COMP 170 Discrete Mathematical Tools for CS
2005 Fall Semester – Assignment # 5 Challenge Problem

Note: This page contains the challenge problem for Assignment 5. The solution to this problem should be submitted with the rest of Assignment 5 on October 20.

Challenge problem for Assignment 5: In assignment 5, Problem 4, you show that, if p and q are prime, then there are exactly $(p-1)(q-1)$ elements in Z_{pq} that are relatively prime to $n = pq$. You also show that if p and q are not prime then the number of elements in Z_{pq} relatively prime to $n = pq$ is not necessarily $(p-1)(q-1)$. In this problem, you try to come up with a general formula for the number of elements in n that are relatively prime to n. In both part (a) and part (b) you need to explain *how* you derived your solution.

(a) First assume that $n = p^i$ where p is some prime number. How many elements of Z_n are relatively prime to $n = p^i$? If possible, express your answer in terms of n and p.

(b) Now let n be an arbitrary number. How many elements of Z_n are relatively prime to n. If possible, express your answer in terms of n and $p_1, p_2, \ldots, p_t$, where the p_i are the primes that divide n.