GRE Math Subject Prep Course: Calculus I

June 2, 2021

1. (Exam VI Prob 56)[1] Let $f(x)$ be a function defined for all real x such that the coordinates of each point of its graph satisfy $|y| = |x^2 - x^3|$. The total number of points at which $f(x)$ must be differentiable is

   (A) none (B) 1 (C) 2

   (D) 3 (E) infinite

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2. (Chapter 2 Prob 13)[2] What is the slope of the tangent line to the curve $xy(x + y) = x + y^4$ at the point $(1, 1)$?

   (A) 2 (B) 1 (C) 0

   (D) -1 (E) -2

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3. (Week 1 Prob 2)[3] Evaluate $\lim_{x\to 0} \dfrac{(1+x)^\alpha - 1}{x}$ for $\alpha \in \mathbb{R}$.

   (A) 0 (B) α (C) 2α

   (D) 3α (E) 1

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4. (Exam VI Prob 1) Let $f(x) = \left(\dfrac{\sin x}{x}\right)^{1/x^2}$. Evaluate $\lim_{x\to 0^+} f(x)$.
   (Note: to reduce the pain of applying l'Hopital's rule again and again, try Taylor expansion.)

   (A) $-1/6$ (B) 0 (C) $e^{-1/6}$

   (D) $\ln(1/6)$ (E) 1

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5. (Exam VI Prob 36) Evaluate $\lim_{x\to\infty} \sqrt{x^2 + 2x} - x$.

   (A) 0 (B) 1 (C) $\sqrt{2}$

   (D) 2 (E) ∞

[1] The problems with "Exam I" – "Exam VI" are taken from the REA book "The Best Test Preparation for the GRE Mathematics Test", 4th edition.

[2] The problems with "Chapter *" are taken from "Cracking the GRE Mathematics Test", 4th Edition.

[3] The problems with "Week *" are taken from the GRE preparation course taught by Christian Parkinson.

6. (Chapter 2 Prob 20) What is the maximum area of a rectangle inscribed in a semicircle of radius a?

(A) $\frac{\sqrt{2}}{2}a^2$ (B) $\frac{\sqrt{2}}{2}a^2$ (C) a^2

(D) $\frac{\pi}{2\sqrt{2}}a^2$ (E) $\sqrt{2}a^2$

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7. (Exam IV Prob 8) Let $f(x) = x^x$ with $x \in [0, 1]$. (here $f(0)$ is defined as $\lim_{x\to 0^+} f(x)$.) Determine where the minimum of f on the interval occurs.

(A) $1/2$ (B) $1/e$ (C) $1/\pi$

(D) $1/4$ (E) 0

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8. (Week 1 Prob 3) Evaluate $\lim_{x\to 0} \frac{\cos(\beta x) - 1}{x^2}$ for $\beta \in \mathbb{R}$.

(A) 0 (B) $\beta/2$ (C) β^2

(D) $-\beta^2/2$ (E) $-\beta^2$

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9. (Week 1 Prob 4) Let $c > 0$. Find the minimum value of $f(x) = e^x - cx$ among $x \in \mathbb{R}$.

(A) c (B) $\ln(c)$ (C) $c + c\ln(c)$

(D) $c - c\ln(c)$ (E) $c\ln(c)$

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10. (Week 1 Prob 5) Let $f(x) = |x| + 3x^2$. What is $f'(x)$?

(A) -3 (B) -4 (C) -5

(D) -6 (E) -7

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11. (Week 1 Prob 6) Evaluate $\lim_{x\to 0}(x^{-2} - \sin^{-2}(x))$.

(A) $-1/2$ (B) $-1/3$ (C) $-1/4$

(D) $-1/5$ (E) $-1/6$

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12. (Week 1 Prob 7) Suppose $f(x) = 3x^2 + bx + c$ has a non-simple root at $x = 2$. What is $f(5)$?

(A) 15 (B) 18 (C) 21

(D) 24 (E) 27

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13. (Week 1 Prob 8) If $f : \mathbb{R} \to \mathbb{R}$ is continuously differentiable on $(-1, 4)$ with $f(3) = 5$ and $f'(x) \geq 1$ for all $x \in (-1, 4)$, what is the greatest possible value of $f(0)$?

(A) 2 (B) 3 (C) 4

(D) 5 (E) 6

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14. (Week 1 Prob 9) Evaluate $\lim_{x \to 0} \dfrac{\sin(2x)}{(1+x)\ln(1+x)}$.

(A) 0 (B) 1 (C) 2

(D) 3 (E) 4

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15. (Week 1 Prob 10) Let $f(x) = e^{g(x)}h(x)$ where $h'(x) = -g'(x)h(x)$ for all $x \in \mathbb{R}$. Which of the following is necessarily true?

(A) f is constant (B) f is non-constant (C) g is constant

(D) g is non-constant (E) none of the above

Answer: ABBAC BCDDE BEACA