ST517 Note Outline 5: Estimation

Lecture 5.1: Point Estimation

Motivating Question: Given a random sample, how can we estimate the population parameter?

- Want a single 'best guess' (based on a sample) for the value of the parameter

- Recall: Sample statistics are random variables and thus have a sampling distribution

- Need some general guidelines for determining good estimators, based on the sampling distribution

Notation:

- θ =

- $\hat{\Theta}$ =

- $\hat{\theta}$ =

1^{st} Desired Property of Estimators

- $\hat{\Theta}$ is an **<u>unbiased estimator</u>** for θ if:
- "Unbiased" implies that the estimator's distribution
- If not:

Example: Show that for a random sample from a population with mean θ, $\hat{\Theta} = \bar{X}$ is unbiased for θ.

2^{nd} Desired Property of Estimators

- **<u>Precision</u>** of an estimator: We would ideally have an estimator that has
- Standard deviation of an estimator is called the
    - $\sigma^2_{\hat{\Theta}} = V(\hat{\Theta}) = E(\hat{\Theta} - E(\hat{\Theta}))^2$
    - $\sigma_{\hat{\Theta}} = \sqrt{V(\hat{\Theta})} = SE(\hat{\Theta})$

Example: For a random sample from a population with mean μ and variance σ^2, what is $SE(\bar{X})$?

Example: Let $X_1,\ldots X_n$ be a random sample from the Uniform distribution U$[0, \theta]$; for this distribution

$$f(x_i) = \frac{1}{\theta} \text{ for } 0 \le x_i \le \theta, i = 1, \ldots n; \quad E(X_i) = \frac{\theta}{2} \text{ and } V(X_i) = \frac{\theta^2}{12}$$

a. Let $\widehat{\Theta}_1 = 2\bar{X}$. Show that $\widehat{\Theta}_1$ is an unbiased estimator of θ.

b. Calculate $V(\widehat{\Theta}_1)$.

Important Points:

- There is a sometimes a tradeoff between bias and variability

- A more general criteria to measure the goodness of an estimator is the

$$MSE(\widehat{\Theta}) = E\left[\left(\widehat{\Theta} - \theta\right)^2\right] = V(\widehat{\Theta}) + \left[Bias(\widehat{\Theta})\right]^2$$

- The best estimator will

Lecture 5.2: Margin of Error

Recall:

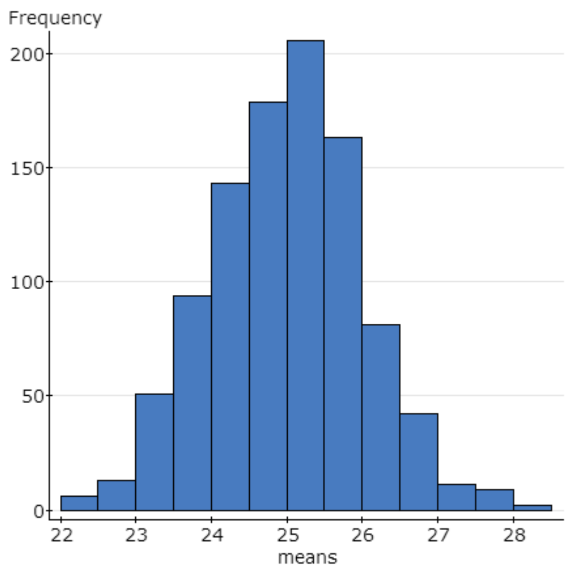
- A point estimate provides a single "best guess" for the true value of the parameter
  - Estimator $\widehat{\Theta}$ is a function of sample statistics
- Sampling variability = variation in sample statistics that results from selecting different random samples
  - Because there are different individuals in each sample
  - So the calculated value of $\widehat{\Theta}$ will vary from sample to sample
- Sampling distribution = distribution of possible values of the statistic
  - Shows predictable pattern of sampling variability
  - Allows us to see what values of a sample statistic are most common

Recall: Sampling Distribution of the Sample Mean $\bar{X}$

- Let $X_1, \ldots, X_n$ be a random sample from a population with mean μ and standard deviation σ
- The sampling distribution of $\bar{X}$:
  - Is well-modeled by a normal distribution **if**:
    - The population follows a normal distribution, OR
    - The sample size is large ($n \geq 30$; this result is called the Central Limit Theorem)
  - Is centered at mean μ
  - Has standard deviation $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

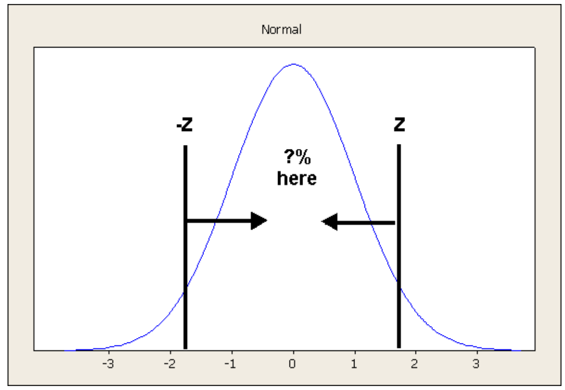
Margin of Error (MOE): Numeric indication of distance a statistic may be from true parameter

- Logic behind the margin of error
    - Determine a distance based on the sampling distribution
    - Put that distance around the statistic



How much should the margin of error be?

- Depends on the standard deviation of the statistic
- Also depends on the percent of the distribution we would have between the two lines



How often do we capture the parameter?

- Sometimes the margin of error will capture true parameter, sometimes it will not

- ____: Percent of possible samples in which the margin of error captures the population parameter.
    - Percentage of possible samples for which margin of error works
    - 90% confidence: Capture the true parameter in 90% of the possible samples
    - Based on sampling distribution of statistic, through the percentiles

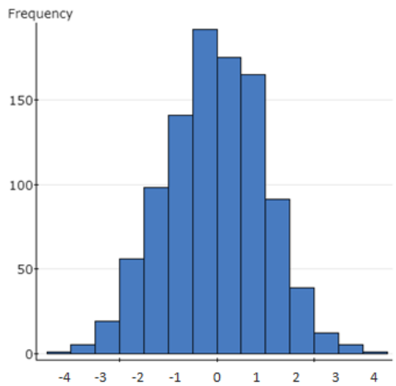
Margin of error (MOE)

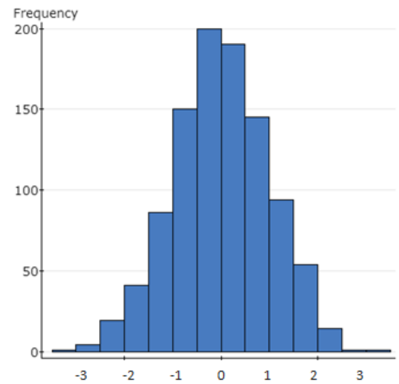
- In general: MOE = Critical value × Standard error

- For a population mean (when σ is known):

- Problem:

- Solution:



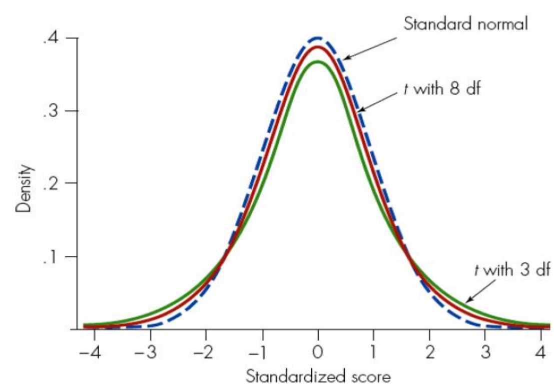


What if we don't know σ?

- If sample is large doesn't make much difference

- When we use *s*, there is added uncertainty in our sampling distribution

The t-distribution



- Similar to the standard normal

- More "squashed" than normal

- Depends on ***degrees of freedom (df)***

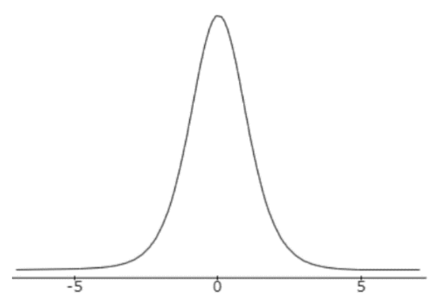
The t-distribution (continued):

- When *df* are large, the t-distribution is equivalent to the normal distribution

The t-distribution: Finding Percentiles

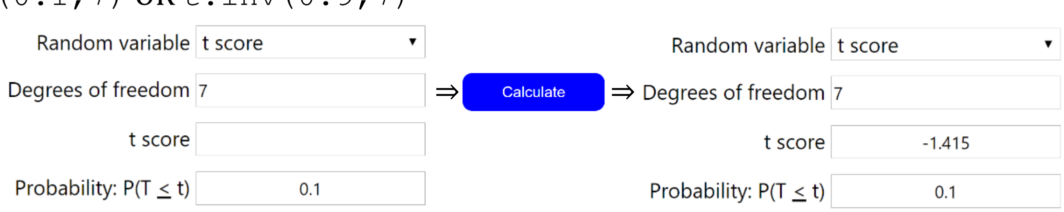
- Confidence coefficients are percentiles of a distribution
- As with finding percentiles under a Normal distribution, focus on conceptual understanding and use technology to calculate the value
    - Graphing calculator (e.g. TI-84): `invT(proportion to left, df)`
    - Software, e.g.
        - SAS: `tinv(proportion to left, df)`
        - Excel: `t.inv(proportion to left, df)`
    - Online calculators
        - E.g. stattrek.com/online-calculator/t-distribution.aspx
        - Fill in: Value of *df* and the proportion to the left; computer will provide the t-score that is the value of the confidence coefficient

Example: For a t-distribution with 7 degrees of freedom, what is the confidence coefficient for 80% confidence?



Using the technology for this example:

- Graphing calc: `invT(0.1,7)` OR `invT(0.9,7)`
- SAS: `tinv(0.1,7)` OR `tinv(0.9,7)`
- Excel: `t.inv(0.1,7)` OR `t.inv(0.9,7)`
- Online calc:



- From each of these, the value of the multiplier is:
    - Don't worry about sign! Just report the positive value

Summary: MOE for a population mean (when σ is unknown)

Lecture 5.3 Confidence Intervals

Confidence Interval (CI)

- In general: Statistic ± Margin of Error

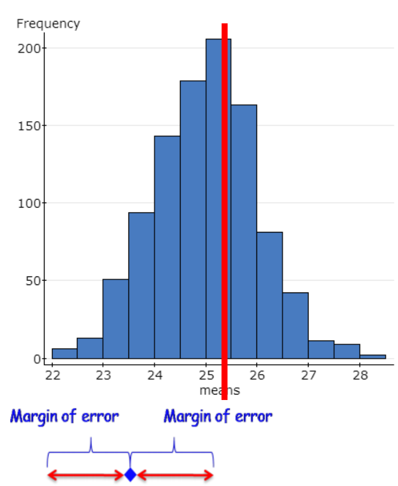
- Common way to present an **interval estimate**

- This interval should contain the population parameter

CI for a population mean (when σ is unknown):

- Conditions required for this interval to be valid:

Visually:



Example: How often do you laugh in a typical day? A study randomly selects 30 adults and records how often they laugh in a day. The sample average is 21 laughs per day with a standard deviation of 13.7 laughs.

a. Calculate a 90% CI for the parameter of interest here and interpret this interval in the context of the problem.

b. Calculate a 95% CI for the parameter of interest here and interpret this interval in the context of the problem.

Lecture 5.4: Important Points about Confidence

- A larger level of confidence produces a larger margin of error.

- A large sample size produces a smaller margin of error.

- News media uses 95% confidence by convention.

- Confidence is arbitrary.

- MOE only accounts for random sampling variability.

- The confidence interval is about the parameter not about the statistic or the individuals.

- Confidence is in the procedure.

Example: Data collected by child development scientists produced the following 90% CI for the average age (in weeks) at which babies begin to crawl: (29.2, 31.8). You can assume that the conditions necessary for inference using this interval were met.

a. What was the sample average age at which babies begin to crawl found by these scientists?

b. What was the margin of error for this interval?

c. Complete the following interpretations within the context of the problem:

   <u>Interpretation of confidence interval</u>: We are _______ confident that the true value of the average age at which babies begin to crawl is between _______ and _______ weeks.

   <u>Interpretation of confidence level</u>: If we take many samples of the same size, about _____ of the resulting confidence intervals would contain the true average age at which babies begin to crawl.

d. Below are three <u>incorrect</u> interpretations for the confidence interval. For each, explain why the interpretation is incorrect.

   1. "90% of all babies begin to crawl between 29.2 and 31.8 weeks of age."

   2. "90% of all samples will have mean ages between 29.2 and 31.8 weeks."

   3. "The mean age at which babies begin to crawl is between 29.2 and 31.8 weeks 90% of the time."