ST503 HW1

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Question 1

a)

The data consists of 9 predictors, lcavol, lweight, age, lbph, svi, lcp, gleason, pgg45, lpsa, gleason and age are composed of integers, and svi is binary data.

```
##       lcavol lweight age      lbph svi      lcp gleason pgg45     lpsa
## 1 -0.5798185  2.7695  50 -1.386294   0 -1.38629       6     0 -0.43078
## 2 -0.9942523  3.3196  58 -1.386294   0 -1.38629       6     0 -0.16252
## 3 -0.5108256  2.6912  74 -1.386294   0 -1.38629       7    20 -0.16252
## 4 -1.2039728  3.2828  58 -1.386294   0 -1.38629       6     0 -0.16252
## 5  0.7514161  3.4324  62 -1.386294   0 -1.38629       6     0  0.37156
## 6 -1.0498221  3.2288  50 -1.386294   0 -1.38629       6     0  0.76547
```

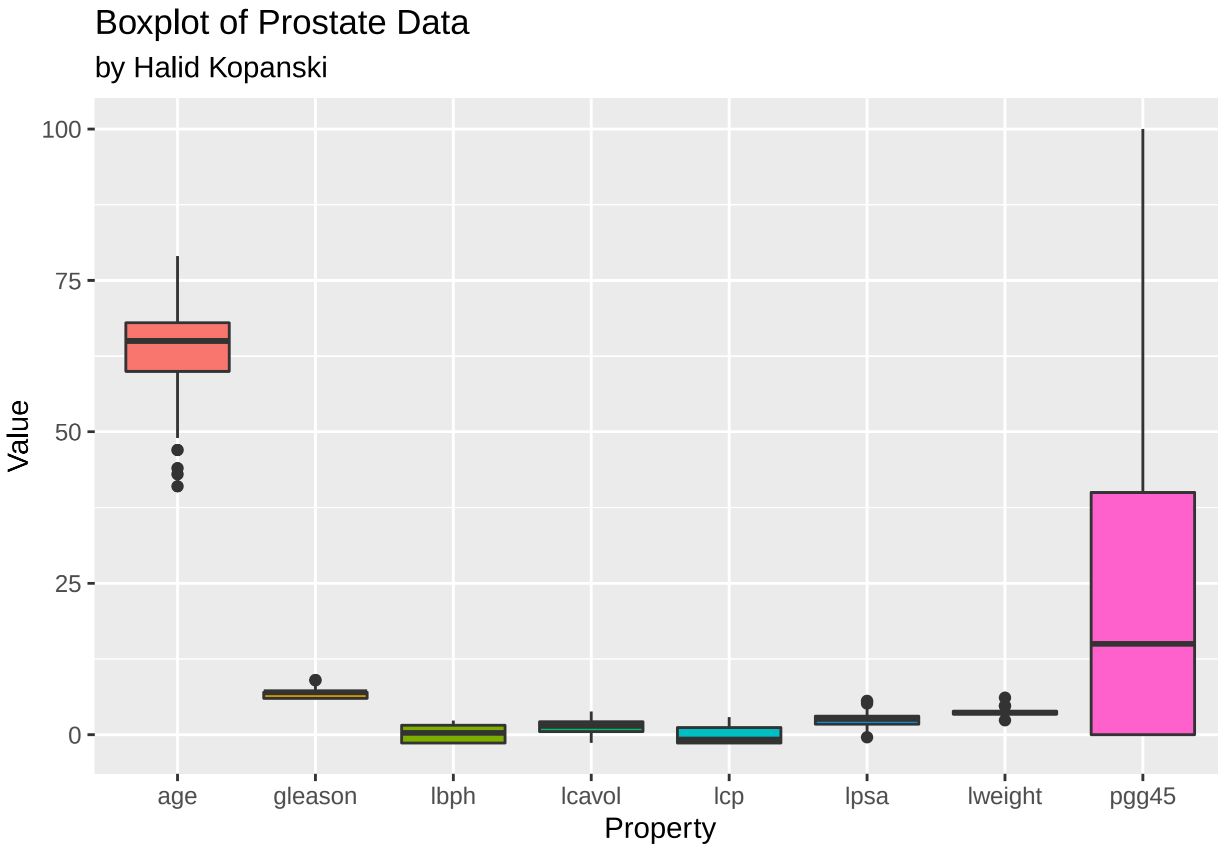
```
## 'data.frame':    97 obs. of  9 variables:
##  $ lcavol : num  -0.58 -0.994 -0.511 -1.204 0.751 ...
##  $ lweight: num  2.77 3.32 2.69 3.28 3.43 ...
##  $ age    : int  50 58 74 58 62 50 64 58 47 63 ...
##  $ lbph   : num  -1.39 -1.39 -1.39 -1.39 -1.39 ...
##  $ svi    : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ lcp    : num  -1.39 -1.39 -1.39 -1.39 -1.39 ...
##  $ gleason: int  6 6 7 6 6 6 6 6 6 6 ...
##  $ pgg45  : int  0 0 20 0 0 0 0 0 0 0 ...
##  $ lpsa   : num  -0.431 -0.163 -0.163 -0.163 0.372 ...
```

```
##      lcavol           lweight           age             lbph
##  Min.   :-1.3471   Min.   :2.375   Min.   :41.00   Min.   :-1.3863
##  1st Qu.: 0.5128   1st Qu.:3.376   1st Qu.:60.00   1st Qu.:-1.3863
##  Median : 1.4469   Median :3.623   Median :65.00   Median : 0.3001
##  Mean   : 1.3500   Mean   :3.653   Mean   :63.87   Mean   : 0.1004
##  3rd Qu.: 2.1270   3rd Qu.:3.878   3rd Qu.:68.00   3rd Qu.: 1.5581
##  Max.   : 3.8210   Max.   :6.108   Max.   :79.00   Max.   : 2.3263
##       svi              lcp             gleason          pgg45
##  Min.   :0.0000   Min.   :-1.3863   Min.   :6.000   Min.   :  0.00
##  1st Qu.:0.0000   1st Qu.:-1.3863   1st Qu.:6.000   1st Qu.:  0.00
##  Median :0.0000   Median :-0.7985   Median :7.000   Median : 15.00
##  Mean   :0.2165   Mean   :-0.1794   Mean   :6.753   Mean   : 24.38
##  3rd Qu.:0.0000   3rd Qu.: 1.1786   3rd Qu.:7.000   3rd Qu.: 40.00
##  Max.   :1.0000   Max.   : 2.9042   Max.   :9.000   Max.   :100.00
##       lpsa
```

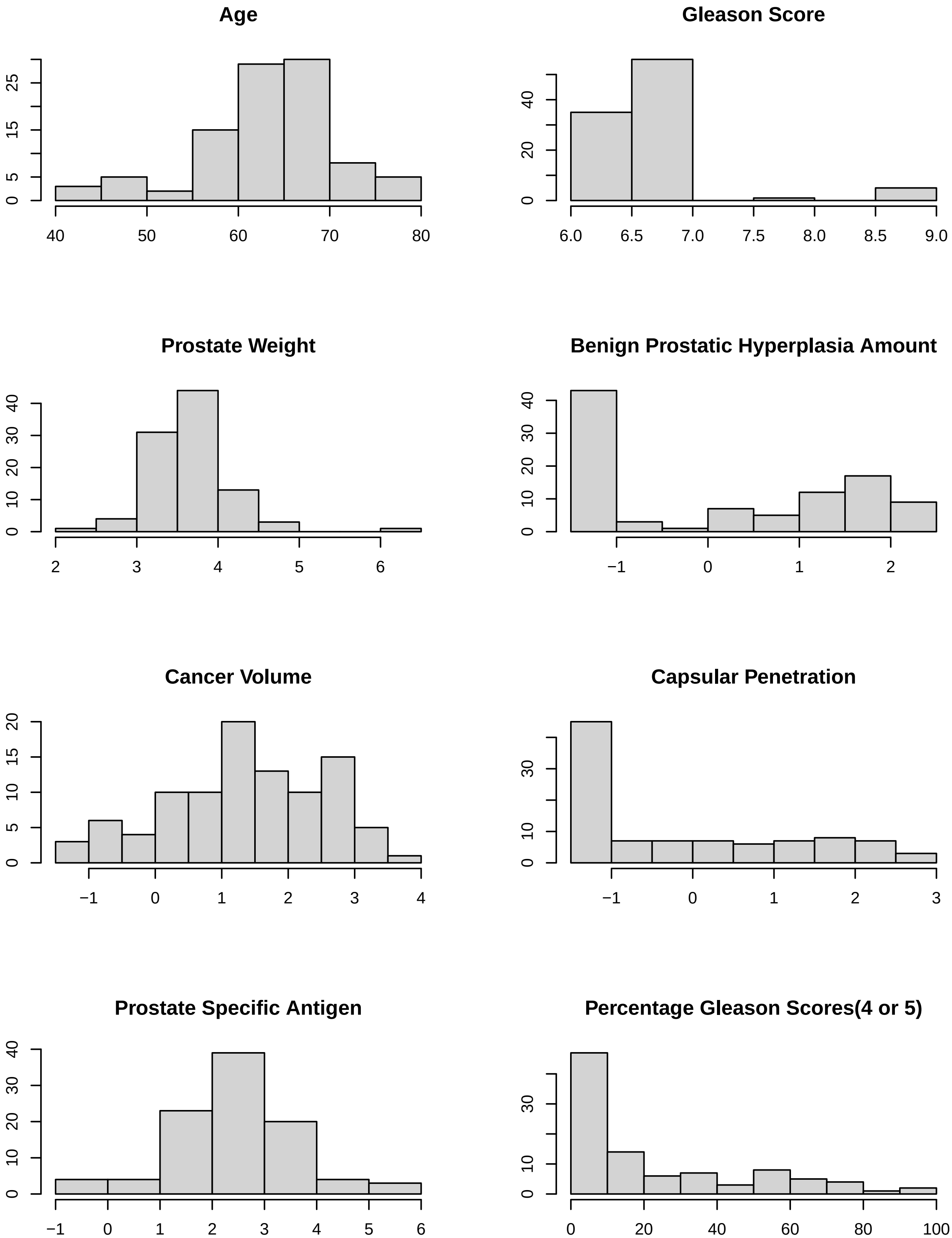
```
##  Min.   :-0.4308
##  1st Qu.: 1.7317
##  Median : 2.5915
##  Mean   : 2.4784
##  3rd Qu.: 3.0564
##  Max.   : 5.5829
```

The summary indicates that some of the data is normal or uniform in the cases where median and mean are nearly.

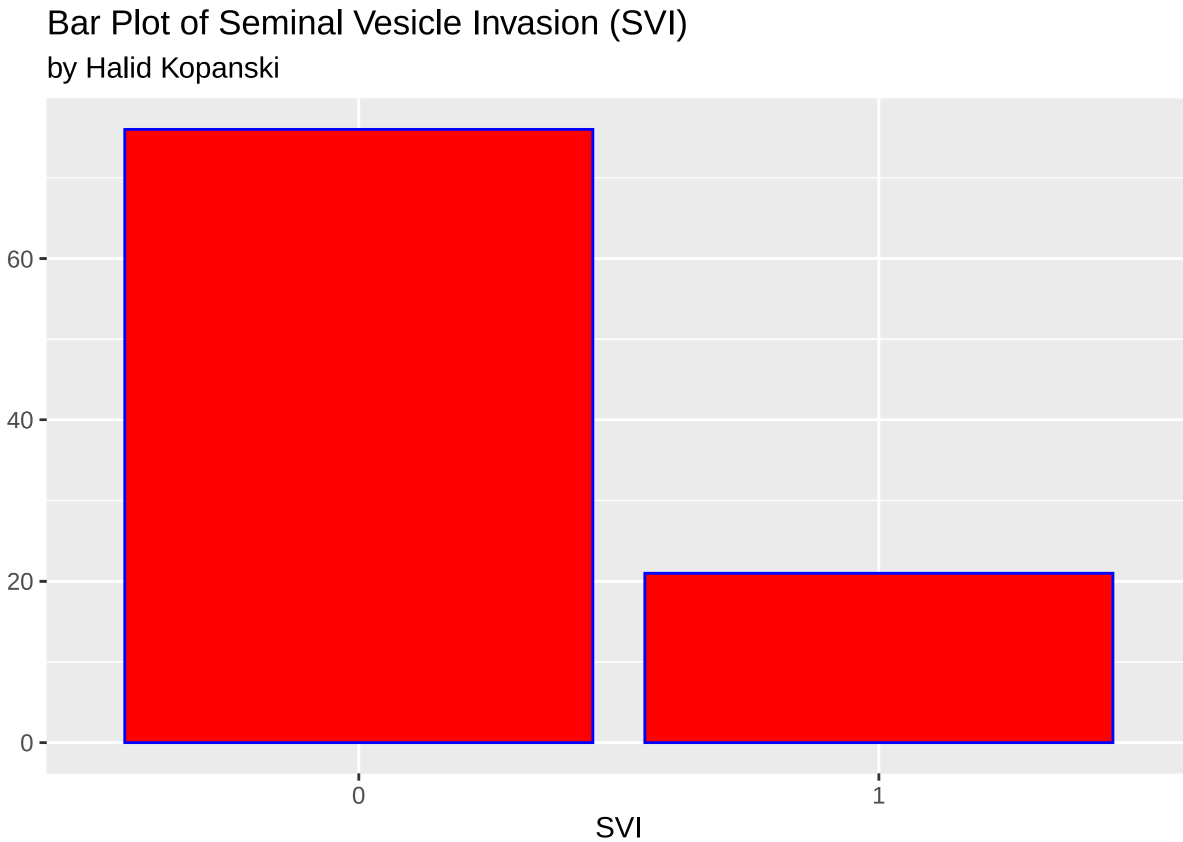
As can be seen in the histograms, the various predictors occupy varying ranges of values. In the cases of age, gleason, lpsa, and lweight a number of outliers can be seen.



The histograms do indicate that age, prostate weight, cancer weight, and prostate specific antigen are normal like in thier distributions. The remaining predictors do not display any known distributions in first indication.



SVI Value	Value Count
0	76
1	21



b)

In the following table, we can see which predictors had outlier values and how many.

Property	mean	iqr	upper	lower	no_outliers
age	63.8659794	8.000000	48.000000	80.000000	5
gleason	6.7525773	1.000000	4.500000	8.500000	5
lbph	0.1003558	2.944439	-5.802952	5.974804	0
lcavol	1.3500096	1.614217	-1.908502	4.548366	0
lcp	-0.1793637	2.564940	-5.233700	5.026060	0
lpsa	2.4783870	1.324700	-0.255390	5.043410	4
lweight	3.6526887	0.502600	2.622000	4.632400	4
pgg45	24.3814433	40.000000	-60.000000	100.000000	0

#2

Normal equations $\qquad Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$

$X^T X\hat{\beta} = X^T y \qquad Q = \sum_{i=1}^{n} \varepsilon_i^2$

a) $\varepsilon^T \varepsilon = \sum_{i=1}^{n} (y - X\hat{\beta})^T (y - X\hat{\beta})$

$= y^T y - 2y^T X\hat{\beta} + \hat{\beta} X^T X\hat{\beta}$

$\frac{\partial \varepsilon^T \varepsilon}{\partial \beta} = -2X^T y + 2X^T X\hat{\beta} = 0$

$X^T y = X^T X\hat{\beta}$

$Q = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_i)^2$

$\frac{\partial Q}{\partial \beta_0} = -2 \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_i) = 0$

$\sum_{i=1}^{n} Y_i - \beta_0 - \beta_1 X_i = 0$

$n\beta_0 = \sum_{i=1}^{n} Y_i - \beta_1 X_i$

$\beta_0 = \frac{1}{n} \sum_{i=1}^{n} Y_i - \beta_1 X_i$

$\beta_0 = \bar{Y} - \beta_1 \bar{X}$

$$\sum_{i=1}^{n} x_i(y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\sum_{i=1}^{n} x_i y_i - \beta_0 x_i - \beta_1 x_i^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i - \beta_0 \sum_{i=1}^{n} x_i - \beta_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i - (\bar{Y} - \beta_1 \bar{x}) \sum_{i=1}^{n} x_i - \beta_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i \left[\frac{1}{n}\sum_{i=1}^{n} y_i - \beta_1 \frac{1}{n}\sum_{i=1}^{n} x_i\right] \Sigma x_i - \beta_1 \sum_{i=0}^{n} x_i^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \Sigma x_i + \beta_1 \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i - \beta_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i + \beta_1\left[\frac{1}{n}\Sigma x_i \Sigma x_i - \Sigma x_i^2\right] = 0$$

$$\beta_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{Y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

#3

a) $y_i = \beta_0 + \beta_1(x_i - \bar{x}) + \varepsilon_i = \beta_0 + \beta_1 x_i - \beta_1\bar{x} + \varepsilon_i$

$$g(b) = \sum (y_i - \beta_0 - \beta_1(x_i - \bar{x}))^2$$

$$\frac{\partial g}{\partial b} = 0$$

$$g(b) = \sum (y_i - \beta_0 - \beta_1 x_i + \beta_1\bar{x})^2$$

$$\frac{\partial g}{\partial \beta_0} = -2\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i + \beta_1 \bar{x}) = 0$$

$$\sum_{i=1}^{n}(y_i - \beta_1 x_i) - n\beta_0 + n\beta_1\bar{x} = 0$$

$$n\beta_0 = \sum_{i=1}^{n}(y_i - \beta_1 x_i) + n\beta_1\bar{x}$$

$$\beta_0 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \beta_1 x_i) + \beta_1\bar{x}$$

$$= \bar{y} - \beta_1\bar{x} + \beta_1\bar{x}$$

$$\beta_0 = \bar{y}$$

$$\frac{\partial g}{\partial b_1} = -2\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \beta_0 - \beta_1(x_i - \bar{x})) = 0$$

$$\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \beta_0 - \beta_1 x_i + \beta_1\bar{x}) = 0$$

$$\sum_{i=1}^{n}(x_i y_i - \beta_0 x_i - \beta_1 x_i^2 + \beta_1\bar{x}x_i - \bar{x}y_i + \bar{x}\beta_0 + \beta_1 x_i\bar{x} - \beta_1\bar{x}^2) = 0$$

$$\sum_{i=1}^{n}(x_i y_i - \beta_0 x_i - \beta_1 x_i^2 + \beta_1\bar{x}x_i - \bar{x}y_i + \beta_1 x_i\bar{x}) = (\beta_1\bar{x}^2 - \beta_0\bar{x})n$$

$$\frac{\partial g}{\partial \beta_1} = -2\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \beta_0 - \beta_1(x_i - \bar{x})) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i y_i - \beta_0 x_i - \beta_1 x_i^2 - \beta_1 \bar{x} x_i - \bar{x} y_i + \beta_0 \bar{x} + \beta_1 x_i \bar{x} - \beta_1 \bar{x}^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i - \beta_0 x_i - \beta_1 x_i^2 - \cancel{\beta_1 \bar{x} x_i} - \bar{x} y_i + \cancel{\beta_1 x_i \bar{x}} = (\beta_1 \bar{x}^2 - \beta_0 \bar{x})n$$

$$-\sum_{i=1}^{n} \beta_1 x_i^2 + \sum_{i=1}^{n} x_i y_i - \beta_0 x_i - \bar{x} y_i = n(\beta_1 \bar{x}^2 - \beta_0 \bar{x})$$

$$-\frac{1}{n}\sum_{i=1}^{n}\left[\beta_1 x_i^2 + x_i y_i\right] - \cancel{\beta_0 \bar{x}} - \bar{x}\bar{y} = \beta_1 \bar{x}^2 - \cancel{\beta_0 \bar{x}}$$

$$\beta_1 \overline{x^2} + \overline{xy} - \bar{x}\bar{y} = \beta_1 \bar{x}^2$$

$$\frac{1}{n}\sum_{i=1}^{n} x_i y_i - \bar{x} y_i = \beta_1 \bar{x}^2 - \beta_1 \overline{x^2}$$

$$\beta_1 = \frac{\frac{1}{n}\sum_{i=1}^{n} x_i y_i - \frac{x_i y_i}{n}}{\bar{x}^2 - \overline{x^2}}$$

Question 4

```
##
## Call:
## lm(formula = gamble ~ ., data = teengamb)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -51.082 -11.320  -1.451   9.452  94.252
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  22.55565   17.19680   1.312   0.1968
## sex         -22.11833    8.21111  -2.694   0.0101 *
## status        0.05223    0.28111   0.186   0.8535
## income        4.96198    1.02539   4.839 1.79e-05 ***
## verbal       -2.95949    2.17215  -1.362   0.1803
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 22.69 on 42 degrees of freedom
## Multiple R-squared:  0.5267, Adjusted R-squared:  0.4816
## F-statistic: 11.69 on 4 and 42 DF,  p-value: 1.815e-06
```

a) The R^2 value is 0.5267234. The predictors explain 52.7% of the variance.

b) Sex has the largest standard error (sum of the residuals) and is calculated to be 8.21

c) Mean of the residuals is 0 and the median is -1.45.

d) The correlation between the fitted and the measured values is 0.7257571.

e) The correlation between the fitted and income level is 0.857142.

f) Females on average spend 22.12 less on gambling per year than males.

Question 5

a)

```
##
## Call:
## lm(formula = taste ~ ., data = cheddar)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -17.390  -6.612  -1.009   4.908  25.449
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -28.8768    19.7354  -1.463  0.15540
## Acetic        0.3277     4.4598   0.073  0.94198
## H2S           3.9118     1.2484   3.133  0.00425 **
## Lactic       19.6705     8.6291   2.280  0.03108 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.13 on 26 degrees of freedom
## Multiple R-squared:  0.6518, Adjusted R-squared:  0.6116
## F-statistic: 16.22 on 3 and 26 DF,  p-value: 3.81e-06
```

b)

This value is calculated to be 0.6517747 and is equal to the R^2 value.

c)

```
##
## Call:
## lm(formula = taste ~ . + 0, data = cheddar)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.4521  -6.5262  -0.6388   4.6811  28.4744
##
## Coefficients:
##        Estimate Std. Error t value Pr(>|t|)
## Acetic   -5.454      2.111  -2.583  0.01553 *
## H2S       4.576      1.187   3.854  0.00065 ***
## Lactic   19.127      8.801   2.173  0.03871 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.34 on 27 degrees of freedom
## Multiple R-squared:  0.8877, Adjusted R-squared:  0.8752
## F-statistic: 71.15 on 3 and 27 DF,  p-value: 6.099e-13
```

The R^2 value without the intercept is 0.888. A more reasonable measure would be the adjusted R^2 which was found to be 0.875

d)

-28.8767696
0.3277413
3.9118411
19.6705434

#6

a) H is a $p \times p$ matrix ($n_{rows} = n_{columns} = p$)

b) $H^T = [X(X^TX)^{-1}X^T]^T = X[(X^TX)^{-1}]^T X^T$ $\quad (ABC)^T = C^TB^TA^T$

$= X[X^TX]^{-1}X^T = H$

c) $H^2 = X(X^TX)^{-1}\underbrace{X^TX(X^TX)^{-1}}_{I}X^T$

$= X(X^TX)^{-1}X^T = H$

d) $tr(H) = tr\left(X(X^TX)^{-1}X^T\right)$

$= tr\left(X^TX(X^TX)^{-1}\right) = tr(I)$

since H is a $p \times p$ matrix

$tr(I) = p$

e)

f) $rank(H) = p$

g) $\hat{y} = y - \hat{\varepsilon} = y - (y - Hy) = y - y + Hy$

$\hat{y} = Hy$

h) $\hat{\varepsilon} = y - \hat{y} = y - Hy = (I - H)y$

i) $(I - H)^T = I^2 - H^T = I - H$

j) $(I-H)(I-H) = I^2 - H - H + H^2 = I^2 - 2H + H = I - H$ $\left(\text{since } \begin{matrix} I^2 = I \\ H^2 = H \end{matrix}\right)$

k) $tr(I - H) = tr(I) - tr(H) = rank(I) - rank(H) = 0$

```r
library(faraway)
library(tidyverse)
library(cowplot)
head(prostate)
str(prostate)
summary(prostate)
prostate %>% pivot_longer(-svi, names_to = "Property", values_to = "Values") %>%
ggplot() + geom_boxplot(aes(x = Property, y = Values, fill = Property)) +
labs(x = "Property", y = "Value", title = "Boxplot of Prostate Data",
     subtitle = "by Halid Kopanski") +
theme(legend.position = "none")
#options(repr.plot.width = 6, repr.plot.height = 6, repr.plot.res = 150)
par(mfrow = c(4, 2))
hist(prostate$age, main = "Age", xlab = "", ylab = "")
hist(prostate$gleason, breaks = 8, main = "Gleason Score", xlab = "", ylab = "")
hist(prostate$lweight, main = "Prostate Weight", xlab = "", ylab = "")
hist(prostate$lbph, main = "Benign Prostatic Hyperplasia Amount", xlab = "", ylab = "")
hist(prostate$lcavol, main = "Cancer Volume", xlab = "", ylab = "")
hist(prostate$lcp, main = "Capsular Penetration", xlab = "", ylab = "")
hist(prostate$lpsa, main = "Prostate Specific Antigen", xlab = "", ylab = "")
hist(prostate$pgg45, main = "Percentage Gleason Scores(4 or 5)", xlab = "", ylab = "")
knitr::kable(table(prostate$svi), col.names = c("SVI Value", "Value Count"))

ggplot(prostate) + geom_bar(aes(as.factor(svi)), fill = "red", col = "blue") +
  labs(title = "Bar Plot of Seminal Vesicle Invasion (SVI)",
  subtitle = "by Halid Kopanski", x = "SVI", y = "")
prostate %>% pivot_longer(-svi, names_to = "Property", values_to = "Values") %>%
  group_by(Property) %>% mutate(outlier = !between(Values,
  as.numeric(quantile(Values)[2]) - 1.5 * IQR(Values),
  as.numeric(quantile(Values)[4]) + 1.5 * IQR(Values))) %>%
summarise(mean = mean(Values),
          iqr = IQR(Values),
          upper = as.numeric(quantile(Values)[2]) - 1.5 * IQR(Values),
          lower = as.numeric(quantile(Values)[4]) + 1.5 * IQR(Values),
          no_outliers = sum(outlier)) %>% knitr::kable()
lin_model <- lm(gamble ~ ., data = teengamb)
summary(lin_model)
r_sqr <- summary(lin_model)$r.squared

mu_res <- round(mean(lin_model$residuals), 2)
med_res <- round(median(lin_model$residuals), 2)

cor_fitted_actual <- cor(lin_model$fitted.values, teengamb$gamble)
cor_fitted_income <- cor(lin_model$fitted.values, teengamb$income)
lin_cheddar <- lm(taste ~ . , cheddar)
summary(lin_cheddar)
r_sqr_emp <- cor(lin_cheddar$fitted.values, cheddar$taste)^2
lin_cheddar2 <- lm(taste ~ . + 0, cheddar)
summary(lin_cheddar2)
qrX <- qr(model.matrix(~ Acetic + H2S + Lactic, cheddar))

Qf <- t(qr.Q(qrX)) %*% as.matrix(cheddar$taste)
```

```
param_reg <- backsolve(qr.R(qrX), Qf)
```