

$$5a) \bar{x}_a = \sum_{i=1}^n a_i x_i$$

$$\sum_{i=1}^n E(a_i x_i) = \sum_{i=1}^n a_i E(x_i)$$

$$= n(a E(x)) = \mu$$

$$\Rightarrow na\mu = \mu$$

$$\Rightarrow a = \frac{1}{n}$$

$$b) \text{Var}(\bar{x}_a) = \text{Var}\left(\sum_{i=1}^n a_i x_i\right)$$

$$= \frac{1}{n^2} \sum_{i=1}^n \text{Var}(x_i)$$

$$= \text{Var}(nax)$$

$$= n^2 \text{Var}(ax)$$

$$= n^2 a^2 \text{Var}(x)$$

$$= n^2 a^2 = 1$$

$$a^2 = \frac{1}{n^2}$$

$$a = \frac{1}{n}$$