COMP 3711H – Fall 2016
Tutorial 2

1. There are n items in an array. It is easy to see that that their minimum can be found using $n - 1$ comparisons and that $n - 1$ are actually required. It is also easy to see that finding the max can similarly be done using $n - 1$ comparisons with $n - 1$ required.

   Design an algorithm that finds *both* the minimum and the maximum using at most $\frac{3}{2}n + c$ comparisons where $c > 0$ can be any constant you want.

   Note: Although it is harder to prove, $\frac{3}{2}n + c$ *comparisons is actually a lower bound.*

2. Prove that insertion in a binary search tree requires at least $O(\log n)$ comparisons (in the worst case) per insertion, where n is the number of items in the search tree.

   Hint: What lower bounds have we learned in class? Suppose you built the search tree using insertions. What can you do with it?

3. Build a Binary Search Tree for the items
   8, 4, 6, 13, 3, 9, 11, 2, 1, 12, 10, 5, 7
   and draw the final tree.
   Now, delete 3, 9, 4 in order and draw the resulting trees.

4. The maximum item in a set of n real-valued keys is well defined. The maximum item in a set of n 2-dimensional real-valued points is not.

   One definition that is used in database theory is that of *skyline vectors.* These are also known as *maximal points* or *maximal vectors.*

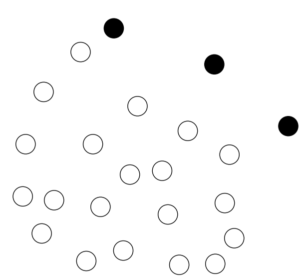
   Let $S = \{p_1, p_2, \ldots, p_n\}$ be a set of 2-d points where $p_i = (x_i, y_i)$. A point $p \in S$ is a *skyline vector* if no other point is bigger than it in both x and y dimensions.

   Formally p_j *dominates* p_i if

   $$x_i < x_j \quad \text{and} \quad y_i < y_j.$$

   $p = (x, y)$ *is a skyline vector in* S if no p_i in S dominates p.

   In the example below, the 3 filled points are the skyline ones.

   (a) Give an algorithm that finds the skyline vectors in a set S of n points in $O(n \log n)$ time.

(b) Suppose that the points all have integer coordinates in the range $[1, \ldots, n^2]$. Give an $O(n)$ algorithm for solving the same problem.