

GRE Math Subject Prep Course: Complex Analysis

July 19, 2021

1. (Chapter 7 Prob 30)¹ What is the polar form of a complex number equal to $(i - \sqrt{3})^6$?

- (A) -2^6 (B) $2^6(-1 + i)$ (C) $2^6(1 - i)$
(D) $2^6\left(\frac{1}{2} - \sqrt{\frac{3}{2}}i\right)$ (E) 2^6
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2. (Exam I Prob 17)² The sum of the 9th roots of unity is

- (A) 0 (B) 1 (C) 9
(D) 10 (E) $1 + i$
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3. (Chapter 7 Prob 20) What is $\text{Log}(-e^3)$?

- (A) -3 (B) 3 (C) $3 + \pi i$
(D) $3 - \pi i$ (E) $3 + 2\pi i$
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4. (Exam I Prob 16) The fixed point(s) of a Mobius transformation $w(z) = \frac{z-2}{z-1}$ is (are)

- (A) $1 + \sqrt{3}$ (B) $1 \pm 2i$ (C) $2i$
(D) $1 \pm i$ (E) $-1 \pm \sqrt{2}i$
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5. (Chapter 7 Prob 25) In the complex plane, the set of all points that satisfy the equation $(\bar{z})^2 = z^2$ is

- (A) a circle (B) a point (C) a ray
(D) a line (E) two lines

¹The problems with “Chapter *” are taken from “Cracking the GRE Mathematics Test”, 4th Edition.

²The problems with “Exam I” – “Exam VI” are taken from the REA book “The Best Test Preparation for the GRE Mathematics Test”, 4th edition.

6. (Chapter 7 Prob 24) What are the complex roots of the equation $e^{2z} = i$?

- (A) $\frac{i}{2} \left(-\frac{\pi}{2} + 2n\pi \right)$ (B) $2i \left(-\frac{\pi}{2} + n\pi \right)$ (C) $\frac{i}{2} \left(\frac{\pi}{2} + n\pi \right)$
(D) $2i \left(\frac{\pi}{2} + 2n\pi \right)$ (E) $\frac{i}{2} \left(\frac{\pi}{2} + 2n\pi \right)$
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7. (Exam V Prob 56) Which of the following functions are analytic?

- I. \bar{z}
II. $\bar{z} \sin z$
III. $z + \sin z$
IV. $z + \bar{z}$
V. ze^z

- (A) I only (B) I and II only (C) III and V only
(D) IV only (E) None of the above
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8. (Exam VI Prob 7) Suppose $u(x, y)$ is harmonic in a domain D , and $v(x, y)$ is the harmonic conjugate of u . Let $f(z) = u(x, y) + iv(x, y)$. Which of the following statements are true?

- I. $g(z) = v - iu$ is analytic in D .
II. $f'(z) = u_x + iv_y$.
III. $v(x, y) + x + y$ satisfies Laplace's equation in D .

- (A) I only (B) II only (C) III only
(D) I and II only (E) I and III only
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9. (Exam VI Prob 51) Suppose $f(z)$ is a nonconstant entire function. Which of the following is always true?

- (A) $\lim_{z \rightarrow \infty} f(z) = 0$
(B) $\lim_{z \rightarrow 0} f(z) = 0$
(C) $f'(z)$ may not be entire.
(D) $\oint f(z) dz = 2\pi i$ for every simple, closed curve in the complex plane.
(E) None of the above
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10. (Chapter 7 Prob 48) Let $f(z)$ be a complex analytic function such that $f(z) = (5x - 3y) + iv(x, y)$, where $v(x, y)$ is a real-valued function and $x, y \in \mathbb{R}$. If $v(4, 1) = 7$, what is $v(3, 2)$?

(A) -10 (B) -9 (C) 1
(D) 9 (E) 14

11. (Chapter 7 Prob 26) Which of the following is a harmonic conjugate $u(x, y)$ of the harmonic function $v = x - 3x^{2y} + y^3$?

(A) $x^3 - 3xy^2 + y$ (B) $-x^3 + 3xy^2 - y$ (C) $-y^3 + 3x^{2y} - x$
(D) $y^3 - 3x^{2y} + x$ (E) $-x^3 + 3xy^2$

12. (Exam V Prob 45) Let C be the circle $|z| = 3$, described in a counterclockwise orientation, and write

$$g(w) = \oint_C \frac{2z^2 - 2 - z}{z - w} dz.$$

Then $g(2)$ is given by

(A) 1 (B) $2\pi i$ (C) 0
(D) $4\pi i$ (E) $8\pi i$

13. (Practice Book Prob 53)³ In the complex plane, let C be the circle $|z| = 2$ with positive (counterclockwise) orientation. Then

$$\int_C \frac{dz}{(z - 1)(z + 3)^2} =$$

(A) 0 (B) $2\pi i$ (C) $\frac{\pi i}{2}$
(D) $\frac{\pi i}{8}$ (E) $\frac{\pi i}{16}$

14. (Exam VI Prob 12) Determine the Laurent series for $f(z) = \frac{1}{z - 2}$ which converges in the annulus $1 \leq |z - 3| < \infty$.

(A) $\sum_{n=0}^{\infty} (z - 3)^n$ (B) $\sum_{n=0}^{\infty} (z - 3)^{-n}$ (C) $\sum_{n=0}^{\infty} (-1)^n (z - 3)^{-n-1}$
(D) $\sum_{n=0}^{\infty} (-1)^n (z - 3)^{-n}$ (E) $\sum_{n=1}^{\infty} (-1)^n (z - 3)^{-n}$

³The problems with "Practice Book" are taken from the mathematics test practice book by ETS, which can be found at <http://www.ets.org/Media/Tests/GRE/pdf/Math.pdf>

Answer: AACDE ECEE DBEDC