Iterative Combinatorial Auctions: Theory and Practice

David C. Parkes and Lyle H. Ungar

AAAI-00, pp. 74-81

Presented by Xi LI
May 2006
COMP670O
HKUST

Why iterative?

- In Generalized Vickrey Auction (GVA)
  - $2^{|G|}$ bundles to evaluate
  - A large winner-determination (WD) problem
  - Optimal
- In iBundle
  - Agents bid for less bundles
  - Each iteration contains a smaller WD problem
  - Also optimal for reasonable agent bidding strategy

Outline

- iBundle – the model
- Optimality
- Computational Analysis

Notation

G denote the set of items to be auctioned,

I denote the set of agents,

$S \subseteq G$ denote a bundle of items.

Bid – each bid contains a *buddle S* and its corresponding *bid price*

For each buddle *S* in round *t*, there are

$p^t_{\mathrm{bid},i}(S)$ - *bid price*, provided by agent i

$p^t_{\mathrm{ask}}(S)$ - *ask price*, announced by the auctioneer

Bidding Price

- $p^t_{bid,i}(S) \geq p^t_{ask}(S)$ - ε, where $\varepsilon > 0$ is the minimal increment of ask price.
- Agent can also bid ε below the ask price, but then it cannot bid a higher price in future rounds.
- Agent will NOT bid a price that higher than the value of the bundle, i.e. $p^t_{bid,i}(S) \leq v_i(S)$

XOR Bids

- Agents can place multiple bids for exclusive-or bundles, e.g. S_1 XOR S_2, to indicate that an agent wants either all items in S_1 or all items in S_2 but not both S_1 and S_2 (The allocation guarantees a agent is satisfied by at most one bid (bundle) or nothing).

i.e.

Agent

S_1: {apple, soap, mushroom}

XOR

S_2: {apple, baseball}

XOR

S_3: {dictionary, beer}

The Auction - iBundle

Winner-determination. The auctioneer solves a winner-determination problem in each round, computing an allocation of bundles to agents that maximizes revenue. The auctioneer must respect agents' XOR bid constraints, and cannot allocate any item to more than one agent. The provisional allocation becomes the final allocation when the auction terminates.

Termination. The auction terminates when: [T1] all agents submit the same bids in two consecutive rounds, or [T2] all agents that bid receive a bundle.

Approximate WD

Approximate Winner-determination. The auctioneer can also use an approximate algorithm for winner-determination, and still maintain the same incentives for myopic agents to follow the same bidding strategy. To achieve this an approximate algorithm must have the *bid-monotonicity* property:

Monotonicity[1] $s \subseteq g_j, \quad s' \subseteq s, \quad v' \geq v \;\Rightarrow\; s' \subseteq g'_j.$

People will not get lose his bid if he propose more money for fewer goods.

---

[1] From Lehmann *et al.*(1999)

Myopic Best-Response Bidding Strategy

- A myopic agent bids to maximize utility at the current ask prices.
- The myopic best-response strategy is to submit an XOR bid for all bundles *S* that maximize utility $u_i(S)=v_i(S)-p(S)$ at the current prices. This maximizes the probability of a successful bid for bid-monotonic WD algorithms.

Dummy item

- When agent i have un-conflict bids, add an dummy item to each bids, so that all those bids will not be compatible with each other (so that the XOR bids don't need special care)

B1: {1, 2, 3}

B2: {4, 5, 6}

add dummy item Xi →

B1: {1, 2, 3, Xi}

B2: {4, 5, 6, Xi}

Outline

- iBundle – the model
- Optimality
- Computational Analysis

Optimality of iBundle

Theorem 1. i*Bundle terminates with an allocation that is within* $3\min\{|G|, |I|\}\epsilon$ *of the optimal solution, for myopic best-response agent bidding strategies.*

Proof of Theorem 1

- IP Model

$$\max_{x_i(S)} \sum_{S \subseteq G} \sum_{i \in I} x_i(S) v_i(S) \qquad \text{[IP]}$$

$$\text{s.t.} \quad \sum_{S \subseteq G} x_i(S) \leq 1, \qquad \forall i \qquad \text{(IP-1)}$$

$$\sum_{S \subseteq G,\ j \in S} \sum_{i} x_i(S) \leq 1, \qquad \forall j \qquad \text{(IP-2)}$$

$$x_i(S) \in \{0, 1\}, \qquad \forall i, S$$

$x_i(S)$ indicate whether or not agent i receives bundle S

$v_i(S)$ denote agent i's value for bundle S

Proof of Theorem 1

- LP Relaxation

$$\max_{x_i(S), y(k)} \sum_{S \subseteq G} \sum_{i \in I} x_i(S) v_i(S) \qquad [\mathrm{LP}_2]$$

$$\text{s.t.} \quad \sum_{S \subseteq G} x_i(S) \leq 1, \qquad \forall i \qquad (\text{LP-1})$$

$$\sum_{i \in I} x_i(S) \leq \sum_{k \in K, S \in k} y(k), \qquad \forall S \qquad (\text{LP-2})$$

$$\sum_{k \in K} y(k) \leq 1 \qquad (\text{LP-3})$$

$$x_i(S), y(k) \geq 0, \qquad \forall i, S, k$$

k correspond to a partition of items intc bundles

K is the set of all possible partitions

Proof of Theorem 1

- LP Dual

$$\min_{p(i),p(S),\pi} \sum_{i\in I} p(i) + \pi \qquad [\mathrm{DLP}_2]$$

$$\text{s.t.} \quad p(i) + p(S) \geq v_i(S), \qquad \forall i, S \qquad (\text{DLP-1})$$

$$\pi - \sum_{S\in k} p(S) \geq 0, \qquad \forall k \qquad (\text{DLP-2})$$

$$p(i), p(S), \pi \geq 0, \qquad \forall i, S$$

$$\max_{x_i(S),y(k)} \sum_{S\subseteq G} \sum_{i\in I} x_i(S) v_i(S) \qquad [\mathrm{LP}_2]$$

$$p(i) \longrightarrow \sum_{S\subseteq G} x_i(S) \leq 1, \qquad \forall i \qquad (\text{LP-1})$$

$$p(S) \longrightarrow \sum_{i\in I} x_i(S) \leq \sum_{k\in K, S\in k} y(k), \qquad \forall S \qquad (\text{LP-2})$$

$$\pi \longrightarrow \sum_{k\in K} y(k) \leq 1 \qquad (\text{LP-3})$$

$$x_i(S), y(k) \geq 0, \qquad \forall i, S, k$$

Proof of Theorem 1

Feasible primal. To construct a feasible primal solution assign $x_i(S_i) = 1$ and $x_i(S') = 0$ for all $S' \neq S_i$. Partition $y(k^*) = 1$ for $k^* = [S_1, \ldots, S_{|I|}]$, and $y(k) = 0$ otherwise.

Feasible dual. To construct a feasible dual solution assign $p(S) = p_{\text{ask}}(S)$. Constraints (DLP-1) and (DLP-2) are satisfied with assignments:

$$p(i) = \max\left\{0, \max_{S \subseteq G}\{v_i(S) - p(S)\}\right\} \qquad (1)$$

$$\pi = \max_{k \in K} \sum_{S \in k} p(S) \qquad (2)$$

$p(i)$ can be interpreted as agent i's maximum utility

π can be interpreted as the maximum revenue

Complementary-slackness conditions

- $x_i(S) > 0 \Rightarrow p(i) + p(S) = v_i(S), \qquad \forall i, S$

$$v_i(S) - p_{\text{ask}}(S) + 2\epsilon \geq \max\left\{0,\ \max_{S'}\{v_i(S') - p_{\text{ask}}(S')\}\right\}$$

$$p(i) + p(S) \leq v_i(S) + 2\epsilon, \qquad \forall i, S$$

- $y(k) > 0 \Rightarrow \pi - \sum_{S \in k} p(S) = 0, \qquad \forall k$

$$\pi - \sum_{S \in k} p(S) \leq \min\{|G|, |I|\}\epsilon, \qquad \forall k$$

$$\min_{p(i),p(S),\pi} \sum_{i \in I} p(i) + \pi \qquad [\text{DLP}_2]$$

$$x_i(S) \longrightarrow \text{s.t.} \quad p(i) + p(S) \geq v_i(S), \qquad \forall i, S \qquad \text{(DLP-1)}$$

$$y(k) \longrightarrow \pi - \sum_{S \in k} p(S) \geq 0, \qquad \forall k \qquad \text{(DLP-2)}$$

$$p(i), p(S), \pi \geq 0, \qquad \forall i, S$$

Optimality

$$\sum_{i \in I} p(i) \;\leq\; \sum_{i \in I} v_i(S_i) - \sum_{i \in I} p(S_i) + 2\min\{|G|, |I|\}\epsilon$$

$$\pi \;\leq\; \sum_{i \in I} p(S_i) + \min\{|G|, |I|\}\epsilon$$

$$\pi + \sum_{i \in I} p(i) \;\leq\; \sum_{i \in I} v_i(S_i) + 3\min\{|G|, |I|\}\epsilon$$

$$V_{\mathrm{DLP}} \;\leq\; V_{\mathrm{LP}} + 3\min\{|G|, |I|\}\epsilon \qquad V^*_{\mathrm{LP}} \leq V_{\mathrm{DLP}}$$

$$V_{\mathrm{LP}} \geq V^*_{\mathrm{LP}} - 3\min\{|G|, |I|\}\epsilon$$

The constructed primal solution is integral, so V_{LP} is feasible and optimal

Termination

Termination. By contradiction, assume the auction never terminates. Informally, [T1] implies that agents must submit different bids in successive rounds, but with myopic best-response bidding this implies that prices must increase, and agents must eventually bid above their values for bundles. We prove a contradiction with myopic best-response bidding strategies.

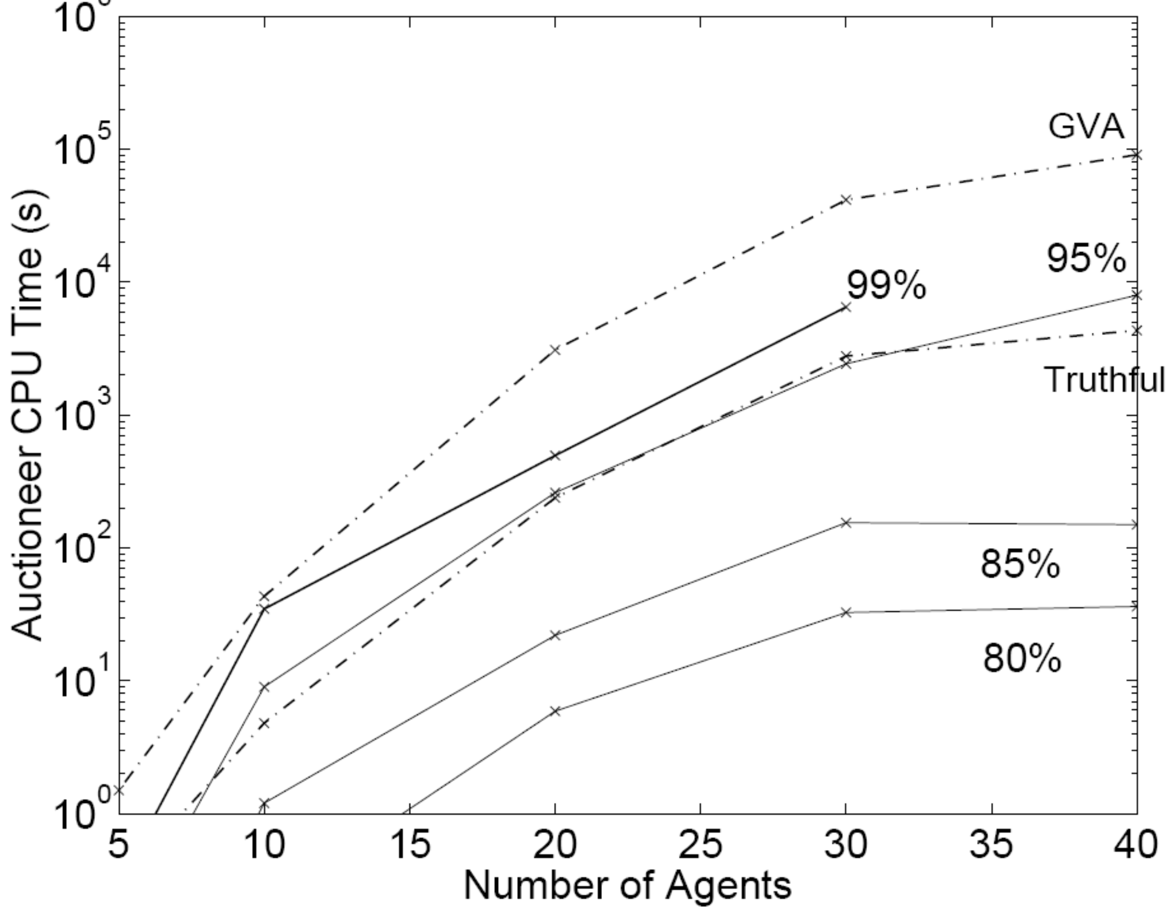
Outline

- iBundle – the model
- Optimality
- Computational Analysis

Computational Analysis

- In iBundle, the worst case gives $O(BV_{max}/\varepsilon)$ to converge
- The value of ε determines
  - The number of rounds to termination
  - The allocative efficiency

Computational Results



Computational Results

Problem		GVA	*i*Bundle			Approx-Bundle
			$\simeq 90\%$	$\simeq 95\%$	$\simeq 99\%$	
Decay	Eff (%)	100	91.5	94.9	98.3	85.1
$67.3\%^d$	WD-time^a (s)	41700	831	2400	5650	0
13.4^e	Pr-time^b (s)	–	26	34.5	44	39.2
	Comm^c (kBit)	18.8	221	306	394	377
WR	Eff (%)	100	90.7	94.9	99.2	79.4
71.5%	WD-time (s)	3	0.6	1.7	6	0
1	Pr-time (s)	–	5.4	11.5	40.9	12.2
	Comm (kBit)	18.1	20.5	52.1	144	53.1
Rand	Eff (%)	100	89.3	97	99	95.8
37.8%	WD-time (s)	68	4.4	7.4	11	0
11.2	Pr-time (s)	–	6.5	9.7	12.1	12.9
	Comm (kBit)	18.7	49.5	66.4	82.6	85.6
Unif	Eff (%)	100	–	95.6	99.1	76.2
58%	WD-time (s)	25	–	6.6	18.7	0
3	Pr-time (s)	–	–	14.7	42.0	46
	Comm (kBit)	18.2	–	56.5	120	124