COMP 170 Discrete Mathematical Tools for CS
2005 Fall Semester – Written Assignment # 3
Distributed: Sept 16, 2005
REVISED & CORRECTED Sept 21, 2053
Due: Sept 22, 2005 *at end of class*

The top of your submission should contain (i) your name, (ii) your student ID #, (ii) your email address and (iv) your tutorial section.

Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. A solution that consists of just a number will be counted as wrong.

2nd Note: Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.

3rd Note: Some of these problems are taken (some modified) from section 1.3 of the textbook

Problem 1: In class we stated that

each row of Pascal's triangle first increases and then decreases.

In this question you will prove this statement.

(a) Using the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ prove that

$$\text{if } 0 < k \le n/2 \text{ then } \binom{n}{k-1} < \binom{n}{k}.$$

(b) Using part (a) and the fact that $\binom{n}{k} = \binom{n}{n-k}$ prove that *each row of Pascal's triangle first increases and then decreases.*

Problem 2: (a) If you have ten distinct chairs to paint, in how many ways can you paint six of them orange and four of them red?

(b) Now, how many ways can you paint two of them green, four of them blue, and four of them red?

Problem 3: In a Cartesian coordinate system, how many paths are there from the origin to the point with integer coordinates (m, n) if the paths are built up of exactly $m + n$ horizontal and vertical line segments, each of length 1?

You should assume that all of the horizontal edges go from left to right and all the vertical edges from bottom up.

That is, you start at point $(0, 0)$ and each edge either goes
(i) vertically, from (i, j) to $(i, j + 1)$ or
(ii) horizontally, from (i, j) to $(i + 1, j)$.

Problem 4: A 25-member club must have a president, vice president, secretary, and treasurer, as well as a three-person nominating committee. If the officers must be different people, and if no officer may be on the nominating committee, in how many ways could the officers and nominating committee be chosen? Answer the same question if officers may be on the nominating committee.

Problem 5: In class we proved, for $0 \leq k \leq n$,

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

using the sum principle. Now prove this equation algebraically (directly) by plugging in the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and showing that the left side equals the right side.

Problem 6: Give two proofs that

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$$

Your first proof should be purely algebraic, i.e., just plug in the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and show that the left side equals the right side. You second proof should be combinatorial, i.e., it should show that the left and right side are just two different ways to count the same thing.

Problem 7: Explain why

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0.$$

An algebraic proof would be sufficient (a combinatorial proof is not necessary).

Challenge problem: (a) Suppose that $n = 2k$ is even. Evaluate $\sum_{i=0}^{k} \binom{n}{2i}$.

(b) Now suppose that $n = 3k$ for some integer k. Evaluate $\sum_{i=0}^{k} \binom{n}{3i}$.