

# **HKN ECE 340 Exam 1 Review (Sanat, Alex, Marwan)**

## **Motivation:**

- Dope content
- Literally why your computer works
- Nothing works without semiconductors
- Multi billion-dollar industry
- Gateway into the NanoFab Lab
- Illini Pride (Bardeen, Holonyak, Kilby)

## **List of Topics:**

1. Crystal Structure and Miller Indices
2. Energy Bands and Charge Carriers in Semiconductors
3. Fermi-Dirac Distribution function and Fermi Level
4. Drift of carriers in Electric Field
5. Optical Absorption
6. Carrier Lifetime and Photoconductivity
7. Diffusion of Carriers

## **Advice and Study tips:**

- READ THE TEXTBOOK!!! It is phenomenal. I still read it to understand concepts that I don't remember. Everything is beautifully worded and weaved.
- How you do in your exams is not an indicator of your knowledge and understanding of semiconductors.
- Go over Homework Solutions and get a general idea of how they want you to solve the questions

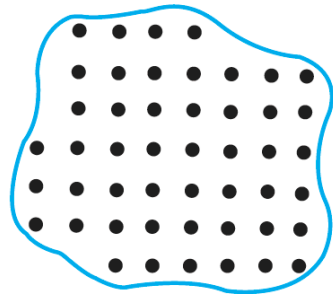
# 1. CRYSTAL/ LATTICE STRUCTURE

Why is this important?

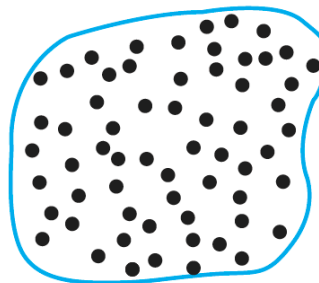
- It depends on how you will process your semiconductor and how it will behave when devices are fabricated on the wafer (ECE 444: IC Theory and Fabrication)
- It is the crystal structure which leads to the formation of the bandgap (ECE 488: Compound Semiconductors)

Three types of **Atomic arrangements**:

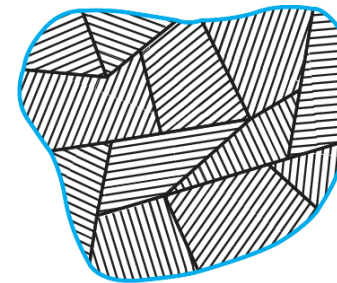
1. *Crystalline*: Periodic, Long range order
2. *Amorphous*: No periodic structure, short range order
3. *Polycrystalline* (many crystalline): small regions of single crystal material



(a) Crystalline



(b) Amorphous

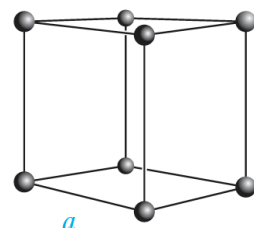


(c) Polycrystalline

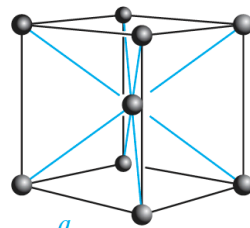
Stack these up → 3D Lattice

## 1.1. Cubic Lattices:

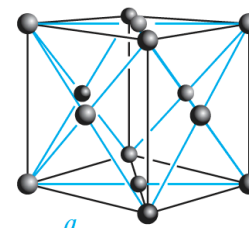
2. Simple Cubic
3. Face Centered
4. Body Centered
5. Diamond/ Zinc-Blende Lattice



Simple cubic



Body-centered cubic



Face-centered cubic

**$a$  = Lattice Constant**

*This is a very important parameter which has to be taken into consideration during the fabrication of LEDs, Solar Cells, LASERs, and any other devices that use multiple different semiconductors*  
(ECE 443: LEDs and Solar Cells; ECE 488: Compound Semiconductors and Devices)

### Practice Question: Example 1-1 Pg. 26

Find (i) the total number of atoms in an fcc unit cell, (ii) the fraction of the fcc unit cell volume filled with hard spheres.

**Answer:** (i) 4 (ii) 74%

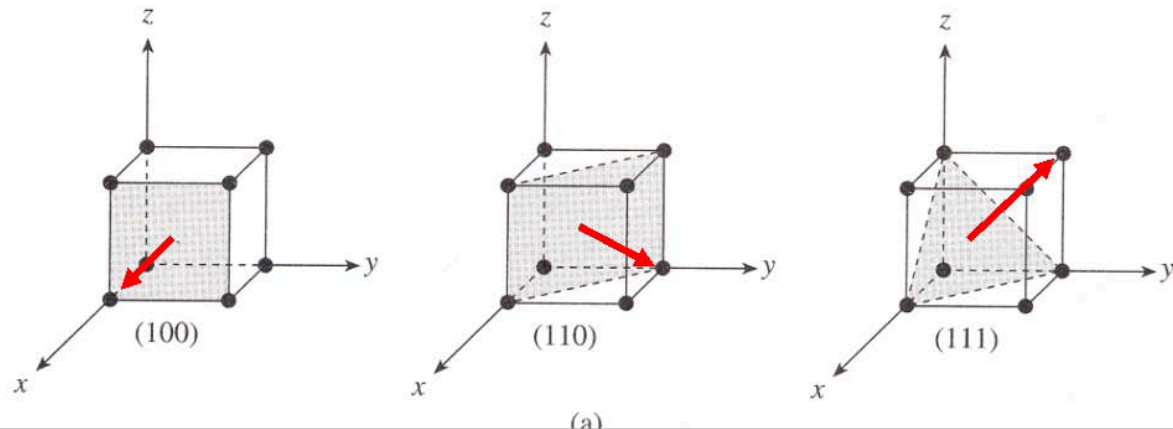
In 400 level semiconductor courses, we look at the primitive cells for these various lattice structures to simplify our own understanding and multiple mathematical operations like the Fourier Transform

## 1.2. Miller Indices ( $hkl$ ):

- Basically, a set of vectors
- Determine how the boule of manufactured Silicon (or semiconductor) must be cut.
- One can't randomly cleave in any direction
- We prefer certain orientations over others for different applications.
- Get comparatively complicated for Hexagonal compared to Cubic.

Steps to find the Miller Indices of a specific plane:

1. List the intercepts in the units of the lattice constants
2. Find the reciprocal of the integer parts
3. Reduce the reciprocals to the smallest set of integers ( $hkl$ )



(100), (110), (111) Miller Index Planes

These planes have different number of atoms per unit area for different lattice structures  
(Ex: 111 of Diamond Lattice contains  $(0.5 \cdot 3 + 3 \cdot (1/6)) = 2$  atoms/  $\text{cm}^2$ )

## 2. ENERGY BANDS AND CHARGE CARRIERS

Discrete (single) atoms exhibit energy levels which are *Quantized* (Bohr H-atom)

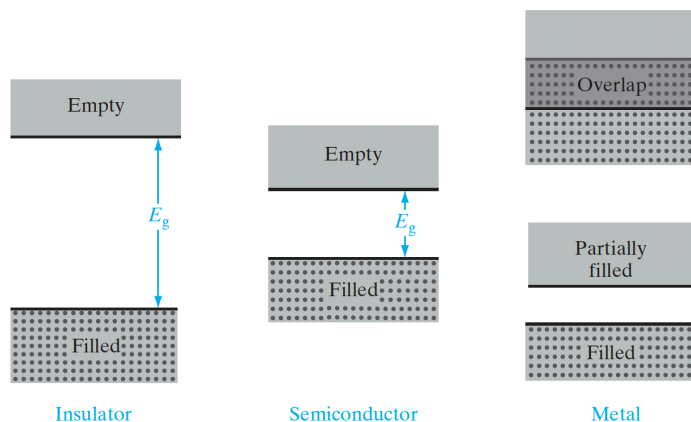
Column IV elements like Si lack 4 electrons in their outermost shells.

14 electrons:

$1s^2 2s^2 2p^6 3s^2 3p^2 \rightarrow$  need 4 electrons to fill the p orbital  $\rightarrow$  Covalent bonds  $\rightarrow$  Crystal Structure  $\rightarrow$  Energy Bands

### 2.1. Formation of Energy Bands

- Isolated (large atomic spacing) Semiconductor atoms have sharp discrete energy levels
- As atoms get closer together, the outer shell electrons overlap, and interact with each other
- Isolated outer shells merge into 3D shared electron probability distribution
- Discrete energy levels broaden into **Bands**
- Large number of atoms present which leads to a large number of energy levels which are closely packed
- These energy levels in reality are discrete but can be assumed to be continuous



Energy bands of Insulators, Semiconductors, and Metals at 0K

The empty band is called the Conduction Band and the filled band is called the Valence Band. For any material to conduct, there must be electrons in the Conduction Band.

The void gap between the Conduction and the Valence band is called the Energy Band Gap ( $E_g$ )

$$E_g = E_C - E_V$$

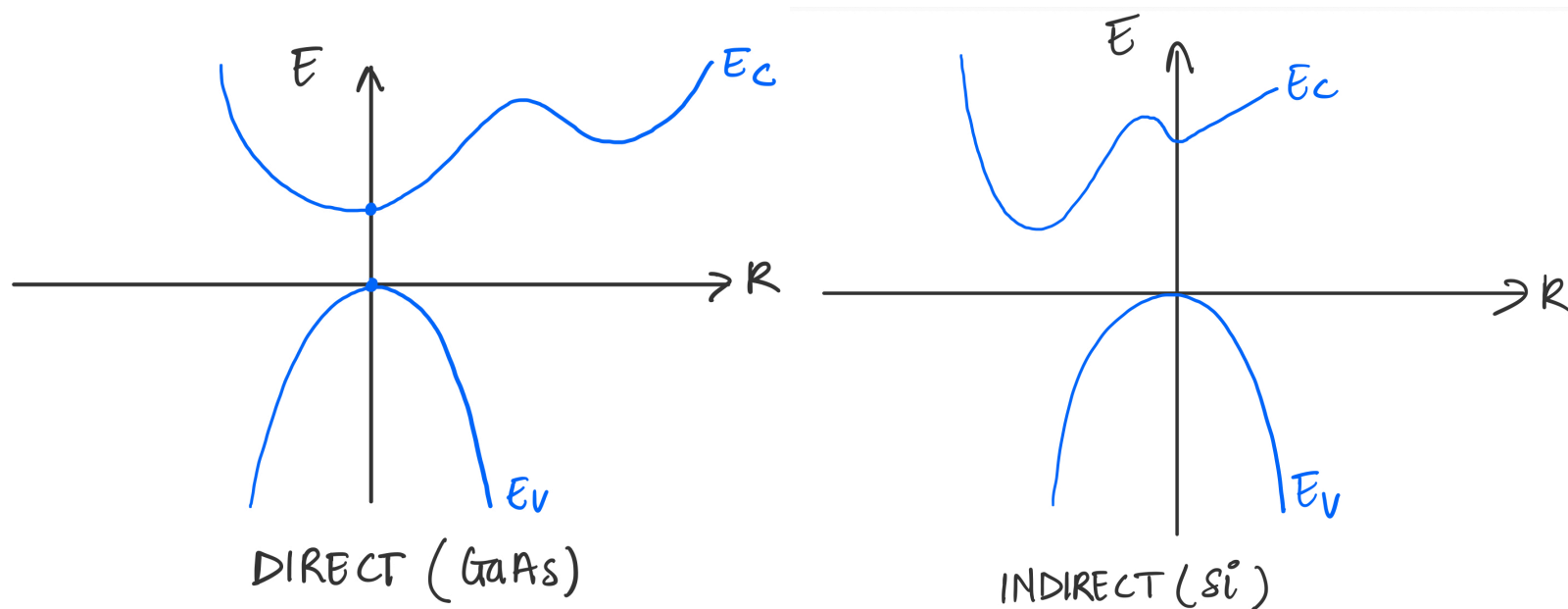
## 2.2. DEFINITION OF A SEMICONDUCTOR:

A semiconductor material is one whose energy bandgap is between 0eV and 5eV.

### *Types of Semiconductors:*

1. Direct Bandgap: lowest value of the conduction and valence band occur at the same value of momentum
2. Indirect Bandgap: lowest value of the conduction and valence band occur at different momentum values

For Photonic/Optoelectronic devices, such as LEDs and Solar Cells, we want to use direct bandgap semiconductors to avoid the lower efficiency (ECE 304: Photonic Devices, ECE 443: LEDs and Solar Cells)



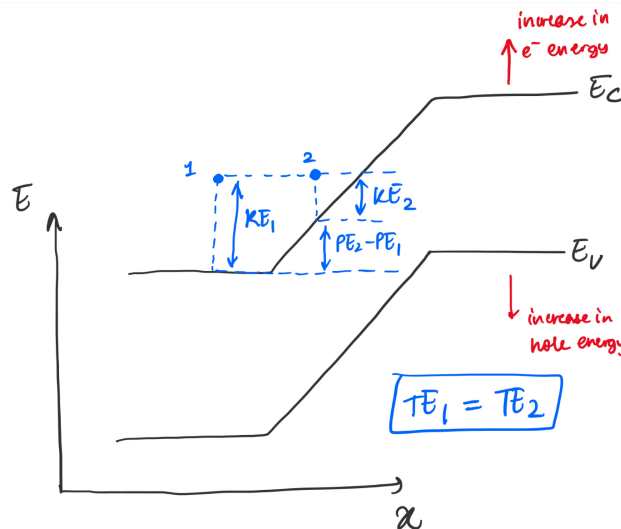
### 2.3. CHARGE CARRIERS IN SEMICONDUCTORS

At 0K there are no electrons in the conduction band. However, as we raise the temperature, we see thermal excitation of electrons from the valence band to the conduction band. This leads to an absence of an electron at the valence band site. This **absence of the electron is called a hole ( $h^+$ )** has an effective positive charge which is equal in magnitude of the charge on the electron( $e^-$ ). To conduct:

Holes  $\rightarrow$  Valence Band  
Electrons  $\rightarrow$  Conduction Band

The creation of the hole due to thermal excitation of the electron is called Electron Hole Pair (**EHP**) Formation. At  $T=300K$ ,  $kT=25.9$  meV: very few holes in valence band and very few electrons in the conduction band.

In an INTRINSIC semiconductor (no impurity), for every electron in the conduction band, there is a hole in the valence band.



## 2.4. EFFECTIVE MASS

Electrons in a crystal are **NOT** completely free meaning that they interact with the periodic potential of the lattice. Therefore, motion cannot be expected to be the same for electrons in free space.

$$m^* = \frac{\hbar^2}{d^2 E / d\mathbf{k}^2}$$

## 2.5. INTRINSIC MATERIAL

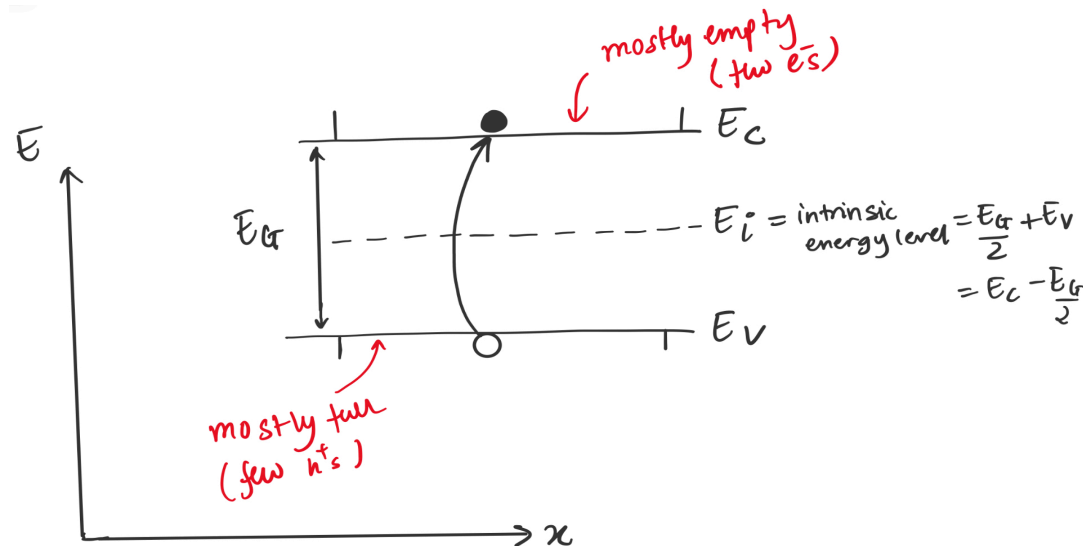
- **NO** impurities
- No charge carriers at 0K

$$n = p = n_i$$

$n$  = Conduction band electron concentration (electrons/ cm<sup>3</sup>)

$p$  = Valence band hole concentration (holes/ cm<sup>3</sup>)

$n_i$  = Intrinsic carrier concentration (carriers/ cm<sup>3</sup>)





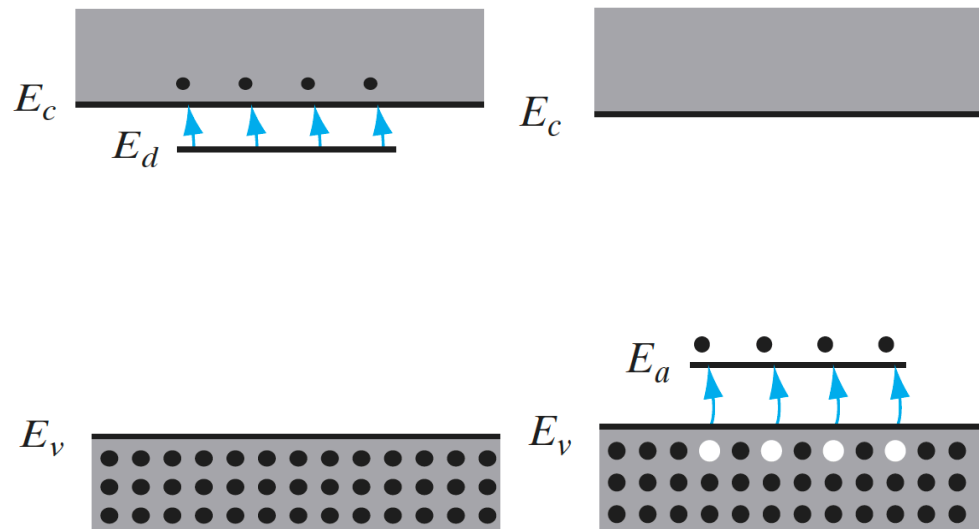
$$np = n_i^2$$

$$n - p + N_A^- - N_D^+ = 0$$

## 2.6. EXTRINSIC MATERIAL

- Makes semiconductors dope because we dope them (get it?)
- Take the site of one atom of the bulk (if we dope Si with B, B will take one Si site)
- Introduce impurities to increase the number of charge carriers without having to raise the temperature and rely on thermal EHP generation
- Two types of dopant:
  - **Acceptors ( $N_A$ ):** electrons are “accepted” or “stolen” from the valence band leading to generation of a hole in the valence band
  - **Donors ( $N_D$ ):** electrons are “donated” to the conduction band by the impurity

**NOTE:** When we talk about Si as semiconductor, if we add B which has one electron less than Si, B here will act as an acceptor. Whereas, if we add P as our dopant, P having one electron more than Si will act as a donor.



Donation of electron  
from the donor level to  
the Conduction band

Acceptance of electron  
from the valence band to  
the Acceptor energy level

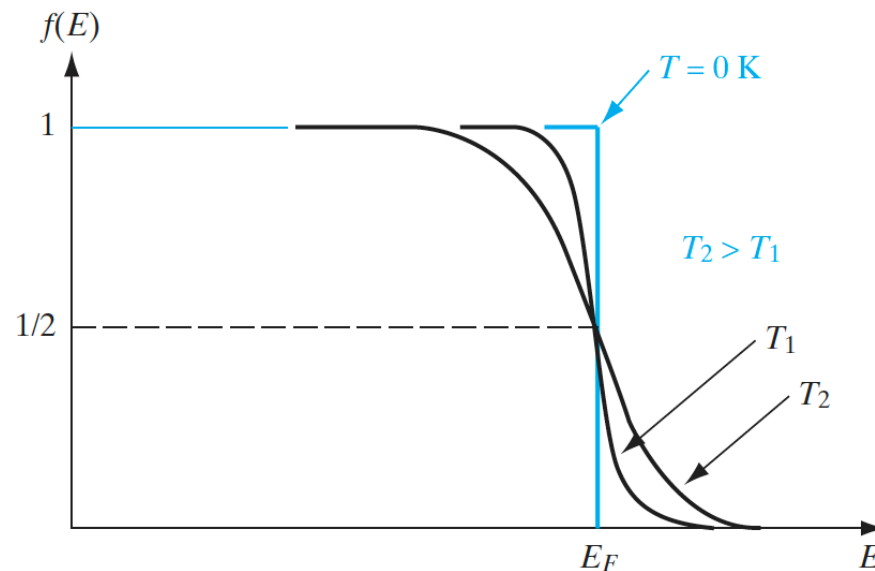
### 3. FERMI DIRAC DISTRIBUTION FUNCTION AND THE FERMI LEVEL

The Fermi Dirac Distribution function ( $f(E)$ ) gives us the probability that an available energy state at  $E$  will be occupied by an electron at absolute temperature  $T$ .

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$f(E_F) = \frac{1}{2}$$

**Fermi Level:** the energy level at which finding an electron is 0.5



The position of the Fermi Level changes as we dope our semiconductor as follows:

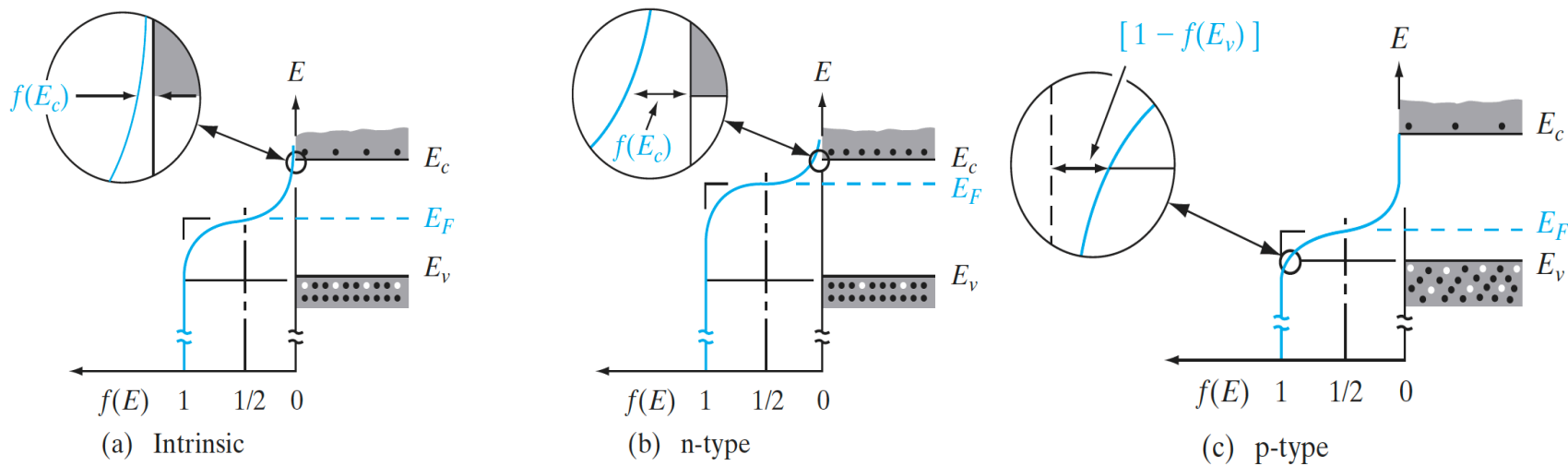
- **p type:**

- $E_F - E_V$  reduces
- More holes are present in the valence band as the electrons have been excited to the impurity energy level. There are very few electrons in the conduction band compared to the acceptor energy level which is fairly close to the valence band. This means that the Fermi level would move to closer to the valence band.

- **n type:**

- $E_C - E_F$  reduces
- Due to the excitation of electrons from the donor energy level to the conduction band, there is a high number of electrons in the conduction band compared to that in the valence band. This would move the Fermi level closer to the conduction band.

The extent to which the Fermi Level moves depends on the doping concentration



## Carrier Concentration at Equilibrium

$$n_0 = \int_{E_C}^{\infty} f(E)N(E) dE = \text{Equilibrium electron concentration}$$
$$p_0 = \int_{-\infty}^{E_V} [1 - f(E)]N(E) dE = \text{Equilibrium hole concentration}$$

Where E is the energy level, N(E) is the number of states available for an electron to occupy, and f(E) is the Fermi Dirac Distribution function at E.

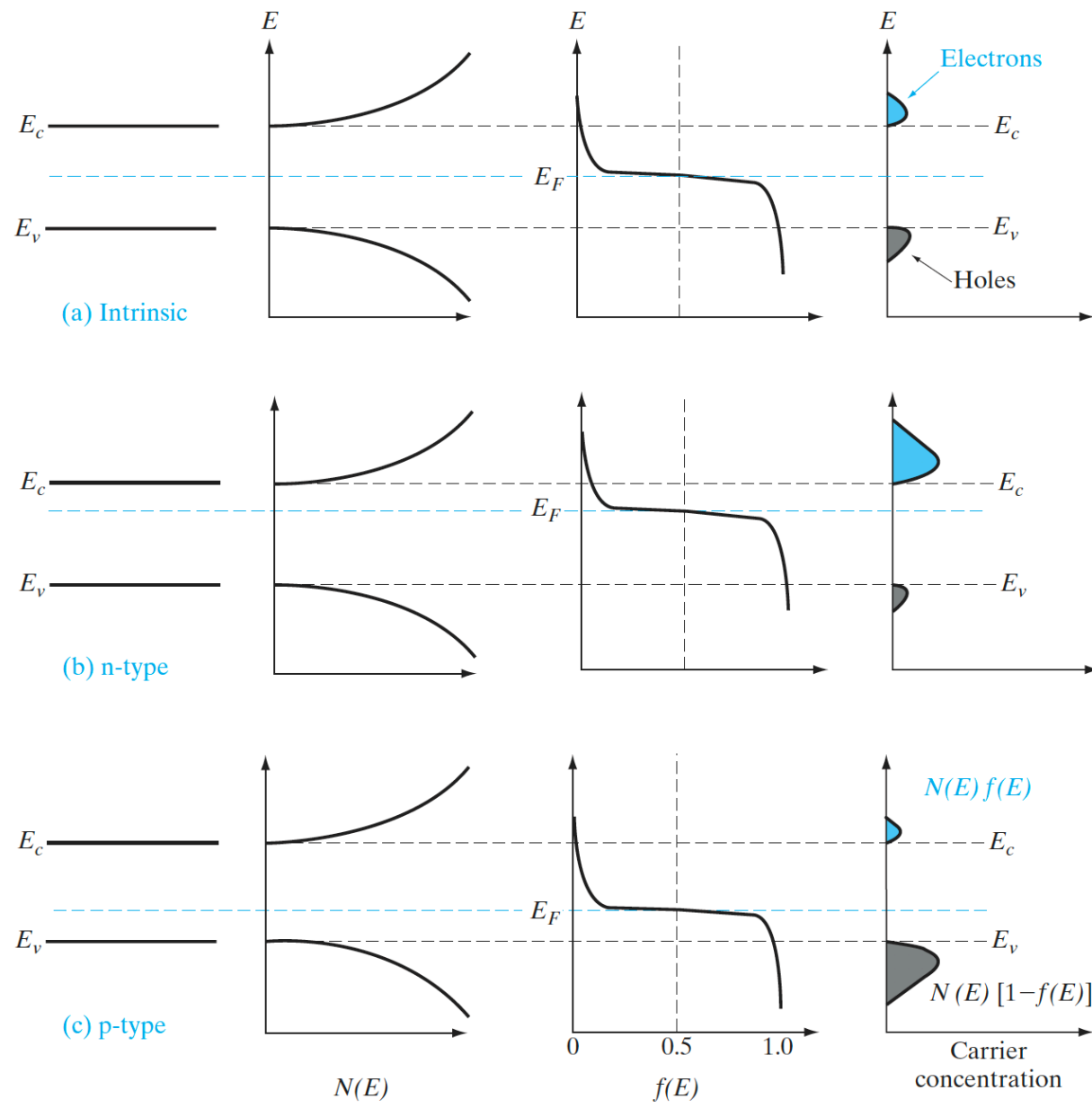
The result of the integration gives us the following:

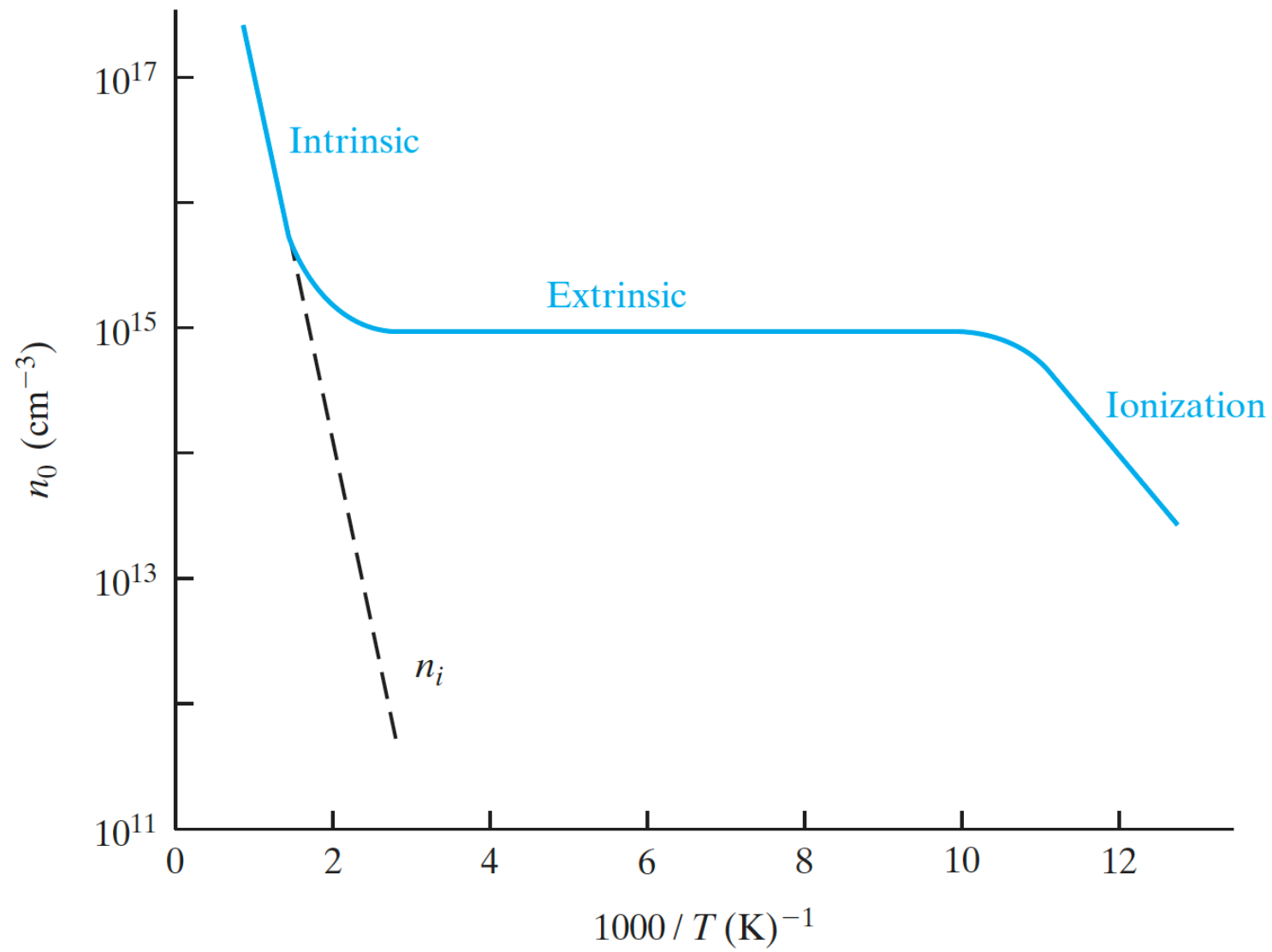
$$n_0 = N_C f(E_C) = N_C \exp\left(-\frac{E_C - E_F}{kT}\right)$$
$$p_0 = N_V [1 - f(E_V)] = N_V \exp\left(-\frac{E_F - E_V}{kT}\right)$$

$N_C$  = effective density of states in the conduction band;  $N_V$  = effective density of states in the valence band

$$n_0 p_0 = n_i^2$$
$$\Rightarrow n_i = \sqrt{N_C N_V} \exp\left(-\frac{E_G}{2kT}\right)$$
$$\Rightarrow n_0 = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$$
$$p_0 = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

$$\boxed{\frac{dE_F}{dx} = 0}$$





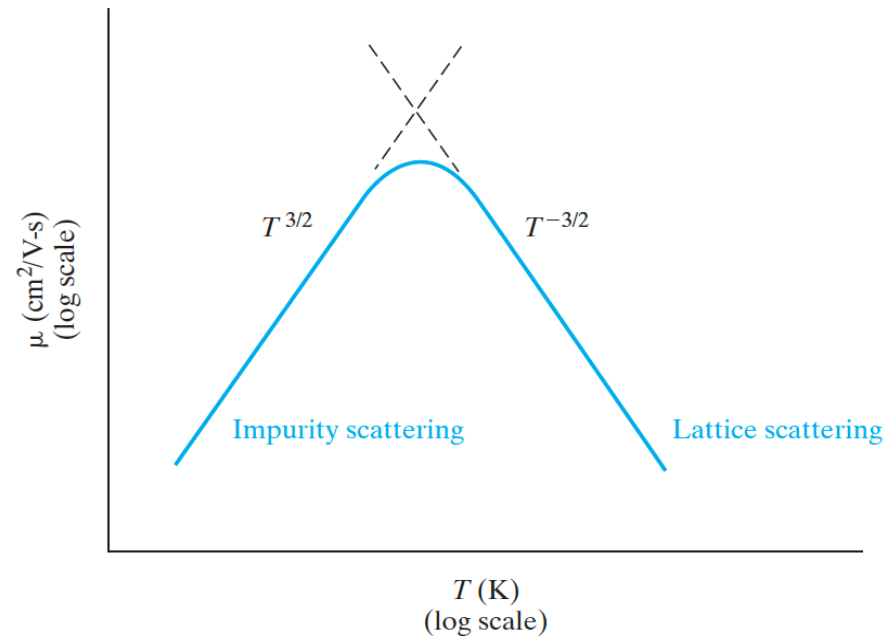
## 4. DRIFT OF CARRIERS IN ELECTRIC FIELD

$$J = \sigma E$$

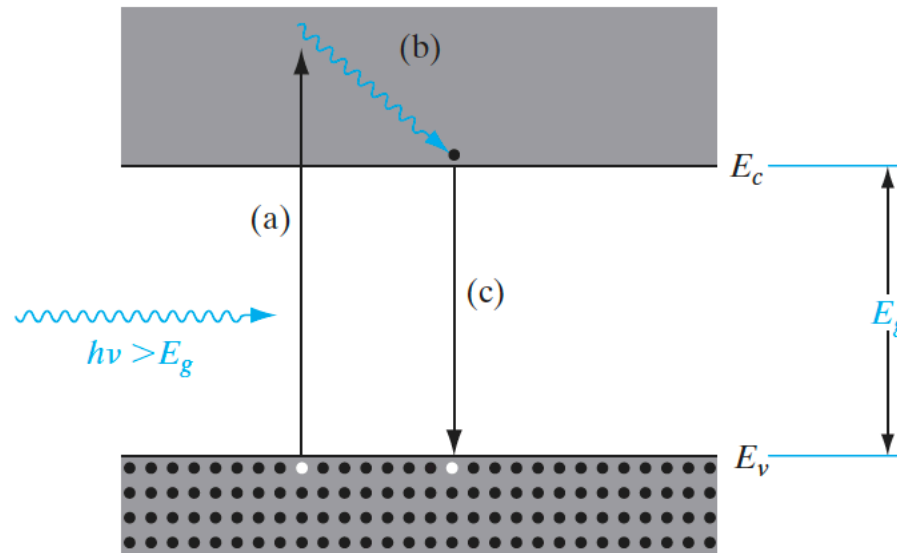
$$\sigma = q(n\mu_n + p\mu_p)$$

$$\rho = \frac{1}{\sigma}$$

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \dots$$



## 5. OPTICAL ABSORPTION



**a:** EHP generated by photon with  $h\nu > E_g$

**b:** Thermalization: the electron gained more energy than it needed to get to the conduction band; therefore, it gives away the extra energy in the form of heat which heats up the lattice

**c:** EHP recombination

When light of intensity ( $I_0$ ) is directed at a sample of thickness  $l$ , given that the energy of the photons is greater than the bandgap of the sample, some of the light is absorbed and the rest is transmitted.

$$-\frac{d\mathbf{I}(x)}{dx} = \alpha\mathbf{I}(x)$$

$$\mathbf{I}(x) = \mathbf{I}_0 e^{-\alpha x}$$

$$\mathbf{I}_t = \mathbf{I}_0 e^{-\alpha l}$$

$\alpha$  = Absorption Coefficient



## 6. CARRIER LIFETIME AND PHOTOCONDUCTIVITY

$$\begin{aligned}\frac{dn(t)}{dt} &= \text{net rate of change in the conduction band electron concentration} \\ &= \text{thermal generation rate} - \text{recombination rate}\end{aligned}$$

$$\frac{dn(t)}{dt} = \alpha_r n_i^2 - \alpha_r n(t)p(t)$$

Now, consider that a flash of light leads to EHP generation. The number of holes and electrons generated are equal. Therefore, we get:

$$\begin{aligned}\frac{d\delta n(t)}{dt} &= \alpha_r n_i^2 - \alpha_r [n_0 + \delta n(t)][p_0 + \delta p(t)] \\ &= -\alpha_r [(n_0 + p_0)\delta n(t) + \delta n^2(t)]\end{aligned}$$

$\delta n(t) = \delta p(t)$

Where  $\delta n(t)$  and  $\delta p(t)$  are the instantaneous excess carrier concentration,  $\Delta n$  and  $\Delta p$  are the values at  $t = 0$ .

If the excess carrier concentration is small, we neglect  $\delta n^2(t)$ . This leaves us with:

$$\begin{aligned}\frac{d\delta n(t)}{dt} &= -\alpha_r (n_0 + p_0)\delta n(t) \\ \delta n(t) &= \Delta n \exp(-t\alpha_r(n_0 + p_0)) = \Delta n \exp\left(-\frac{t}{\tau}\right)\end{aligned}$$

Where  $\tau$  is known as the carrier lifetime. Depending on the dopant concentrations, we can approximate  $\tau$  which is known as the minority carrier lifetime  $\tau_n (= -\alpha_r p_0)$  or  $\tau_p (= -\alpha_r n_0)$

We can say that the general case for the optical generation we get the following formulae where  $g_{\text{op}}$  is the optical generation rate:

$$\delta n = \tau_n g_{\text{op}} \quad \text{and} \quad \delta p = \tau_p g_{\text{op}}$$

$$\Delta \sigma = q g_{\text{op}} (\tau_n \mu_n + \tau_p \mu_p)$$

## 7. DIFFUSION OF CARRIERS

Diffusion: random motion from high concentration to low concentration

Key factor needed for diffusion to take place: **Concentration Gradient**

$$\phi_n(x) = -D_n \frac{dn(x)}{dx}$$

$$\phi_p(x) = -D_p \frac{dp(x)}{dx}$$

$\phi_n$  = Electron Flux Density  
 $\phi_p$  = Hole Flux Density  
 $D_n$  = Diffusion Coefficient for electrons  
 $D_p$  = Diffusion Coefficient for holes  
 The carrier fluxes are proportional to the concentration gradient

We know that the current density (current per unit area) is the carrier flux density multiplied by the charge of the carriers  $\rightarrow J = q\phi$ . This gives us the following result for the **Diffusion Current Density**:

$$J_n(\text{diff.}) = -(-q)D_n \frac{dn(x)}{dx} = +qD_n \frac{dn(x)}{dx}$$

$$J_p(\text{diff.}) = -(+q)D_p \frac{dp(x)}{dx} = -qD_p \frac{dp(x)}{dx}$$

## Diffusion and Drift of Carriers

$$J(x) = J_n(x) + J_p(x)$$

The total current density is the sum of the current density due to holes and electrons. The current density of the carriers are equal to the sum of the diffusion current density and the drift current density.

$$J_n(x) = \underbrace{q\mu_n n(x)\mathcal{E}(x)}_{\text{drift}} + \underbrace{qD_n \frac{dn(x)}{dx}}_{\text{diffusion}}$$
$$J_p(x) = \underbrace{q\mu_p p(x)\mathcal{E}(x)}_{\text{drift}} - \underbrace{qD_p \frac{dp(x)}{dx}}_{\text{diffusion}}$$

$$\mathcal{E}(x) = -\frac{dV(x)}{dx}$$

## Diffusion and Recombination

The Einstein Relation gives us an essential relation between the Diffusion Coefficient and the Mobility of our carriers.

$$\frac{D}{\mu} = \frac{kT}{q}$$

Consider diffusion taking place alongside recombination:

$$\left. \frac{\partial p}{\partial t} \right|_{x \rightarrow x + \Delta x} = \frac{1}{q} \frac{J_p(x) - J_p(x + \Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

Rate of hole buildup = increase of hole concentration in  $\delta x A$  per unit time - recombination rate

We get the same equation for electrons. Simplifying further:

$$\frac{\partial p(x, t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

$$\frac{\partial \delta n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

Plugging the calculated values for the Diffusion currents:

$$\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}$$

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

## Steady State Carrier Injection and Diffusion Length

In many problems, steady state carrier distribution is maintained making the time derivatives are zero resulting in:

$$\frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} \equiv \frac{\delta n}{L_n^2}$$

$$\frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} \equiv \frac{\delta p}{L_p^2}$$

Where  $L_n$  and  $L_p$  are the diffusion lengths for electrons and holes which is basically the distance the carriers diffuse before they recombine. This gives us the injected carrier distribution as follows:

$$\delta p(x) = \Delta p e^{-x/L_p}$$

