COMP 170 Discrete Mathematical Tools for CS
2006 Fall Semester – Written Assignment # 6
Distributed: Oct 24, 2006 – Due: Oct 31, 2006 *at end of class*

The top of your submission should contain (i) your name, (ii) your student ID #, (ii) your email address and (iv) your tutorial section.

Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. A solution that consists of just a number will be counted as wrong.

2nd Note: Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.

3rd Note: Most of these problems are taken (some modified) from sections 3.1, 3.2 and 3.3 of the textbook.

4th Note: Your assignment can either be submitted at the end of your Tuesday lecture session or before 5PM in the collection bin in front of room 4213A.

Problem 1: Show that the statements $s \Rightarrow t$ and $\neg s \vee t$ are equivalent.

Problem 2: Prove the DeMorgan's law that states $\neg(p \wedge q) = \neg p \vee \neg q$.

Problem 3: Show that $p \oplus q$ is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$.

Problem 4: (Distributive "laws")
(a) Is $w \wedge (u \oplus v)$ equivalent to $(w \wedge u) \oplus (w \wedge v)$?
(a) Is $w \vee (u \oplus v)$ equivalent to $(w \vee u) \oplus (w \vee v)$?

Problem 5: Which of the following statements (in which Z^+ stands for the positive integers and Z stands for all integers) is true and which is false? Don't forget to explain why.
(a) $\forall z \in Z^+ \, (z^2 + 6z + 10 > 20)$
(b) $\forall z \in Z \, (z^2 - z \geq 0)$
(c) $\exists z \in Z^+ \, (z - z^2 > 0)$
(d) $\exists z \in Z \, (z^2 - z = 6)$

Problem 6: Consider the statement: "For all primes p, either p is odd or p is 2."
(a) Use symbolic statements and a universal quantifier to express the above statement.
(b) Express the negation of the statement in (a) using an existential quantifier.

Problem 7: Let $p(x)$ stand for "x is a prime," $q(x)$ for "x is even," and $r(x, y)$ stand for "$x = y$". Use these three symbolic statements and appropriate logical notation to write the statement "There is one and only one even prime." (Use the set Z^+ of positive integers for your universe.)

Problem 8: (a) Construct a contrapositive proof that for all real numbers x, if $x^2 - 2x \neq -1$ then $x \neq 1$.

(b) Construct a proof by contradiction that for all real numbers x, if $x^2 - 2x \neq -1$ then $x \neq 1$.