# ST502 Project

Halid Kopanski and Justin Feathers

# Part I

### Framingham Heart Study

The Framingham data set contains the diastolic blood pressure of 300 smokers and nonsmokers. For this study we will assume the following:

$H_0$: $\mu_1$ - $\mu_2 = 0$

$H_A$: $\mu_1$ - $\mu_2 \neq 0$

The null hypothesis being there is no difference in the blood pressure between smokers and non smokers. We believe that there is a difference and this paper will prove if we have enough evidence to reject the null hypothesis.

### Normality of Data

Plotting the density of total data (see Appendix I: Figure 1A), we can that it closely follows a normal distribution, Density plots of the split data exhibit similar forms (see Appendix I: Figure 1B). We can also calculate the kurtosis and the skew of the data, which are 3.812, 0.88 for kurtosis and skew respectively, and see that they are quite close to values typically found in normally distributed data. Additionally, the calculated values of sample standard deviation, $\frac{IQR}{1.35}$, and $\frac{MAD}{0.675}$ are 22.89, 22.22, and 21.48. The similarities between the three values indicate that the data is not influenced by outliers. Therefore, we will assume the data and the subsequent split data to be normal.

### Statistical Analysis

In the first analysis we will be assuming equal variances, thereby allowing us to use pooled sample variance (see Appendix I: Equation 1).

The pooled sample variance of the data is 510 using a degree of freedom value 298. In this case, we reject the null hypothesis because the calculated p value, 0.0041, is smaller than the chosen $\alpha$ value of 0.05. In terms of t values, our observed t value of -3.04 is smaller than the t value, -1.97 for a two sided $\alpha$ of 0.05.

When computing the observed t value using the assumption that the population variances are not equivalent (variance smoker is 352.2 and variance nonsmoker is 562.1), a value of -2.9 is obtained. Comparing that the two sided $\alpha$ of -1.98 with 158 degrees of freedom (as computed using the Satterthwaite Approximation see Appendix I: Equation 4). The observed t value is less than the chosen $\alpha$ value of 0.05. In case, there is sufficient evidence to reject the null hypothesis in favor of the alternative.

The observed 95% confidence intervals are -15.08 to -3.23 for pooled sample variance and -15.4 to -2.91 for non pooled sample variance. It can be seen that 0 does not fall into the 95% confidence interval in either case. Therefore the null hypothesis can be rejected.

In all three case, there is sufficient evidence to reject the null hypothesis and to support the alternative at an $\alpha$ level of 0.05.

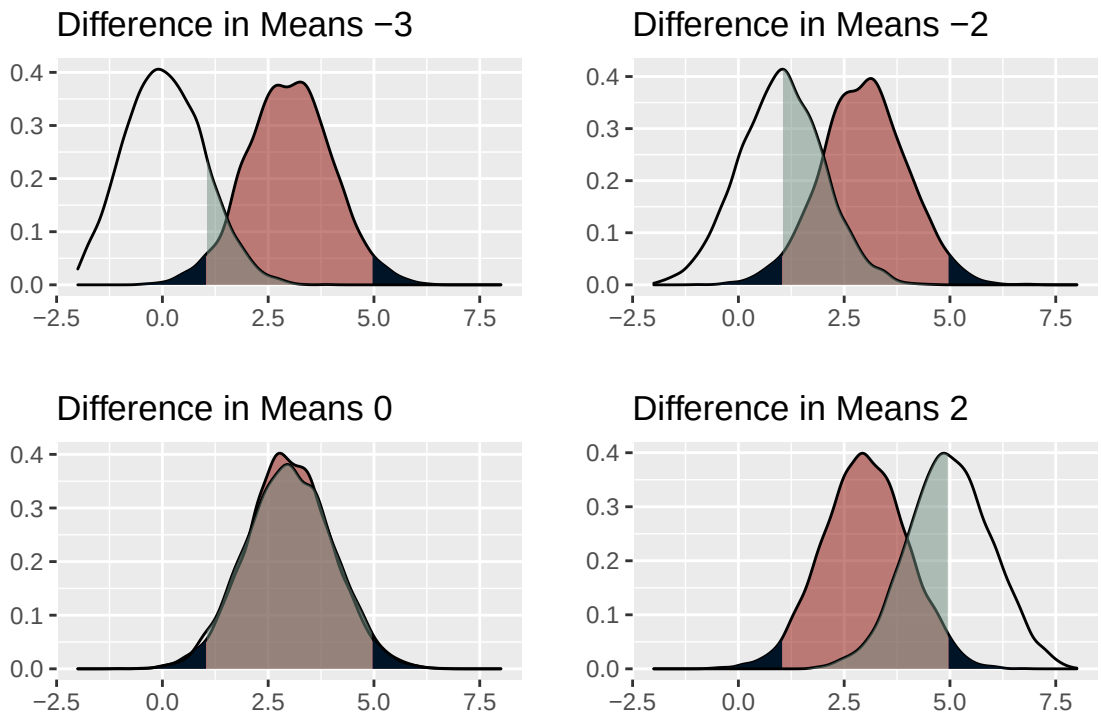
## Part II

### Introduction

In Part I we were able to see how hypothesis testing allowed to find enough evidence to challenge the "status quo" argument that smoking has no affect on the blood pressure of a person. What we found, through a number of robust statistical methods, that the difference in means, calculated to be 9.1577778, was significant to not have arisen by chance. That within an $\alpha$ of 5%, we are confident that the two groups do exhibit measurable differences. In the following sections, we will explore, through simulated data, the concept of randomness in data and what steps we can take to mitigate that randomness. All simulated date will be generated using the built in rnorm() function in R.

### Power of Hypothesis Testing

It is important to briefly discuss the concept of power in hypothesis testing. Power describes the probability of not committing a Type II error, which is to not reject the null hypothesis when there was in actuality enough evidence to do so. The probability of a Type II error is represented by $\beta$. Power is the complement to that probability (1 - $\beta$). The value of $\beta$ is the portion of the alternate distribution that is within the null hypothesis non rejection region limits. Below are a number of plots to help depict this concept. In each of the plots the shaded red is the null distribution and its location stays constant (mean = 3). The dark tails represent the rejection region of the null distribution. The outline distribution represents the "true" alternative distribution. Within that distribution is the shaded light blue region which is equal to $\beta$. It can be seen that as the difference between the two distributions shrinks the value of $\beta$ increases and the power shrinks. Power reaches minimum and $\beta$ reaches maximum when the two distributions are the same.




### Simulation Study

Below, various scenarios were simulated by creating 2 normally distributed data sets of various mean, variances, and sample sizes. Each scenario was repeated a thousand times. For each of those repeats a hypothesis

test was conducted and the number of times the null hypothesis was rejected was recorded. See Appendix III for results. All hypothesis testing was using pooled and unpooled variance, regardless of the actual values of variance.

The following table includes the values of each parameter used in the simulation:

| Parameter | Possible Values |
|---|---|
| $\mu_0$ | 0, 4, 5, 6, 10 |
| $\sigma_0^2$ | 1, 4, 9 |
| $n_0$ | 10, 30, 70 |
| $\mu_A$ | 5 |
| $\sigma_A^2$ | 1 |
| $n_A$ | 10, 30, 70 |

It was noted that the smaller difference between the alternative and null mean values were, the less likely the null hypothesis was rejected. Larger values of variance also reduced the number of rejected null hypothesis. In both cases, the more overlap between the two distributions, the less powerful the test. One way that can increase the power of the t-test is to increase the sample size. For a specific example, we can look at test cases #9 and #129. In both cases the distribution means were 6 and 5 for the null and alternative hypotheses respectively, along with the respective variances of 4 and 1. The only difference between the two cases was the sample size ($n_9 = 10$ and $n_{129} = 70$). This increase in sample size resulted in almost an 4 fold increase in power (286/952 (pooled) or 265/948 (unpooled) rejected nulls). Test cases 69 and 114 use different combinations of, but lower than 70, sample sizes than the aforementioned two. While they exhibited increased signs of power, they did not get as high as test case 129. It stands to reason that large variances and small deltas in mean can be mitigated by increasing the sample size accordingly.

Below are summaries of (1 - $\beta$) calculations for test cases #9 and #129:

| Power |
|---|
| Min. :0.0500 |
| 1st Qu.:0.1169 |
| Median :0.2979 |
| Mean :0.3793 |
| 3rd Qu.:0.6115 |
| Max. :1.0000 |

| Power |
|---|
| Min. :0.07074 |
| 1st Qu.:0.86108 |
| Median :0.96526 |
| Mean :0.89540 |
| 3rd Qu.:0.99472 |
| Max. :1.00000 |

The pattern further.....

## Appendix I: Equations and Figures

### Figures

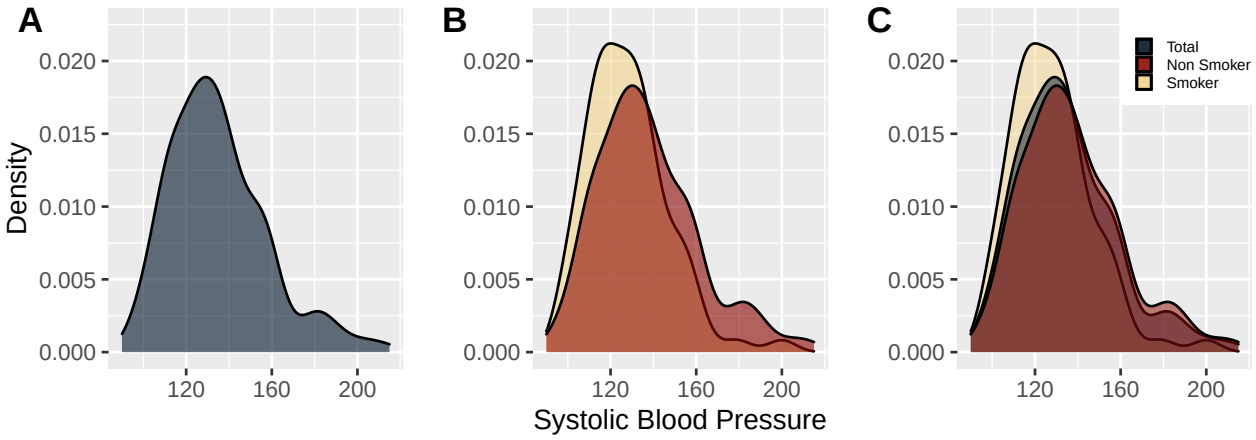



**Figure 1: Data plots**

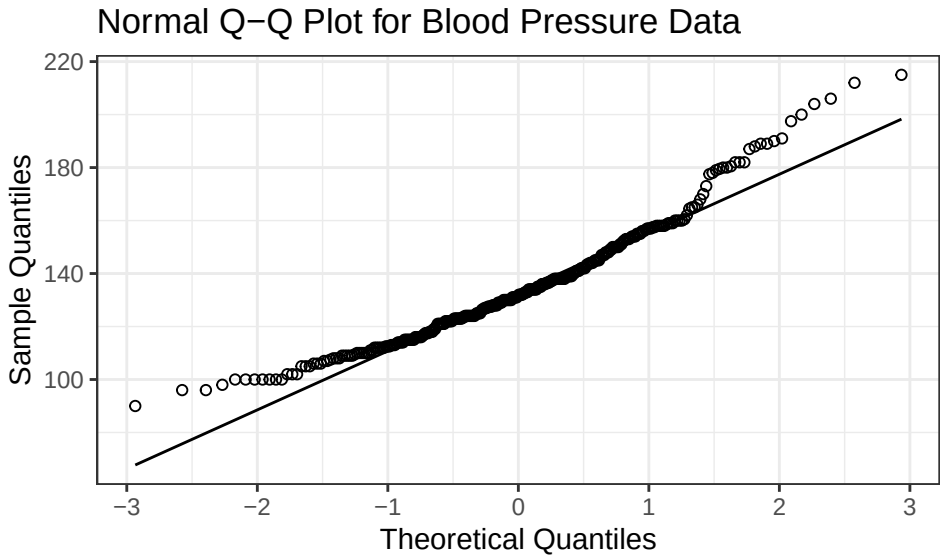



**Figure 2: Q-Q Plot**

---

### Equations:

**Equation 1: Pooled Sample Variance:**

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

**Equation 2: Observed T Statistic (pooled variance):**

$$t_{obs} = \frac{\mu_1 - \mu_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

**Equation 3: Observed T Statistic (distinct variance):**

$$t_{obs} = \frac{\mu_1 - \mu_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

**Equation 4: Satterthwaite Approximation:**

$$\nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$

## Appendix II: Part I Results

Data Mean: 134.935
Data Sample Variance: 524.0852258

Smoker Mean: 128.0666667
Smoker Variance: 352.2117117

Nonsmoker Mean: 137.2244444
Nonsmoker Variance: 562.1447123

Difference in mean: -9.1577778

Pooled Sample Variance: 510.0136987
Nonpooled Sample Variance: 9.9936838

CI Pooled: -15.08, -3.23
CI Nonpooled: -15.4, -2.91

## Appendix III: Part II Results

| Test Case | $\mu_1$ | $\sigma_1^2$ | $n_1$ | $\mu_2$ | $\sigma_2^2$ | $n_2$ | Test Results Unpooled | Test Results Pooled | Difference in Mean |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 10 | 5 | 1 | 10 | 1000 | 1000 | -5 |
| 2 | 4 | 1 | 10 | 5 | 1 | 10 | 590 | 562 | -1 |
| 3 | 5 | 1 | 10 | 5 | 1 | 10 | 58 | 39 | 0 |
| 4 | 6 | 1 | 10 | 5 | 1 | 10 | 535 | 546 | 1 |
| 5 | 10 | 1 | 10 | 5 | 1 | 10 | 1000 | 1000 | 5 |
| 6 | 0 | 4 | 10 | 5 | 1 | 10 | 1000 | 1000 | -5 |
| 7 | 4 | 4 | 10 | 5 | 1 | 10 | 288 | 259 | -1 |
| 8 | 5 | 4 | 10 | 5 | 1 | 10 | 45 | 55 | 0 |
| 9 | 6 | 4 | 10 | 5 | 1 | 10 | 286 | 265 | 1 |
| 10 | 10 | 4 | 10 | 5 | 1 | 10 | 1000 | 1000 | 5 |
| 11 | 0 | 9 | 10 | 5 | 1 | 10 | 996 | 995 | -5 |
| 12 | 4 | 9 | 10 | 5 | 1 | 10 | 143 | 156 | -1 |
| 13 | 5 | 9 | 10 | 5 | 1 | 10 | 59 | 50 | 0 |
| 14 | 6 | 9 | 10 | 5 | 1 | 10 | 156 | 200 | 1 |
| 15 | 10 | 9 | 10 | 5 | 1 | 10 | 992 | 999 | 5 |
| 16 | 0 | 1 | 30 | 5 | 1 | 10 | 1000 | 1000 | -5 |
| 17 | 4 | 1 | 30 | 5 | 1 | 10 | 781 | 776 | -1 |
| 18 | 5 | 1 | 30 | 5 | 1 | 10 | 58 | 52 | 0 |
| 19 | 6 | 1 | 30 | 5 | 1 | 10 | 767 | 740 | 1 |
| 20 | 10 | 1 | 30 | 5 | 1 | 10 | 1000 | 1000 | 5 |
| 21 | 0 | 4 | 30 | 5 | 1 | 10 | 1000 | 1000 | -5 |
| 22 | 4 | 4 | 30 | 5 | 1 | 10 | 278 | 260 | -1 |
| 23 | 5 | 4 | 30 | 5 | 1 | 10 | 10 | 6 | 0 |
| 24 | 6 | 4 | 30 | 5 | 1 | 10 | 274 | 281 | 1 |
| 25 | 10 | 4 | 30 | 5 | 1 | 10 | 1000 | 1000 | 5 |
| 26 | 0 | 9 | 30 | 5 | 1 | 10 | 1000 | 1000 | -5 |
| 27 | 4 | 9 | 30 | 5 | 1 | 10 | 84 | 82 | -1 |
| 28 | 5 | 9 | 30 | 5 | 1 | 10 | 5 | 3 | 0 |
| 29 | 6 | 9 | 30 | 5 | 1 | 10 | 81 | 87 | 1 |
| 30 | 10 | 9 | 30 | 5 | 1 | 10 | 1000 | 1000 | 5 |
| 31 | 0 | 1 | 70 | 5 | 1 | 10 | 1000 | 1000 | -5 |
| 32 | 4 | 1 | 70 | 5 | 1 | 10 | 831 | 812 | -1 |
| 33 | 5 | 1 | 70 | 5 | 1 | 10 | 61 | 54 | 0 |
| 34 | 6 | 1 | 70 | 5 | 1 | 10 | 848 | 842 | 1 |
| 35 | 10 | 1 | 70 | 5 | 1 | 10 | 1000 | 1000 | 5 |
| 36 | 0 | 4 | 70 | 5 | 1 | 10 | 1000 | 1000 | -5 |
| 37 | 4 | 4 | 70 | 5 | 1 | 10 | 240 | 242 | -1 |
| 38 | 5 | 4 | 70 | 5 | 1 | 10 | 3 | 4 | 0 |
| 39 | 6 | 4 | 70 | 5 | 1 | 10 | 257 | 244 | 1 |
| 40 | 10 | 4 | 70 | 5 | 1 | 10 | 1000 | 1000 | 5 |
| 41 | 0 | 9 | 70 | 5 | 1 | 10 | 1000 | 1000 | -5 |
| 42 | 4 | 9 | 70 | 5 | 1 | 10 | 36 | 29 | -1 |
| 43 | 5 | 9 | 70 | 5 | 1 | 10 | 0 | 1 | 0 |
| 44 | 6 | 9 | 70 | 5 | 1 | 10 | 39 | 31 | 1 |
| 45 | 10 | 9 | 70 | 5 | 1 | 10 | 1000 | 1000 | 5 |
| 46 | 0 | 1 | 10 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 47 | 4 | 1 | 10 | 5 | 1 | 30 | 784 | 734 | -1 |
| 48 | 5 | 1 | 10 | 5 | 1 | 30 | 48 | 53 | 0 |
| 49 | 6 | 1 | 10 | 5 | 1 | 30 | 768 | 753 | 1 |

| Test Case | $\mu_1$ | $\sigma_1^2$ | $n_1$ | $\mu_2$ | $\sigma_2^2$ | $n_2$ | Test Results Unpooled | Test Results Pooled | Difference in Mean |
|---|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 1 | 10 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 51 | 0 | 4 | 10 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 52 | 4 | 4 | 10 | 5 | 1 | 30 | 549 | 534 | -1 |
| 53 | 5 | 4 | 10 | 5 | 1 | 30 | 148 | 161 | 0 |
| 54 | 6 | 4 | 10 | 5 | 1 | 30 | 531 | 523 | 1 |
| 55 | 10 | 4 | 10 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 56 | 0 | 9 | 10 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 57 | 4 | 9 | 10 | 5 | 1 | 30 | 421 | 417 | -1 |
| 58 | 5 | 9 | 10 | 5 | 1 | 30 | 220 | 203 | 0 |
| 59 | 6 | 9 | 10 | 5 | 1 | 30 | 414 | 404 | 1 |
| 60 | 10 | 9 | 10 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 61 | 0 | 1 | 30 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 62 | 4 | 1 | 30 | 5 | 1 | 30 | 968 | 968 | -1 |
| 63 | 5 | 1 | 30 | 5 | 1 | 30 | 62 | 46 | 0 |
| 64 | 6 | 1 | 30 | 5 | 1 | 30 | 967 | 963 | 1 |
| 65 | 10 | 1 | 30 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 66 | 0 | 4 | 30 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 67 | 4 | 4 | 30 | 5 | 1 | 30 | 664 | 696 | -1 |
| 68 | 5 | 4 | 30 | 5 | 1 | 30 | 65 | 65 | 0 |
| 69 | 6 | 4 | 30 | 5 | 1 | 30 | 687 | 665 | 1 |
| 70 | 10 | 4 | 30 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 71 | 0 | 9 | 30 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 72 | 4 | 9 | 30 | 5 | 1 | 30 | 403 | 383 | -1 |
| 73 | 5 | 9 | 30 | 5 | 1 | 30 | 55 | 56 | 0 |
| 74 | 6 | 9 | 30 | 5 | 1 | 30 | 387 | 397 | 1 |
| 75 | 10 | 9 | 30 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 76 | 0 | 1 | 70 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 77 | 4 | 1 | 70 | 5 | 1 | 30 | 995 | 997 | -1 |
| 78 | 5 | 1 | 70 | 5 | 1 | 30 | 42 | 42 | 0 |
| 79 | 6 | 1 | 70 | 5 | 1 | 30 | 998 | 994 | 1 |
| 80 | 10 | 1 | 70 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 81 | 0 | 4 | 70 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 82 | 4 | 4 | 70 | 5 | 1 | 30 | 765 | 763 | -1 |
| 83 | 5 | 4 | 70 | 5 | 1 | 30 | 11 | 8 | 0 |
| 84 | 6 | 4 | 70 | 5 | 1 | 30 | 783 | 764 | 1 |
| 85 | 10 | 4 | 70 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 86 | 0 | 9 | 70 | 5 | 1 | 30 | 1000 | 1000 | -5 |
| 87 | 4 | 9 | 70 | 5 | 1 | 30 | 392 | 396 | -1 |
| 88 | 5 | 9 | 70 | 5 | 1 | 30 | 4 | 7 | 0 |
| 89 | 6 | 9 | 70 | 5 | 1 | 30 | 376 | 422 | 1 |
| 90 | 10 | 9 | 70 | 5 | 1 | 30 | 1000 | 1000 | 5 |
| 91 | 0 | 1 | 10 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 92 | 4 | 1 | 10 | 5 | 1 | 70 | 832 | 825 | -1 |
| 93 | 5 | 1 | 10 | 5 | 1 | 70 | 45 | 47 | 0 |
| 94 | 6 | 1 | 10 | 5 | 1 | 70 | 836 | 838 | 1 |
| 95 | 10 | 1 | 10 | 5 | 1 | 70 | 1000 | 1000 | 5 |
| 96 | 0 | 4 | 10 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 97 | 4 | 4 | 10 | 5 | 1 | 70 | 627 | 628 | -1 |
| 98 | 5 | 4 | 10 | 5 | 1 | 70 | 243 | 242 | 0 |
| 99 | 6 | 4 | 10 | 5 | 1 | 70 | 665 | 623 | 1 |
| 100 | 10 | 4 | 10 | 5 | 1 | 70 | 1000 | 1000 | 5 |

| Test Case | $\mu_1$ | $\sigma_1^2$ | $n_1$ | $\mu_2$ | $\sigma_2^2$ | $n_2$ | Test Results Unpooled | Test Results Pooled | Difference in Mean |
|---|---|---|---|---|---|---|---|---|---|
| 101 | 0 | 9 | 10 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 102 | 4 | 9 | 10 | 5 | 1 | 70 | 542 | 544 | -1 |
| 103 | 5 | 9 | 10 | 5 | 1 | 70 | 353 | 323 | 0 |
| 104 | 6 | 9 | 10 | 5 | 1 | 70 | 531 | 544 | 1 |
| 105 | 10 | 9 | 10 | 5 | 1 | 70 | 1000 | 1000 | 5 |
| 106 | 0 | 1 | 30 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 107 | 4 | 1 | 30 | 5 | 1 | 70 | 993 | 993 | -1 |
| 108 | 5 | 1 | 30 | 5 | 1 | 70 | 42 | 47 | 0 |
| 109 | 6 | 1 | 30 | 5 | 1 | 70 | 997 | 991 | 1 |
| 110 | 10 | 1 | 30 | 5 | 1 | 70 | 1000 | 1000 | 5 |
| 111 | 0 | 4 | 30 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 112 | 4 | 4 | 30 | 5 | 1 | 70 | 856 | 872 | -1 |
| 113 | 5 | 4 | 30 | 5 | 1 | 70 | 125 | 121 | 0 |
| 114 | 6 | 4 | 30 | 5 | 1 | 70 | 853 | 872 | 1 |
| 115 | 10 | 4 | 30 | 5 | 1 | 70 | 1000 | 1000 | 5 |
| 116 | 0 | 9 | 30 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 117 | 4 | 9 | 30 | 5 | 1 | 70 | 618 | 655 | -1 |
| 118 | 5 | 9 | 30 | 5 | 1 | 70 | 165 | 186 | 0 |
| 119 | 6 | 9 | 30 | 5 | 1 | 70 | 650 | 639 | 1 |
| 120 | 10 | 9 | 30 | 5 | 1 | 70 | 1000 | 1000 | 5 |
| 121 | 0 | 1 | 70 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 122 | 4 | 1 | 70 | 5 | 1 | 70 | 1000 | 1000 | -1 |
| 123 | 5 | 1 | 70 | 5 | 1 | 70 | 58 | 45 | 0 |
| 124 | 6 | 1 | 70 | 5 | 1 | 70 | 999 | 1000 | 1 |
| 125 | 10 | 1 | 70 | 5 | 1 | 70 | 1000 | 1000 | 5 |
| 126 | 0 | 4 | 70 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 127 | 4 | 4 | 70 | 5 | 1 | 70 | 959 | 969 | -1 |
| 128 | 5 | 4 | 70 | 5 | 1 | 70 | 46 | 53 | 0 |
| 129 | 6 | 4 | 70 | 5 | 1 | 70 | 952 | 948 | 1 |
| 130 | 10 | 4 | 70 | 5 | 1 | 70 | 1000 | 1000 | 5 |
| 131 | 0 | 9 | 70 | 5 | 1 | 70 | 1000 | 1000 | -5 |
| 132 | 4 | 9 | 70 | 5 | 1 | 70 | 747 | 738 | -1 |
| 133 | 5 | 9 | 70 | 5 | 1 | 70 | 53 | 57 | 0 |
| 134 | 6 | 9 | 70 | 5 | 1 | 70 | 766 | 732 | 1 |
| 135 | 10 | 9 | 70 | 5 | 1 | 70 | 1000 | 1000 | 5 |

## Appendix IV: Code

```
knitr::opts_chunk$set(echo = TRUE)

#Use required packages
library(tidyverse) #for plots and data manipulation
library(cowplot) #aligning plots
library(gridExtra)
library(scales)

df_data <- read_csv("framingham_data.csv") # Read in data
df_data$index <- seq(nrow(df_data)) # Add an index column

#df_data %>% summary # Summarize Data

# Split data into smoker and nonsmoker
df_smoker <- df_data %>% filter(currentSmoker == 1)
df_nonsmoker <- df_data %>% filter(currentSmoker == 0)

#Create a sample variance function to ensure proper calculation
sample_variance <- function(x, sampling = TRUE){
    if (sampling == TRUE){
        sum((x - mean(x))^2) / (length(x) - 1)
    } else if(sampling == FALSE) {
        sum((x - mean(x))^2) / (length(x))
    }
}
#Create pooled sample variance function
f_pooled_variance <- function(x, y){
    ((length(x) - 1) * sample_variance(x) +
     (length(y) - 1) * sample_variance(y)) /
    (length(x) + length(y) - 2)
}

# Skewness function
skew_function <- function(x){
    mean((x - mean(x))^3) / sqrt(sample_variance(x))^3
}

# kurtosis function
kurt_function <- function(x){
    mean((x - mean(x))^4) / sqrt(sample_variance(x))^4
}

# Create a Satterthawaite Approximation Function

satterth <- function(s1, s2, n1, n2){
    term1 <- s1/n1
    term2 <- s2/n2
    nu <- (term1 + term2)^2 / ((term1^2/(n1 - 1)) + (term2^2/(n2 - 1)))
    return(floor(nu))
}
```

```r
#Plot and compare split data

#options(repr.plot.width = 6, repr.plot.height = 4, repr.plot.res = 150)

plot_colors <- c("#001427","#708d81","#f4d58d","#bf0603","#8d0801")
y_limits <- c(0, 0.0225)

total_data <- ggplot(df_data) + geom_density(aes(sysBP),
                                             fill = plot_colors[1],
                                             alpha = 0.6) +
                        ylim(y_limits) + ylab("Density") + xlab("")

sep_data <- ggplot() + geom_density(data = df_smoker, aes(sysBP),
                                    fill = plot_colors[3], alpha = 0.6) +
                       geom_density(data = df_nonsmoker, aes(sysBP),
                                    fill = plot_colors[5], alpha = 0.6) +
                       ylim(y_limits) + ylab("") + xlab("Systolic Blood Pressure")

plot_3 <- ggplot() + geom_density(data = df_smoker, aes(sysBP,
                                    fill = plot_colors[3]), alpha = 0.5) +
                       geom_density(data = df_data, aes(sysBP,
                                    fill = plot_colors[1]), alpha = 0.5) +
                       geom_density(data = df_nonsmoker, aes(sysBP,
                                    fill = plot_colors[5]), alpha = 0.5) +
                       ylim(y_limits) + ylab("") + xlab("") +
                       scale_fill_manual("",
                                         values = plot_colors[c(1, 5, 3)],
                                         labels = c("Total", "Non Smoker", "Smoker")) +
                       theme(legend.position = c(0.8, 0.9),
                             legend.text = element_text(size = 6),
                             legend.key.height = unit(0.25, 'cm'),
                             legend.key.width = unit(0.25, 'cm'))

#plot_grid(total_data, sep_data, plot_3, align = 'vh',
          #hjust = -1,nrow = 2, ncol = 2)

data_kurtosis <- kurt_function(df_data$sysBP)
data_skew <- skew_function(df_data$sysBP)
data_IQR <- as.numeric(quantile(df_data$sysBP, probs = 0.75)) -
       as.numeric(quantile(df_data$sysBP, probs = 0.25))
data_MAD <- median(abs(df_data$sysBP - median(df_data$sysBP)))
data_samVar <- sample_variance(df_data$sysBP)

eIQR <- data_IQR / 1.35
eMAD <- data_MAD / 0.675

# Q-Q Plot

data_qqplot <-
ggplot(df_data, aes(sample = sysBP)) +
stat_qq(shape = 1) + stat_qq_line() +
ggtitle("Normal Q-Q Plot for Blood Pressure Data") +
xlab("Theoretical Quantiles") +
```

```
ylab("Sample Quantiles")

# Common values for analysis

alpha <- 0.05

mu_smoker <- mean(df_smoker$sysBP)
var_smoker <- sample_variance(df_smoker$sysBP)
n_smoker <- length(df_smoker$sysBP)

mu_nonsmoker <- mean(df_nonsmoker$sysBP)
var_nonsmoker <- sample_variance(df_nonsmoker$sysBP)
n_nonsmoker <- length(df_nonsmoker$sysBP)

# Two Sample T-test - Pooled Sample Variance - P-value

dof_1 <- (n_smoker + n_nonsmoker - 2)

p_sample_var_1 <- f_pooled_variance(df_smoker$sysBP,
                                    df_nonsmoker$sysBP)


t_obs_1 <- (mu_smoker - mu_nonsmoker) / (sqrt(p_sample_var_1) * sqrt(1/n_smoker + 1/n_nonsmoker))

t_stat_1 <- qt(alpha / 2, dof_1)

p_value_obs_1 <- dt(t_obs_1, dof_1)

#Two Sample T-test - Difference Variance Sample Variance - P-value

dof_2 <- satterth(var_smoker, var_nonsmoker, n_smoker, n_nonsmoker)

np_sample_var_2 <- (var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)

t_obs_2 <- (mu_smoker - mu_nonsmoker) / (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker))

t_stat_2 <- qt(alpha / 2, dof_2)

p_value_obs_2 <- dt(t_obs_2, dof_2)

# Confidence Limits

diff_mu <- mu_smoker - mu_nonsmoker

#Pooled Sample variance

CL_pooled <- t_stat_1 * (sqrt(p_sample_var_1/n_smoker + p_sample_var_1/n_nonsmoker))

#Non pooled Sample variance

CL_nonpooled <- t_stat_2 * (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker))

CI_pooled <- round(c(diff_mu + CL_pooled, diff_mu - CL_pooled), 2)
```

```r
CI_nonpooled <-round(c(diff_mu + CL_nonpooled, diff_mu - CL_nonpooled), 2)

#Power Calculation assuming delta means is the true delta

cv_lo_p <- qnorm(alpha / 2, 0, sqrt(p_sample_var_1/n_smoker +
                                    p_sample_var_1/n_nonsmoker))
cv_hi_p <- qnorm(1 - alpha / 2, 0, sqrt(p_sample_var_1/n_smoker +
                                        p_sample_var_1/n_nonsmoker))

power1 <- pnorm(cv_lo_p, (mu_smoker - mu_nonsmoker),
                sqrt(p_sample_var_1/n_smoker + p_sample_var_1/n_nonsmoker))
power2 <- 1 - pnorm(cv_hi_p, (mu_smoker - mu_nonsmoker),
                    sqrt(p_sample_var_1/n_smoker + p_sample_var_1/n_nonsmoker))

power_pooled <- sum(power1, power2)

cv_lo_non <- qnorm(alpha / 2, 0,
               (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)))
cv_hi_non <- qnorm(1 - alpha / 2, 0,
               (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)))

power1 <- pnorm(cv_lo_non, (mu_smoker - mu_nonsmoker),
                (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)))
power2 <- 1 - pnorm(cv_hi_non, (mu_smoker - mu_nonsmoker),
                    (sqrt(var_nonsmoker/n_smoker + var_nonsmoker/n_nonsmoker)))

power_nonpooled <- sum(power1, power2)


#Part II
#Introduction

options(repr.plot.width = 12, repr.plot.height = 5, repr.plot.res = 150)
set.seed(100)

null_mean <- 3
alt_means <- c(0, 1, 3, 5)
plot_list <- list()

#plot_colors <- c("#072ac8","#1e96fc","#a2d6f9","#fcf300","#ffc600")

for(i in 1:length(alt_means)){

    sim1 <- rnorm(5000, null_mean, sqrt(1))
    sim2 <- rnorm(5000, alt_means[i], sqrt(1))

    alpha1 <- qnorm(0.025, null_mean, sqrt(1))
    alpha2 <- qnorm(0.975, null_mean, sqrt(1))

    df_set <- tibble("H0" = sim1, "HA" = sim2)

    title_string <- sprintf("Difference in Means %i", (alt_means[i] - null_mean))
```

```
    plot_list[[i]]  <-
    ggplot(data = df_set) + geom_density(aes(H0), alpha = 0.5, fill = plot_colors[5]) +
                            geom_area(
                                aes(x = stage(H0, after_scale = oob_censor(x, c(-Inf, alpha1)
                                                                           )
                                              )
                                   ),
                                stat = "density", fill = plot_colors[1]
                                 ) +
                            geom_area(
                                aes(x = stage(H0, after_scale = oob_censor(x, c(alpha2, Inf)
                                                                           )
                                              )
                                   ),
                                stat = "density", fill = plot_colors[1]
                                     ) +
                            geom_density(aes(HA), alpha = 0.5) +
                            geom_area(
                                aes(x = stage(HA, after_scale = oob_censor(x, c(alpha1, alpha2)
                                                                           )
                                              )
                                   ),
                                stat = "density", fill = plot_colors[2], alpha = 0.5
                                     ) +
                            xlim(-2, 8) + xlab("") + ylab("") + ggtitle(title_string)

                                }

do.call(grid.arrange, plot_list)
#Part II
set.seed(1)

alpha <- 0.05

test_function <- function (x, y, pooled = FALSE){

    mu_1 <- mean(x)
    var_1 <- sample_variance(x, sampling = TRUE)

    mu_2 <- mean(y)
    var_2 <- sample_variance(y, sampling = TRUE)

    #Calculate the pooled sample variance
    pooled_sample <- ((length(x) - 1) * var_1 +
                          (length(y) - 1) * var_2) / (length(x) + length(y) - 2)

    #calculate the observed t statistic
    if (pooled == TRUE){

        cal_sigma <- (sqrt(pooled_sample/length(x) + pooled_sample/length(y)))

        ttest <- (mu_1 - mu_2) / cal_sigma
        dof <- length(x) + length(y) - 2 #Determine degrees of freedom
```

```
        } else {

        cal_sigma <- (sqrt(var_1/length(x) + var_2/length(y)))

        ttest <- (mu_1 - mu_2) / cal_sigma
        dof <- satterth(var_1, var_2, length(x), length(y))
    }

    # Determine whether or not the null hypothesis
    # can be rejected (1 = rejected, 0 = not rejected)
    verdict <- !between(ttest, qt(alpha / 2, dof), qt(1 - alpha / 2, dof))

    #Power calculation assuming calculated difference in means is Ha
    cv_lo <- qnorm(alpha / 2, 0, cal_sigma)
    cv_hi <- qnorm(1 - alpha / 2, 0, cal_sigma)

    power1 <- pnorm(cv_lo, (mu_1 - mu_2), cal_sigma)
    power2 <- 1 - pnorm(cv_hi, (mu_1 - mu_2), cal_sigma)

    power <- sum(power1, power2)

    #Return calculated values
    return(c(mu_1, var_1, mu_2, var_2, ttest, cal_sigma, dof, verdict, power))
}



mu1 <- c(0, 4, 5, 6, 10)
var1 <- c(1, 4, 9)
n1 <- c(10, 30, 70)

mu2 <- 5
var2 <- 1
n2 <- c(10, 30, 70)

sim_test <- function(x_mu, x_var, x_n, y_mu, y_var, y_n, pooled = TRUE){

    sim_data_results <- matrix(rep(0, 9), ncol = 9)

    for (i in 1:1000){

        sim_set1 <- rnorm(x_n, x_mu, sqrt(x_var))
        sim_set2 <- rnorm(y_n, y_mu, sqrt(y_var))

        sim_data_results <- rbind(sim_data_results,
                                  test_function(sim_set1, sim_set2, pooled))

        #print(sim_data_results)
    }

    df_sim_data <- data.frame(sim_data_results[2 : nrow(sim_data_results),])
    colnames(df_sim_data) = c("Null Mean", "Null Variance", "Alternate Mean",
                              "Alternate Variance", "T statistic",
```

```
                              "Calculated Variance", "DoF", "Null Reject",
                              "Power")
    return(df_sim_data)
}

# HA: mean = 5, var = 1
df_combo <- expand.grid(mu1, var1, n1, mu2, var2, n2)
df_combo2 <- tibble(cbind(1:nrow(df_combo), df_combo,
                          matrix(rep(0, 2 * nrow(df_combo)), ncol = 2)))
colnames(df_combo2) <- c("Test_Case", "mu1", "var1", "n1", "mu2", "var2",
                         "n2", "Test_Results_up", "Test_Results_po")

test_results <- list()
test_results2 <- list()

for (i in 1:nrow(df_combo)){
    test_results[[i]] <- do.call(sim_test,
                                 as.list(as.numeric(df_combo[i,])))
    test_results2[[i]] <- do.call(sim_test,
                                  as.list(as.numeric(c(df_combo[i,],
                                                       pooled = TRUE))))
    df_combo2[i, 8] <- sum(as.data.frame(test_results[i])[,8])
    df_combo2[i, 9] <- sum(as.data.frame(test_results2[i])[,8])
    }

df_combo2 <- df_combo2 %>% mutate(diff = mu1 - mu2)
#df_combo2 %>%  head()

knitr::kable(summary(test_results[[9]]["Power"]))

knitr::kable(summary(test_results[[129]]["Power"]))

# Plotting
options(repr.plot.width = 6, repr.plot.height = 4, repr.plot.res = 150)

plot_grid(total_data, sep_data, plot_3, align = 'vh',
          hjust = -1,nrow = 1, ncol = 3, labels = c("A", "B", "C"))
data_qqplot + theme_bw()
knitr::kable(df_combo2, col.names = c("Test Case",
                                      "$\\mu_1$",
                                      "$\\sigma_1^2$",
                                      "$n_1$",
                                      "$\\mu_2$",
                                      "$\\sigma_2^2$",
                                      "$n_2$",
                                      "Test Results Unpooled",
                                      "Test Results Pooled",
                                      "Difference in Mean"),
             escape = FALSE)
```