1.a)

$$\bar{Y}_n = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

$Y_1, \ldots, Y_n \overset{iid}{\sim} Poisson(\lambda)$

$E(Y) = \lambda$

$Var(Y) = \lambda$

$$\bar{Y}_n = E(Y_n) = \frac{1}{n} E\left(\sum_{i=1}^{n} Y_i\right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} E(Y_i)$$

$$= \frac{1}{n}\, n \quad \lambda$$

$$= \lambda$$

$$Var(Y_n) = Var\left(\frac{1}{n} \sum_{i=1}^{n} Y_i\right)$$

$$= \frac{1}{n^2} Var\left(\sum_{i=1}^{n} Y_i\right)$$

$$= \frac{1}{n^2} \sum_{i=1}^{n} Var(Y_i)$$

$$= \cancel{n} \frac{1}{n^{\cancel{2}}} \sigma^2$$

$$= \frac{\lambda}{n}$$

λ is the mean of the population while $\bar{Y}_n$ is the sample mean. λ is fixed and unknown while the sample is mean is a random variable. In this case $\bar{Y}_n - \lambda = 0$ therefore this is an unbiased statistic.

Problem 4

$X_1 \ldots\ldots X_n \sim \Gamma(n, \lambda)$

Since multiple random samples from an exponential distribution have a joint distribution of Gamma where $\alpha = n$ and $\lambda = \lambda$.

$P(X_n \geq 2)$

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} \qquad x \geq 0$$

$\alpha = n$

$$f(x) = \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x}$$

$$P(x \geq 2) = \int_0^2 \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x} dx$$

$$= \frac{\lambda^n}{\Gamma(n)} \int_0^2 x^{n-1} e^{-\lambda x} dx$$

$$= \frac{\lambda^n}{\Gamma(n)} \left[\frac{(n-1)!}{\lambda^n} \left(1 - e^{-2\lambda} \sum_{i=0}^{n} \frac{(2\lambda)^i}{i!}\right) \right]$$

$$= \frac{(n-1)!}{\Gamma(n)} \left[1 - e^{-2\lambda} \sum_{i=0}^{n} \frac{(2\lambda)^i}{i!} \right]$$

5 a) $\bar{X}_a = \sum_{i=1}^{n} a_i x_i$

$\sum_{i=1}^{n} E(a_i x_i) = \sum_{i=1}^{n} a_i \, E(x_i)$

$= n(a_i E(x)) = \mu$

$\Rightarrow n a \mu = \mu$

$\Rightarrow a = \frac{1}{n}$

b) $Var(\bar{X}_a) = Var\left(\sum_{i=1}^{n} a_i x_i\right)$

$= \frac{1}{n^2} \sum_{i=1}^{n} Var(x_i)$

$= Var(naX)$

$= n^2 \, Var(aX)$

$= n^2 a^2 \, Var(X)$

$= n^2 a^2 = 1$

$a^2 = \frac{1}{n^2}$

$a = \frac{1}{n}$

6) a) $y_i = X_i + e_i$

$$\frac{1}{n}\sum_{i=1}^{n} Y_i = \frac{1}{n}\sum_{i=1}^{n}(X_i + e_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} X_i + \frac{1}{n}\sum_{i=1}^{n} e_i$$

$$= E(X_i) + E(e_i)$$

$$= \mu + \mu_e$$

b) $$E\left\{\frac{1}{n}\sum_{i=1}^{n} Y_i\right\}$$

$$= \frac{1}{n} E\left(\sum_{i=1}^{n} y_i\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(y_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(X_i + e_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(X_i) + \frac{1}{n}\sum_{i=1}^{n} E(e_i)$$

$$= \mu + \mu_e$$

c) $\frac{1}{n}\sum_{i=1}^{n} Y_i$

$E(Y_i) = \mu + \mu_e$

only if μ_e is zero

d) See Code