Erratum to
"Vickrey Pricing and Shortest Paths: What is an edge worth?"

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In a paper published in FOCS 2001, titled "Vickrey Pricing and Shortest Paths: What is an edge worth?", we considered the following *replacement paths problem:*

> Given a graph G with non-negative edge weights, and a pair of nodes x, y, let $P = (e_1, e_2, \ldots, e_p)$ denote the sequence of edges in a shortest path from x to y in G. The replacement paths problem is to compute the shortest path from x to y, in each of the graphs $G \setminus e_i$, that is, G with edge e_i removed, for $i = 1, 2, \ldots, p$.

A naïve algorithm for the replacement paths problem runs in $O(n(m + n \log n))$ time, executing the single-source shortest path algorithm up to n times.

In our paper [3], we claimed that the replacement paths problem can be solved in $O(m+n \log n)$ time for both undirected and directed graphs. However, there is a flaw that invalidates the algorithm for directed graphs. (The algorithm for undirected graphs remains valid.) The same bound for undirected graphs is also achieved by Nardelli, Proietti, and Widmayer [6], who solve the *most vital node* problem with an algorithm that also solves the replacement paths problem. Their work is in turn based on earlier work by Malik, Mittal and Gupta [5], Ball, Golden, and Vohra [1] and Bar-Noy, Khuller, and Schieber [2].

The error in our algorithm for directed graphs led us to investigate the hardness of the replacement paths problem and other related problems. We have recently established lower bounds that show that no replacement paths algorithm of a certain class (including all known algorithms) can achieve the running time of the algorithm for undirected graphs [4].

The mistake in the directed graph algorithm of [3] occurs on page 257, just after Lemma 2, where we say "A simple corollary of this lemma is the fact that if (u, v) is the single edge of $path(x, y;\ G \setminus e)$ in $E(V_x, V_y)$, then $path(v, y;\ G \setminus e) = path(v, y)$." In other words, we claimed that one can compute the shortest path from a node v to y in a graph $G \setminus e$ by computing the shortest path in G, then counting its cost as infinite if it happens to use e. The simple graph in Figure 1 demonstrates the error. In this figure, the shortest path from v to y goes through the edge e, which causes our algorithm to ignore the replacement edge candidate (x, v). This is clearly wrong, since the correct solution is to use the *second* shortest path from v to y, which does not go through e. Because of this error, our algorithm fails to find the replacement path for edge e in this graph.

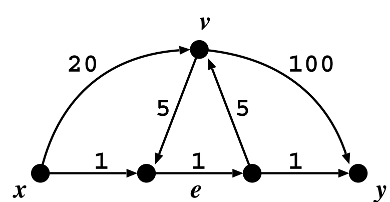

Figure 1: A directed graph for which the replacement paths algorithm of [3] fails.

References

[1] M. O. Ball, B. L. Golden, and R. V. Vohra. Finding the most vital arcs in a network. *Oper. Res. Letters*, 8:73–76, 1989.

[2] A. Bar-Noy, S. Khuller, and B. Schieber. The complexity of finding most vital arcs and nodes. Technical Report CS-TR-3539, Institute for Advanced Studies, Univ. Maryland, 1995.

[3] J. Hershberger and S. Suri. Vickrey prices and shortest paths: What is an edge worth? In *Proc. 42nd Annu. IEEE Symp. Found. Comput. Sci.*, pages 252–259, 2001.

[4] J. Hershberger, S. Suri, and A. Bhosle. On the difficulty of some shortest path problems. Manuscript, 2002.

[5] K. Malik, A. K. Mittal, and S. K. Gupta. The k most vital arcs in the shortest path problem. *Oper. Res. Letters*, 8:223–227, 1989.

[6] E. Nardelli, G. Proietti, and P. Widmayer. Finding the most vital node of a shortest path. In *Proc. COCOON*, 2001.

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