GRE Math Subject Prep Course: Calculus I

June 14, 2021

1. (Chapter 2 Prob 46) The smallest positive integer x for which the power series $\sum_{n=1}^{\infty} \frac{n!(2n)!}{(3n)!}x^n$ does *not* converge is

   (A) 4 (B) 6 (C) 7

   (D) 8 (E) 9

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2. (Chapter 2 Prob 47) In the Taylor series expansion (in powers of x) of the function $f(x) = e^{x^2-x}$, what is the coefficient of x^3?

   (A) -7 (B) $-\frac{3}{2}$ (C) $-\frac{7}{6}$

   (D) $\frac{7}{6}$ (E) $\frac{3}{2}$

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3. (Week 3 Prob 1) Suppose $f(x) = \sum_{n=1}^{\infty}(-1)^n x^{3n}$ for $x \in (-1, 1)$. Find a closed form of $f'(x)$.

   (A) $3x^2$ (B) $-3x^2$ (C) $\frac{3x^2}{1+x^3}$

   (D) $\frac{3x^2}{(1+x^3)^2}$ (E) $-\frac{3x^2}{(1+x^3)^2}$

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4. (Week 3 Prob 2) For which values of x does $\sum_{n=1}^{\infty} \frac{n!x^{2n}}{n^n(1+x^{2n})}$ converge?

   (A) It diverges for any $x \in \mathbb{R}$

   (B) It converges for any $x \in \mathbb{R}$

   (C) It converges only for $|x| < e$

   (D) It converges only for $|x| < 2e$

   (E) It converges only for $|x| < \frac{1}{e}$

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5. (Week 3 Prob 3) Compute $\int_0^{\infty} \lfloor x \rfloor e^{-x} dx$ where $\lfloor x \rfloor$ denotes the largest integer smaller than x.

   (A) $\frac{1}{e-1}$ (B) $\frac{1}{e+1}$ (C) $\frac{1}{e}$

   (D) $e - 1$ (E) $e + 1$

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6. (Week 3 Prob 6) For which x does $\sum_{n=1}^{\infty} nx^n$ converge?

(A) $|x| < 1$ (B) $|x| < 2$ (C) $|x| < e$

(D) $|x| < 2e$ (E) $|x| < e^2$

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7. (Week 3 Prob 6) For which x does $\sum_{n=1}^{\infty} n!x^n$ converge?

(A) $|x| < 1$ (B) $|x| < 2$ (C) $|x| < e$

(D) $|x| < 2e$ (E) $|x| < e^2$

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8. (Week 3 Prob 7) Find the Taylor series for $f(x) = \int_0^x \frac{\sin(t)}{t} dt$ around $x = 0$.

(A) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!(2n)}$ (B) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!(2n+1)}$ (C) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!(2n+1)}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!(2n+1)}$ (E) $\sum_{n=1}^{\infty} \frac{x^{2n+1}}{(2n+1)!(2n+1)}$

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9. (Week 3 Prob 9) Find $f^{100}(2)$ for $f(x) = \frac{3}{x^2+5x+4}$.

(A) $100!$ (B) $100!(\frac{1}{3^{101}} - \frac{1}{6^{101}})$ (C) $100!(\frac{1}{6^{101}} - \frac{1}{3^{101}})$

(D) $\frac{1}{3^{101}} - \frac{1}{6^{101}}$ (E) $\frac{1}{6^{101}} - \frac{1}{3^{101}}$

Answer: CCEBA AACB