COMP170
Discrete Mathematical Tools for Computer Science
Big O Notation

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A quick and dirty Introduction to big O Notation

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(You'll see more details in COMP171 and COMP271.)

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Answer depends upon value of n.

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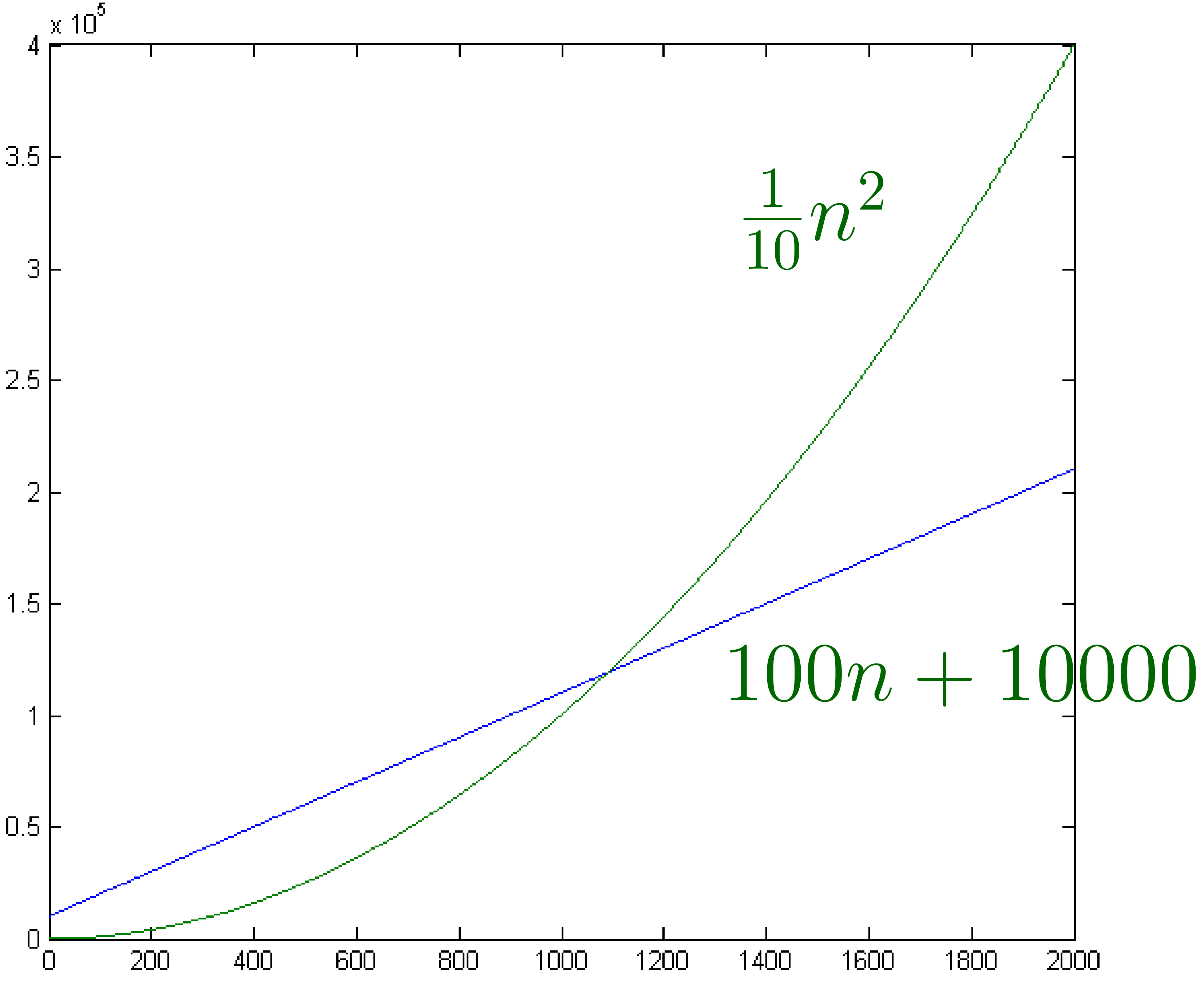
In Computer Science we are usually interested in what happens when our problem input size gets large.



x 10^5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000
$\frac{1}{10}n^2$
$100n + 10000$

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such that

(ii) There is some positive $c \in R$

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If (i) There is some positive $x_0 \in R$

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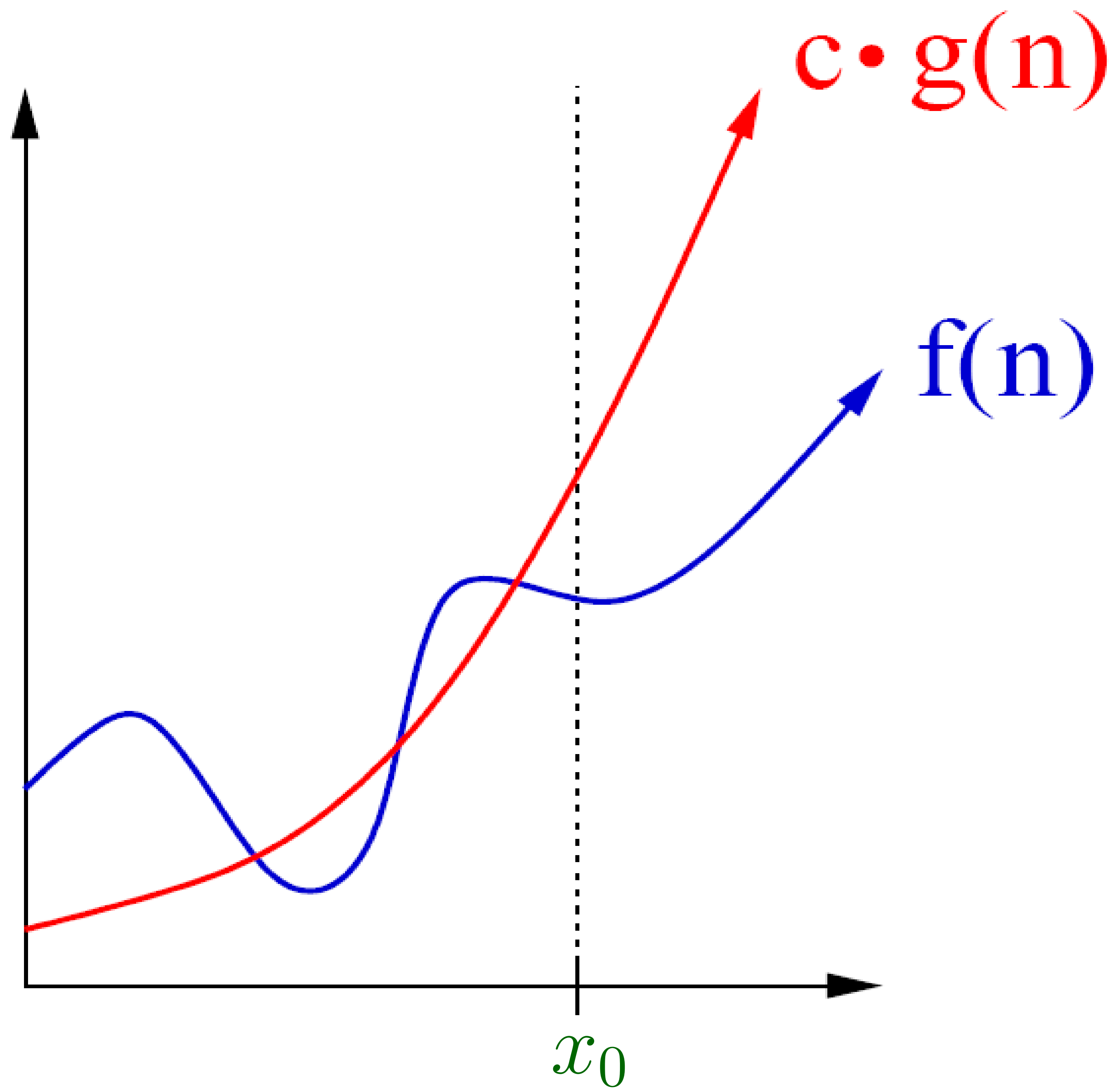
such that

$$\forall x \geq x_0 \qquad f(x) \leq cg(x).$$


c • g(n)
f(n)
x_0

Let $x_0 = 1091$.

Can verify, $\forall n > x_0,\ 100n + 10000 \le \frac{1}{10}n^2$.

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More Examples:

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More Examples:

$$4n^2$$

$$8n^2 + 2n - 3$$

$$n^2/5 + \sqrt{n} - 10\log n$$

$$n(n-3)$$

are all $O(n^2)$.

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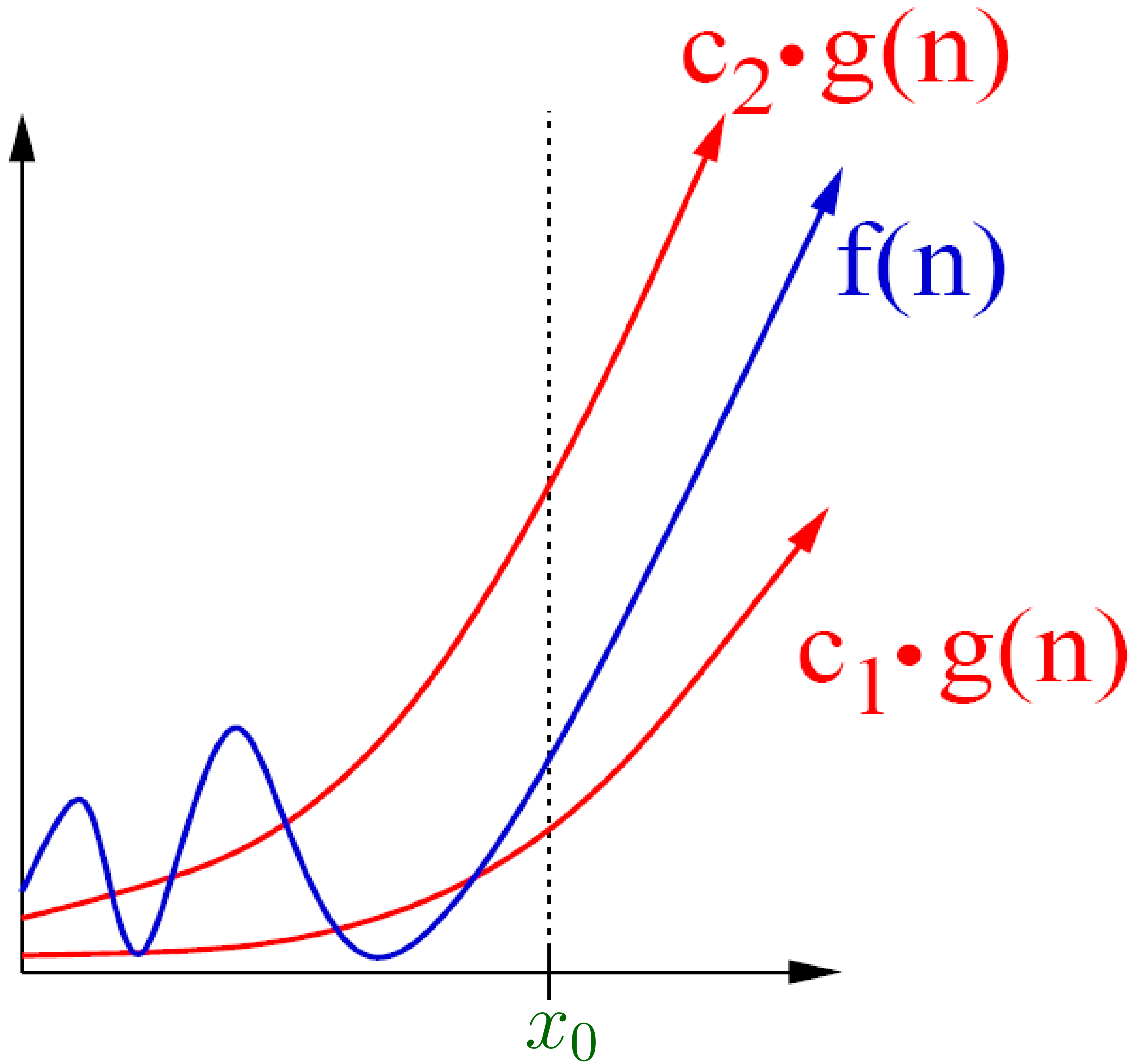
In this case we say

$$f(n) = \Theta(g(n))$$

which is the same as

$$g(n) = \Theta(f(n))$$


c2•g(n)
f(n)
c1•g(n)
x0

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