

**COMP 170 Discrete Mathematical Tools for CS**  
**2010 Spring Semester – Written Assignment # 3**  
**Distributed: February 18, 2010 – Due: February 25, 2010**

Your solutions should contain (i) your name, (ii) your student ID #, (iii) your email address and (iv) your tutorial section.

Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions should be submitted before 5PM of the due date in the collection bin in front of Room 4213A (this is near the TA labs).

**Problem 1:** In class we stated that  
*each row of Pascal's triangle first increases and then decreases.*  
In this question you will prove this statement.

(a) Using the formula  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$  prove that  
if  $0 < k \leq n/2$  then  $\binom{n}{k-1} < \binom{n}{k}$ .

(b) Using part (a) and the fact that  $\binom{n}{k} = \binom{n}{n-k}$  prove that  
*each row of Pascal's triangle first increases and then decreases.*

**Problem 2:** (a) If you have twelve distinct chairs to paint, in how many ways can you paint seven of them orange and five of them red?  
(b) Now, how many ways can you paint four of them green, three of them blue, and five of them red?

**Problem 3:** In a Cartesian coordinate system, how many paths are there from the origin to the point with integer coordinates  $(m, n)$  if the paths are built up of exactly  $m + n$  horizontal and vertical line segments, each of length 1?

You should assume that all of the horizontal edges go from left to right and all the vertical edges from bottom up. You should also assume that  $m, n > 0$ .

That is, you start at point  $(0, 0)$  and each edge either goes

- (i) vertically, from  $(i, j)$  to  $(i, j + 1)$  or
- (ii) horizontally, from  $(i, j)$  to  $(i + 1, j)$ .

**Problem 4:** In class we proved, for  $0 < k < n$ ,

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

using the sum principle. Now prove this equation algebraically (directly) by plugging in the formula  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$  and showing that the left side equals the right side.

**Problem 5:** Give two proofs that

$$\binom{n}{k} \binom{k}{j} = \binom{n}{j} \binom{n-j}{k-j}$$

Your first proof should be purely algebraic, i.e., just plug in the formula  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$  and show that the left side equals the right side. Your second proof should be combinatorial, i.e., it should show that the left and right sides are just two different ways to count the same thing.

**Problem 6:** Explain why

$$\sum_{i=0}^n 2^i \binom{n}{i} = 3^n.$$

An algebraic proof would be sufficient (a combinatorial proof is not necessary). *Hint: use the binomial theorem.*