

Recall that **(\*\*)  $y[i-1] = x[i]$  and  $(*)$   $x[i-1] = y[i] - q[i-1]x[i]$**   
**and we want  $j[i]x[i] + k[i]y[i] = \gcd(k[i], j[i])$**

**Example:  $k = 184, j = 69$**

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$i$	$k[i]$	$=$	$j[i]q[i]$	$+$	$r[i]$	$k[i]$	$j[i]$	$r[i]$	$q[i]$	$y[i]$	$x[i]$
0	184	$=$	$69(2)$	$+$	46	184	69	46	2		
1	69	$=$	$46(1)$	$+$	23	69	46	23	1		
2	46	$=$	$23(2)$	$+$	0	46	23	0	2		

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- 2) Then calculate  $y[2] = 0, x[2] = 1$

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- 4) We are done! Note that  $184(-1) + 69(3) = 23 = \gcd(184, 69)$ .

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0	99	$=$	$63(1)$	$+$	36	99	63	36	1		
1	63	$=$	$36(1)$	$+$	27	63	36	27	1		
2	36	$=$	$27(1)$	$+$	9	36	27	9	1		
3	27	$=$	$9(3)$	$+$	0	27	9	0	3		

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- 4) We are done! Note that  $99(2) + 63(-3) = 9 = \gcd(99, 63)$ .