ASSIGNMENT 5: COMP2711H

FALL 2015

Q1 A base-10 numeral is randomly chosen from the range 000...999. What is the probability that the numeral contains at most one of the elements in the set $\{3, 5\}$? (10 marks)

Q2 Use the axioms for probability and mathematical induction to prove that for all integers $n \geq 2$, if $A_1, A_2, A_3, \cdots, A_n$ are any mutually disjoint events in a probability space (S, p), then

$$p\left(\cup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} p(A_i).$$

(10 marks)

Q3 The *standard loaded coin* has probability $p(H) = 0.8$ and $p(T) = 0.2$. Flip a standard loaded coin 20 times. What is the probability of get k heads? (10 marks)

Q4 A fair coin is tossed until either a head comes up or four tails are obtained. What is the expected number of tosses? (10 marks)

Q5 Two fair dice are rolled. At least one is a four. What is the probability that both are fours? (10 marks)

Q6 A fair coin is tossed three times. Consider the following three events:

- A is the event that the first toss is a head.
- B is the event that the second toss is a head.
- C is the event that exactly two heads are in a row.

Answer the following three questions:

(a) Are A and B independent? (4 marks)

(b) Are A and C independent? (4 marks)

(c) Are B and C independent? (4 marks)

Q7 Suppose a fair coin is tossed six times. Let X denote the number of heads. (12 marks)

- What is the distribution of the random variable X?
- What is the expected value of the random variable X?
- What are the variance and the standard deviation of X?

Q8 Suppose that we have two bags each containing black and white balls. One bag contains three times as many white balls as blacks. The other bag contains three times as many black balls as white. Suppose we choose one of these bags at random. For this bag we select five balls at random, replacing each ball after it has been selected (i.e., in the selected bag, put the same ball as the selected one back into the bag). The result is that we find 4 white balls and one black. What is the probability that we were using the bag with mainly white balls? (16 marks)

Q9 Let X and Y be two independent random variables on a probability space (S, p). Prove that

$$E(XY) = E(X)E(Y).$$

(10 marks)

Date: Handed out on 18 October, due on 28 October at the *beginning* of the lecture.