COMP 170 Discrete Mathematical Tools for CS
2008 Fall Semester – Written Assignment # 6
Distributed: Oct 30, 2008 – Due: November 6, 2008

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.
Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions can either be submitted at the end of your Thursday lecture section or, before 5PM, in the collection bin in front of Room 4213A.

Problem 1: (a) Construct a contrapositive proof that for all real numbers x, if $x^2 - 2x \neq 3$ then $x \neq 3$.
(b) Construct a proof by contradiction that for all real numbers x, if $x^2 - 2x \neq 3$ then $x \neq 3$.

Problem 2: Prove that

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

for all integers $n \geq 1$.

Problem 3: For what values of $n \geq 1$ is $n! \geq 5 \cdot 2^n$? Use mathematical induction to show that your answer is correct.

Problem 4: Prove that every integer greater than 17 is a sum of a nonnegative integer multiple of 4 and a nonnegative integer multiple of 7.
(Hint: first prove the four base cases *of* $n = 18, 19, 20, 21$ *and then prove the inductive step assuming that* $n > 21$*.)*

Problem 5: Find the error in the following "proof" that all positive integers n are equal: *Let $p(n)$ be the statement that all numbers in an n-element set of positive integers are equal. Then $p(1)$ is true. Now assume $p(n-1)$ is true, and let N be the set of the first n integers. Let N' be the set of the first $n-1$ integers, and let N'' be the set of the last $n-1$ integers. By $p(n-1)$, all members of N' are equal, and all members of N'' are equal. Thus, the first $n-1$ elements of N are equal and the last $n-1$ elements of N are equal, and so all elements of N are equal. Therefore, all positive integers are equal.*

Challenge problem Note: this is a logic problem. You can use any tools you want (not necessarily from our class) to solve it.

- Assume that every tree in a forest (size unknown) has at least one leaf.
- It is known that the number of trees in the forest is greater than the number of leaves on any given tree.

From the above two facts we conclude that there exist at least two trees in the forest with the same number of leaves.

Is the above conclusion valid?