Problem 4

$X_1 \ldots\ldots X_n \sim \Gamma(n, \lambda)$

Since multiple random samples from an exponential distribution have a joint distribution of Gamma where $\alpha = n$ and $\lambda = \lambda$.

$P(X_n \geq 2)$

$$f(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} \qquad x \geq 0$$

$\alpha = n$

$$f(x) = \frac{\lambda^{n}}{\Gamma(n)} x^{n-1} e^{-\lambda x}$$

$$P(x \geq 2) = \int_0^2 \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x} \, dx$$

$$= \frac{\lambda^n}{\Gamma(n)} \int_0^2 x^{n-1} e^{-\lambda x} \, dx$$

$$= \frac{\lambda^n}{\Gamma(n)} \left[\frac{(n-1)!}{\lambda^n} \left(1 - e^{-2\lambda} \sum_{i=0}^{n} \frac{(2\lambda)^i}{i!} \right) \right]$$

$$= \frac{(n-1)!}{\Gamma(n)} \left[1 - e^{-2\lambda} \sum_{i=0}^{n} \frac{(2\lambda)^i}{i!} \right]$$