

# COMP170

# Discrete Mathematical Tools for Computer Science

## Lecture 8

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*Discrete Math for Computer Science*

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*Section 3.2, pp. 104-114*

## 3.2 Variables and Quantifiers

- Variables and Universes
- Quantifiers
- Standard Notation for Quantification
- Statements about Variables
- Proving Quantified Statements True or False
- Negation of Quantified Statements
- Implicit Quantification

# Variables and Universes

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In statements such as  $m^2 > m$ , variable  $m$  is *not constrained*.  
Unconstrained variables are called *free variables*.

Each possible value of a free variable gives a *new* statement.  
The **Truth** or **Falsehood** of this new statement, is determined  
by substituting in the new value for the variable.

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Again consider the statement:  $(*) \quad m^2 > m$

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Two main points:

- Clearly state the universe
- A statement about a variable can be **True** for some values of a variable and **False** for others.

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- While  $m^2 > m$  is True for values such as  $m = -3$  or  $m = 9$  it is False  $m = 0$  or  $m = 1$ .
- Thus, it is not True that  $m^2 > m$  for every integer  $m$ , so (\*) is False



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- A phrase like for every integer  $m$  that converts a symbolic statement about potentially any member of our universe into a statement about the universe is called a **quantifier**
- A quantifier that asserts a statement about a variable is true for every value of the variable in its universe, is called a **universal quantifier**.

# Examples of universal quantifiers

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(\*\*) For every real number  $m$ ,  $2m$  is even

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- To show that a statement with an existential quantifier is **True**, we need only exhibit *one* value of the variable being quantified that makes the statement **True**.
  - Example for (\*\*\*) : set  $m = 2$

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**Examples:** Use  $Z$  for universe of all integers

- For all integers  $n$ ,  $n^2 \geq n$  becomes  $\forall n \in Z (n^2 \geq n)$
- There exists an integer  $n$  is such that  $n^2 \not\geq n$  becomes  
$$\exists n \in Z (n^2 \not\geq n)$$

Using the given notation let's rewrite the part of Euclid's division theorem (Theorem 2.12) that states

For every positive integer  $n$  and every nonnegative integer  $m$ , there are integers  $q$  and  $r$ , with  $0 \leq r < n$ , such that  $m = qn + r$

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$$\forall n \in \mathbb{Z}^+ \quad (\forall m \in \mathbb{N} \quad (\exists q \in \mathbb{N} \quad (\exists r \in \mathbb{N} \quad ((r < n) \wedge (m = qn + r)) \quad ))))$$

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Let  $p(m, n, q, r)$  denote  $m = nq + r$  with  $0 \leq r < n$ .

Leave out references to universes to clearly see the order in which the quantifiers occur.

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$$\forall n (\forall m (\exists q (\exists r p(m, n, q, r) )))$$

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Consider the following two statements.

$$\text{a) } \forall x \in R^+ (x > 1)$$

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$$\text{a')} \quad \forall x \in R ( (x > 0) \Rightarrow (x > 1) )$$

$$\text{b')} \quad \exists x \in R ( (x > 0) \wedge (x > 1) )$$

### Theorem 3.2:

Let  $U_1, U_2$  be two universes with  $U_1 \subseteq U_2$ .

Suppose that  $q(x)$  is a statement such that

$$(*) \quad U_1 = \{x \in U_2 \mid q(x) \text{ is true}\}.$$

Then, if  $p(x)$  is a statement about  $U_2$ , it may also be interpreted as a statement about  $U_1$ , and

a.  $\forall x \in U_1 (p(x))$  is equiv. to  $\forall x \in U_2 (q(x) \Rightarrow p(x))$ ,  
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## Proof:

By  $(*)$ ,  $q(x)$  must be **True** for all  $x \in U_1$  and  
**False** for all  $x \in U_2$  but  $x \notin U_1$ .

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c)  $\forall x \in R (\exists y \in R (y > x))$

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- d)  $\forall x \in R (\forall y \in R (y > x))$  **F**. Let  $x = 1, y = 0$
- e)  $\exists x \in R ((x \geq 0) \wedge \forall y \in R^+ (y > x))$   
**T**. Let  $x = 0$ .

## Principle 3.2

### (The Meaning of Quantified Statements)

- The statement  $\exists x \in U (p(x))$  is **True** if there exists at least one value of  $x \in U$  for which the statement  $p(x)$  is **True**.
- The statement  $\exists x \in U (p(x))$  is **False** if there is **no**  $x \in U$  for which  $p(x)$  is **True**.
- The statement  $\forall x \in U (p(x))$  is **True** if  $p(x)$  is **True** for every value of  $x \in U$ .
- The statement  $\forall x \in U (p(x))$  is **False** if  $p(x)$  is **False** for at least one value of  $x \in U$ .

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i.e., there exists some integer  $n$  s.t.  $n^2 \leq 0$ ,

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Thus, the negation of our for all ( $\forall$ ) statement is a  
there exists ( $\exists$ ) statement.

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**Proof:**

$p(x)$	$\neg p(x)$	$\forall x \in U(p(x))$	$\neg \forall x \in U(p(x))$	$\exists x \in U(\neg p(x))$
always true	always false	true	false	false
not always true	not always false	false	true	true



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**Example:** Let  $p(x)$  be the statement  $x^2 > x$ . Then

$$\neg \forall n \in Z (n^2 > 0) \quad \text{is equivalent to} \quad \exists n \in Z (n^2 \leq 0)$$

**Corollary 3.4:** The statements

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Now, setting  $q(x) = \neg p(x)$  gives

$\forall x \in U(\neg p(x))$  and  $\neg \exists x \in U(p(x))$  are equivalent,

and proves the corollary.

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Let  $p(x)$  be  $2x$  is odd.

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The corollary then says that

$$\neg \exists x \in Z (2x \text{ is odd})$$

is equivalent to

$$\forall x \in Z (2x \text{ is even})$$

## 3.2 Variables and Quantifiers

- Variables and Universes
- Quantifiers
- Standard Notation for Quantification
- Statements about Variables
- Proving Quantified Statements True or False
- Negation of Quantified Statements
- Implicit Quantification

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In symbols

$$\forall m \in \mathbb{Z} \left( \forall n \in \mathbb{Z} \left( (p(m) \wedge p(n)) \Rightarrow p(m + n) \right) \right)$$