

COMP 170 – Discrete Mathematical Tools for CS
2006 Fall Semester – Practice Assignment #2
Distributed: Dec 5, 2006

This handout is meant as a *practice* assignment to let you practice the material taught during the last weeks of class. Solutions to this assignment *should not* be handed in.

Most of these problems are taken (modified) from the back of Section 5.7 of the textbook.

- Problem 1:** Suppose a student who knows 60% of the material covered in a chapter of a textbook is going to take a five-question objective (each answer is either right or wrong, not multiple choice or true-false) quiz. Let X be the random variable that gives the number of questions the student answers correctly for each quiz in the sample space of all quizzes the instructor could construct.
- (a) What is the expected value of the random variable $X - 3$?
 - (b) What is the expected value of $(X - 3)^2$?
 - (c) What is the variance of X ?
- Problem 2:** If the quiz in Problem 1 has 100 questions;
- (a) what is the expected number of right answers?
 - (b) what is the variance of the expected number of right answers?
- Problem 3:** Show that if X and Y are independent and b and c are constant, then $X - b$ and $Y - c$ are independent.
(Note: This simplifies the proof of Theorem 5.29)
- Problem 4:** A cup contains three coins; one \$1 coin; one \$2 coin and one \$5 coin. Withdraw two coins, first one and then the second, without replacement.
- (a) What is the expected amount of money and variance for the first draw?
 - (b) For the second draw?
 - (c) For the sum of both draws?
- Problem 5:** What are the expected number of failures and the variance of the number of failures in n independent trials with probability p of success?
Compare your answers with the corresponding results for successes.
- Problem 6:** Let X be a random variable and c a constant number. What is $Var(cX)$ (as a function of $Var(X)$)?
- Problem 7:**
- (a) Roll a fair die and let X be the number of dots showing on top. What are $E(X)$ and $Var(X)$?
 - (b) What are $E(2X)$ and $Var(2X)$?
 - (c) Now roll another die and let Y be the number of dots showing. What are $E(X + Y)$ and $Var(X + Y)$?

Problem 8: Flip four fair coins. Let X be the number of heads showing. Now flip four $\frac{1}{3}$ -biased coins (that is, they have $P(H) = \frac{1}{3}$) and let Y be the number of heads showing.

(a) What is $E(X + Y)$?

(b) What is $Var(X + Y)$?

Problem 9: You have a fair 10-sided die with the numbers 1 to 10 painted on its sides.

(a) Roll the die until you see a **1**. Then roll it until you see a **2**. Keep on rolling until you have seen all of the numbers in the order **1,2,3,4,5,6,7,8,9,10**. What is the expected number of times that you have rolled the die?

(b) Now start rolling the die again. What is the expected number of times you have to roll it until you have seen all of the numbers **1,2,3,4,5,6,7,8,9,10** at least once each?

Problem 10: A standard *deck* contains 52 cards, 4 each of **2,3,4,5,6,7,8,9,J,Q,K,A**. Now start the following process. Pick a random card from the deck, show it, and then return it to the deck. Continue repeating this process, stopping when each type of card, **2,3,4,5,6,7,8,9,J,Q,K,A**, has been seen at least once. What is the expected number of cards that you will have drawn?