HKN ECE 329 Exam 2 Review session

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Magnetostatics ($\frac{\partial I}{\partial t} = 0$)

- Lorentz Force: $\vec{F} = (q\vec{v} \times \vec{B}) + q\vec{E}$
- Biot-Savart Law: $d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{l} \times \hat{a}_R}{R^2}$ $\quad \overrightarrow{dF} = \overrightarrow{I_1 dl} \times \frac{\overrightarrow{I_2 dl} \times \vec{r}}{R^2}\frac{\mu}{4\pi} = \overrightarrow{Idl} \times \overrightarrow{dB}$
  - Useful for finding differential B at a point and the force on one wire due to another

Ampere's Law

- Current Density (J): Amount of current flowing over a given area $\vec{I}_{enclosed} = \iint_S \vec{J} \cdot \vec{dS}$
- Magnetic Field Intensity (H): $\vec{B} = \mu \vec{H}$

- Ampere's Law: Used to find the magnetic field around current carrying devices.
  - Use RHR to find direction on field
  - Wire: $\vec{H} = \frac{I}{2\pi r}\phi$
  - Sheet of current: $\vec{H} = -\frac{\vec{J}_s}{2}\operatorname{sgn}(x)\hat{z}$
  - Solenoid: (N is the coil density) $H = NI$

$$\oint_C \vec{H} \cdot \vec{dl} = \iint_S \vec{J} \cdot \vec{dS}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

Continuity Equation and Maxwell's Correction

- The amount of charge in the universe is a constant and must be conserved in isolated systems
  - This leads to the continuity correction for charge carrying systems: $$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$
- In order to satisfy continuity, we must add a displacement current to Ampere's Law:

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

- So, our 4 final Maxwell equations are:

$$1. \nabla \cdot D = \rho$$

$$2. \nabla \cdot B = 0$$

$$3. \nabla \times E = -\frac{\partial B}{\partial t}$$

$$4. \nabla \times H = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Non-Conservative Fields

- Integral of E·dl around a closed path is no longer zero!
- Magnetic Flux: Amount of magnetic field lines penetrating a surface

$$\psi \equiv \iint_S \vec{B} \cdot \vec{dS}$$

- Electromotive Force (emf): Change in voltage between a point and itself which gives rise to a current in the wire.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \neq 0$$

$$\oint_C \vec{E} \cdot \vec{dl} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\varepsilon_{MF} = -\frac{\partial \psi}{\partial t} = -N \frac{\partial \psi}{\partial t}$$

How do we get non-zero flux?

1. Area or B·dS changes
   - Example: Wire entering a uniform magnetic field, wire rotating in a constant magnetic field
2. Time varying B
3. Position dependent B and v≠0
   - Example: Wire loop moving away from a current carrying wire

- Current through the wire:
  - Negative sign is used to indicate that the current **opposes** changes in flux

$$I = \frac{\varepsilon_{mf}}{R} = -\frac{1}{R}\frac{\partial \psi}{\partial t}$$

Inductance (L)

- The tendency of a device to resist changes in current. Measured in Henry's

$$L = \frac{\psi}{I}$$

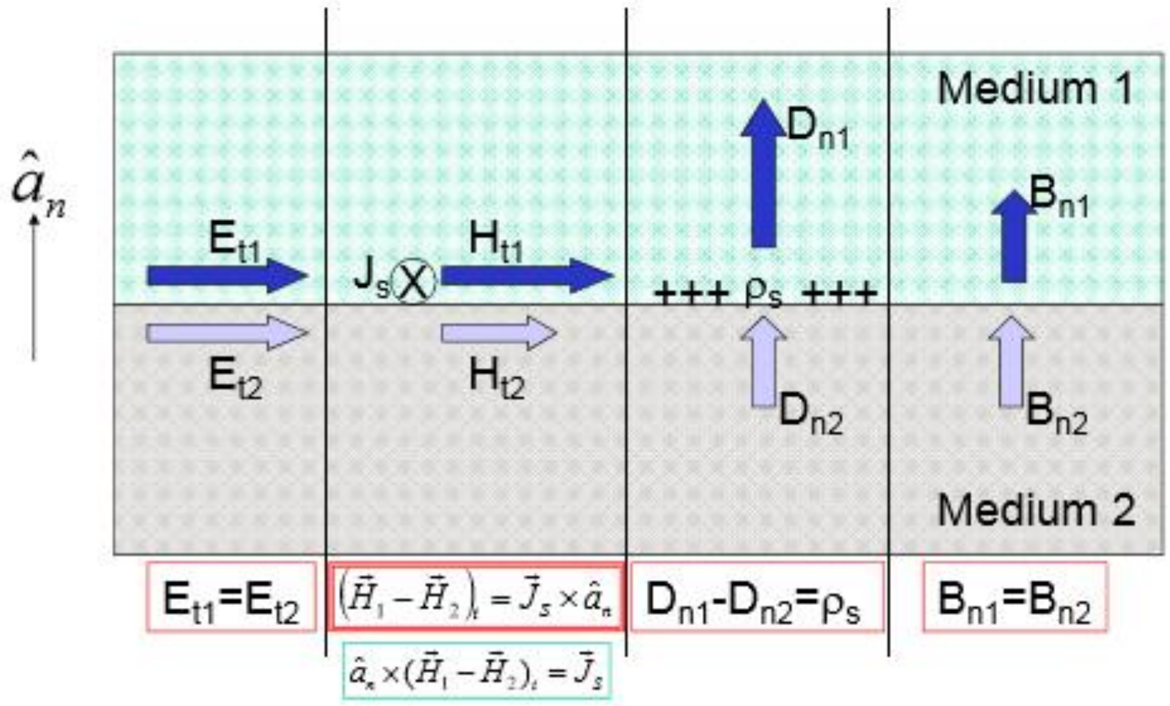
$$\ell = \frac{L}{l}$$

$$\varepsilon_{MF} = -L\frac{\partial I}{\partial t}$$

$$\zeta\ell = \mu_0\varepsilon_0$$

$$E = \frac{1}{2}I^2L$$

Boundary Conditions



Materials

Diamagnetic ($X_m < 0$): magnetic dipole opposes external field.

Ex: Water, Copper

Paramagnetic ($X_m > 0$): magnetic dipole points in same direction as external field.

Ex: Aluminum

Ferromagnetic ($X_m >> 0$): Incredibly strong atomic dipole.

Ex: Iron

$$\vec{B}_{total} = \mu_0(\vec{H}_{ext} + \vec{M})$$
$$\vec{M} = \chi_m \vec{H}_{ext}$$

$$\vec{D} = \varepsilon_0 \vec{E}_{tot} + \vec{P}$$
$$\vec{P} = \varepsilon_0 \chi_e \vec{E}_{tot}$$

$$\boxed{\begin{aligned} \vec{B}_{total} &= \mu_0(1+\chi_m)\vec{H}_{ext} \\ \vec{B} &= \mu\vec{H} \\ \mu &= \mu_0(1+\chi_m) = \mu_0\mu_r \\ \mu_r &= 1+\chi_m \end{aligned}}$$

$$\boxed{\begin{aligned} \vec{D} &= \varepsilon_0(1+\chi_e)\vec{E}_{tot} \\ \vec{D} &= \varepsilon\vec{E}_{tot} \\ \varepsilon &= \varepsilon_0(1+\chi_e) = \varepsilon_0\varepsilon_r \\ \varepsilon_r &= 1+\chi_e \end{aligned}}$$

Wave Equation

In a charge free region with 0 conductivity:

- Found by combining Faraday's Law and Ampere's Law (assuming ρ=0, σ=0, ε and μ are constants)
- Solved by the sine and cosine function therefore it can be solved by any Fourier Series
  - Follow D'Alembert solutions

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$E = f(t - \frac{z}{v_p})x$$

$$H = \pm\frac{1}{\eta} f(t - \frac{z}{v_p})y$$

Useful relationships:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$

Poynting's Theorem

Poynting Vector: $\vec{S} = \vec{E} \times \vec{H}$

- S has units of W/m^2

Poynting's Theorem: $\nabla \bullet \vec{S} = -\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 E^2\right) - \frac{\partial}{\partial t}\left(\frac{1}{2}\mu_0 H^2\right) - \vec{E} \bullet \vec{J}$

- If E·J is positive, the area is absorbing power
- If E·J is negative, the area is supplying power

Power relation: $P = \oiint_S \vec{S} \cdot \vec{dS}$

Average Poynting:

$$\langle S \rangle = \frac{1}{2}\mathrm{Re}\left\{E \times H^*\right\} = \frac{|E|^2}{2\eta} = \frac{|H|^2 \eta}{2}$$

$$E = E_0 e^{\mp i\beta z} x$$

$$H = \pm \frac{E_0}{\eta} e^{\mp i\beta z} y$$

Plane Wave Sources

1. Direction of H is given by the RHR, magnitude given by: $|H| = \frac{|J_s|}{2}$
   - Direction is different on the other side of the source!!!
2. E points opposite of J_s
   - Direction is the same on the other side of the source!!!
3. Wave propagates away from source
4. Relate magnitudes of E and H: $|E| = \eta |H|$
5. Solve for Poynting Vector: $\vec{S} = \vec{E} \times \vec{H}$
   - S points in the direction of propagation (perpendicular to source)

Previous Exam Questions

Spring 2016 #1

1. (25 points) For parts (a)-(c), you must **show your work or state your reasoning** to receive full credit. For parts (d)-(h), **circle the correct answer and give an explanation.** No credit will be given for correct answers without explanation.

   An infinite sheet of current $\mathbf{J_s}$ at $z = 0$ generates a monochromatic wave. For $z < 0$, the monochromatic wave generated propagates in a homogeneous dielectric material with $\mu = \mu_0$ and $\epsilon = \epsilon_r\epsilon_0$, and is described by

$$\mathbf{E}(z,t) = 4\cos\left[\left(2\pi \times 10^{14}\right)t + \left(\pi \times 10^{6}\right)z\right]\hat{y}, \quad z < 0$$

   a) (4 points) For the region $z < 0$, give the unit vector directions associated with the magnetic field $\mathbf{H}$ and the Poynting vector $\mathbf{S}$.

   b) (4 points) What is the propagation velocity v of the wave?

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c) (4 points) What is the intrinsic impedance η and the relative permittivity ϵ_r of the medium in $z < 0$?

d) (2 points) What is the correct phasor expression for the electric field $\tilde{E}$ for $z < 0$?

   i. $\tilde{E} = 4\cos(\beta z)\hat{y}\,\frac{\mathrm{V}}{\mathrm{m}}$

   ii. $\tilde{E} = 4e^{-j\beta z}\hat{y}\,\frac{\mathrm{V}}{\mathrm{m}}$

   iii. $\tilde{E} = 4e^{j\beta z}\hat{z}\,\frac{\mathrm{V}}{\mathrm{m}}$

   iv. $\tilde{E} = 4e^{j\beta z}\hat{y}\,\frac{\mathrm{V}}{\mathrm{m}}$

   v. None of the above

e) (2 points) What is the correct phasor expression for the magnetic field $\tilde{H}$ for $z < 0$?

   i. $\tilde{H} = 4e^{j\beta z}\hat{x}\,\frac{\mathrm{A}}{\mathrm{m}}$

   ii. $\tilde{H} = \frac{4}{\eta}e^{j\beta z}\hat{x}\,\frac{\mathrm{A}}{\mathrm{m}}$

   iii. $\tilde{H} = 4\eta e^{j\beta z}\hat{x}\,\frac{\mathrm{A}}{\mathrm{m}}$

   iv. $\tilde{H} = -\frac{4}{\eta}e^{j\beta z}\hat{x}\,\frac{\mathrm{A}}{\mathrm{m}}$

   v. None of the above

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$$\mathbf{E}(z,t) = 4\cos\left[\left(2\pi \times 10^{14}\right)t + \left(\pi \times 10^{6}\right)z\right]\hat{y}, \quad z < 0$$

f) (3 points) If the region $z > 0$ is vacuum, what is the phasor expression for the electric field $\tilde{E}^+$ for $z > 0$? **Hint:** Use boundary conditions.

i. $\tilde{E}^+ = 4e^{-j\beta z}\hat{y}\,\frac{\text{V}}{\text{m}}$

ii. $\tilde{E}^+ = 4e^{j\beta z}\hat{y}\,\frac{\text{V}}{\text{m}}$

iii. $\tilde{E}^+ = -4e^{j\beta z}\hat{y}\,\frac{\text{V}}{\text{m}}$

iv. $\tilde{E}^+ = -4e^{-j\beta z}\hat{y}\,\frac{\text{V}}{\text{m}}$

v. None of the above

g) (3 points) If the region $z > 0$ is vacuum, what is the phasor expression for the magnetic field $\tilde{H}^+$ for $z > 0$?

i. $\tilde{H}^+ = -\frac{4}{\eta_0}e^{j\beta z}\hat{x}\,\frac{\text{A}}{\text{m}}$

ii. $\tilde{H}^+ = -\frac{4}{\eta_0}e^{-j\beta z}\hat{x}\,\frac{\text{A}}{\text{m}}$

iii. $\tilde{H}^+ = 4e^{j\beta z}\hat{x}\,\frac{\text{A}}{\text{m}}$

iv. $\tilde{H}^+ = \frac{4}{\eta}e^{j\beta z}\hat{x}\,\frac{\text{A}}{\text{m}}$

v. None of the above

h) (3 points) What is the phasor expression for the surface current density $\tilde{J}_s$? **Hint:** Use boundary conditions again.

i. $\tilde{J}_s = -\frac{8}{\eta}\hat{y}\,\frac{\text{A}}{\text{m}}$

ii. $\tilde{J}_s = \frac{8}{\eta}\hat{y}\,\frac{\text{A}}{\text{m}}$

iii. $\tilde{J}_s = -4(\frac{1}{\eta_0} + \frac{1}{\eta})\hat{y}\,\frac{\text{A}}{\text{m}}$

iv. $\tilde{J}_s = 4(\frac{1}{\eta_0} + \frac{1}{\eta})\hat{y}\,\frac{\text{A}}{\text{m}}$

v. None of the above

Spring 2016 #2

2. (25 points) A long solenoid is wound on a cylinder core made of iron. The relative permeability of iron is $\mu_r = 5000$. The solenoid has radius $r = 2\,\mathrm{cm}$ and is wound with a density of 50 loops per meter. The axis of the solenoid is on the z−axis and a current of $I = 1\,\mathrm{A}$ is flowing in the wire in the $\hat{\theta}$−direction (counter-clockwise when viewed from above).

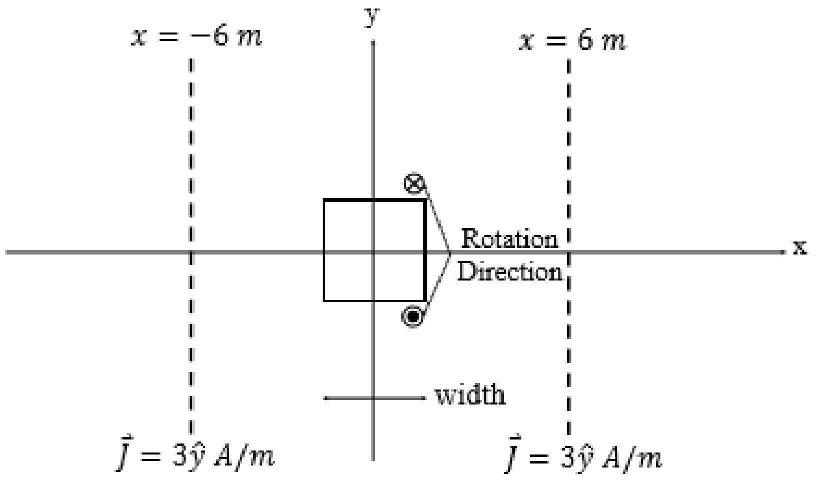
   a) (8 points) Assuming that $\mathbf{H} = 0$ outside the solenoid and that the solenoid is long enough so the field is independent of z. What is the magnetic field $\mathbf{H}$ and the magnetic flux density $\mathbf{B}$ in the interior of the solenoid?

   b) (8 points) What is the per-unit-length inductance $\mathcal{L}$ of the solenoid?

   c) (9 points) Now the iron core is hollowed out by drilling a hole of radius $r = 1\,\mathrm{cm}$ through its center axis. What is the new per-unit-length inductance $\mathcal{L}$ of the solenoid?

Spring 2016 #4

4. (25 points) Two current sheets are oriented and positioned as shown in the figure below (dashed lines). They are surrounded by free space. A square loop of wire is located at the origin (on the $xy-$ plane) as shown, with resistance of 2Ω. The loop has an area of 1m^2.



a) (8 points) Determine the magnetic field strength and magnetic flux density everywhere in space due to the current sheets.

b) (8 points) Determine the induced EMF $\mathcal{E}$ and current on the loop if it is rotated about the $x-axis$ at a rate of 1 revolution per second. Use $\hat{z}$ as the starting direction of the surface vector $d\mathbf{S}$. Be sure to get the signs correct. The top of the loop is moving into the plane of the paper as shown in the figure.

c) (9 points) Repeat (b) if the loop were instead positioned at $x = 9\,\text{m}$ $(y = 0)$ and still on the $xy-$ plane.

Summer 2015 #1

1. **Magnetic field and inductance problems:**

   a) Consider the DC current density function $\mathbf{J}(x, y, z) = \hat{y}[4\delta(x)\delta(z) + A\delta(x - x_o)\delta(z)]$ A/m^2 where coordinates x, y, and z are measured in meter units.

      i. (2 pts) What are the units of parameter A? Justify your answer.

      ii. (5 pts) If $x_o = 4$ m what is the numerical value of scalar A that leads to $\mathbf{B}(x_o/4, 0, 0) = 0$? Show your work.

Summer 2015 #1

1. **Magnetic field and inductance problems:**

b) I have a rod of some solid with an unknown permeability μ. To determine μ experimentally I go to the lab and insert the rod within a solenoid of many turns having a diameter exactly matching the diameter of the rod. I observe that the time constant of exponential decay of the solenoid current in an RL circuit that I construct decreases by 0.1% with the rod inserted replacing the air core of the solenoid.

   i. (4 pts) What is the differential equation for the RL circuit loop current that exhibits the exponential decay that I observed? Justify the equation in terms of simple circuit principles.

   ii. (4 pts) Determine μ in terms of μ_o. Show reasoning.

   iii. (4 pts) Is the rod diamagnetic or paramagnetic? Explain.

Summer 2015 #1

1. **Magnetic field and inductance problems:**

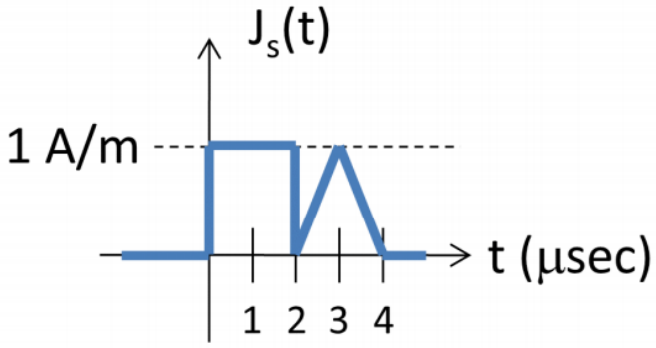
c) I have cylindrical shaped current sheet of length $\ell = 2$ m and radius $a = 20$ cm on which a surface current of $\mathbf{J}_s = \hat{\phi}2$ A/m is flowing in the azimuthal direction $\hat{\phi}$ around the cylinder in counter-clockwise direction when viewed from above the cylinder.

   i. (2 pts) Sketch the cylinder with the directions of $\mathbf{J}_s$ and the resulting magnetic flux density $\mathbf{B}$ within the interior of the cylinder unambiguously indicated.

   ii. (4 pts) What is the numerical value of $|\mathbf{B}|$ right at the center of the cylinder assuming that the cylinder is air filled? Justify your answer.

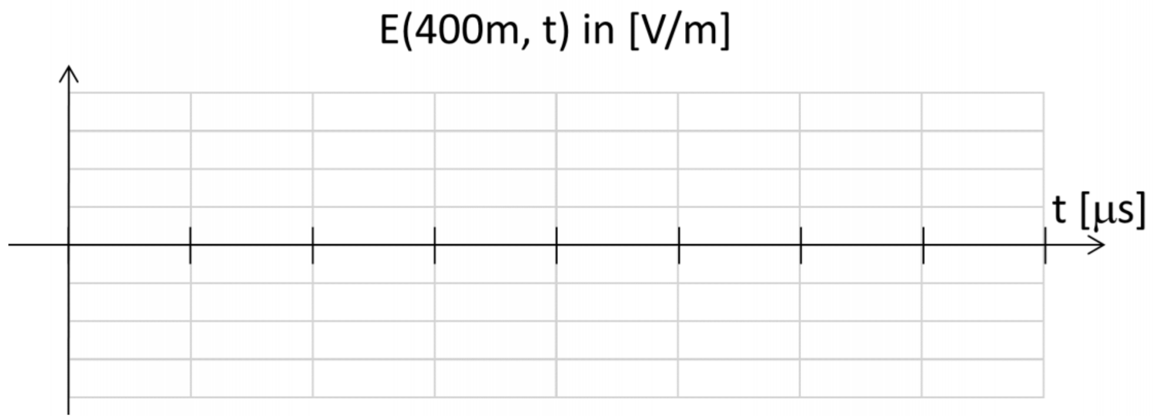
Spring 2017 #5

5. (24 points) Consider a sheet current on the xz-plane of the form $\mathbf{J}_s = \hat{x}(rect(\frac{t-1}{2}) + \triangle(\frac{t-3}{2}))\,[\frac{A}{m}]$ with time in microseconds. Wave propagation occurs on both sides of the sheet in media where $\epsilon = 9\epsilon_0$ and $\mu = 4\mu_0$. Note that $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{6}c \approx 50\,[\frac{m}{\mu s}]$ and $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{2}{3}\eta_0 \approx 60\pi\,[\Omega]$



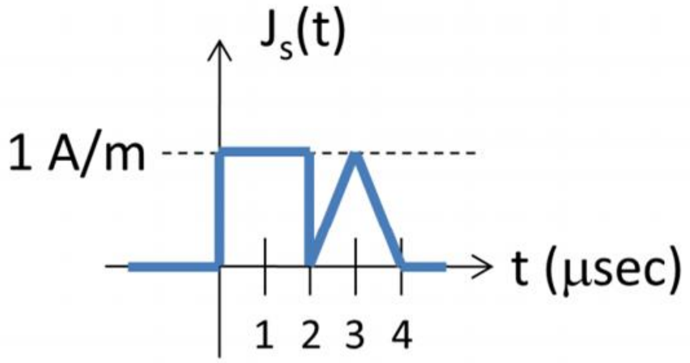
a) (4 pts) What are $\mathbf{E}(x, y, z, t)$ and $\mathbf{H}(x, y, z, t)$? Include units and vector directions to earn full credit.

b) (7 pts) Plot the relevant vector component of $\mathbf{E}$ versus t at $y = 400m$. Label the tick marks on the axes with quantitative values to earn full credit.



Spring 2017 #5

5. (24 points) Consider a sheet current on the xz-plane of the form $\mathbf{J}_s = \hat{x}(rect(\frac{t-1}{2}) + \triangle(\frac{t-3}{2}))\,[\frac{A}{m}]$ with time in microseconds. Wave propagation occurs on both sides of the sheet in media where $\epsilon = 9\epsilon_0$ and $\mu = 4\mu_0$. Note that $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{6}c \approx 50\,[\frac{m}{\mu s}]$ and $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{2}{3}\eta_0 \approx 60\pi\,[\Omega]$



c) (13 pts) Plot the relevant vector component of $\mathbf{H}$ versus y for $t = 6\mu s$. Label the tick marks on the axes with quantitative values to earn full credit.