Note: Submission of solutions to challenge problems is purely voluntary. Submitting correct solutions could raise your grade but not submitting will not lower your grade. Please see class website for more information on Challenge Problem marking

Question 1. A *n x n* grid consists of *n* non-overlapping horizontal lines and *n* non-overlapping vertical lines. Here is a 4 x 4 grid:
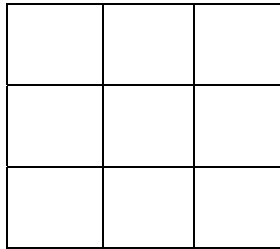
 How many rectangles are there in a *n x n* grid? Note that a square is a special rectangle and, for the sake of this question, a rectangle must have non-zero area. Answer this question using the Bijection Principle. Clearly spell out the bijection used.

Question 2: Answer the following questions using the Product Principle:

a) Consider coloring cells in a table with one row and *n* columns using the three colors red, green and blue. A coloring is *legal* if neighboring cells have different colors. Here is an example of legal coloring:

| RED | GREEN | BLUE | GREEN | BLUE | RED | GREEN |

Here is an example of illegal coloring:

| RED | RED | BLUE | GREEN | BLUE | BLUE | GREEN |

How many legal colorings are there? Your solution should be given in terms of the parameter *n*.

b) In the previous sub-question, what if there are *m (m>3)* colors? Your solution should be given in terms of the parameters *m* and *n*.

c) Consider coloring cells in a table with two rows and *n* columns using *m (m>=3)* colors. A coloring is *legal* if horizontally neighboring cells have different colors and vertically neighboring cells also have different colors. Here is an example of legal coloring:

| RED | GREEN | BLUE | GREEN | BLUE | RED | GREEN |
| GREEN | RED | GREEN | BLUE | RED | GREEN | RED |

Here is an example of illegal coloring:

| BLUE | RED | BLUE | RED | BLUE | RED | GREEN |
| GREEN | RED | GREEN | GREEN | RED | GREEN | RED |

How many legal colorings are there? Your solution should be given in terms of the parameters *m* and *n*. (Hint: Consider the two cells in each column together.)

d) In the previous sub-question, what if *diagonally* neighboring cells are also not allowed to have the same color?