ST502 Project

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Part I

Framingham Heart Study

The Framingham data set contains the systolic blood pressure of 300 smokers and nonsmokers. For this study we will assume the data are random samples from normal distributions (see Appendix I for more details) where μ_1 and σ_1^2 are the mean and variance for nonsmokers and μ_2 and σ_2^2 are the mean and variance for smokers. The null hypothesis, H_0: μ_1 - $\mu_2 = 0$, states there is no significant difference in blood pressure between smokers and nonsmokers. We wish to prove the alternative hypothesis, H_A: μ_1 - $\mu_2 \neq 0$, which states there is a significant difference in the means. We will accomplish this through hypothesis testing to determine if enough evidence does, indeed, exist to reject the null hypothesis.

Methods

We conducted the analysis under two conditions: the first assumed equal population variance (using pooled sample variance, S_p^2) and the second assumed unequal population variance. The results will be presented and any discrepancies between the two outcomes will be discussed.

Analysis

We will conduct the analysis under two conditions; the first assuming equal population variance (using pooled sample variance, S_p) and the second assuming unequal population variance. The results will be presented and any discrepancies between the two outcomes will be discussed.

To begin, we first found the difference in mean blood pressure between smokers and nonsmokers in this dataset; this was calculated to be -9.16. Despite this value being nonzero, we cannot safely assume the difference occurred other than by random chance. In the first analysis we assumed equal variances, thereby allowing us to use pooled sample variance (see Appendix I: Equation 1) which was calculated to be 510 using 298 degrees of freedom. Under these conditions, the p value (probability of obtaining the observed results) was calculated to be 0.0041, which is smaller than (probability of committing a Type I error) of 0.05. In terms of t values, our observed t value of -3.04 is smaller than the t value, -1.97 for a two sided of 0.05.

Additionally, the 95% confidence limits were calculated for the pooled variance assumption. The observed 95% confidence interval is -15.08 to -3.23. One can see that zero does not fall into this interval, which allows the null hypothesis to be rejected. In other words, at an level of 0.05, there is significant evidence to suggest this group of smokers and nonsmokers have a different range of systolic blood pressure when using pooled variance.

We then conducted the same analysis as before, but now assuming unequal population variances. In this case, the computed sample variances of the two groups were 352.2 for smokers and 562.1 for nonsmokers. Using equation 3 from Appendix 1, an observed t value of -2.9 was obtained. Comparing that the two-sided α of -1.98 with 158 degrees of freedom (as computed using the Satterthwaite Approximation; see Appendix I: Equation 4), the observed t value was less than the chosen α value of 0.05, as in the case of pooled sample variance.

Furthermore, the 95% confidence limit in the non pooled case was found to be -15.4 to -2.91. It can be noted that zero is absent from this range, as in the case of the pooled sample variance. In this second scenario, there is sufficient evidence to reject the null hypothesis and to support the alternative at an α level of 0.05.

We have shown that there is sufficient evidence that the blood pressure between smokers and nonsmokers are different. This statement holds true in all cases described in this paper. It should be noted, that while the observed systolic blood pressure values vary between smoker and non smokers, we do not know the underlying cause based on the provided data.

Test Preference

Here, both tests allow us to reject the null hypothesis, so either one could be useful. In this case, we must allow the computed statistics to dictate which test we ought to prefer. The standard deviation (pooled variance) was calculated from the denominator of Equation 2 (see Appendix I) which returned a value of 3.01. Similarly, the standard deviation (unequal variance) was calculated from the denominator of Equation 3 which returned a value of 9.99. We want to have data that is as consistent as possible, in other words, with the smallest spread, i.e., variance; therefore, we prefer the test with pooled variance.

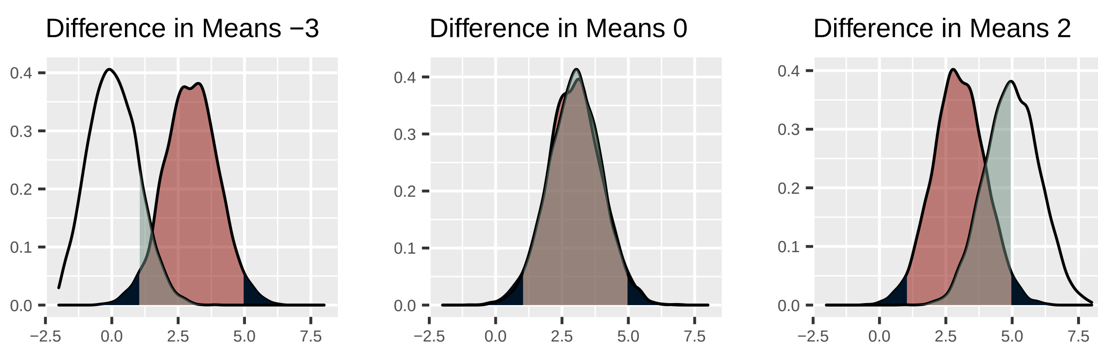
Part II

Introduction

In Part I, we were able to see how hypothesis testing allowed us to produce enough evidence to challenge the status quo argument that "smoking has no effect on the blood pressure of a person." We found, through a number of robust statistical methods, the difference in means (calculated to be -9.16) was significant enough to not have arisen merely by chance. Within an α of 5%, we are confident that the two groups do exhibit measurable differences. In the following sections, we will explore, through simulated data, the concept of randomness in data and what steps we can take to mitigate that randomness. All simulated data will be generated using the built-in rnorm() function in R.

Power of Hypothesis Testing

It is important to briefly discuss the concept of power in hypothesis testing. Power describes the probability of not committing a Type II error, which is to not reject the null hypothesis when there was in actuality enough evidence to do so. The probability of a Type II error is represented by β. Power is the complement to that probability (1 - β) and can be thought of as test sensitivity. The more powerful the test the more sensitive it is to differences between distributions. The value of β is the portion of the alternate distribution that is within the null hypothesis non rejection region limits. Below are a number of plots to help depict this concept. In each of the plots the shaded red is the null distribution and its location stays constant (mean = 3). The dark tails represent the rejection region of the null distribution and correspond to the size of α (in this case 2.5% on each side. α also indicates the probability of committing a Type I error, rejecting the null hypothesis incorrectly. The outline distribution represents the "true" alternative distribution. Within that distribution is the shaded light blue region which is equal to β. It can be seen that as the difference between the two distributions shrinks the value of β increases and the power shrinks. Power reaches minimum and β reaches maximum when the two distributions are the same.



Simulation Study

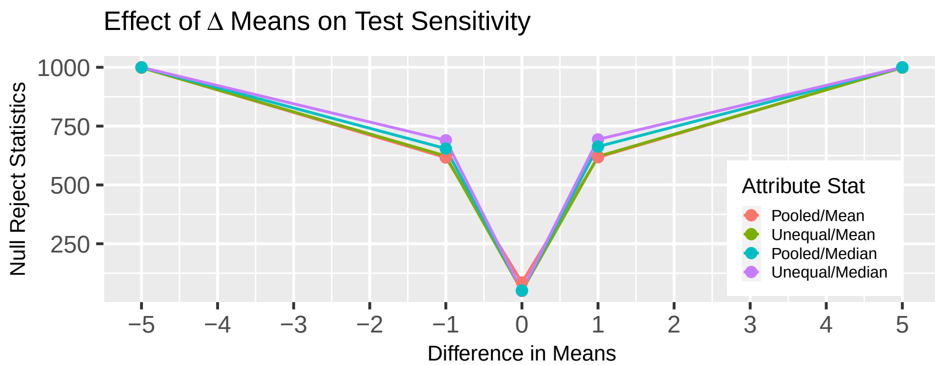
Multiple scenarios were simulated by creating 2 normally distributed data sets of various mean, variances, and sample sizes. Each scenario was repeated a thousand times. For each of those repeats a hypothesis test was conducted and the number of times the null hypothesis was rejected was recorded. See Appendix III for results. All hypothesis testing was using pooled and unequal variance, regardless of the actual values of variance.

The following table includes the values of each parameter used in the simulation:

Parameter	Possible Values
μ_1	0, 4, 5, 6, 10
σ_1^2	1, 4, 9
n_1	10, 30, 70
μ_2	5
σ_2^2	1
n_2	10, 30, 70

Observations

It was noted that the smaller difference between the alternative and null mean values were, the less likely the null hypothesis was rejected (see plot below and for more detail Appendix IV Item A). Larger values of variance also reduced the number of rejected null hypothesis (see Appendix IV Item B). In both cases, the more overlap between the two distributions, the less powerful the test. It was observed that increasing the sample size increased power (see Appendix IV Item C, D, and E). For a specific example, we can look at test cases #9 and #129 where $\mu_1 = 6$, $\mu_2 = 5$, $\sigma_1^2 = 4$ and $\sigma_2^2 = 1$. The two cases differed in sample size where $n_9 = 10$ and $n_{129} = 70$. This increase in sample size resulted in almost a 4 fold increase in power (271 vs 951 (pooled) or 265 vs 948 (unequal) rejected nulls). It stands to reason that large variances and small deltas in mean can be mitigated by increasing the sample size accordingly.



Type I errors can be clearly observed in the simulated data. All test cases where $\Delta\mu$ was zero, roughly 50 out of the 1000 simulations resulted in rejected null hypotheses (see Appendix IV Item A table). These erroneous results, in roughly 5% of the cases, are due to the difference in variance between the distribution. That percentage value closely matches that of α.

One final observation, for each of the test cases (1 - β) was calculated under the assumption that the observed $\Delta\mu$ was the true distribution. It was found that (1 - β) values followed a similar pattern to that of rejected null hypotheses indicating the relationship between power and sensitivity. Below is the calculations for test cases 9 and 129:

	Test #9 (pooled)	Test #9 (unequal)	Test #129 (pooled)	Test #129 (unequal)
1st Quartile	0.1169414	0.1211982	0.7310973	0.5215600
Median	0.2979222	0.3113339	0.9137144	0.7240296
Mean	0.3793437	0.3775096	0.8223957	0.6850268
3rd Quartile	0.6114560	0.5889342	0.9808740	0.8777601

Conclusion

In Part I, we sought to reject the null hypothesis and determine which testing procedure was preferable by performing t-tests in terms of both p-values and confidence intervals with pooled and unequal variances. The results showed that there is a measurable difference in systolic blood pressure and a test with lower variance is preferable in order to reject the null hypothesis. These results, however, were the end-product of two tests run in two scenarios. To verify we came to the correct conclusions, it was important to run a numerical simulation study with many more differing variables which would, in turn, create many more scenarios. If the results led to the same conclusions, it would be safe to assume (with high probability) that our conclusions from Part I were correct.

After performing 1,000 trials for each of 135 different scenarios, the data showed (as described above) that the probability of rejecting the null hypothesis is maximized with large differences in means or low variances. The data also showed that this ideal scenario can be “forced” by increasing sample sizes when the differences in means are small and variances are large. Importantly, increasing the sample size greatly increases the power, or probability of rejecting the null hypothesis when it should, in fact, be rejected. The results from Parts I and II agree, therefore, we can assume our conclusions were correct.

Appendix I: Equations and Figures

Normality of Data

Plotting the density of total data (Figure 1A), we can see that it closely follows a normal distribution. Density plots of the split data exhibit similar forms (Figure 1B and 1C). We calculated the kurtosis and the skew of the data and found them to 3.812, 0.88 for kurtosis and skew respectively. These values are quite close to those typically found in normally distributed data. Additionally, the calculated values of sample standard deviation, $\frac{IQR^*}{1.35}$, and $\frac{MAD^*}{0.675}$ are 22.89, 22.22, and 21.48. The similarities between the three values indicate that the data is not heavily influenced by outliers and the Q-Q plot confirms this (See Figure 2). Therefore, we will assume the data and the subsequent split data to be normal.

*Interquartile Range (IQR), Median Absolute Deviation (MAD)

Figure 1: Data plots

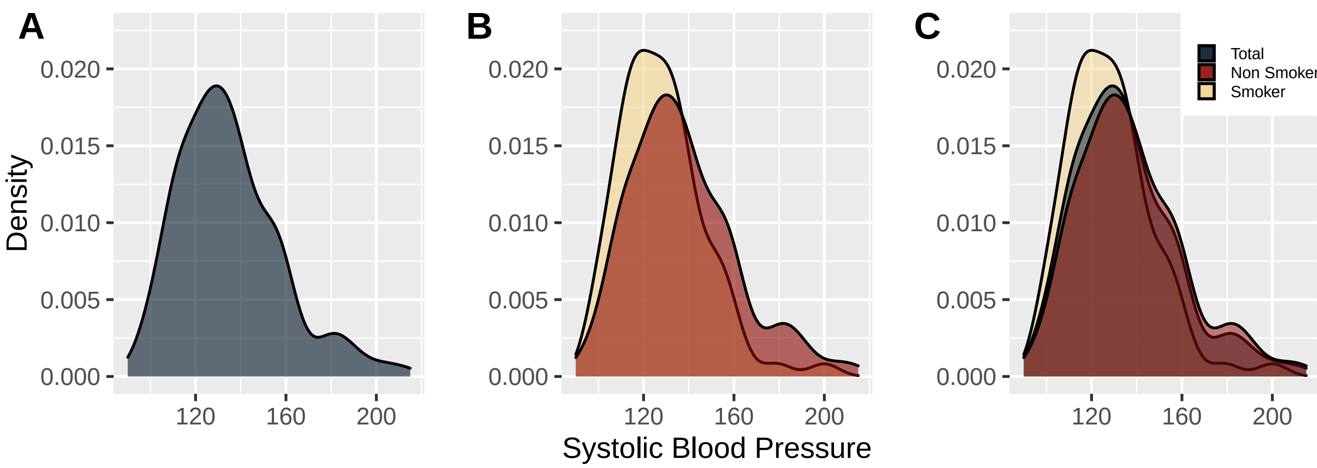
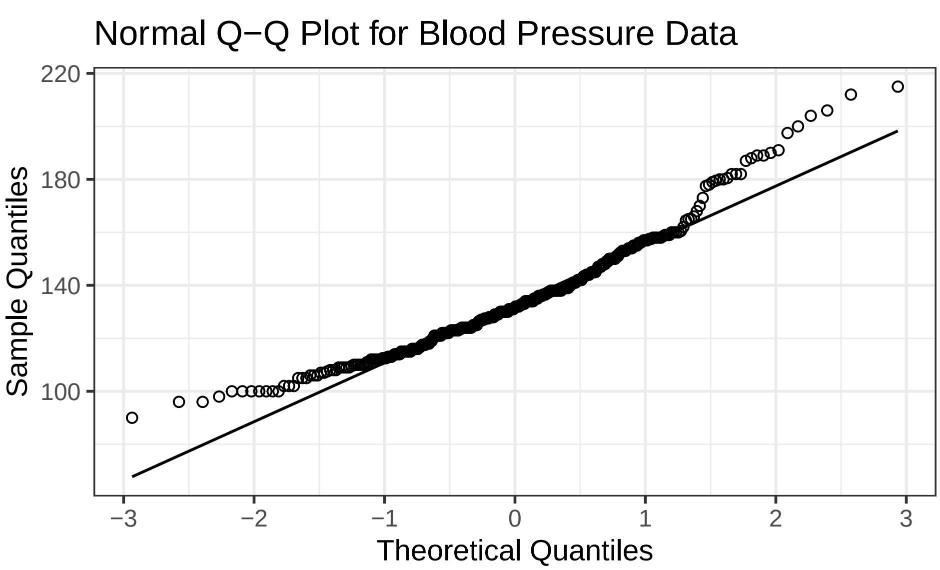


Figure 2: Q-Q Plot



Equations:

Equation 1: Pooled Sample Variance:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Equation 2: Observed T Statistic (pooled variance):

$$t_{obs} = \frac{\mu_1 - \mu_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Equation 3: Observed T Statistic (distinct variance):

$$t_{obs} = \frac{\mu_1 - \mu_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Equation 4: Satterthwaite Approximation:

$$\nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Appendix II: Part I Results

	Attribute Value
Data Mean	134.935000
Data Sample Variance	524.085226
Smoker Mean	128.066667
Smoker Variance	352.211712
Nonsmoker Mean	137.224444
Nonsmoker Variance	562.144712
Difference in Mean	-9.157778
Pooled Sample Varance	510.013699
T-Statistic Std Deviation (Pooled)	3.011131
T-Statistic Std Deviation (Unequal)	9.993684
CI Pooled Lower	-15.080000
CI Pooled Upper	-3.230000
CI Nonpooled Lower	-15.400000
CI Nonpooled Upper	-2.910000

Appendix III: Part II Results

Simulation Data

Test Case	μ_1	σ_1^2	n_1	μ_2	σ_2^2	n_2	# of Rejected Null (Unequal σ^2)	# of Rejected Null (Pooled σ^2)	Difference in Mean
1	0	1	10	5	1	10	1000	1000	-5
2	4	1	10	5	1	10	578	562	-1
3	5	1	10	5	1	10	54	39	0
4	6	1	10	5	1	10	527	546	1
5	10	1	10	5	1	10	1000	1000	5
6	0	4	10	5	1	10	1000	1000	-5
7	4	4	10	5	1	10	268	259	-1
8	5	4	10	5	1	10	43	55	0
9	6	4	10	5	1	10	271	265	1
10	10	4	10	5	1	10	1000	1000	5
11	0	9	10	5	1	10	992	995	-5
12	4	9	10	5	1	10	126	156	-1
13	5	9	10	5	1	10	53	50	0
14	6	9	10	5	1	10	133	200	1
15	10	9	10	5	1	10	992	999	5
16	0	12	10	5	1	10	967	985	-5
17	4	12	10	5	1	10	121	151	-1
18	5	12	10	5	1	10	39	47	0
19	6	12	10	5	1	10	125	136	1
20	10	12	10	5	1	10	979	988	5
21	0	1	30	5	1	10	1000	1000	-5
22	4	1	30	5	1	10	739	747	-1
23	5	1	30	5	1	10	57	47	0
24	6	1	30	5	1	10	733	759	1
25	10	1	30	5	1	10	1000	1000	5
26	0	4	30	5	1	10	1000	1000	-5
27	4	4	30	5	1	10	527	247	-1
28	5	4	30	5	1	10	46	3	0
29	6	4	30	5	1	10	511	245	1
30	10	4	30	5	1	10	1000	1000	5
31	0	9	30	5	1	10	1000	1000	-5
32	4	9	30	5	1	10	350	65	-1
33	5	9	30	5	1	10	49	2	0
34	6	9	30	5	1	10	326	71	1
35	10	9	30	5	1	10	1000	1000	5
36	0	12	30	5	1	10	1000	1000	-5
37	4	12	30	5	1	10	272	50	-1
38	5	12	30	5	1	10	43	3	0
39	6	12	30	5	1	10	290	46	1
40	10	12	30	5	1	10	1000	1000	5
41	0	1	70	5	1	10	1000	1000	-5
42	4	1	70	5	1	10	772	840	-1
43	5	1	70	5	1	10	50	57	0
44	6	1	70	5	1	10	780	838	1
45	10	1	70	5	1	10	1000	1000	5
46	0	4	70	5	1	10	1000	1000	-5
47	4	4	70	5	1	10	662	273	-1
48	5	4	70	5	1	10	53	2	0
49	6	4	70	5	1	10	676	227	1
50	10	4	70	5	1	10	1000	1000	5
51	0	9	70	5	1	10	1000	1000	-5

Test Case	μ_1	σ_1^2	n_1	μ_2	σ_2^2	n_2	# of Rejected Null (Unequal σ^2)	# of Rejected Null (Pooled σ^2)	Difference in Mean
52	4	9	70	5	1	10	531	30	-1
53	5	9	70	5	1	10	50	0	0
54	6	9	70	5	1	10	527	44	1
55	10	9	70	5	1	10	1000	1000	5
56	0	12	70	5	1	10	1000	1000	-5
57	4	12	70	5	1	10	457	19	-1
58	5	12	70	5	1	10	45	0	0
59	6	12	70	5	1	10	446	16	1
60	10	12	70	5	1	10	1000	1000	5
61	0	1	100	5	1	10	1000	1000	-5
62	4	1	100	5	1	10	797	862	-1
63	5	1	100	5	1	10	50	50	0
64	6	1	100	5	1	10	783	842	1
65	10	1	100	5	1	10	1000	1000	5
66	0	4	100	5	1	10	1000	1000	-5
67	4	4	100	5	1	10	700	261	-1
68	5	4	100	5	1	10	52	2	0
69	6	4	100	5	1	10	693	251	1
70	10	4	100	5	1	10	1000	1000	5
71	0	9	100	5	1	10	1000	1000	-5
72	4	9	100	5	1	10	612	26	-1
73	5	9	100	5	1	10	36	0	0
74	6	9	100	5	1	10	613	11	1
75	10	9	100	5	1	10	1000	1000	5
76	0	12	100	5	1	10	1000	1000	-5
77	4	12	100	5	1	10	569	13	-1
78	5	12	100	5	1	10	52	0	0
79	6	12	100	5	1	10	556	3	1
80	10	12	100	5	1	10	1000	1000	5
81	0	1	10	5	1	30	1000	1000	-5
82	4	1	10	5	1	30	711	786	-1
83	5	1	10	5	1	30	36	49	0
84	6	1	10	5	1	30	729	767	1
85	10	1	10	5	1	30	1000	1000	5
86	0	4	10	5	1	30	1000	1000	-5
87	4	4	10	5	1	30	261	529	-1
88	5	4	10	5	1	30	51	151	0
89	6	4	10	5	1	30	281	524	1
90	10	4	10	5	1	30	1000	1000	5
91	0	9	10	5	1	30	996	1000	-5
92	4	9	10	5	1	30	138	398	-1
93	5	9	10	5	1	30	47	240	0
94	6	9	10	5	1	30	155	437	1
95	10	9	10	5	1	30	998	1000	5
96	0	12	10	5	1	30	978	1000	-5
97	4	12	10	5	1	30	118	385	-1
98	5	12	10	5	1	30	49	225	0
99	6	12	10	5	1	30	128	373	1
100	10	12	10	5	1	30	977	1000	5
101	0	1	30	5	1	30	1000	1000	-5
102	4	1	30	5	1	30	978	983	-1
103	5	1	30	5	1	30	52	50	0
104	6	1	30	5	1	30	967	963	1
105	10	1	30	5	1	30	1000	1000	5
106	0	4	30	5	1	30	1000	1000	-5
107	4	4	30	5	1	30	666	662	-1

Test Case	μ_1	σ_1^2	n_1	μ_2	σ_2^2	n_2	# of Rejected Null (Unequal σ^2)	# of Rejected Null (Pooled σ^2)	Difference in Mean
108	5	4	30	5	1	30	53	43	0
109	6	4	30	5	1	30	676	680	1
110	10	4	30	5	1	30	1000	1000	5
111	0	9	30	5	1	30	1000	1000	-5
112	4	9	30	5	1	30	368	422	-1
113	5	9	30	5	1	30	44	59	0
114	6	9	30	5	1	30	371	424	1
115	10	9	30	5	1	30	1000	1000	5
116	0	12	30	5	1	30	1000	1000	-5
117	4	12	30	5	1	30	305	313	-1
118	5	12	30	5	1	30	56	58	0
119	6	12	30	5	1	30	313	326	1
120	10	12	30	5	1	30	1000	1000	5
121	0	1	70	5	1	30	1000	1000	-5
122	4	1	70	5	1	30	991	994	-1
123	5	1	70	5	1	30	53	43	0
124	6	1	70	5	1	30	999	996	1
125	10	1	70	5	1	30	1000	1000	5
126	0	4	70	5	1	30	1000	1000	-5
127	4	4	70	5	1	30	906	775	-1
128	5	4	70	5	1	30	40	12	0
129	6	4	70	5	1	30	894	768	1
130	10	4	70	5	1	30	1000	1000	5
131	0	9	70	5	1	30	1000	1000	-5
132	4	9	70	5	1	30	686	379	-1
133	5	9	70	5	1	30	51	6	0
134	6	9	70	5	1	30	696	437	1
135	10	9	70	5	1	30	1000	1000	5
136	0	12	70	5	1	30	1000	1000	-5
137	4	12	70	5	1	30	586	287	-1
138	5	12	70	5	1	30	52	5	0
139	6	12	70	5	1	30	584	261	1
140	10	12	70	5	1	30	1000	1000	5
141	0	1	100	5	1	30	1000	1000	-5
142	4	1	100	5	1	30	997	1000	-1
143	5	1	100	5	1	30	51	58	0
144	6	1	100	5	1	30	996	996	1
145	10	1	100	5	1	30	1000	1000	5
146	0	4	100	5	1	30	1000	1000	-5
147	4	4	100	5	1	30	960	809	-1
148	5	4	100	5	1	30	43	6	0
149	6	4	100	5	1	30	953	823	1
150	10	4	100	5	1	30	1000	1000	5
151	0	9	100	5	1	30	1000	1000	-5
152	4	9	100	5	1	30	817	380	-1
153	5	9	100	5	1	30	43	2	0
154	6	9	100	5	1	30	792	386	1
155	10	9	100	5	1	30	1000	1000	5
156	0	12	100	5	1	30	1000	1000	-5
157	4	12	100	5	1	30	694	245	-1
158	5	12	100	5	1	30	47	1	0
159	6	12	100	5	1	30	739	268	1
160	10	12	100	5	1	30	1000	1000	5
161	0	1	10	5	1	70	1000	1000	-5
162	4	1	10	5	1	70	761	841	-1
163	5	1	10	5	1	70	44	40	0

Test Case	μ_1	σ_1^2	n_1	μ_2	σ_2^2	n_2	# of Rejected Null (Unequal σ^2)	# of Rejected Null (Pooled σ^2)	Difference in Mean
164	6	1	10	5	1	70	771	844	1
165	10	1	10	5	1	70	1000	1000	5
166	0	4	10	5	1	70	1000	1000	-5
167	4	4	10	5	1	70	280	647	-1
168	5	4	10	5	1	70	51	235	0
169	6	4	10	5	1	70	287	637	1
170	10	4	10	5	1	70	1000	1000	5
171	0	9	10	5	1	70	995	1000	-5
172	4	9	10	5	1	70	153	581	-1
173	5	9	10	5	1	70	68	312	0
174	6	9	10	5	1	70	157	543	1
175	10	9	10	5	1	70	999	1000	5
176	0	12	10	5	1	70	980	1000	-5
177	4	12	10	5	1	70	122	532	-1
178	5	12	10	5	1	70	38	372	0
179	6	12	10	5	1	70	122	545	1
180	10	12	10	5	1	70	977	1000	5
181	0	1	30	5	1	70	1000	1000	-5
182	4	1	30	5	1	70	992	997	-1
183	5	1	30	5	1	70	50	53	0
184	6	1	30	5	1	70	991	995	1
185	10	1	30	5	1	70	1000	1000	5
186	0	4	30	5	1	70	1000	1000	-5
187	4	4	30	5	1	70	706	846	-1
188	5	4	30	5	1	70	54	112	0
189	6	4	30	5	1	70	694	859	1
190	10	4	30	5	1	70	1000	1000	5
191	0	9	30	5	1	70	1000	1000	-5
192	4	9	30	5	1	70	437	643	-1
193	5	9	30	5	1	70	50	154	0
194	6	9	30	5	1	70	383	628	1
195	10	9	30	5	1	70	1000	1000	5
196	0	12	30	5	1	70	1000	1000	-5
197	4	12	30	5	1	70	319	547	-1
198	5	12	30	5	1	70	42	171	0
199	6	12	30	5	1	70	325	564	1
200	10	12	30	5	1	70	1000	1000	5
201	0	1	70	5	1	70	1000	1000	-5
202	4	1	70	5	1	70	1000	1000	-1
203	5	1	70	5	1	70	58	59	0
204	6	1	70	5	1	70	1000	1000	1
205	10	1	70	5	1	70	1000	1000	5
206	0	4	70	5	1	70	1000	1000	-5
207	4	4	70	5	1	70	971	957	-1
208	5	4	70	5	1	70	64	47	0
209	6	4	70	5	1	70	957	977	1
210	10	4	70	5	1	70	1000	1000	5
211	0	9	70	5	1	70	1000	1000	-5
212	4	9	70	5	1	70	730	753	-1
213	5	9	70	5	1	70	43	53	0
214	6	9	70	5	1	70	761	727	1
215	10	9	70	5	1	70	1000	1000	5
216	0	12	70	5	1	70	1000	1000	-5
217	4	12	70	5	1	70	633	621	-1
218	5	12	70	5	1	70	51	61	0
219	6	12	70	5	1	70	607	651	1

Test Case	μ_1	σ_1^2	n_1	μ_2	σ_2^2	n_2	# of Rejected Null (Unequal σ^2)	# of Rejected Null (Pooled σ^2)	Difference in Mean
220	10	12	70	5	1	70	1000	1000	5
221	0	1	100	5	1	70	1000	1000	-5
222	4	1	100	5	1	70	1000	1000	-1
223	5	1	100	5	1	70	58	57	0
224	6	1	100	5	1	70	1000	1000	1
225	10	1	100	5	1	70	1000	1000	5
226	0	4	100	5	1	70	1000	1000	-5
227	4	4	100	5	1	70	993	980	-1
228	5	4	100	5	1	70	41	23	0
229	6	4	100	5	1	70	989	983	1
230	10	4	100	5	1	70	1000	1000	5
231	0	9	100	5	1	70	1000	1000	-5
232	4	9	100	5	1	70	870	829	-1
233	5	9	100	5	1	70	54	22	0
234	6	9	100	5	1	70	864	801	1
235	10	9	100	5	1	70	1000	1000	5
236	0	12	100	5	1	70	1000	1000	-5
237	4	12	100	5	1	70	794	663	-1
238	5	12	100	5	1	70	58	17	0
239	6	12	100	5	1	70	778	675	1
240	10	12	100	5	1	70	1000	1000	5
241	0	1	10	5	1	100	1000	1000	-5
242	4	1	10	5	1	100	801	832	-1
243	5	1	10	5	1	100	48	58	0
244	6	1	10	5	1	100	799	875	1
245	10	1	10	5	1	100	1000	1000	5
246	0	4	10	5	1	100	1000	1000	-5
247	4	4	10	5	1	100	267	631	-1
248	5	4	10	5	1	100	51	238	0
249	6	4	10	5	1	100	284	649	1
250	10	4	10	5	1	100	1000	1000	5
251	0	9	10	5	1	100	1000	1000	-5
252	4	9	10	5	1	100	144	572	-1
253	5	9	10	5	1	100	59	408	0
254	6	9	10	5	1	100	154	619	1
255	10	9	10	5	1	100	996	1000	5
256	0	12	10	5	1	100	977	1000	-5
257	4	12	10	5	1	100	142	573	-1
258	5	12	10	5	1	100	49	414	0
259	6	12	10	5	1	100	113	572	1
260	10	12	10	5	1	100	979	1000	5
261	0	1	30	5	1	100	1000	1000	-5
262	4	1	30	5	1	100	998	995	-1
263	5	1	30	5	1	100	43	58	0
264	6	1	30	5	1	100	997	997	1
265	10	1	30	5	1	100	1000	1000	5
266	0	4	30	5	1	100	1000	1000	-5
267	4	4	30	5	1	100	740	887	-1
268	5	4	30	5	1	100	49	174	0
269	6	4	30	5	1	100	722	897	1
270	10	4	30	5	1	100	1000	1000	5
271	0	9	30	5	1	100	1000	1000	-5
272	4	9	30	5	1	100	399	727	-1
273	5	9	30	5	1	100	53	234	0
274	6	9	30	5	1	100	420	726	1
275	10	9	30	5	1	100	1000	1000	5

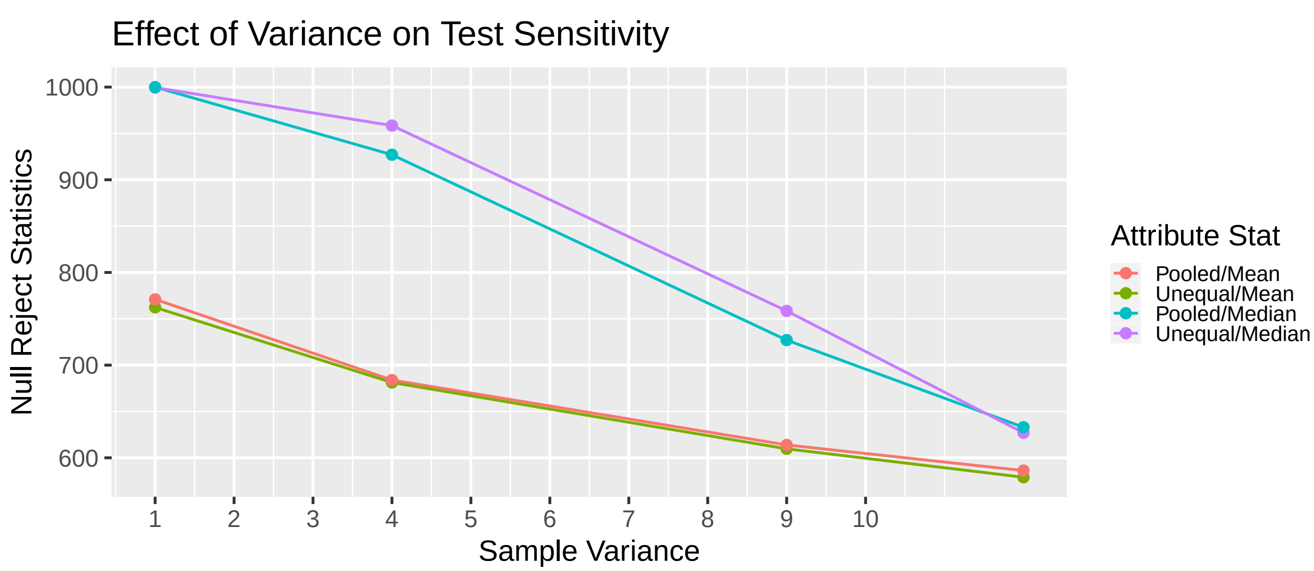
Test Case	μ_1	σ_1^2	n_1	μ_2	σ_2^2	n_2	# of Rejected Null (Unequal σ^2)	# of Rejected Null (Pooled σ^2)	Difference in Mean
276	0	12	30	5	1	100	1000	1000	-5
277	4	12	30	5	1	100	302	645	-1
278	5	12	30	5	1	100	57	245	0
279	6	12	30	5	1	100	326	605	1
280	10	12	30	5	1	100	1000	1000	5
281	0	1	70	5	1	100	1000	1000	-5
282	4	1	70	5	1	100	1000	1000	-1
283	5	1	70	5	1	100	36	53	0
284	6	1	70	5	1	100	1000	1000	1
285	10	1	70	5	1	100	1000	1000	5
286	0	4	70	5	1	100	1000	1000	-5
287	4	4	70	5	1	100	967	981	-1
288	5	4	70	5	1	100	45	74	0
289	6	4	70	5	1	100	966	974	1
290	10	4	70	5	1	100	1000	1000	5
291	0	9	70	5	1	100	1000	1000	-5
292	4	9	70	5	1	100	785	831	-1
293	5	9	70	5	1	100	44	78	0
294	6	9	70	5	1	100	756	841	1
295	10	9	70	5	1	100	1000	1000	5
296	0	12	70	5	1	100	1000	1000	-5
297	4	12	70	5	1	100	646	744	-1
298	5	12	70	5	1	100	49	91	0
299	6	12	70	5	1	100	621	766	1
300	10	12	70	5	1	100	1000	1000	5
301	0	1	100	5	1	100	1000	1000	-5
302	4	1	100	5	1	100	1000	1000	-1
303	5	1	100	5	1	100	57	53	0
304	6	1	100	5	1	100	1000	1000	1
305	10	1	100	5	1	100	1000	1000	5
306	0	4	100	5	1	100	1000	1000	-5
307	4	4	100	5	1	100	993	993	-1
308	5	4	100	5	1	100	46	47	0
309	6	4	100	5	1	100	995	993	1
310	10	4	100	5	1	100	1000	1000	5
311	0	9	100	5	1	100	1000	1000	-5
312	4	9	100	5	1	100	877	885	-1
313	5	9	100	5	1	100	69	46	0
314	6	9	100	5	1	100	864	878	1
315	10	9	100	5	1	100	1000	1000	5
316	0	12	100	5	1	100	1000	1000	-5
317	4	12	100	5	1	100	794	802	-1
318	5	12	100	5	1	100	47	43	0
319	6	12	100	5	1	100	787	774	1
320	10	12	100	5	1	100	1000	1000	5

Appendix IV: Part II Summary

Item A

Δ Means	Average # Rejects (Unequal)	Median # Rejects (Unequal)	Average # Rejects (Pooled)	Median # Rejects (Pooled)
-5	998.20312	1000.0	999.68750	1000.0
-1	623.10938	690.0	616.29688	654.5
0	49.46875	50.0	85.42188	51.5
1	621.14062	693.5	617.56250	663.0
5	998.39062	1000.0	999.79688	1000.0

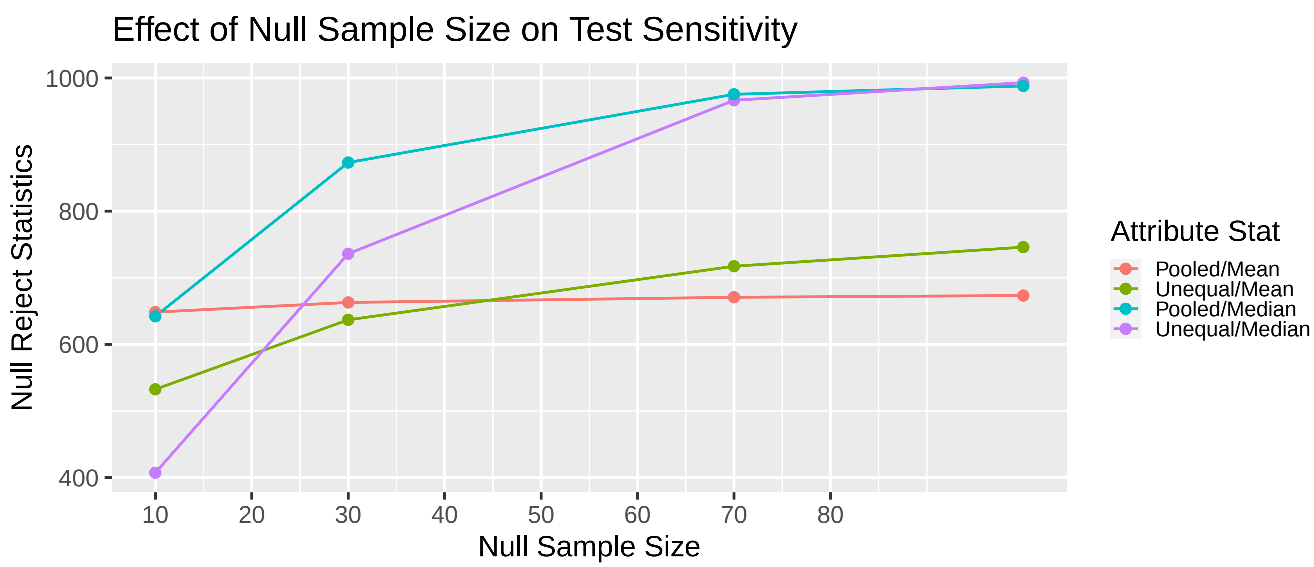
Item B



Sample Variance	Average # Rejects (Unequal)	Median # Rejects (Unequal)	Average # Rejects (Pooled)	Median # Rejects (Pooled)
1	762.300	999.5	771.0125	1000
4	681.225	958.5	683.9125	927
9	609.700	758.5	613.8750	727

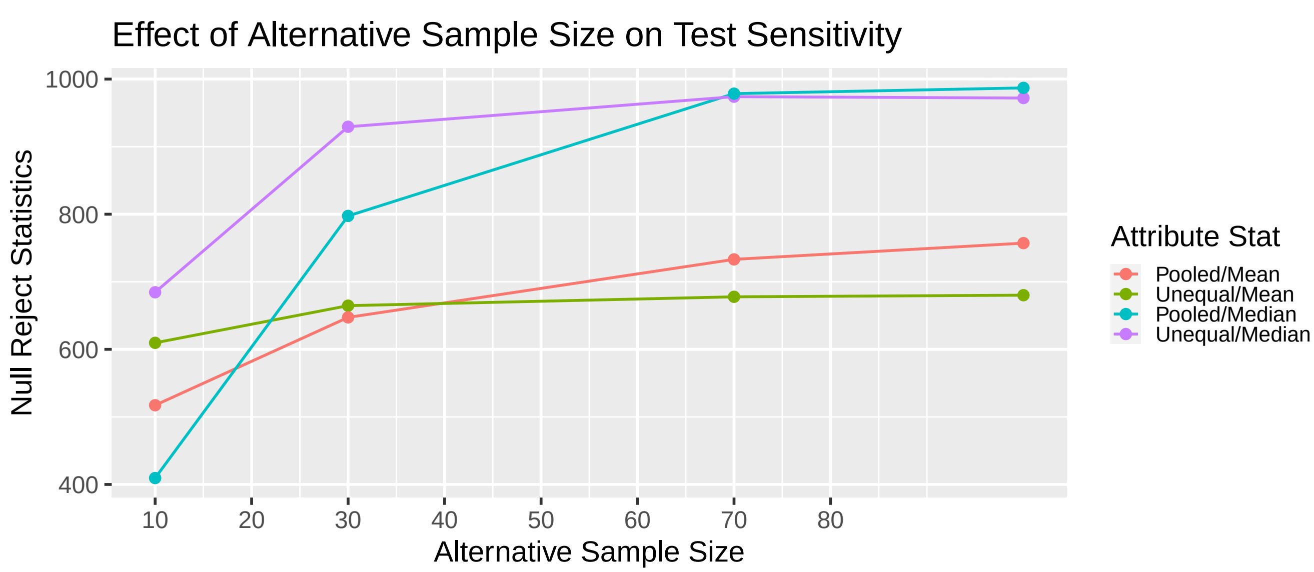
Sample Variance	Average # Rejects (Unequal)	Median # Rejects (Unequal)	Average # Rejects (Pooled)	Median # Rejects (Pooled)
12	579.025	627.0	586.2125	633

Item C



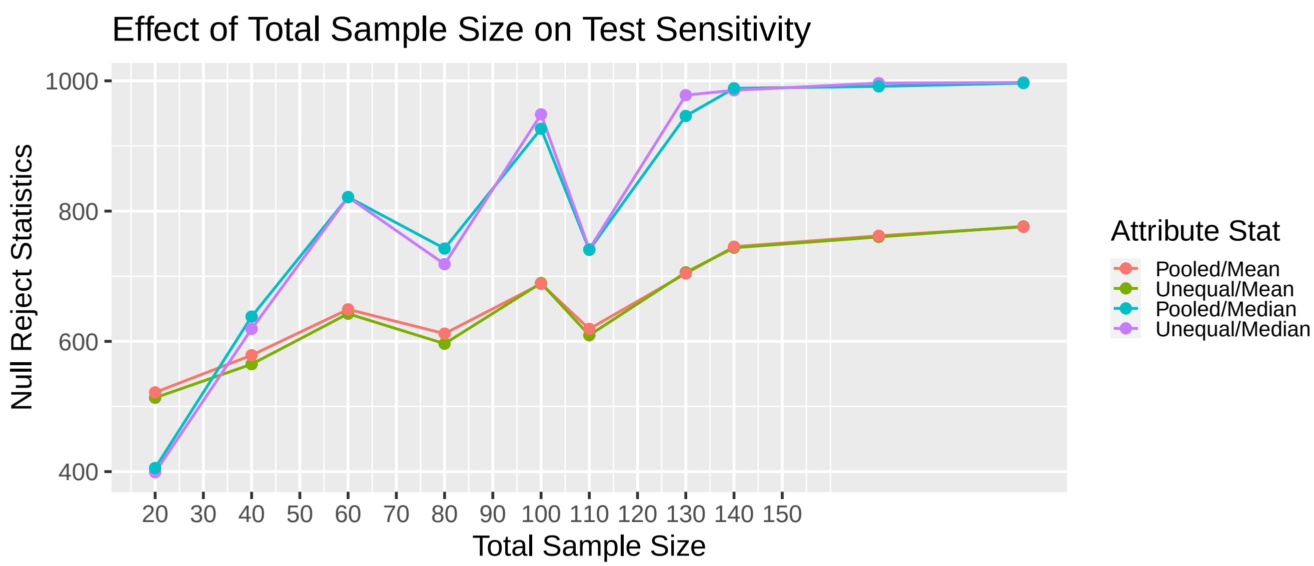
Null Sample Size	Average # Rejects (Unequal)	Median # Rejects (Unequal)	Average # Rejects (Pooled)	Median # Rejects (Pooled)
10	532.3625	407.0	648.3375	642.0
30	636.7625	736.0	662.8375	873.0
70	717.2125	966.5	670.6000	975.5
100	745.9125	993.0	673.2375	988.0

Item D



Alternative Sample Size	Average # Rejects (Unequal)	Median # Rejects (Unequal)	Average # Rejects (Pooled)	Median # Rejects (Pooled)
10	609.6625	684.5	517.3125	409.5
30	664.6500	929.5	647.3000	797.5
70	677.7750	974.0	733.1750	978.5
100	680.1625	972.0	757.2250	987.0

Item E



Combined Sample Size	Average # Rejects (Unequal)	Median # Rejects (Unequal)	Average # Rejects (Pooled)	Median # Rejects (Pooled)
20	513.400	399.0	521.650	405.5
40	564.900	619.0	578.725	638.0
60	642.450	821.5	649.150	821.5
80	596.350	718.5	611.875	742.5
100	689.525	948.5	688.300	926.5
110	609.400	741.5	619.050	740.5
130	705.950	978.0	704.100	946.0
140	743.750	985.5	745.300	988.5
170	760.350	996.5	762.075	991.5
200	776.450	997.5	775.700	996.5

Appendix V: Code

```
knitr::opts_chunk$set(echo = TRUE)

#Use required packages
library(tidyverse) #for plots and data manipulation
library(cowplot) #aligning plots
library(gridExtra)
library(scales)

df_data <- read_csv("framingham_data.csv") # Read in data
df_data$index <- seq(nrow(df_data)) # Add an index column

#df_data %>% summary # Summarize Data

# Split data into smoker and nonsmoker
df_smoker <- df_data %>% filter(currentSmoker == 1)
df_nonsmoker <- df_data %>% filter(currentSmoker == 0)

#Create a sample variance function to ensure proper calculation
sample_variance <- function(x, sampling = TRUE){
    if (sampling == TRUE){
        sum((x - mean(x))^2) / (length(x) - 1)
    } else if(sampling == FALSE) {
        sum((x - mean(x))^2) / (length(x))
    }
}
#Create pooled sample variance function
f_pooled_variance <- function(x, y){
    ((length(x) - 1) * sample_variance(x) +
     (length(y) - 1) * sample_variance(y)) /
    (length(x) + length(y) - 2)
}

# Skewness function
skew_function <- function(x){
    mean((x - mean(x))^3) / sqrt(sample_variance(x))^3
}

# kurtosis function
kurt_function <- function(x){
    mean((x - mean(x))^4) / sqrt(sample_variance(x))^4
}

# Create a Satterthawaite Approximation Function

satterth <- function(s1, s2, n1, n2){
    term1 <- s1/n1
    term2 <- s2/n2
    nu <- (term1 + term2)^2 / ((term1^2/(n1 - 1)) + (term2^2/(n2 - 1)))
    return(floor(nu))
}


#Plot and compare split data

#options(repr.plot.width = 6, repr.plot.height = 4, repr.plot.res = 150)
```

```r
plot_colors <- c("#001427","#708d81","#f4d58d","#bf0603","#8d0801")
y_limits <- c(0, 0.0225)

total_data <- ggplot(df_data) + geom_density(aes(sysBP),
                                             fill = plot_colors[1],
                                             alpha = 0.6) +
                       ylim(y_limits) + ylab("Density") + xlab("")

sep_data <- ggplot() + geom_density(data = df_smoker, aes(sysBP),
                                    fill = plot_colors[3], alpha = 0.6) +
                       geom_density(data = df_nonsmoker, aes(sysBP),
                                    fill = plot_colors[5], alpha = 0.6) +
                       ylim(y_limits) + ylab("") + xlab("Systolic Blood Pressure")

plot_3 <- ggplot() + geom_density(data = df_smoker, aes(sysBP,
                                    fill = plot_colors[3]), alpha = 0.5) +
                       geom_density(data = df_data, aes(sysBP,
                                    fill = plot_colors[1]), alpha = 0.5) +
                       geom_density(data = df_nonsmoker, aes(sysBP,
                                    fill = plot_colors[5]), alpha = 0.5) +
                       ylim(y_limits) + ylab("") + xlab("") +
                       scale_fill_manual("",
                                         values = plot_colors[c(1, 5, 3)],
                                         labels = c("Total", "Non Smoker", "Smoker")) +
                       theme(legend.position = c(0.8, 0.9),
                             legend.text = element_text(size = 6),
                             legend.key.height = unit(0.25, 'cm'),
                             legend.key.width = unit(0.25, 'cm'))

#plot_grid(total_data, sep_data, plot_3, align = 'vh',
          #hjust = -1,nrow = 2, ncol = 2)

data_kurtosis <- kurt_function(df_data$sysBP)
data_skew <- skew_function(df_data$sysBP)
data_IQR <- as.numeric(quantile(df_data$sysBP, probs = 0.75)) -
       as.numeric(quantile(df_data$sysBP, probs = 0.25))
data_MAD <- median(abs(df_data$sysBP - median(df_data$sysBP)))
data_samVar <- sample_variance(df_data$sysBP)

eIQR <- data_IQR / 1.35
eMAD <- data_MAD / 0.675

# Q-Q Plot

data_qqplot <-
ggplot(df_data, aes(sample = sysBP)) +
stat_qq(shape = 1) + stat_qq_line() +
ggtitle("Normal Q-Q Plot for Blood Pressure Data") +
xlab("Theoretical Quantiles") +
ylab("Sample Quantiles")

# Common values for analysis

alpha <- 0.05

mu_smoker <- mean(df_smoker$sysBP)
var_smoker <- sample_variance(df_smoker$sysBP)
```

```r
n_smoker <- length(df_smoker$sysBP)

mu_nonsmoker <- mean(df_nonsmoker$sysBP)
var_nonsmoker <- sample_variance(df_nonsmoker$sysBP)
n_nonsmoker <- length(df_nonsmoker$sysBP)

# Two Sample T-test - Pooled Sample Variance - P-value

dof_1 <- (n_smoker + n_nonsmoker - 2)

p_sample_var_1 <- f_pooled_variance(df_smoker$sysBP,
                                    df_nonsmoker$sysBP)
p_sample_var_w <- sqrt(p_sample_var_1/n_smoker + p_sample_var_1/n_nonsmoker)

t_obs_1 <- (mu_smoker - mu_nonsmoker) / (p_sample_var_w )

t_stat_1 <- qt(alpha / 2, dof_1)

p_value_obs_1 <- dt(t_obs_1, dof_1)

#Two Sample T-test - Difference Variance Sample Variance - P-value

dof_2 <- satterth(var_smoker, var_nonsmoker, n_smoker, n_nonsmoker)

np_sample_var_2 <- (var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)

t_obs_2 <- (mu_smoker - mu_nonsmoker) / (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker))

t_stat_2 <- qt(alpha / 2, dof_2)

p_value_obs_2 <- dt(t_obs_2, dof_2)

# Confidence Limits

diff_mu <- mu_smoker - mu_nonsmoker

#Pooled Sample variance

CL_pooled <- t_stat_1 * p_sample_var_w

#Non pooled Sample variance

CL_nonpooled <- t_stat_2 * (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker))

CI_pooled <- round(c(diff_mu + CL_pooled, diff_mu - CL_pooled), 2)

CI_nonpooled <-round(c(diff_mu + CL_nonpooled, diff_mu - CL_nonpooled), 2)

#Power Calculation assuming delta means is the true delta

cv_lo_p <- qnorm(alpha / 2, 0, sqrt(p_sample_var_1/n_smoker +
                                      p_sample_var_1/n_nonsmoker))
cv_hi_p <- qnorm(1 - alpha / 2, 0, sqrt(p_sample_var_1/n_smoker +
                                          p_sample_var_1/n_nonsmoker))

power1 <- pnorm(cv_lo_p, (mu_smoker - mu_nonsmoker),
                sqrt(p_sample_var_1/n_smoker + p_sample_var_1/n_nonsmoker))
power2 <- 1 - pnorm(cv_hi_p, (mu_smoker - mu_nonsmoker),
```

```r
                    sqrt(p_sample_var_1/n_smoker + p_sample_var_1/n_nonsmoker))

power_pooled <- sum(power1, power2)

cv_lo_non <- qnorm(alpha / 2, 0,
               (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)))
cv_hi_non <- qnorm(1 - alpha / 2, 0,
               (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)))

power1 <- pnorm(cv_lo_non, (mu_smoker - mu_nonsmoker),
                (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)))
power2 <- 1 - pnorm(cv_hi_non, (mu_smoker - mu_nonsmoker),
                    (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)))

power_nonpooled <- sum(power1, power2)


#Part II
#Introduction

#options(repr.plot.width = 12, repr.plot.height = 3, repr.plot.res = 150)
set.seed(100)

null_mean <- 3
alt_means <- c(0, 3, 5)
plot_list <- list()

#plot_colors <- c("#072ac8","#1e96fc","#a2d6f9","#fcf300","#ffc600")

for(i in 1:length(alt_means)){

    sim1 <- rnorm(5000, null_mean, sqrt(1))
    sim2 <- rnorm(5000, alt_means[i], sqrt(1))

    alpha1 <- qnorm(0.025, null_mean, sqrt(1))
    alpha2 <- qnorm(0.975, null_mean, sqrt(1))

    df_set <- tibble("H0" = sim1, "HA" = sim2)

    title_string <- sprintf("Difference in Means %i", (alt_means[i] - null_mean))

    plot_list[[i]]  <-
    ggplot(data = df_set) + geom_density(aes(H0), alpha = 0.5, fill = plot_colors[5]) +
                            geom_area(
                                aes(x = stage(H0, after_scale = oob_censor(x, c(-Inf, alpha1)
                                                                             )
                                                 )
                                   ),
                                stat = "density", fill = plot_colors[1]
                                 ) +
                            geom_area(
                                aes(x = stage(H0, after_scale = oob_censor(x, c(alpha2, Inf)
                                                                             )
                                                 )
                                   ),
                                stat = "density", fill = plot_colors[1]
                                     ) +
                            geom_density(aes(HA), alpha = 0.5) +
```

```r
                            geom_area(
                                aes(x = stage(HA, after_scale = oob_censor(x, c(alpha1, alpha2)
                                                                          )
                                             )
                                   ),
                                stat = "density", fill = plot_colors[2], alpha = 0.5
                                     ) +
                            xlim(-2, 8) + xlab("") + ylab("") + ggtitle(title_string) +
                            theme(text = element_text(size = 8))

                                }

do.call(grid.arrange, c(plot_list, ncol = 3, heights = 0.5))
#Part II
set.seed(1)

alpha <- 0.05

test_function <- function (x, y, pooled = FALSE){

    mu_1 <- mean(x)
    var_1 <- sample_variance(x, sampling = TRUE)

    mu_2 <- mean(y)
    var_2 <- sample_variance(y, sampling = TRUE)

    #Calculate the pooled sample variance
    pooled_sample <- ((length(x) - 1) * var_1 +
                          (length(y) - 1) * var_2) / (length(x) + length(y) - 2)

    #calculate the observed t statistic
    if (pooled == TRUE){

        cal_sigma <- (sqrt(pooled_sample/length(x) + pooled_sample/length(y)))

        ttest <- (mu_1 - mu_2) / cal_sigma
        dof <- length(x) + length(y) - 2 #Determine degrees of freedom

        } else {

        cal_sigma <- (sqrt(var_1/length(x) + var_2/length(y)))

        ttest <- (mu_1 - mu_2) / cal_sigma
        dof <- satterth(var_1, var_2, length(x), length(y))
    }

    # Determine whether or not the null hypothesis
    # can be rejected (1 = rejected, 0 = not rejected)
    verdict <- !between(ttest, qt(alpha / 2, dof), qt(1 - alpha / 2, dof))

    #Power calculation assuming calculated difference in means is Ha
    cv_lo <- qnorm(alpha / 2, 0, cal_sigma)
    cv_hi <- qnorm(1 - alpha / 2, 0, cal_sigma)

    power1 <- pnorm(cv_lo, (mu_1 - mu_2), cal_sigma)
    power2 <- 1 - pnorm(cv_hi, (mu_1 - mu_2), cal_sigma)

    power <- sum(power1, power2)
```

```r
    #Return calculated values
    return(c(mu_1, var_1, mu_2, var_2, ttest, cal_sigma, dof, verdict, power))
}



mu1 <- c(0, 4, 5, 6, 10)
var1 <- c(1, 4, 9, 12)
n1 <- c(10, 30, 70, 100)

mu2 <- 5
var2 <- 1
n2 <- c(10, 30, 70, 100)

sim_test <- function(x_mu, x_var, x_n, y_mu, y_var, y_n, pooled){

    sim_data_results <- matrix(rep(0, 9), ncol = 9)

    for (i in 1:1000){

        sim_set1 <- rnorm(x_n, x_mu, sqrt(x_var))
        sim_set2 <- rnorm(y_n, y_mu, sqrt(y_var))

        sim_data_results <- rbind(sim_data_results,
                                  test_function(sim_set1, sim_set2, pooled))

        #print(sim_data_results)
    }

    df_sim_data <- data.frame(sim_data_results[2 : nrow(sim_data_results),])
    colnames(df_sim_data) = c("Null Mean", "Null Variance", "Alternate Mean",
                              "Alternate Variance", "T statistic",
                              "Calculated Variance", "DoF", "Null Reject",
                              "Power")
    return(df_sim_data)
}

# HA: mean = 5, var = 1
df_combo <- expand.grid(mu1, var1, n1, mu2, var2, n2)
df_combo2 <- tibble(cbind(1:nrow(df_combo), df_combo,
                          matrix(rep(0, 2 * nrow(df_combo)), ncol = 2)))
colnames(df_combo2) <- c("Test_Case", "mu1", "var1", "n1", "mu2", "var2",
                         "n2", "Test_Results_up", "Test_Results_po")

test_results <- list()
test_results2 <- list()

for (i in 1:nrow(df_combo)){
    test_results[[i]] <- do.call(sim_test,
                                 as.list(as.numeric(c(df_combo[i,],
                                                      pooled = FALSE))))
    test_results2[[i]] <- do.call(sim_test,
                                  as.list(as.numeric(c(df_combo[i,],
                                                       pooled = TRUE))))
    df_combo2[i, 8] <- sum(as.data.frame(test_results[i])[,8])
    df_combo2[i, 9] <- sum(as.data.frame(test_results2[i])[,8])
    }
```

```r
df_combo2 <- df_combo2 %>% mutate(diff = mu1 - mu2)
#df_combo2 %>%  head()

# Some summary statitics to look for relationships in the attributes
av_med_stats_by_diff <-
    df_combo2 %>% group_by(diff) %>% summarise(avg_up = mean(Test_Results_up),
                                              med_up = median(Test_Results_up),
                                              avg_po = mean(Test_Results_po),
                                              med_po = median(Test_Results_po))


av_med_stats_by_var <-
    df_combo2 %>% group_by(var1) %>% summarise(avg_up = mean(Test_Results_up),
                                              med_up = median(Test_Results_up),
                                              avg_po = mean(Test_Results_po),
                                              med_po = median(Test_Results_po))

av_med_stats_by_nulln <-
    df_combo2 %>% group_by(n1) %>% summarise(avg_up = mean(Test_Results_up),
                                            med_up = median(Test_Results_up),
                                            avg_po = mean(Test_Results_po),
                                            med_po = median(Test_Results_po))

av_med_stats_by_altn <-
    df_combo2 %>% group_by(n2) %>% summarise(avg_up = mean(Test_Results_up),
                                            med_up = median(Test_Results_up),
                                            avg_po = mean(Test_Results_po),
                                            med_po = median(Test_Results_po))

av_med_stats_by_comn <-
    df_combo2 %>% mutate(comn = n1 + n2) %>% group_by(comn) %>%
    summarise(avg_up = mean(Test_Results_up),
              med_up = median(Test_Results_up),
              avg_po = mean(Test_Results_po),
              med_po = median(Test_Results_po))

av_med_stats_by_diff %>% gather(key = "stat", value = "calc", avg_up:med_po) %>%
                         ggplot() + geom_line(aes(x = diff, y = calc, color = stat)) +
                                    geom_point(aes(x = diff, y = calc, color = stat)) +
                                    xlab("Difference in Means") +
                                    ylab("Null Reject Statistics") +
                                    scale_x_continuous(breaks = seq(-5, 5, 1)) +
                                    ggtitle("Effect of" ~ Delta ~ "Means on Test Sensitivity") +
                                    scale_color_discrete(name = "Attribute Stat",
                                                         labels = c("Pooled/Mean",
                                                                    "Unequal/Mean",
                                                                    "Pooled/Median",
                                                                    "Unequal/Median")) +
                                    theme(legend.position = c(0.85, 0.3),
                                          title = element_text(size = 8),
                                          legend.text = element_text(size = 6),
                                          legend.key.height = unit(0.25, 'cm'),
                                          legend.key.width = unit(0.25, 'cm'))

pow_tab_9a <- test_results[[9]] %>% pull(Power) %>% summary() %>% unname() %>% matrix(ncol = 1)
pow_tab_9b <- test_results2[[9]] %>% pull(Power) %>% summary() %>% unname() %>% matrix(ncol = 1)
pow_tab_129a <- test_results[[129]] %>% pull(Power) %>% summary() %>% unname() %>% matrix(ncol = 1)
```

```r
pow_tab_129b <- test_results2[[129]] %>% pull(Power) %>% summary() %>% unname() %>% matrix(ncol = 1)

d <- cbind(pow_tab_9a, pow_tab_9b, pow_tab_129a, pow_tab_129b)[2:5,]

colnames(d) = c("Test #9 (pooled)", "Test #9 (unequal)", "Test #129 (pooled)", "Test #129 (unequal)")
rownames(d) = c("1st Quartile", "Median", "Mean", "3rd Quartile")

knitr::kable(d)

# Plotting
options(repr.plot.width = 6, repr.plot.height = 4, repr.plot.res = 150)

plot_grid(total_data, sep_data, plot_3, align = 'vh',
          hjust = -1,nrow = 1, ncol = 3, labels = c("A", "B", "C"))
data_qqplot + theme_bw()
d1 <- matrix(c(mean(df_data$sysBP), sample_variance(df_data$sysBP),
               mu_smoker, var_smoker,
               mu_nonsmoker, var_nonsmoker,
               diff_mu, p_sample_var_1, p_sample_var_w,
               np_sample_var_2, CI_pooled, CI_nonpooled), ncol = 1)


rownames(d1) <- c("Data Mean", "Data Sample Variance", "Smoker Mean",
                  "Smoker Variance",
                  "Nonsmoker Mean", "Nonsmoker Variance",
                  "Difference in Mean", "Pooled Sample Varance",
                  "T-Statistic Std Deviation (Pooled)",
                  "T-Statistic Std Deviation (Unequal)",
                  "CI Pooled Lower", "CI Pooled Upper",
                  "CI Nonpooled Lower", "CI Nonpooled Upper")

d1 %>% knitr::kable(col.names = c("Attribute Value"))
knitr::kable(df_combo2, col.names = c("Test Case",
                                      "$\\mu_1$",
                                      "$\\sigma_1^2$",
                                      "$n_1$",
                                      "$\\mu_2$",
                                      "$\\sigma_2^2$",
                                      "$n_2$",
                                      "# of Rejected Null (Unequal $\\sigma^{2}$)",
                                      "# of Rejected Null (Pooled $\\sigma^{2}$)",
                                      "Difference in Mean"),
             escape = FALSE)
av_med_stats_by_diff %>% gather(key = "stat", value = "calc", avg_up:med_po) %>%
                         ggplot() + geom_line(aes(x = diff, y = calc, color = stat)) +
                                    geom_point(aes(x = diff, y = calc, color = stat)) +
                                    xlab("Difference in Means") +
                                    ylab("Null Reject Statistics") +
                                    ggtitle("Effect of" ~ Delta ~ "Means on Test Sensitivity") +
                                    scale_x_continuous(breaks = seq(-5, 5, 1)) +
                                    scale_color_discrete(name = "Attribute Stat",
                                                         labels = c("Pooled/Mean",
                                                                    "Unequal/Mean",
                                                                    "Pooled/Median",
                                                                    "Unequal/Median")
                                                         ) +
                                    theme(#legend.position = c(0.85, 0.3),
                                          legend.text = element_text(size = 8),
```

```r
                                          legend.key.height = unit(0.25, 'cm'),
                                          legend.key.width = unit(0.4, 'cm'))

knitr::kable(av_med_stats_by_diff, col.names = c("$\\Delta$ Means",
                                                 "Average # Rejects (Unequal)",
                                                 "Median # Rejects (Unequal)",
                                                 "Average # Rejects (Pooled)",
                                                 "Median # Rejects (Pooled)"),
             escape = FALSE)
av_med_stats_by_var %>% gather(key = "stat", value = "calc", avg_up:med_po) %>%
                         ggplot() + geom_line(aes(x = var1, y = calc, color = stat)) +
                                    geom_point(aes(x = var1, y = calc, color = stat)) +
                                    xlab("Sample Variance") +
                                    ylab("Null Reject Statistics") +
                                    ggtitle("Effect of Variance on Test Sensitivity") +
                                    scale_x_continuous(breaks = seq(-5, 10, 1)) +
                                    theme(#legend.position = c(0.85, 0.8),
                                          legend.text = element_text(size = 8),
                                          legend.key.height = unit(0.25, 'cm'),
                                          legend.key.width = unit(0.4, 'cm')) +
                                    scale_color_discrete(name = "Attribute Stat",
                                                         labels = c("Pooled/Mean",
                                                                    "Unequal/Mean",
                                                                    "Pooled/Median",
                                                                    "Unequal/Median")
                                                         )

knitr::kable(av_med_stats_by_var, col.names = c("Sample Variance",
                                                 "Average # Rejects (Unequal)",
                                                 "Median # Rejects (Unequal)",
                                                 "Average # Rejects (Pooled)",
                                                 "Median # Rejects (Pooled)"),
             escape = FALSE)

av_med_stats_by_nulln %>% gather(key = "stat", value = "calc", avg_up:med_po) %>%
                          ggplot() + geom_line(aes(x = n1, y = calc, color = stat)) +
                                     geom_point(aes(x = n1, y = calc, color = stat)) +
                                     xlab("Null Sample Size") +
                                     ylab("Null Reject Statistics") +
                                     ggtitle("Effect of Null Sample Size on Test Sensitivity") +
                                     scale_x_continuous(breaks = seq(10, 80, 10)) +
                                     theme(#legend.position = c(0.85, 0.65),
                                           legend.text = element_text(size = 8),
                                           legend.key.height = unit(0.25, 'cm'),
                                           legend.key.width = unit(0.4, 'cm')) +
                                     scale_color_discrete(name = "Attribute Stat",
                                                         labels = c("Pooled/Mean",
                                                                    "Unequal/Mean",
                                                                    "Pooled/Median",
                                                                    "Unequal/Median"))



knitr::kable(av_med_stats_by_nulln, col.names = c("Null Sample Size",
                                                 "Average # Rejects (Unequal)",
                                                 "Median # Rejects (Unequal)",
                                                 "Average # Rejects (Pooled)",
                                                 "Median # Rejects (Pooled)"),
```

```
             escape = FALSE)

av_med_stats_by_altn %>% gather(key = "stat", value = "calc", avg_up:med_po) %>%
                          ggplot() + geom_line(aes(x = n2, y = calc, color = stat)) +
                                     geom_point(aes(x = n2, y = calc, color = stat)) +
                                     xlab("Alternative Sample Size") +
                                     ylab("Null Reject Statistics") +
                                     ggtitle("Effect of Alternative Sample Size on Test Sensitivity") +
                                     scale_x_continuous(breaks = seq(10, 80, 10)) +
                                     theme(#legend.position = c(0.85, 0.65),
                                           legend.text = element_text(size = 8),
                                           legend.key.height = unit(0.25, 'cm'),
                                           legend.key.width = unit(0.4, 'cm')) +
                                     scale_color_discrete(name = "Attribute Stat",
                                                         labels = c("Pooled/Mean",
                                                                    "Unequal/Mean",
                                                                    "Pooled/Median",
                                                                    "Unequal/Median"))


knitr::kable(av_med_stats_by_altn, col.names = c("Alternative Sample Size",
                                                  "Average # Rejects (Unequal)",
                                                  "Median # Rejects (Unequal)",
                                                  "Average # Rejects (Pooled)",
                                                  "Median # Rejects (Pooled)"),
             escape = FALSE)


av_med_stats_by_comn %>% gather(key = "stat", value = "calc", avg_up:med_po) %>%
                          ggplot() + geom_line(aes(x = comn, y = calc, color = stat)) +
                                     geom_point(aes(x = comn, y = calc, color = stat)) +
                                     xlab("Total Sample Size") +
                                     ylab("Null Reject Statistics") +
                                     ggtitle("Effect of Total Sample Size on Test Sensitivity") +
                                     scale_x_continuous(breaks = seq(10, 150, 10)) +
                                     theme(#legend.position = c(0.85, 0.7),
                                           legend.text = element_text(size = 8),
                                           legend.key.height = unit(0.25, 'cm'),
                                           legend.key.width = unit(0.4, 'cm')) +
                                     scale_color_discrete(name = "Attribute Stat",
                                                         labels = c("Pooled/Mean",
                                                                    "Unequal/Mean",
                                                                    "Pooled/Median",
                                                                    "Unequal/Median"))

knitr::kable(av_med_stats_by_comn, col.names = c("Combined Sample Size",
                                                  "Average # Rejects (Unequal)",
                                                  "Median # Rejects (Unequal)",
                                                  "Average # Rejects (Pooled)",
                                                  "Median # Rejects (Pooled)"),
             escape = FALSE)
```