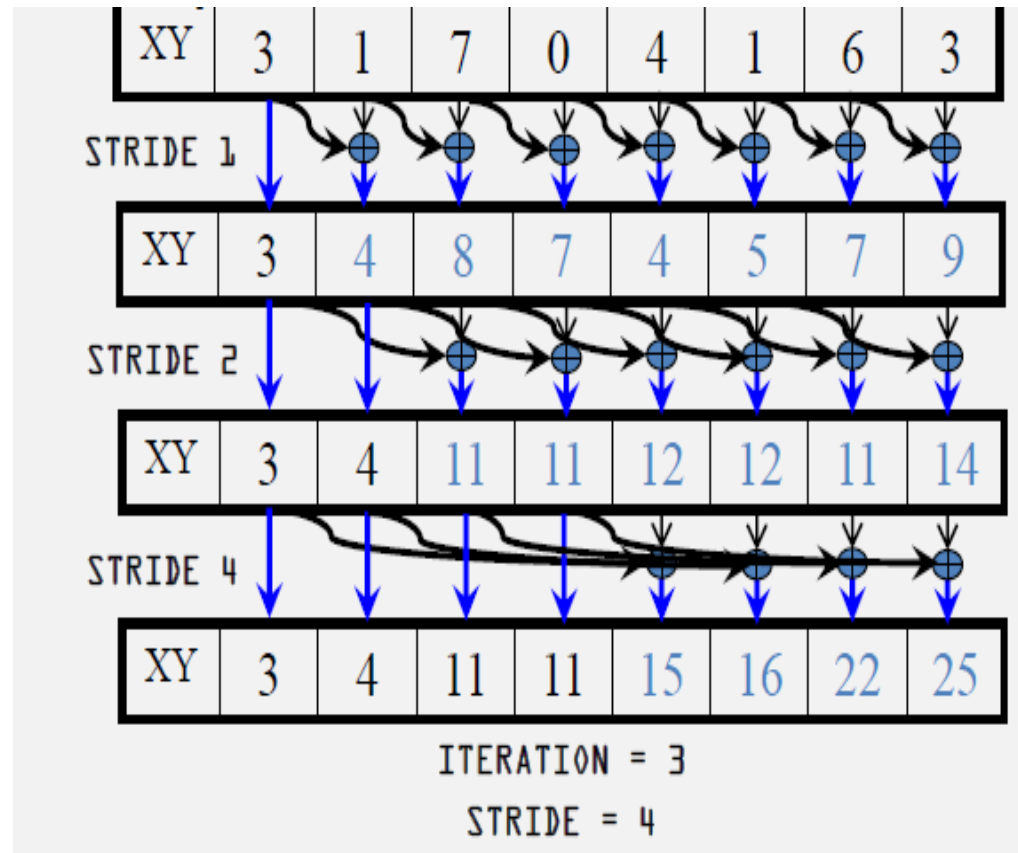
A Better Parallel Algorithm (cont.)

- Active threads stride to $n-1$ (n -stride threads)
- Thread j adds elements j and j -stride from shared memory and writes result into element j in shared memory
- Requires barrier synchronization, once before read and once before write

A Better Parallel Algorithm (cont.)



A Better Parallel Algorithm (cont.)



Handling Dependencies

- During every iteration, each thread can overwrite the input of another thread
- Barrier synchronization to ensure all input have been properly generated
- All threads secure input operand that can be overwritten by another thread
- Barrier synchronization to ensure that all threads have secured their input
- All threads perform addition and write output

The Better Scan Kernel

```
1. __global__ void work_inefficient_scan_kernel(float *X, float *Y,
int InputSize) {
2.   __shared__ float XY[SECTION_SIZE];
3.   int i = blockIdx.x*blockDim.x + threadIdx.x;
4.   if (i < InputSize) {XY[threadIdx.x] = X[i];}
   // the code below performs iterative scan on XY
5.   for (unsigned int stride = 1; stride <= threadIdx.x; stride *= 2) {
6.     __syncthreads();
7.     float in1 = XY[threadIdx.x - stride];
8.     __syncthreads();
9.     XY[threadIdx.x] += in1;
10. }
```


Work Efficiency Considerations

- This Scan executes $\log(n)$ parallel iterations
 - An iteration performs $(n-1), (n-2), (n-4), \dots, n/2$ adds
 - Total adds: $n * \log(n) - (n-1) \rightarrow O(n * \log(n))$ work
- This scan algorithm is not work efficient
 - Sequential scan algorithm does n adds
 - A factor of $\log(n)$ can hurt: 10x for 1024 elements!
- A parallel algorithm can be slower than a sequential one when execution resources are saturated from low work efficiency

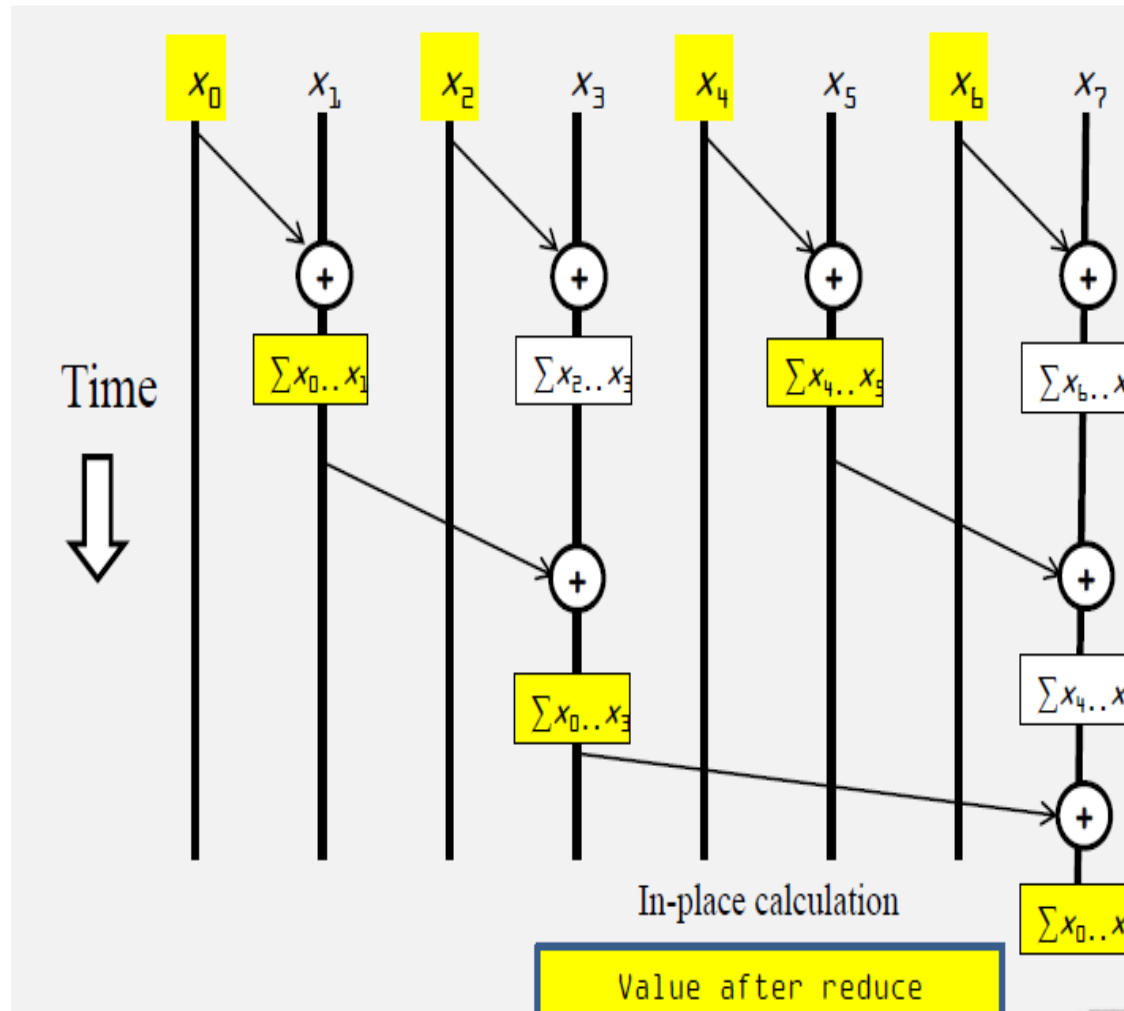
How To Improve Efficiency

- Two-phased balanced tree traversal
- Aggressive reuse of computation results
- Reducing control divergence with more complex thread index to data index mapping

Balanced Tree for Scan

- Form a balanced binary tree on the input data and sweep it to and from the root
- Tree is not an actual data structure, but a concept to determine what each thread does at each step
- For scan:
 - Traverse down from leaves to root building partial sums at internal nodes in the tree
 - Root holds sum of all leaves
 - Traverse back up the tree building the output from the partial sums

Parallel Scan – Reduction Phase

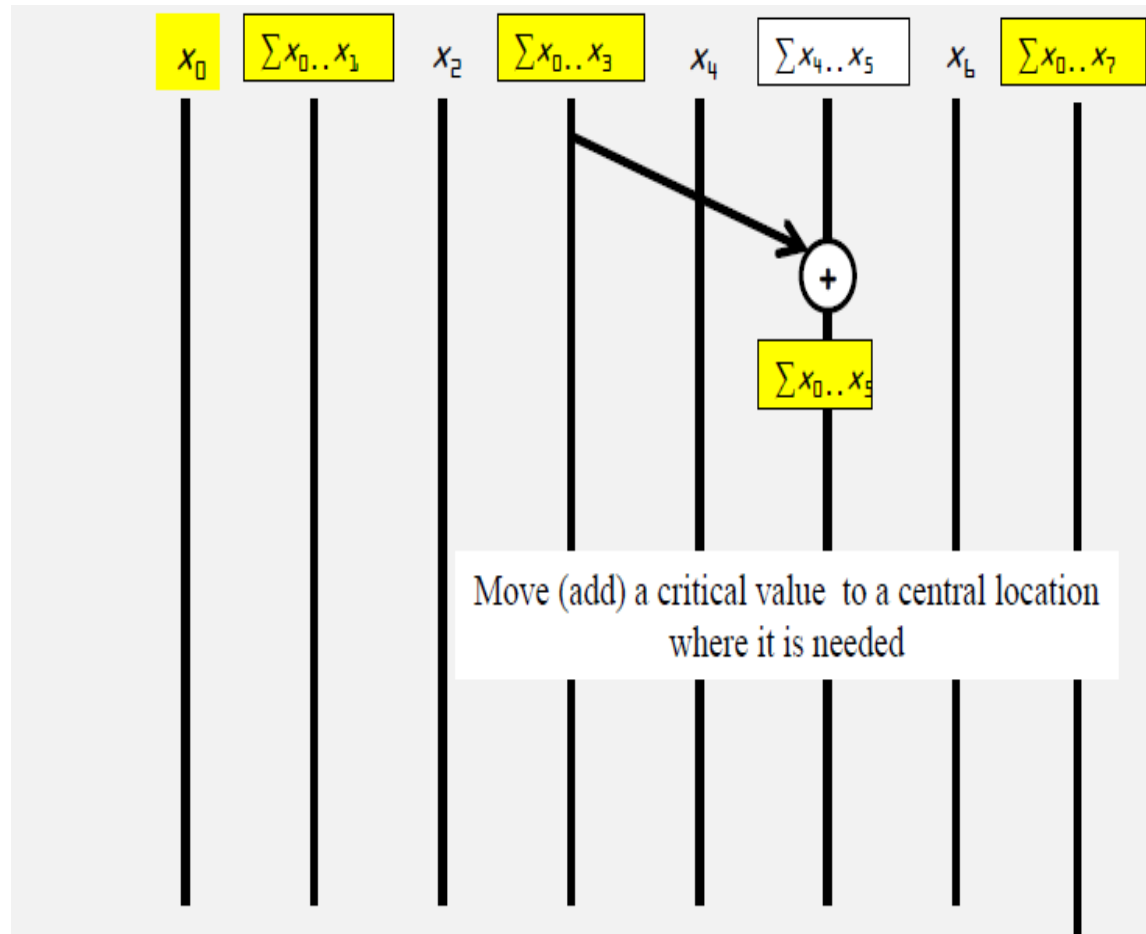


Reduction Phase Kernel Code

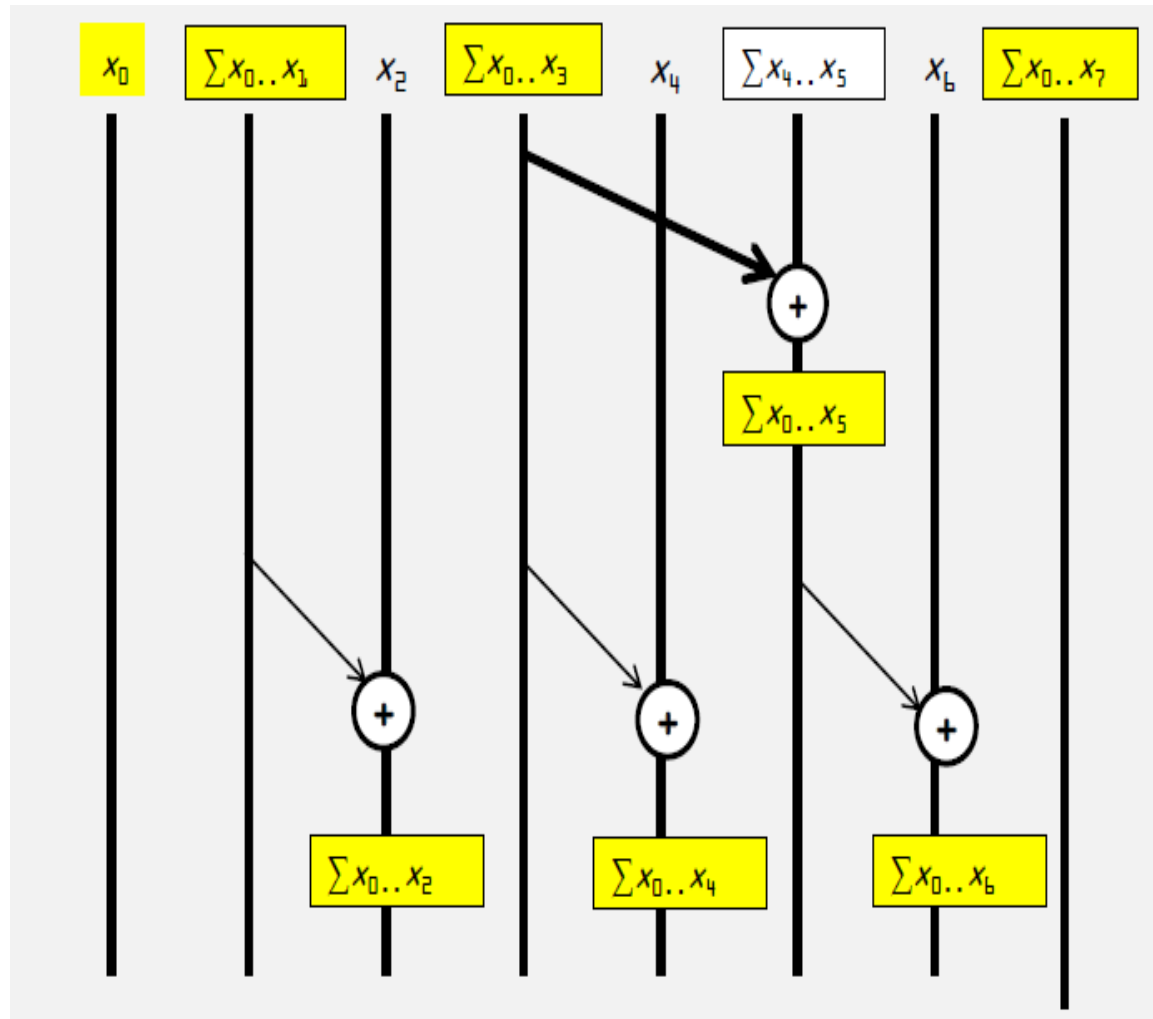
```
// XY[2*BLOCK_SIZE] is in shared memory
```

```
for (int stride = 1; stride <= BLOCK_SIZE; stride *= 2) {  
    int index = (threadIdx.x+1)*stride*2 - 1;  
    if(index < 2*BLOCK_SIZE)  
        XY[index] += XY[index-stride];  
    __syncthreads()  
}
```

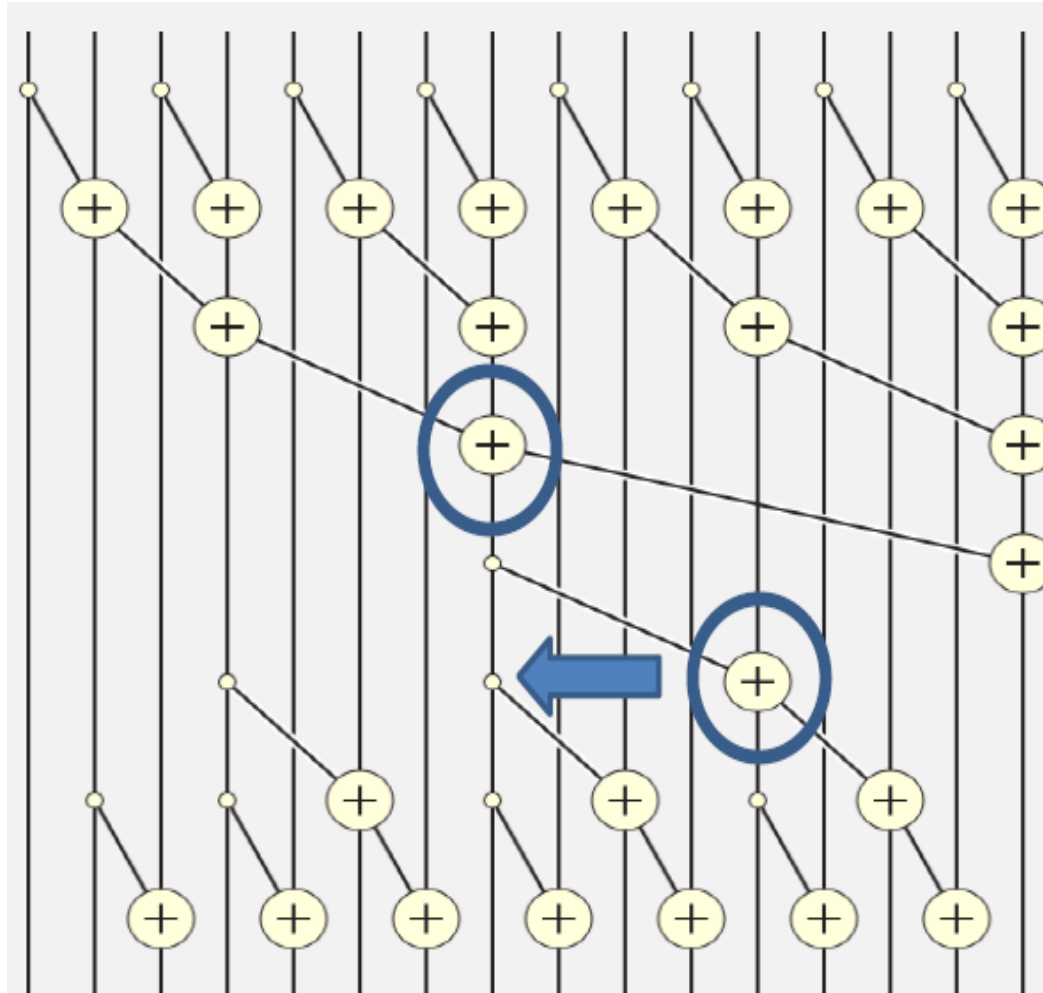
Parallel Scan – Post Reduction Reverse Phase



Parallel Scan – Post Reduction Reverse Phase



Putting it all together



Post Reduction Reverse Phase Kernel Code

```
for (int stride = BLOCK_SIZE/2; stride > 0; stride /= 2) {  
    __syncthreads();  
    int index = (threadIdx.x+1)*stride*2 - 1;  
    if(index+stride < 2*BLOCK_SIZE) {  
        XY[index + stride] += XY[index];  
    }  
}  
__syncthreads();  
if (i < InputSize) Y[i] = XY[threadIdx.x];
```

Efficiency Analysis

- The work efficient kernel executes $\log(n)$ parallel iterations in the reduction step
 - The iterations do $n/2, n/4, \dots, 1$ adds
 - Total adds: $(n-1) \rightarrow O(n)$ work
- It executes $\log(n)-1$ parallel iterations in the post reduction reverse step
 - The iterations do $2-1, 4-1, \dots, n/2-1$ adds
 - Total adds: $(n-2) - (\log(n)-1) \rightarrow O(n)$ work
- Both phases perform up to no more than $2*(n-1)$ adds
- The total number of adds is no more than twice of that done in the efficient sequential algorithm

Tradeoffs

- The work efficient scan kernel is normally more desirable
 - Better Energy efficiency
 - Less execution resource requirement
- However, the work inefficient kernel could be better for absolute performance due to its single-step nature if there is sufficient execution resource

Summary

- Prefix scan is a common data-parallel primitive.
- A naïve parallel implementation is work inefficient.
- A work-efficient implementation takes a two phase balanced tree approach.
- There are tradeoffs between the two implementations.