EECS 16A Midterm 1 Review Session

Presented by <NAMES >(HKN)

Disclaimer

Although some of the presenters may be course staff, the material covered in the review session may not be an accurate representation of the topics covered in and difficulty of the exam.

Slides are posted at — on Piazza.

HKN Drop-In Tutoring

- These details should be edited

Systems of Equations and Gaussian Elimination

Vectors

Conceptually, a vector is a collection of numbers that each represent a variable. If there are n variables, then the vector is n-dimensional.

Example:
A point in 3D can be represented as (x, y, z) In vector form, this would be represented as $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Matrices

- Collection of **vectors**
- 2D table for **storing data**
  - ◎ Systems of equations for imaging observations
- Notable/useful matrices
  - ◎ Identity matrix
  - ◎ Augmented matrices
  - ◎ Rotation matrix
  - ◎ Many others!

Augmented: $$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 2 & 1 & 1 & 4 \\ -3 & 2 & 3-2 & 10 \end{array}\right]$$

Rotation: $$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Matrix Transformations

Matrices are often used to perform transformations, especially in $\mathbb{R}^2$

Two important transformations:

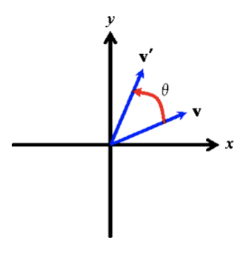

Figure 1: Rotation

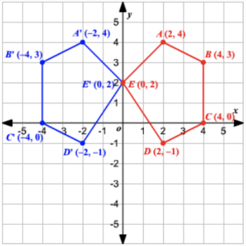

Figure 2: Reflection

Rotation Matrix

The rotation matrix rotates points by a specified angle, theta:

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Use this matrix by plugging in desired rotation angle, then multiply with vector.

Note: Rotation matrices also preserve the length of a vector.

Example: Rotation Matrix that rotates vector by 90°

$$R(90^\circ) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

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Reflection Matrix

The reflection matrix reflects vectors across a line (Notice that such matrix also preserves the length of a vector)

Notable reflection matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Reflection across x-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

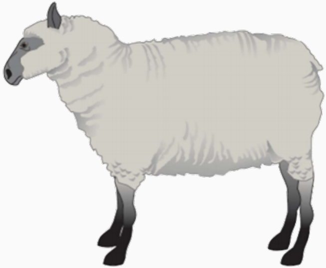
Reflection across y-axis

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Reflection across $y = x$

Matrix Transformations

All linear transformations can be expressed as a matrix



sheep

sheared sheep

Question

What is the resulting vector after the following (non-subsequent) transformations are applied to the vector $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

1. Rotate by 45°

2. Reflect across $y = x$

Example

Solution

What is the resulting vector after the following (non-subsequent) transformations are applied to the vector $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

1. Rotate by 45°

$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{5\sqrt{2}}{2} \end{bmatrix}$$

2. Reflect across $y = x$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$