**COMP 170 Discrete Mathematical Tools for CS**
**2005 Fall Semester – Written Assignment # 2**
**Distributed: Sept 9, 2005 – Due: Sept 15, 2005 *at end of class***

The top of your submission should contain (i) your name, (ii) your student ID #, (ii) your email address and (iv) your tutorial section.

Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. A solution that consists of just a number will be counted as wrong.

2nd Note: Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.

3rd Note: Some of these problems are taken (some modified) from section 1.2 of the textbook

**Problem 1:** There are more functions from the real numbers to the real numbers than most of us can imagine. In discrete mathematics, however, we often work with functions from a finite set $S$ with $s$ elements to a finite set $T$ with $t$ elements. Thus, there are only a finite number of functions from $S$ to $T$. How many functions are there from $S$ to $T$ in this case?

**Problem 2:** Consider the sets $S_3 = \{a, b, c\}$ and $S_4 = \{1, 2, 3, 4\}$?
(a) How many functions are there from the set $S_3$ to $S_4$?
(b) How many *one-to-one* functions are there from the set $S_3$ to $S_4$?
(c) How many *onto* functions are there from the set $S_3$ to $S_4$?
(d) How many *bijections* are there from the set $S_3$ to $S_4$?
(e) How many functions are there from the set $S_4$ to $S_3$?
(f) How many *one-to-one* functions are there from the set $S_4$ to $S_3$?
(g) How many *onto* functions are there from the set $S_4$ to $S_3$?
(h) How many *bijections* are there from the set $S_4$ to $S_3$?
(i) How many functions are there from the set $S_4$ to $S_4$?
(j) How many *one-to-one* functions are there from the set $S_4$ to $S_4$?
(k) How many *onto* functions are there from the set $S_4$ to $S_4$?
(l) How many *permutations* are there from the set $S_4$ to $S_4$?

**Problem 3:**
(a) Assuming $k \leq n$, in how many ways can we pass out $k$ distinct pieces of fruit to $n$ children if each child may get at most one piece?
(b) What if $k > n$?
Assume for both questions that we pass out all the fruit.

**Problem 4:**

(a) Assuming $k \leq n$, in how many ways can we pass out $k$ identical pieces of fruit to $n$ children if each child may get at most one?
(b)What if $k > n$?
Assume for both questions that we pass out all the fruit

**Problem 5:** A base ten number is string of five digits, where the first digit is not 0 and each digit is in the set $\{0, 1, ..., 9\}$ (so 52375 is a base ten number with five digits but 02323 and 2323 are not base ten number with five digits).
(a) How many base ten numbers have five digits?
(b)How many five-digit numbers have no two consecutive digits equal?
(c) How many have at least one pair of consecutive digits equal?

**Problem 6:** Suppose you are choosing participants for a panel discussion on allowing karaoke on campus. You must choose 4 administrators from a group of 10 and 4 students from a group of 20. In how many ways can this be done?

**Problem 7:** Suppose you are organizing a panel discussion on allowing on campus. Participants will sit behind a table in the order in which you list them. You must choose 4 administrators from a group of 10 and 4 students from a group of 20.
(a) If the administrators must sit together in a group and the students must sit together in a group, in how many ways can you choose and list the 8 people?
(b) If you must alternate students and administrators, in how many ways can you choose and list them?

**Problem 8:** A basketball team has 12 players. However, only 5 players play at any given time during a game.
(a) In how may ways can the coach choose the 5 players?
(b) To be more realistic, the 5 players playing a game normally consist of 2 guards, 2 forwards, and 1 center. If there are 5 guards, 4 forwards, and 3 centers on the team, in how many ways can the coach choose 2 guards, 2 forwards, and 1 center?
(c) What if one of the centers is equally skilled at playing forward?

**Challenge Problem:** *This question will not be marked and does not have to be done. If you can do it properly, then you can be sure that you really understand this section. The best submitted solutions to this problem will be given extra-credit.*

A tennis club has $2n$ members. We want to pair up the members by twos for singles matches. In how many ways can we pair up all the members of the club? Suppose that in addition to specifying who plays whom, we also determine who serves first for each pairing. Now in how many ways can we specify our pairs?