The contact mapping class group of lens spaces

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01.
The contact mapping class group

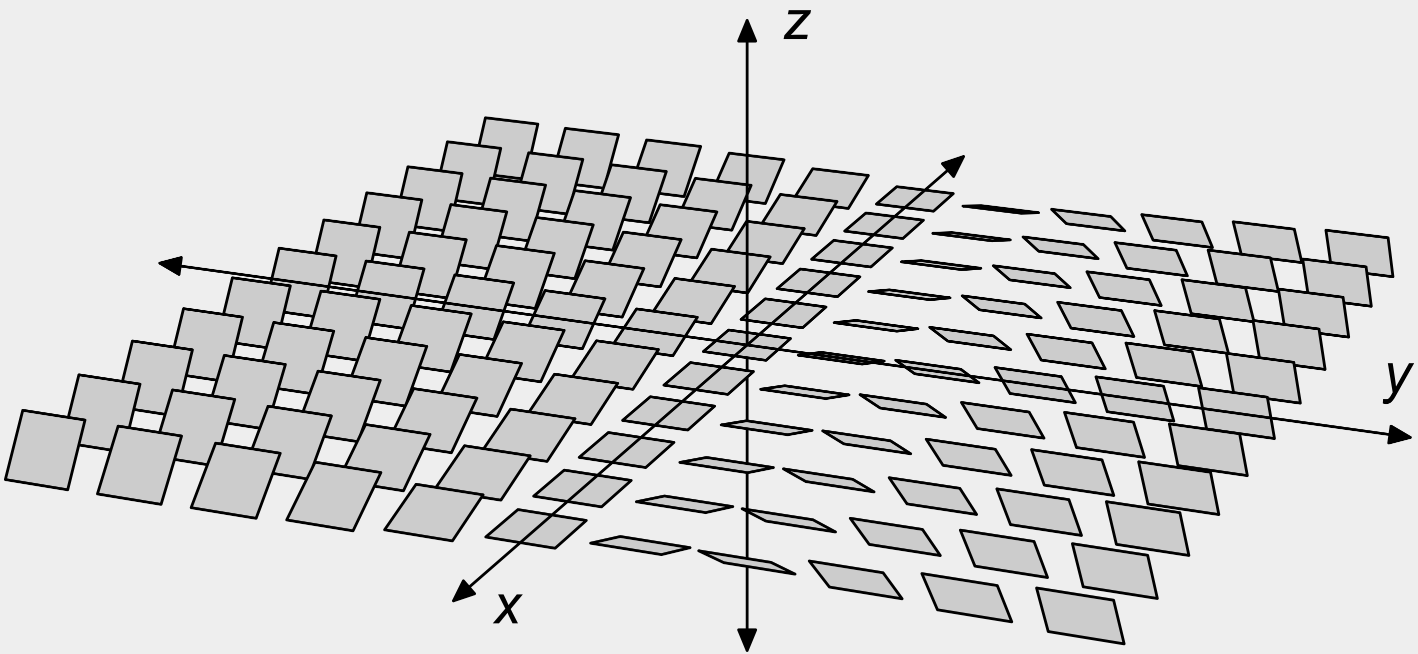
Definitions

- A contact form of a 3-manifold M

- A contact structure of α

Examples

- The standard contact structure of $\mathbb{R}^3$



The standard contact structure $(\mathbb{R}^3, \xi_{std})$

Definitions

- A strict contactomorphism
- A coorientation preserving contactomorphism
- A coorientation reversing contactomorphism

Examples

- $\alpha_1 = dz - ydx, \;\; \alpha_2 = dz + xdy$
- coorientation preserving contactomorphism
- coorientation reserving contactomorphism

Today

- We only consider coorientation preserving contactomorphisms
- We focus on contact structures, not contact forms
- Strict contactomorphisms depend on the choice of a contact form.
- Coorientation reversing contactomorphisms are confusing.

Definitions

- The group of contactomorphism : $\mathrm{Cont}(M, \xi)$
- The contact mapping class group: $\pi_0(\mathrm{Cont}(M, \xi))$

Exotic phenomena

- $i : \mathrm{Cont}(M, \xi) \to \mathrm{Diff}_+(M)$

- An exotic contactomorphism : $\ker i_* \neq 0$

Exotic phenomena

- $(S^1 \times S^2, \xi_{std})$
- f: a Dehn twist about $\{p\} \times S^2$
- (Gompf) $f^n \nsim f^m$ if $m \neq n$

The exact sequence from a fibration

- $\Xi(M, \xi)$: the space of contact structures on M isotopic to ξ.
- $\mathrm{Diff}_0(M)$: the connected component of $\mathrm{Diff}(M)$ containing id.
- $\mathrm{Cont}_0(M, \xi) = \mathrm{Diff}_0(M) \cap \mathrm{Cont}(M, \xi)$

The exact sequence from a fibration

- $\mathrm{Cont}_0(M,\xi) \hookrightarrow \mathrm{Diff}_0(M) \to \Xi(M,\xi)$

The contact mapping class group

- Almost nothing is known

General strategy

- Fix a submanifold
- Determine the contact mapping class group of the complement

General strategy

- (S^3, ξ_{std})

02.

Lens spaces

Main theorem

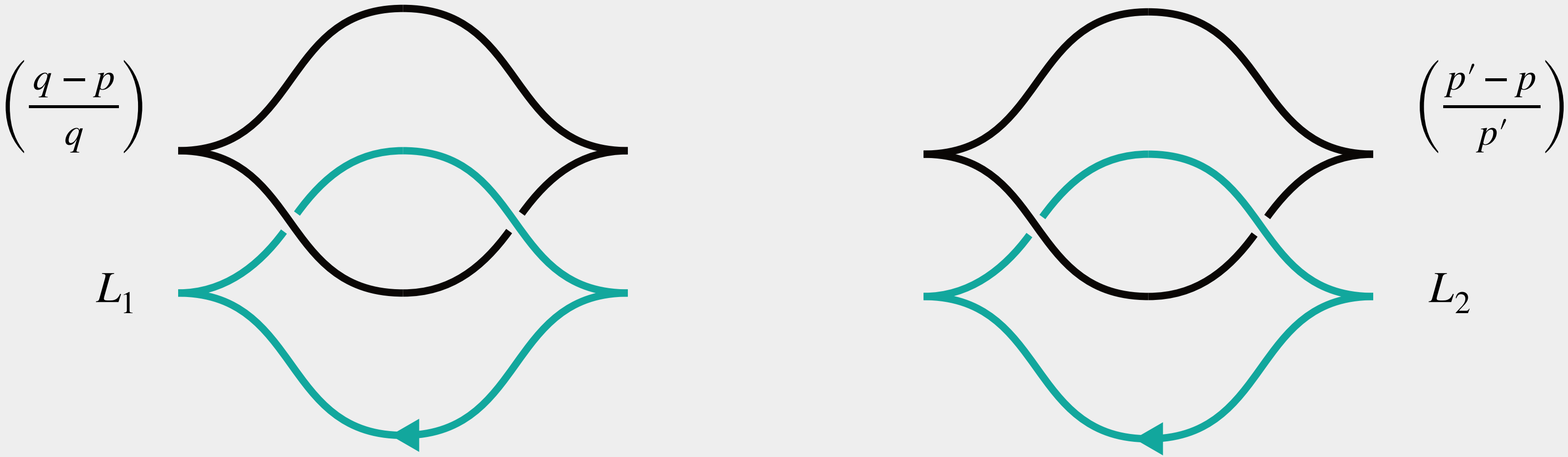
$$\pi_0(\mathrm{Cont}(L(p,q),\xi_{std})) = \begin{cases} \mathbb{Z}_2 & p \neq 2 \text{ and } q \equiv -1 \pmod{p} \\ \mathbb{Z}_2 & q \not\equiv \pm 1 \pmod{p} \text{ and } q^2 \equiv 1 \pmod{p} \\ 1 & \text{otherwise} \end{cases}$$

Applications

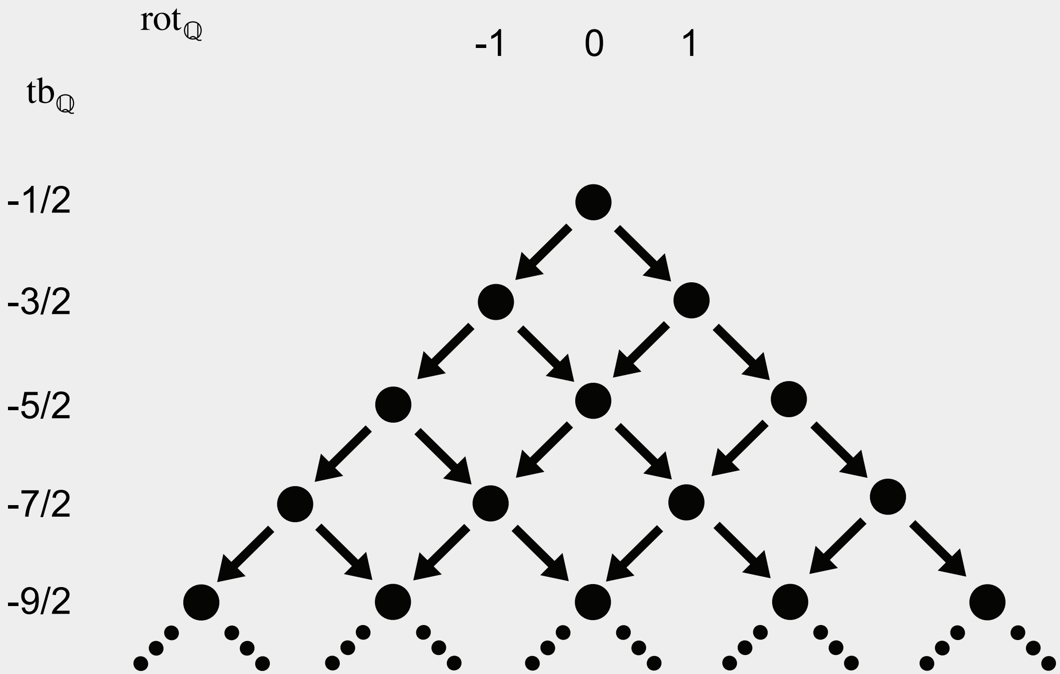
- (Baker-Etnyre-M-Onaran, work in progress)
  Classification of Legendrian torus knots in $L(p, q)$

Strategy

- Classify Legendrian rational unknots in $L(p, q)$
- Perturb contactomorphism to fix a neighborhood of a Legendrian rational unknot
- Determine the contact mapping class group of the complement.



Legendrian rational unknots in $(\mathbb{RP}^3, \xi_{std})$



Legendrian rational unknots in $(\mathbb{RP}^3, \xi_{std})$

Thank you!