



COMP 2012H Honors Object-Oriented Programming and Data Structures

Topic 17: Hashing

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Motivation

- How would you **find** a student record given just the student's name?
- How does an electronic dictionary **look up** for a word, say, "computer"?
- Each machine has an IP address in the Internet. How will an internet service **look up** an IPv6 address?



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Part I

Hashing



= 79054025
255fb1a2
6e4bc422
aef54eb4

General Idea

0	
1	
2	data item
3	john 25000
4	phil 31250
5	
6	dave 27500
7	mary 28200
8	
9	

- A **hash table** is an **array** of some **fixed size**, containing all the data items.
- Each item has a **key**; **search** is performed based on the keys.
- Each key is mapped into some position in the array in the range **0** to **$m - 1$** , where **m** is the array size.
- The mapping is called **hash function**.
- Example applications:
 - Compilers use hash tables, called **symbol tables**, to keep track of declared identifiers in a program.
 - On-line spell checkers: After **hashing** the entire dictionary, one can check each word in **constant time** and print out the mis-spelled words in order of their appearance in the document.

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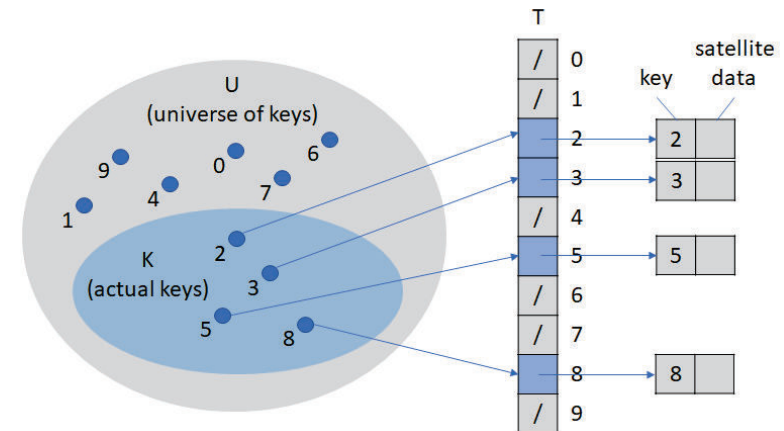
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Hash Table

- **Hash table** is a data structure that supports: **search**, **insertion**, and **deletion** (deletion may be unnecessary in some applications).
- The implementation of **hash tables** is called **hashing**.
- **Hashing** is a technique which allows the executions of above operations in **constant time** on average.
- Unlike other data structures such as linked lists or binary trees, data items are generally **not** ordered in **hash tables**.
- As a consequence, **hash tables** don't support the following operations
 - ▶ **find_min** and **find_max**
 - ▶ finding successor and predecessor
 - ▶ reporting data within a given range
 - ▶ listing out the data in order

Unrealistic Solution



- Universe of **keys** U is the set of **all possible values** of the keys.
- Each position, also called a **slot**, in the **hash table** T corresponds to a key in U .
 - ▶ $T[k]$ corresponds to a data item with key k .
 - ▶ If the table contains no data with key k , then $T[k] = \text{nil}$.

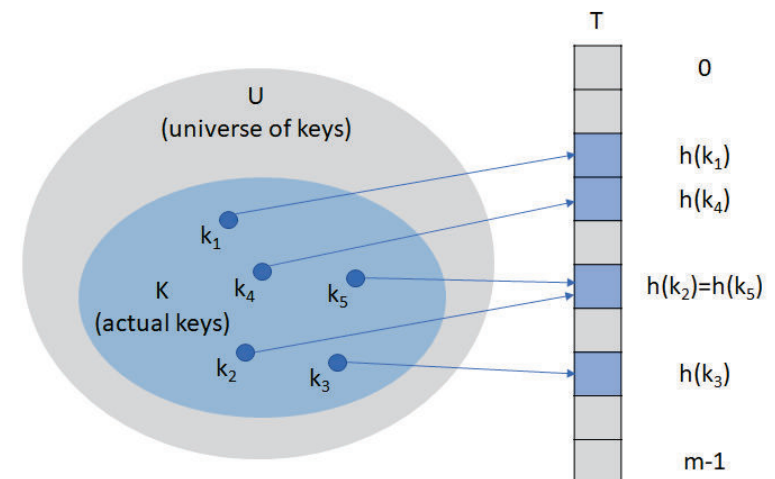
Unrealistic Solution ..

- **Insertion**, **deletion** and **search** all take **constant time**.
- **Problem**: it wastes too much space if the universe of keys is **large** compared with the actual number of data to be stored.

E.g., in HKUST, student IDs are 8-digit integers. So the key universe has a size of 10^8 , but we only have ~ 7000 students (not counting the alumni)!

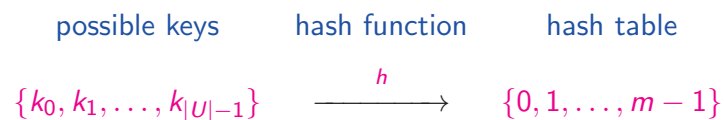


Hash Function



- **Hash function**, h maps the universe of keys U into the slots of a hash table $T[0, 1, \dots, m-1]$.
- Several **keys** may be mapped to the **same slot**.

Hash Function ..



- Usually, $m \ll |U|$.
- The keys k_i are assumed to be natural numbers.
- If they are not, they can always be converted or interpreted in natural numbers.
- $h(k_i)$ = an integer in $[0, \dots, m-1]$ is called the hash value of k_i .
- With hashing, an item of key k is stored in $T[h(k)]$.

Collision Problem



- Two keys may be hashed to the same slot.
- Question: Can we ensure that any two distinct keys are hashed to different slots?
- No! If $N > m$, where
 - ▶ m = size of the hash table, and
 - ▶ N = number of data

Solution:

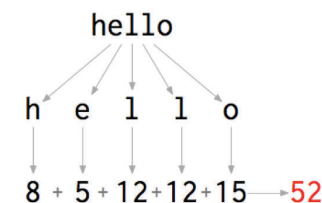
1. Design a good hash function that is
 - ▶ fast to compute, and
 - ▶ can minimize the number of collisions
2. Design a method to resolve the collisions when they occur.

Hash Function Design

- A simple and reasonable strategy: $h(k) = k \bmod m$.
- e.g. $m = 12, k = 100, h(k) = 4$
- It requires only a single division operation (quite fast).
- Certain values of m should be avoided: e.g., if $m = 2^p$, then $h(k)$ is just the p lowest-order bits of k ; thus, the hash function does not depend on all the bits.
- Similarly, if the keys are decimal numbers, m should not be set to be a power of 10.
- It's a good practice to set the table size m to a prime number.
- Good values for m : primes not too close to exact powers of 2
 - ▶ e.g., for a hash table to hold 2,000 numbers, if we don't mind an average of 3 numbers being hashed to the same slot, choose $m = 701$.

How to Deal with String Keys: Method 1

Add up the ASCII values of the characters in the string.
(For simplicity, we only use their positions in the alphabets here.)



- Most hash functions assume that keys are natural numbers. If keys are not natural numbers, a way must be found to interpret them as natural numbers.
- Problems:
 - ▶ Different permutations of the same set of characters would have the same hash value.
 - ▶ If the table size is large, the keys do not distribute well.
 - ▶ e.g., $m = 10,007$ (a prime number) and all string keys have ≤ 8 characters. Since ASCII values ≤ 127 , their numeric keys are in the range between 0 and $127 \times 8 = 1,016$.

How to Deal with String Keys: Method 2

$$h(\text{key}) = (\text{key}[0] + 27 \cdot \text{key}[1] + 27^2 \cdot \text{key}[2]) \bmod m$$

where m is hash table size.

- If the first 3 characters are random and the table size m is 10,007, then it is a reasonably equitable distribution.
- Problems:
 - ▶ letters in an English word are not random;
 - ▶ according to some dictionary, there are only 2,851 different combinations for the first 3 letters of English words;
 - ▶ therefore, only at most 28% of the table can actually be hashed to (if $m = 10,007$).

How to Deal with String Keys: Method 3

$$\begin{aligned} h(\text{key}) &= \left(\sum_{i=0}^{L-1} 37^{(L-1-i)} \cdot \text{key}[i] \right) \bmod m \\ &= \left(37^{(L-1)} \cdot \text{key}[0] + 37^{(L-2)} \cdot \text{key}[1] + \dots + \text{key}[L-1] \right) \bmod m \end{aligned}$$

where L is the length of a key.

- This hash function involves all characters in the key and the hash values are expected to distribute well.

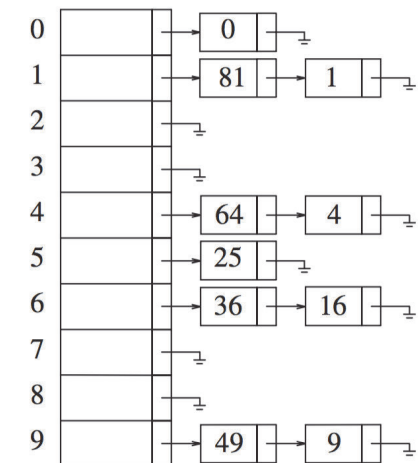
Part II

Collision Handling



Separate Chaining

- Keys: the set of squared numbers $\{1, 4, 9, 16, \dots\}$.
- Hash function: $h(k) = k \bmod 10$.
- Using the idea of equivalence classes.



- The hash table is more than a simple array, but a table of linked lists.
- Keys having the same hash values are chained on a separate linked list.

Separate Chaining Operations

To insert a key k :

- Compute $h(k)$.
- If $T[h(k)]$ contains a null pointer, the list (or chain) is empty. Initialize this table entry to point to a linked list with a single node containing k alone.
- If $T[h(k)]$ points to a non-empty list, add k to the beginning of the list.

To delete a key k

- Compute $h(k)$ to determine which list to traverse.
- Search for the key k in the list that $T[h(k)]$ points to.
- Delete the item with key k if it is found.

Separate Chaining Features

- If the hash function works well, the number of keys in each linked list will be a small constant.
- Therefore, we expect that each search, insertion, and deletion can be done in constant time.
- Disadvantage: Memory allocations and de-allocations in linked list manipulations slow down the operations.
- Advantage: deletion is easy.

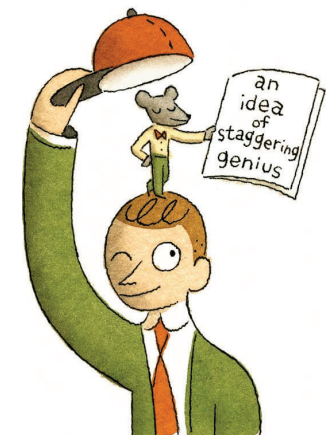


Open Addressing

- Instead of putting keys of the same hash value into a chain, open addressing will relocate the key k to be inserted if it collides with an existing key.
- Open addressing needs to determine the sequence of slots to be examined for key relocation.
- Key k may be stored at an entry different from $T[h(k)]$.
- Two issues arise:
 - ▶ what is the relocation scheme?
 - ▶ how to search for k later?
- Three common methods for resolving collisions in open addressing
 1. linear probing
 2. quadratic probing
 3. double hashing

Basic Strategy of Open Addressing

- To insert a key k , compute $h_0(k)$.
- If $T[h_0(k)]$ is empty, insert it there.
- If collision occurs, probe alternative cell in the following order: $h_1(k), h_2(k), \dots$, until an empty cell is found.
- $h_i(k) = (\text{hash}(k) + f(i)) \bmod m$, where the function f determines the collision resolution strategy and $f(0) = 0$.
- Different open addressing methods differ in the definition of $f()$.



Linear Probing: Insertion

$$f(i) = i$$
$$h_i(k) = (\text{hash}(k) + i) \bmod m$$

- Basic strategy: Table cells are probed **sequentially** (with wrap-around) until an **empty slot** is found.
- Again let m be the table size and N be the number of items.
- Let k be the new key to be inserted; compute $\text{hash}(k)$.
- For $i = 0$ to $m - 1$, compute $j = (\text{hash}(k) + i) \bmod m$.
- If $T[j]$ is **empty**, then we put k there and stop.
- If **no** empty slot can be found to put k , the table is full; report an error.

Linear Probing: Example

- $\text{hash}(k) = k \bmod 10$
- Insert the following keys: 89, 18, 49, 58, 69
- To insert 58, probe $T[8]$, $T[9]$, $T[0]$, $T[1]$
- To insert 69, probe $T[9]$, $T[0]$, $T[1]$, $T[2]$

Table Index	Insert 89	Insert 18	Insert 49	Insert 58	Insert 69
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					

Primary Clustering

- We call a block of **contiguously** occupied table entries a **cluster**.
- On average, when we insert a new key k , we may hit the **middle** of a cluster. Therefore, the time to insert k would be proportional to **half** the size of a cluster.
 \Rightarrow the larger the cluster, the slower the performance.
- Linear probing** has the following **disadvantages**:
 - Once $h(k)$ falls into a cluster, this cluster will definitely **grow** in size by one. Thus, this may worsen the performance of insertions in the future.
 - If two clusters are only separated by one entry, they may be **merged** together by an insertion to that entry.
 \Rightarrow cluster size can increase drastically by 1 insertion.
 - This means that the performance of insertion can deteriorate drastically after a single insertion.
 - Large clusters are easy targets for **collisions**.

Quadratic Probing

$$f(i) = i^2$$
$$h_i(k) = (\text{hash}(k) + i^2) \bmod m$$

Table Index	Insert 89	Insert 18	Insert 49	Insert 58	Insert 69
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					

Quadratic Probing: Example

- Example:
 - ▶ $\text{hash}(k) = k \bmod 10$
 - ▶ Insert the following keys: 89, 18, 49, 58, 69
 - ▶ To insert 58, probe $T[8]$, $T[9]$, $T[(8+4) \bmod 10]$
 - ▶ To insert 69, probe $T[9]$, $T[(9+1) \bmod 10]$, $T[(9+4) \bmod 10]$
- Two keys with different home positions will have different probe sequences. E.g.,
 - ▶ $m = 101$, $h(k_1) = 30$, $h(k_2) = 29$
 - ▶ probe sequence for k_1 : $30, 30+1, 30+4, 30+9$
 - ▶ probe sequence for k_2 : $29, 29+1, 29+4, 29+9$
- If the table size is prime, then a new key can always be inserted if the table is at least half empty (see proof in reference book by Weiss).

Secondary Clustering

- Keys that hashed to the same home position will probe the same alternative cells.
- Simulation results suggest that it generally causes less than an extra half probe per search.
- To avoid secondary clustering, the probe sequence need to be a function of the original key value, not the home position.



Double Hashing

- To alleviate the problem of clustering, the sequence of probes for a key should be independent of its primary home position.
- Thus, use two hash functions: $\text{hash}()$ and $\text{hash}_2()$.

$$f(i) = i \times \text{hash}_2(k)$$

$$h_i(k) = (\text{hash}(k) + i \times \text{hash}_2(k)) \bmod m$$

- e.g., $\text{hash}_2(k) = R - (k \bmod R)$, where $R < m$ and R is prime.



Double Hashing: Example

- $h_i(k) = (\text{hash}(k) + i \times \text{hash}_2(k)) \bmod m$
- $m = 10$, $R = 7$, $\text{hash}(k) = k \bmod 10$, $\text{hash}_2(k) = 7 - (k \bmod 7)$
- Insert the following keys: 89, 18, 49, 58, 69
- 2nd probe for 49: $T[(9+7) \bmod 10]$; 58: $T[(8+5) \bmod 10]$;
- 2nd probe for 69: $T[(9+1) \bmod 10]$

Table Index	Insert 89	Insert 18	Insert 49	Insert 58	Insert 69
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					

Double Hashing: Choice of hash_2

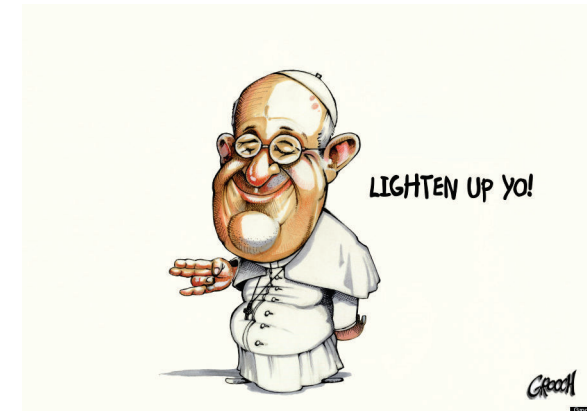
- $\text{hash}_2()$ must **never** evaluate to **zero**.
- For any key k , $\text{hash}_2(k)$ must be **relatively prime** to the table size m . Otherwise, we will only be able to examine a **fraction** of the table entries.
- E.g., if $\text{hash}(k) = 0$ and $\text{hash}_2(k) = m/2$, then we can only examine the entries $T[0]$, $T[m/2]$, and nothing else!
- One solution is to make m **prime**, and choose R to be a **prime number** smaller than m , and set

$$\text{hash}_2(k) = R - (k \bmod R)$$

- **Quadratic probing**, however, does not require the use of a second hash function, and thus is likely to be **simpler** and **faster** in practice.

Part III

Final Remarks



Deletion in Open Addressing

- Actual **deletion** **cannot** be performed in **open addressing** hash tables, otherwise the probing sequence will be broken.
- **Solution**: Add an extra field to each table entry, and mark it as
 - **EMPTY**
 - **ACTIVE**
 - **DELETED**
- It is also called **lazy deletion**.



Re-hashing

- **Load factor** $\alpha = N/m$, where N is the number of actually hashed items in the hash table.
- The operations in a **hash table** will become slower drastically when α becomes large.
- When α becomes large, (roughly) **double** the table size and **re-hash** all data items with a new **hash function**.
- Obviously, **re-hashing** is a very expensive operation. Fortunately, it usually happens infrequently in a well-designed **hash table**.



That's all!
Any questions?

