GRE Math Subject Prep Course: Calculus I

June 7, 2021

1. (Chapter 2 Prob 28)[1] Integrate $\int \frac{x^2}{\sqrt{1-x^2}}dx$.

   (A) $\frac{1}{2}(\arcsin x - \sqrt{1-x^2})$ (B) $\frac{1}{2}(\arcsin x + x\sqrt{1-x^2})$ (C) $\frac{1}{2}(x\arcsin x - \sqrt{1-x^2})$

   (D) $\frac{1}{2}(\arcsin x - x\sqrt{1-x^2})$ (E) $\frac{1}{2}(x\arcsin x + \sqrt{1-x^2})$

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2. (Exam III Prob 13)[2] The value of $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}dx$ is

   (A) 1 (B) $\frac{\pi}{2}$ (C) 0

   (D) $\frac{\pi}{4}$ (E) π

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3. (Chapter 2 Prob 34) The region bounded by the graphs of $y = x^2$ and $y = 6 - |x|$ is revolved around the y-axis. What is the volume of the generated solid?

   (A) $\frac{32}{3}\pi$ (B) 9π (C) 8π

   (D) $\frac{20}{3}\pi$ (E) $\frac{16}{3}\pi$

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4. (Exam V Prob 50) Suppose $\int_0^{\infty} f(x)dx$ exists. Which statements are FALSE?

   I. $\lim_{x\to\infty} f(x) = 0$

   II. $\int_0^{\infty} |f(x)|dx$ exists

   III. $\int_0^{\infty} [f(x)]^2 dx$ exists

   (A) I only (B) II only (C) I and II only

   (D) II and III only (E) I, II and III

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[1] The problems with "Chapter *" are taken from "Cracking the GRE Mathematics Test", 4th Edition.

[2] The problems with "Exam I" – "Exam VI" are taken from the REA book "The Best Test Preparation for the GRE Mathematics Test", 4th edition.

5. (Week 1 Prob 17)[3] For continuous functions $f, g : \mathbb{R} \to (0, \infty)$, define the relation $\sim$ by $f \sim g$ if and only if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1.$$

Suppose that $f \sim g$. Which of these does NOT necessarily follow:

(A) $f^2 \sim g^2$ (B) $\sqrt{f} \sim \sqrt{g}$ (C) $e^f \sim e^g$

(D) $f + g \sim 2g$ (E) $g \sim f$

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6. (Week 1 Prob 20) Compute the derivative $\dfrac{d}{dx}\displaystyle\int_0^{x^2} e^{-t^2}\,dt$

(A) e^{-x^4} (B) $2e^{-x^4}$ (C) xe^{-x^4}

(D) $2xe^{-x^4}$ (E) e^{-x^2}

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7. (Week 1 Prob 22) Calculate $\displaystyle\lim_{n\to\infty} \sum_{k=n+1}^{2in} \frac{1}{k}$.

(A) e^2 (B) 0 (C) $\sin 2$

(D) $\ln 2$ (E) $\cos 2$

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8. (Week 2 Prob 1) What is the length of the curve $(x(t), y(t)) = (\cos(t), \sin(t))$ for $0 \le t \le \pi$?

(A) 1 (B) 2 (C) π

(D) 2π (E) π^2

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9. (Week 2 Prob 3) For $n \in \mathbb{N}$, evaluate $\displaystyle\int_0^{\infty} x^n e^{-x}\,dx$

(A) $\ln(n)$ (B) e^{n-1} (C) e^n

(D) $(n-1)!$ (E) $n!$

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10. (Week 2 Prob 5) Compute $\displaystyle\int \frac{x+2}{x^3 - x^2 + 2x - 2}$.

(A) $\ln(x-1)$ (B) $\ln(x^2+2)$ (C) $\ln(x-1) - \ln(x^2+2)$

(D) $\ln(x-1) - 2\ln(x^2+2)$ (E) $\ln(x-1) - \frac{1}{2}\ln(x^2+2)$

[3]The problems with "Week *" are taken from the GRE preparation course taught by Christian Parkinson.

11. (Week 2 Prob 8) Compute $\displaystyle\int_0^{\pi/2} \frac{dx}{1+\tan(x)^{2021}}$.

(A) 2021 (B) $\tan(2021)$ (C) 2021π

(D) 4π (E) $\frac{\pi}{4}$

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12. (Week 2 Prob 10) Let $f(x) = \frac{1}{x}$, for $x \in [1, \infty)$. Find the area and volume of the shape which results from rotating the graph of f about the $x - axis$

(A) $A = \infty$, $V = \pi$ (B) $A = \pi$, $V = \infty$ (C) $A = \pi$, $V = \pi$

(D) $A = \infty$, $V = \infty$ (E) $A = \pi$, $V = 4\pi$

Answer: DDAE CDDC EEEA