Computer Security

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COMP4631

Lecture 08: Key Management for One-key Ciphers

Topics of this Lecture

1. The generation and distribution of secret keys.
2. A key distribution protocol with a key distribution center.
3. The Diffie-Hellman key exchange protocol.

Secret Key Generation

Question: How to generate a secret key for a one-key cipher?

Answer: It depends on the specific cryptosystem.

Case I: The secret key k is a binary string $k_1 k_2 \cdots k_n$, where k_i are independent of each other.

Solution 1: If n is not long, say 128, flipping a coin n times.

Solution 2: Use a pseudorandom number generator.

Case II: Key bits must satisfy certain relations.

In this case, no general approach exits. It differs from system to system.

Key Generation in a Cipher: Example

- The message and ciphertext spaces: $\mathcal{M} = \mathcal{C} = \{0,1\}^*$.
- $\mathcal{K}$ consisting of all binary 128×128 invertible matrices.
- Encryption is block by block (block size 128 bits). For a secret key $K \in \mathcal{K}$ and a message block m_i, the encryption is

$$E_K(m_i) = m_i K = c_i.$$

  The decryption function is

$$D_K(c_i) = c_i K^{-1} = m_i.$$

Question: How do you generate a binary 128×128 invertible matrix K?

Remark: Flipping a coin 128×128 times does not work!

Key Distribution: Necessity

- For conventional encryption, the two parties must share the same key.
- The key must be protected from access by others.
- The key should be changed regularly (an adversary or enemy may learn the key in some way).

Key distribution: delivering a key to both parties, without allowing others to see the key.

Key agreement: agreeing on a key by parties involved, without allowing others to see the key.

Key Distribution: some General Approaches

- A selects a key, and physically delivers it to B.
- A third party can select the key and and physically deliver it to both A and B.
- If A and B have previously and recently used a key, one party can transmit the new key to the other, encrypted using the old key.
- If A and B each has an encrypted connection to a third party C, C can deliver a key on the encrypted links to A and B.

Key Distribution: more General Approaches

- Secret key distribution using a "public key cipher".
  (It will be introduced later.)
- Other key distribution protocols.

Remark: As an example of protocols for key distribution, we introduce a key distribution protocol using a key distribution center.

A Key Distribution Protocol

Parties involved: A key distribution center (KDC), a group of people to communicate with each other.

Requirements: Whenever A wants to communicate with B, the KDC should generate a temporary key (called **session key**) and distribute it to A and B. Both confidentiality and authenticity must be achieved.

Remark: The session key (temporary key) is established only for this communications between A and B.

A Key Distribution Protocol – Continued

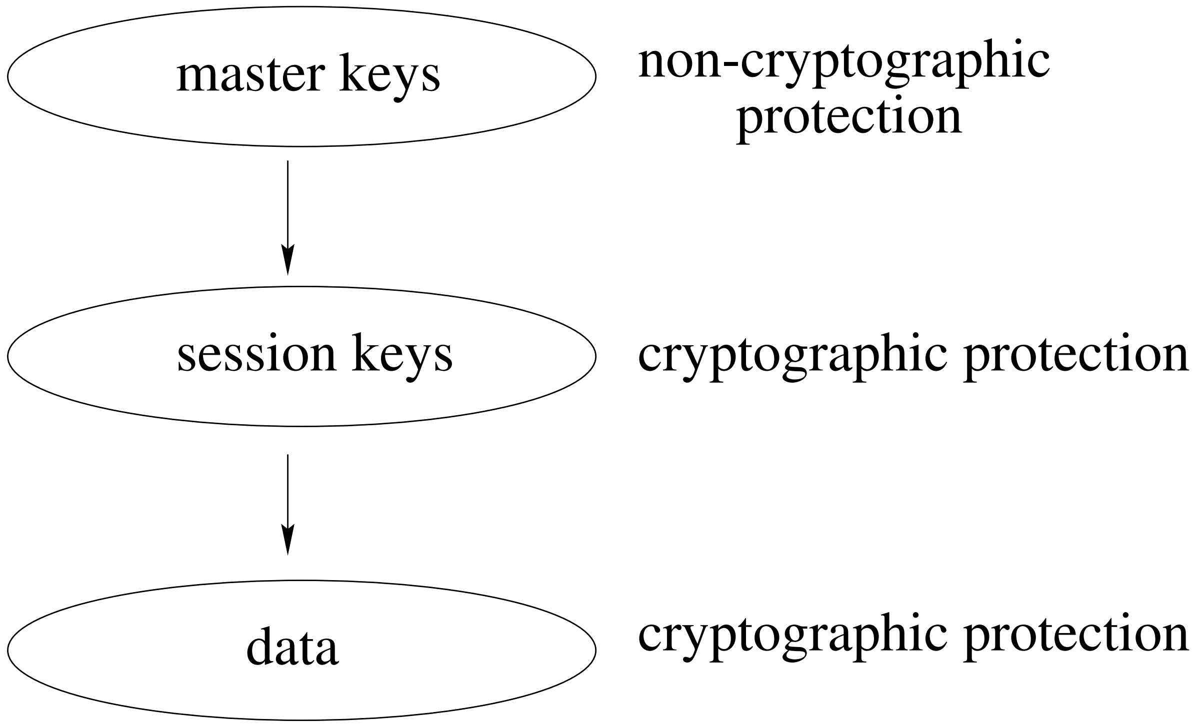
Building blocks needed:

- The KDC and all parties involved in this communication system use a one-key block cipher.
- The KDC and each party A share a secret key k_a, which is called a **master key**.

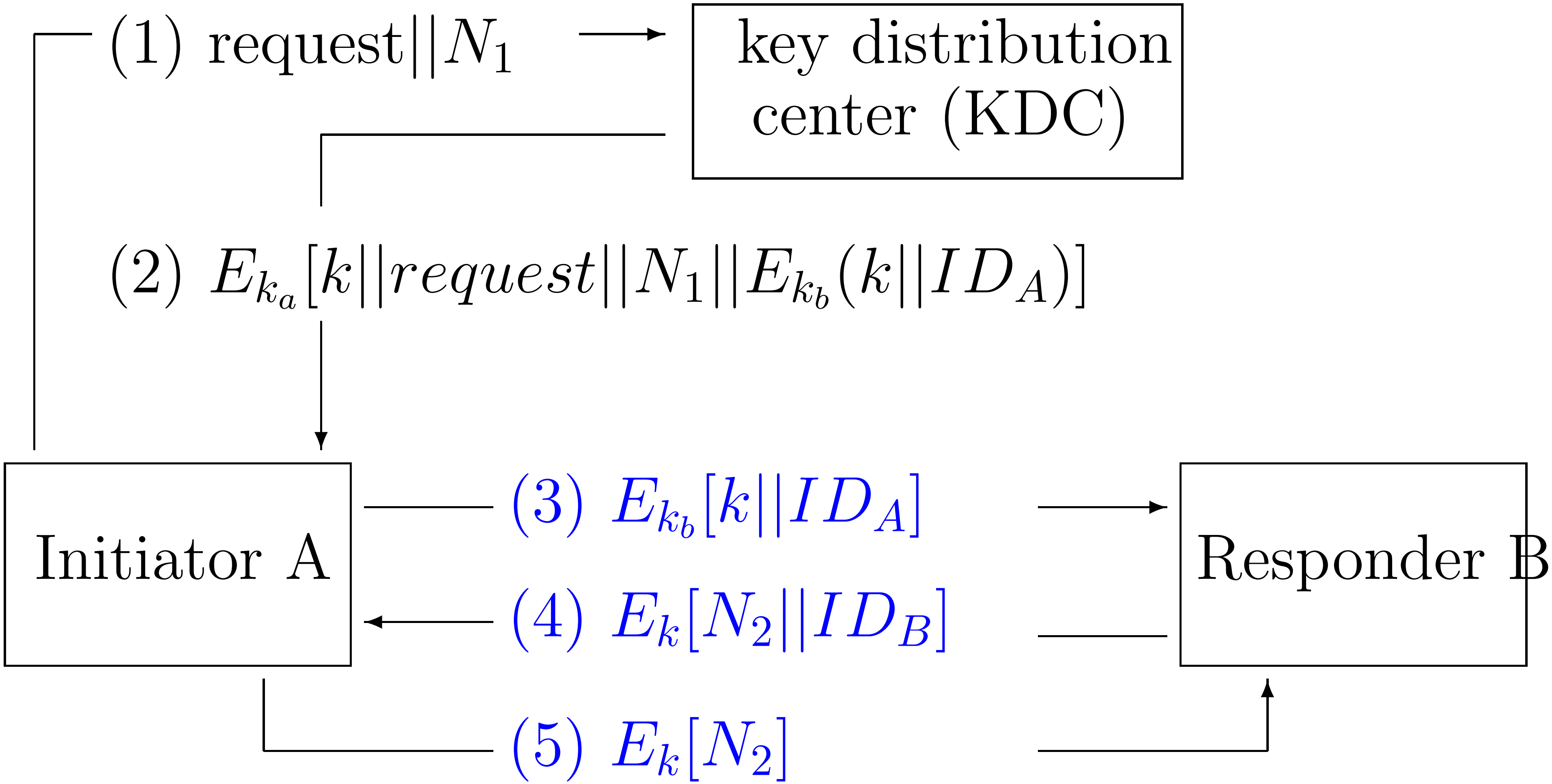
Remark: The master keys are used to protect the sessions keys when they are distributed.

A Key Distribution Protocol – Continued

Pictorial description of use of the key hierarchy:



A Key Distribution Protocol



Parameters in the Key Distribution Protocol

- N_i is a nonce, used as identifier for that transaction.
- k_a, k_b master keys, k secret key.

Question: Which steps are for authentication?

Question: Does it provide mutual authentication or authentication in one direction?

Explaining the Key Distribution Protocol (1)

- The nonce may be a timestamp, a counter, or a random number. The minimum requirement is that it differs with each request. Also it should be hard for an opponent to guess it. So random number is a good choice.

- When A receives (2), A can verify that its original request was not altered before reception by the KDC. Because of the nonce, that is not a reply of some previous request.

  The message (2) also includes two items intended for B: the one-time session key k, and an identifier of A (i.e., its network address), ID_A.

Explaining the Key Distribution Protocol (2)

- After Step (3), a session key has been securely delivered to A and B. They may begin their protected exchange.
- Steps (4) and (5) assure B that the original message received in Step (3) was not a replay of an earlier one by a third person.

  Question: Why?
- Steps (4), (5) and (3) are for authentication.

Discrete logarithms

Primitive roots: Let p be a prime. An integer α is called a **primitive root** of p if each nonzero element $a \in \mathbf{F}_p$ can be uniquely expressed as

$$a = \alpha^i \bmod p$$

for some integer i, where $0 \leq i \leq p-2$.

Discrete logarithm: The exponent i is referred to as the **discrete logarithm**, or **index**, of a for the base α, and is denoted $\log_\alpha a$ or $\mathrm{ind}_\alpha(a)$.

Discrete logarithm problem:
Given p, α, and a, find $\log_\alpha a$.
This is in general very hard.
Brute force solution: compute $b = \alpha^i \bmod p$ for all i, $0 \leq i \leq p-2$ and check if $b = a$.

Primitive roots

Example: 2 is a primitive root of the prime 11. Also we have $\log_2(6) = 9$.

i	0	1	2	3	4	5	6	7	8	9
$2^i \bmod 11$	1	2	4	8	5	10	9	7	3	6

Theorem: Every prime p has at least one primitive root.

To find primitive roots

Rule of tumb: Most primes p have a small primitive root. For example, for the primes less than 100000, approximately 37.5% have 2 as a primitive root, and approximately 87.4% have a primitive root of value 7 or less.

For primes of reasonable size, many programming languages for mathematics have commands for finding primitive roots.

Diffie-Hellman Key Exchange Protocol

User A		User B
Generate random $X_A < p$ calculate $Y_A = \alpha^{X_A} \bmod p$	$\xrightarrow{Y_A}$	Generate random $X_B < p$ Calculate $Y_B = \alpha^{X_B} \bmod p$
Calculate $k = (Y_B)^{X_A} \bmod p$	$\xleftarrow[Y_B]{}$	Calculate $k = (Y_A)^{X_B} \bmod p$

Diffie-Hellman Key Exchange Protocol

- It is for two users to exchange a key securely that can then be used for subsequent encryption of message.
- $k = \alpha^{X_A X_B} \bmod p$. Also p and α are publicly known. But X_A and X_B must be kept secret.
- The security with respect to **passive attacks** is based on the belief that solving the discrete logarithm problem is hard in general. It is vulnerable to an **active attack** if an adversary has control over the communication cannel.