GRE Math Subject Prep Course: Linear Algebra

June 21, 2011

1. (Chapter 5 Prob 5)[1] If the matrices

$$\begin{pmatrix} 3 & -2 & -2 \\ 1 & -1 & -1 \\ 3 & -1 & -2 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & a & 0 \\ -1 & b & 1 \\ 2 & c & -1 \end{pmatrix}$$

are inverses of each other, what is the value of c?

(A) -3 (B) -2 (C) 0

(D) 2 (E) 3

---

2. (Exam II Prob 18)[2] In an homogeneous system of 5 linear equations with 7 unknowns, the rank of the coefficient matrix is 4. The maximum number of independent solution vectors is

(A) 5 (B) 2 (C) 4

(D) 1 (E) 3

---

3. (Exam II Prob 40) If A is a square matrix of order $n \geq 4$, and $a_{ij} = i + j$ represents the entry in row i and column j, then the rank of A is always

(A) 1 (B) 2 (C) $n-2$

(D) $n-1$ (E) none of these

---

4. (Exam II Prob 15) Given that S and T are subspaces of a vector space, which of the following is also a subspace?

(A) $S \cap T$ (B) $S \cup T$ (C) $2S$

(D) Both (A) and (C) (E) Both (B) and (C)

---

[1] The problems with "Chapter *" are taken from "Cracking the GRE Mathematics Test", 4th Edition.

[2] The problems with "Exam I" – "Exam VI" are taken from the REA book "The Best Test Preparation for the GRE Mathematics Test", 4th edition.

5. (Exam IV Prob 35) If T is a linear transformation mapping vectors $(1,0,0),(0,1,0)$ and $(0,0,1)$ to the vectors $(1,2,3),(2,3,1)$ and $(1,1,-2)$ respectively which vector is the image of the vector $(3,-2,1)$ under T?

(A) (1,1,7) (B) (1,0,5) (C) (0,1,5)

(D) (0,1,9) (E) (1,7,0)

---

6. (Chapter 5 Prob 16) Define linear operator S and T on the $xy-$plane ($\mathbb{R}^2$) as follows: S rotates each vector 90° counterclockwise, and T reflects each vector through the $y-$axis. If ST and TS denote the compositions $S \circ T$ and $T \circ S$, respectively, and I is the identity map, which of the following is true?

(A) $ST = I$ (B) $ST = -I$ (C) $TS = I$

(D) $ST = TS$ (E) $ST = -TS$

---

7. (Chapter 5 Prob 15) Let $T : \mathbb{R}^5 \to \mathbb{R}^3$ be a linear transformation whose kernel is a three-dimensional subspace of $\mathbb{R}^5$. The set $\{T(x) : x \in \mathbb{R}^5\}$ is

(A) the trivial subspace

(B) a line through the origin

(C) a plane through the origin

(D) all of $\mathbb{R}^3$

(E) Cannot be determined from the information given

---

8. (Chapter 5 Prob 3) Let A, B and C be real 2×2 matrices, and let 0 denote the 2×2 zero matrix. Which of the following statements is/are true?

I. $A^2 = 0 \;\Rightarrow\; A = 0$

II. $AB = AC \;\Rightarrow\; B = C$

III. A is invertible and $A = A^{-1} \;\Rightarrow\; A = I$ or $A = -I$

(A) I only (B) I and III only (C) II and III only

(D) III only (E) none of the above

---

9. (Week 4 Prob 11) If V, W are 2-dimensional subspaces of $\mathbb{R}^4$, what are the possible dimensions of $V \cap W$?

(A) 0 (B) $0, 1$ (C) $0, 1, 2$

(D) $1, 2$ (E) 2

10. (Week 4 Prob 12) Suppose that V is the vector space of real 2×3 matrices. If T is a linear transformation from V onto $\mathbb{R}^4$, what is the dimension of the null space of T?

(A) 0 (B) 1 (C) 2

(D) 3 (E) 4

Answer: AEBD CECE CC