Expanding $(x+y)^n$

$$(x+y)(x+y)$$

$$= xx + xy + yx + yy \qquad 2^2 \text{ terms}$$

$$\underline{(x+y)(x+y)}(x+y)$$

$$= [xx + xy + yx + yy](x+y)$$

$$= xxx + \underline{xyx} + \underline{yxx} + yyx + \underline{xxy} + xyy + yxy + yyy \qquad 2^3 \text{ terms}$$

$\boxed{\text{Binomial Coefficient}}$

3 x^2y

= # of monomial terms with 2 x's & 1 y

3 xy^2

= # of monomial terms with 1 x & 2 y's

Expanding $(x+y)^n$ result in

2^n monomial terms

Binomial Theorem

# of terms in the expansion

with n x's is $\underline{\binom{n}{0}}$

# of terms in the expansion

with $n-1$ x's & 1 y is $\underline{\binom{n}{1}}$

# of terms in the expansion

with $n-i$ x's & i y's is $\underline{\binom{n}{i}}$

In the expansion of $(x+y)^n$,

# of terms with $\underline{n-i}$ x's, $\underline{i}$ y's

$$= \binom{n}{i}$$

proof:

# of terms with $\underline{n-i}$ x's, $\underline{i}$ y's

= # of lists of length $\underline{n}$ with y at $\underline{i}$ positions

= # of ways to choose $\underline{i}$ positions in a list with $\underline{n}$ positions

$$= \binom{n}{i}$$

proved.