ST_518 Project

MA, HK, BA, JF

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Executive Summary

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Introduction

The "Effervescent Experiment" was conducted by dissolving two different brands of cold medicine tablets at three different, equally spaced water temperatures using a complete block design. We will fit several different models to determine the one that best fits the data and hope to discover the effects, if any, of temperature and brand on dissolving time.

Experimental Design

For this study, we are presented with data from an 'Effervescent Experiment'. The data contains the dissolving times of two different brands of cold medicine tablets that were obtained under various conditions. Those conditions include varying water temperatures (6°, 23°, 40°) and the presence of stirring (magnetic stir bar at 350 rpm). This was a complete block design with stirring acting as the blocking effect.

Exploratory Data Analysis

Summary

The provided dataset contains 48 rows and 6 columns. The 6 columns include 3 explanatory variables (Brand, Temp, Stirred categorical factors), a single response variables (Time, a continuous variable), and 1 descriptor (sample order). Prior to starting any analysis, we will explore the data to gain an understanding of what to expect and to check for violations of any assumptions.

From the summary statistics table (see Appendix 1, Table 1), we can see that each group has exactly 4 entries, eliminating concerns with respect to design imbalance.

There are insights to extract from this table. We can see that there does appear to be a disparity between the mean dissolving times of store brand cold medications when compared to name brand. Store brand cold medicines dissolve in a shorter amount of time as a whole over all effects than does the name brand medicine. This disparity becomes even more pronounced as the effect of temperature is introduced and as the temperature increases. When the brand type is store the means between each of the same temperature effects are much closer in similarity than the between temperature of name brand medicines. The differences between means store brand medicines of 6°, 23° and 40° are 2.15, 1.32 and 0.162 respectively. The differences between means name brand medicines of 6°, 23° and 40° are 2.79, 3.67, and 4.70 respectively. This may indicate an interaction effect on dissolving times by brand as the average difference in dissolving times is 1.21 for store brand and 3.72 for name brand.

Next, consider the range in variability at each factor and level. The range in values of the variance is 6.9458; an interesting result considering nine of the twelve observable variances fall within a range of 0.04 and 1.69. The variability between observations of name brand cold medicines are elevated, especially in cases when the observation was stirred at 23° and 40°. Variance of non-stirred observations, particularly within temperature values of 23° and 40°, are noticeably lower than their stirred counterparts within the same brand. The variance seems to jump by a large amount between the groups. Contrast analysis might be a concern due to the small sample size.

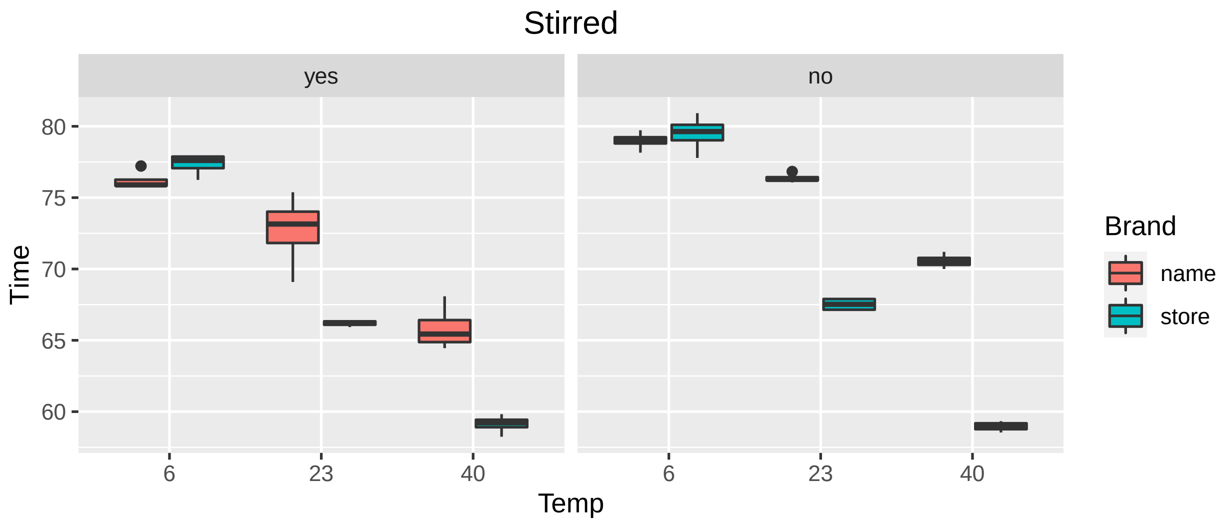
Further contextualizing central tendency and spread, we can see this illustrated in a different perspective by grouping effects together. Take note that temperature has the smallest set of ranges, both at each individual level and as a whole when compared to brand type and stirring effect. There is not a lot of varability among temperature compared to the other effects though they are different enought to identify an irregular increases as temperature increases. The range of values provides additional context to the data's story; data points when focused only on temperature groups tends to be within a smaller range of each other indicating less variability; meanwhile brand and stirring effects have a wider distribution and more variability–outliers withstanding.

Interactions

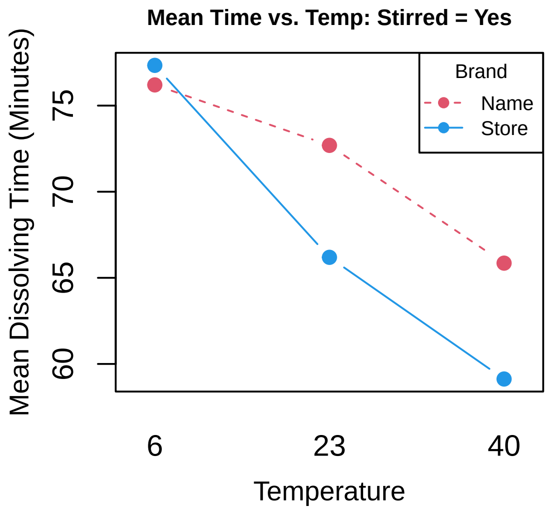
From the boxplots below, we can immediately see that stirring seems to increase the variance of the name-brand medicine–while also decreasing the mean differences of brand within each temperature grouping. An interaction effect between temperature and brand can be deduced if lines are drawn through the centers of the boxes. Earlier, we had introduce an insight from the summary statistics output indicating an inverse relationship between temperature and dissolve time–as temperature increases dissolve time decreases. The boxplot reinforces this idea. In fact, we can also make the claim that temperature has an inverse effect on dissolving times whether stirring is present or not–indications of temperature having a strong effect on dissolving time by itself. Stirring might have an additive effect regardless of temperature.

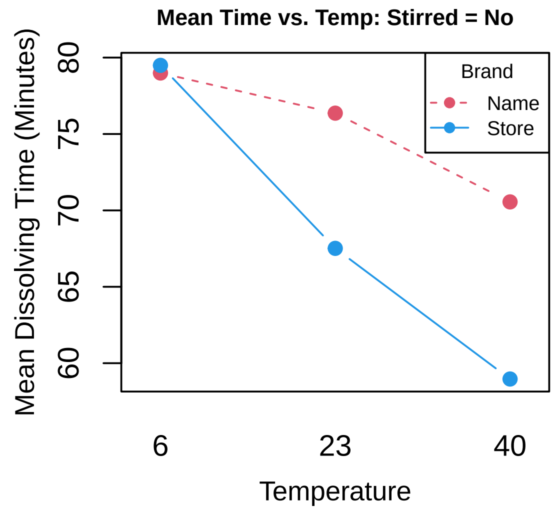
It is simply conjecture at this point, however we noticed that observations of dissolve time while stirring the water seems to have increased the name brand variability, while not stirring the water seems to have increased the store brand variability. Perhaps worth looking into the blocking effects of stirred on variability at various temperatures.

Outliers are present in our boxplots and we will address these data points when looking at assumptions.



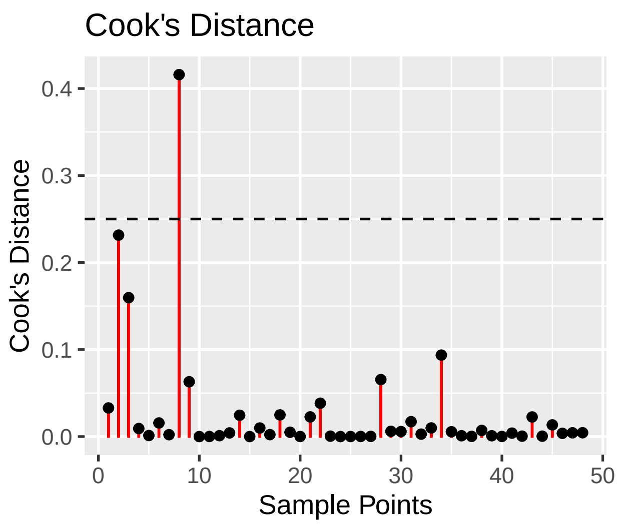
The possible interaction between brand and temperature becomes even more noticeable in the preceding three-factor interaction plots. Specifically, the brand and temperature interaction can be seen when the temperature increases. The slope for the store brand has a more pronounced negative slope than the slope of the name brand. In addition, there might be a slight three-factor interaction between brand, temperature, and stirring as the name and store brand lines appear to be closer together in the stirred=yes plot than the stirred=no plot.

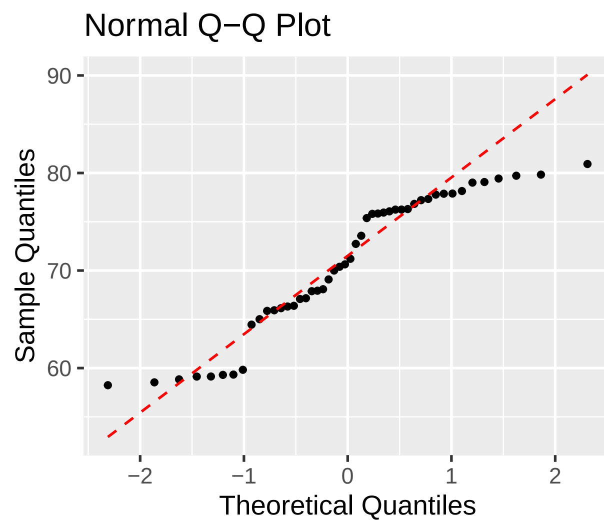




Assumptions and Violations

From the boxplots, we were able to see a small number of outliers. To confirm if there is any concern we plotted the Cook's Distance for each point based on a full linear model. Point 8 has a higher Cook's distance than the rest of the points which may require removal for analysis if it is suspected of causing issues in the analysis. This would have to be weighed against the risks caused by introducing imbalances.





Finally, we check the normality of the data. Here a Q-Q plot is generated for the full model residuals. The data appears to suffer from heavy tails, multimodality and/or gaps in data between the left tail and the center. Since downstream analysis hinges on the assumption that our data is normally distributed, these issues may pose a problem.

Analysis and Results

Contrasts

Conducting a linear contrast analysis on each of the explanatory variables reveals that there are significant differences between groups based on factors, see Appendix 1 Table 2, 3, 4, and 5 for full results.

In the first case, we contrasted the means of stirred versus not stirred. Here the difference in means is -2.41 with an upper 95% confidence limit of -3.04 and a lower 95% CI limit of -1.78. In other words, on average stirring medicine reduces dissolving time by between 3.04 and 1.78 minutes regardless of brand or temperature. When looking only at brand, name brand dissolving times were on average between 4.71 and 5.97 (95% CI) minutes slower than store brand. Since neither of the intervals contained zero we can conclude that there is a difference between brands and between the presence of stirring.

While significant for both store and name brands, stirring had more of an impact to dissolving times for name brand than it did for the store brand. Stirring reduced name brand dissolving times by 2.83 and 4.61 minutes whereas for the store brand that interval was 0.22 and 2 minutes.

A similar analysis was completed for the three levels of temperature. Completing a contrast analysis using a Bonferroni correction we found that in pairwise cases each level was significantly different from the other. The 95% confidence limits were (6.25, 8.38), (13.32, 15.44), and (6.00, 8.13) for the pair wise comparisons of $6°C\ vs\ 23°C$, $6°C\ vs\ 40°C$, and $23°C\ vs\ 40°C$, respectively. Zero did not fall in any of those ranges. When comparing individual levels versus the remainder of the group, $23°C$ was found not to be significantly different from the rest of the levels. That confidence interval ranged from -1.04 to 0.80 minutes of dissolving time. Due to that, we do not have enough evidence to say $23°C$ is different from either $6°C$ or $40°C$.

Model Development

The following models were developed and analyzed for this paper:

$Model\ 1 : Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\gamma\beta)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijkl}$
$Model\ 2 : Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + \epsilon_{ijkl}$
$Model\ 3 : Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijkl}$

Where α is brand effect, β is temperature effect, γ is stir effect. i, j, k are (1, 2), (1,2,3), and (1,2), respectively. ϵ_{ijkl} is assumed to be normally distributed with a μ_ϵ of 0 and a variance of σ^2_ϵ. μ is the overall mean and is an unknown value.

Mixed Effects models:
$Model\ 4 : Y_{ijk} = \mu + \alpha_i + B_j + (\alpha B)_{ij} + \epsilon_{ijk}$
Where brand is fixed and temperature is random.
$Model\ 5 : Y_{ijk} = \mu + A_i + \beta_j + (A\beta)_{ij} + \epsilon_{ijk}$
Where brand is random and temperature is fixed.
$Model\ 6 : Y_{ijk} = \mu + A_i + B_j + (AB)_{ij} + \epsilon_{ijk}$
Where both brand and temperature are random.

With order as a factor:
$Model\ 7 : Y_{ijklm} = \mu + \alpha_i + \beta_j + \gamma_k + \nu_l + (\alpha\beta)_{ij} + \epsilon_{ijklm}$
Similar to model 2, but with an introduced effect, ν, to represent order.

$Model\ 8 : Y_{ijklm} = \mu + \alpha_i + \beta_j + \gamma_k + \nu_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\gamma\beta)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijklm}$
Similar to model 1, but with an introduced effect, ν, to represent order.

Model Selection

Model	Root MSE	R^2	$adj\ R^2$	AIC	BIC
1	1.074835	0.9824498	0.9770873	155.3374101	179.6630232
2	1.3185414	0.9699208	0.965519	171.1984162	186.1680243
3	2.655107	0.8720833	0.860184	236.6806137	247.9078198
7	1.302961	0.9713439	0.966329	170.8720441	187.7128532
8	1.0868873	0.9825525	0.9765706	157.0556795	183.2524936

The mixed/random effects Models 4, 5, 6 all had Root MSE=1.833 (see Appendix 1, Table 13).

Results

Conclusion

Appendix I: Analysis Tables and Figures

Table 2: Data Summary Table

Brand	Temp	Stirred	Min	25%	Mean	Median	75%	Max	Range	Var	n
name	6	yes	75.80973	75.83358	76.20241	75.89223	76.26107	77.21547	1.4057377	0.4593492	4
name	6	no	78.15246	78.79910	78.99061	79.04435	79.23586	79.72130	1.5688327	0.4146440	4
name	23	yes	69.08937	71.82180	72.69145	73.14894	74.01859	75.37855	6.2891789	6.9869087	4
name	23	no	76.06895	76.20492	76.36351	76.27622	76.43481	76.83265	0.7636940	0.1078134	4
name	40	yes	64.45156	64.87321	65.85343	65.43863	66.41886	68.08492	3.6333543	2.5499751	4
name	40	no	69.99943	70.28754	70.55511	70.50947	70.77705	71.20207	1.2026434	0.2544033	4
store	6	yes	76.24402	77.06561	77.33703	77.60659	77.87801	77.89089	1.6468708	0.5964884	4
store	6	no	77.78345	79.01994	79.49240	79.63219	80.10465	80.92176	3.1383169	1.6942517	4
store	23	yes	65.92809	66.08831	66.19126	66.22629	66.32923	66.38436	0.4562787	0.0411024	4
store	23	no	67.08353	67.14393	67.51552	67.52360	67.89520	67.93138	0.8478521	0.2060739	4
store	40	yes	58.24407	58.90895	59.12529	59.21659	59.43293	59.82388	1.5798100	0.4320148	4
store	40	no	58.53920	58.76884	58.96347	58.99050	59.18513	59.33370	0.7945066	0.1202191	4

Table 3: Least Squares Means

Brand	Temp	Stirred	emmean	SE	df	lower.CL	upper.CL
name	6	yes	76.20241	0.5374175	36	75.11248	77.29235
store	6	yes	77.33703	0.5374175	36	76.24709	78.42696
name	23	yes	72.69145	0.5374175	36	71.60152	73.78138
store	23	yes	66.19126	0.5374175	36	65.10132	67.28119
name	40	yes	65.85343	0.5374175	36	64.76350	66.94337
store	40	yes	59.12529	0.5374175	36	58.03535	60.21522
name	6	no	78.99061	0.5374175	36	77.90068	80.08055
store	6	no	79.49240	0.5374175	36	78.40247	80.58233
name	23	no	76.36351	0.5374175	36	75.27358	77.45344
store	23	no	67.51552	0.5374175	36	66.42559	68.60546
name	40	no	70.55511	0.5374175	36	69.46518	71.64505
store	40	no	58.96347	0.5374175	36	57.87354	60.05341

Table 4: Contrast Stirred and Brand

contrast	estimate	SE	df	lower.CL	upper.CL
stirred	-2.413294	0.3102781	36	-3.042567	-1.784021
branding	5.338595	0.3102781	36	4.709322	5.967868

Table 5: Contrast Stirred versus Brand

contrast	estimate	SE	df	lower.CL	upper.CL
stirredbrand	-3.720646	0.4387996	36	-4.610573	-2.8307197
stirredstore	-1.105942	0.4387996	36	-1.995869	-0.2160151

Table 6: Contrast Temperatures

contrast	estimate	SE	df	lower.CL	upper.CL
temp6__23	7.3151767	0.3800116	36	6.254195	8.3761584
temp6__40	14.3812861	0.3800116	36	13.320305	15.4422678
temp23__40	7.0661094	0.3800116	36	6.005128	8.1270910
temp6__rest	10.8482314	0.3290997	36	9.929394	11.7670685
temp23__rest	-0.1245337	0.3290997	36	-1.043371	0.7943034
temp40__rest	-10.7236978	0.3290997	36	-11.642535	-9.8048607

Table 7: Model 1: ANOVA Table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Brand	1	342.007	342.007	296.041	0.000
Temp	2	1654.737	827.368	716.169	0.000
Stirred	1	69.888	69.888	60.495	0.000
Brand:Temp	2	231.852	115.926	100.345	0.000
Brand:Stirred	1	20.510	20.510	17.753	0.000
Temp:Stirred	2	0.125	0.062	0.054	0.948
Brand:Temp:Stirred	2	9.056	4.528	3.919	0.029
Residuals	36	41.590	1.155	NA	NA

Table 8: Model 1: Type III ANOVA Table

	Sum Sq	Df	F value	Pr(>F)
(Intercept)	23227.231	1	20105.451	0.000
Brand	2.575	1	2.229	0.144
Temp	221.582	2	95.901	0.000
Stirred	15.548	1	13.458	0.001
Brand:Temp	80.110	2	34.672	0.000
Brand:Stirred	0.400	1	0.347	0.560
Temp:Stirred	3.668	2	1.588	0.218
Brand:Temp:Stirred	9.056	2	3.919	0.029
Residuals	41.590	36	NA	NA

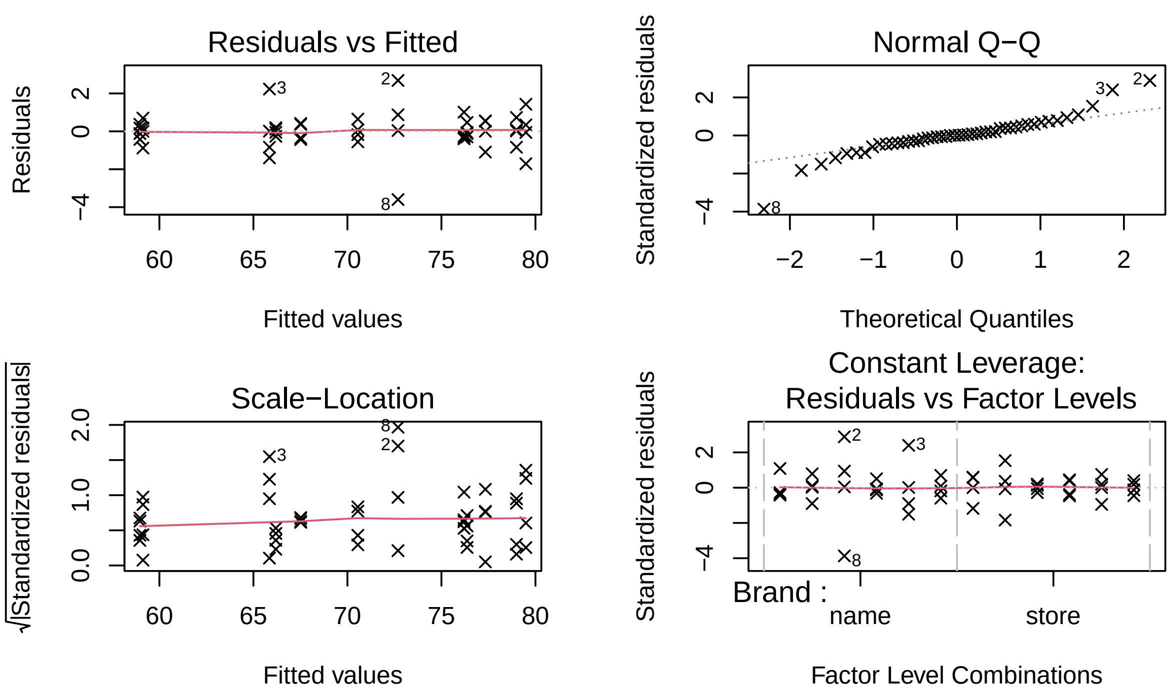


Table 9: Model 2: ANOVA Table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Brand	1	342.00715	342.007154	196.71962	0e+00
Temp	2	1654.73655	827.368276	475.89522	0e+00
Stirred	1	69.88787	69.887866	40.19891	1e-07
Brand:Temp	2	231.85191	115.925956	66.67963	0e+00
Residuals	41	71.28061	1.738551	NA	NA

Table 10: Model 2: Type III ANOVA Table

	Sum Sq	Df	F value	Pr(>F)
(Intercept)	40014.251	1	23015.858	0.000
Brand	2.678	1	1.540	0.222
Temp	366.976	2	105.541	0.000
Stirred	69.888	1	40.199	0.000
Brand:Temp	231.852	2	66.680	0.000
Residuals	71.281	41	NA	NA

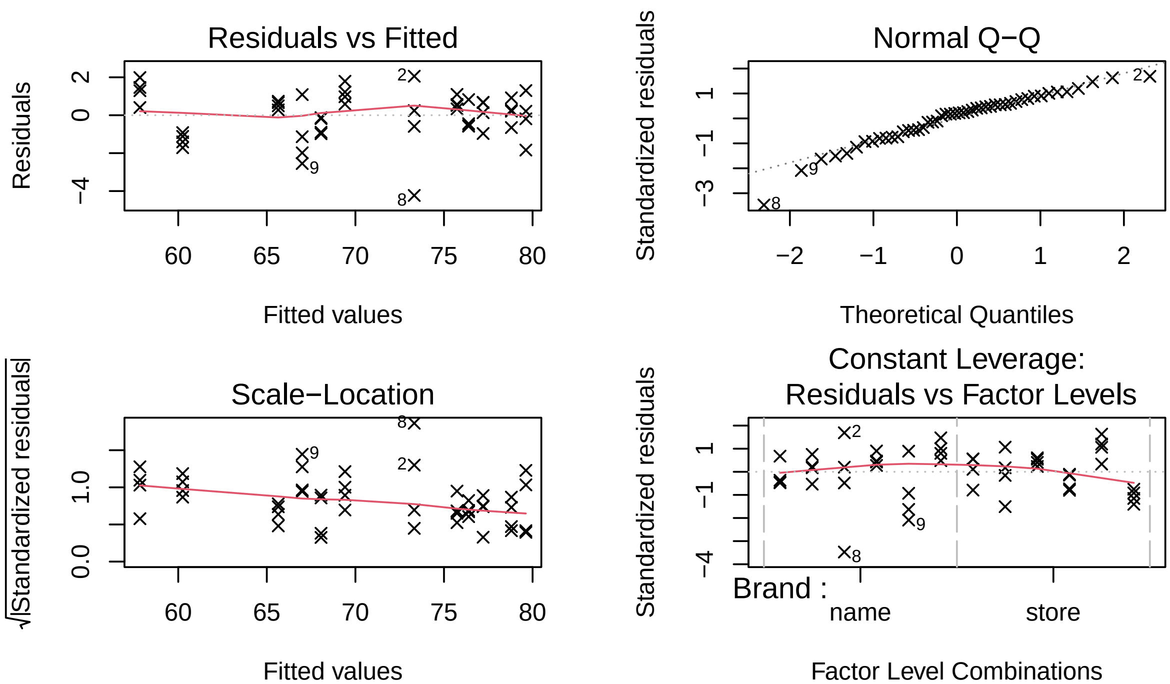


Table 11: Model 3: ANOVA Table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Brand	1	342.00715	342.007154	48.514451	0.0000000
Temp	2	1654.73655	827.368276	117.363970	0.0000000
Stirred	1	69.88787	69.887866	9.913744	0.0029802
Residuals	43	303.13252	7.049593	NA	NA

Table 12: Model 3: Type III ANOVA Table

	Sum Sq	Df	F value	Pr(>F)
(Intercept)	60625.967	1	8599.924	0.000
Brand	342.007	1	48.514	0.000
Temp	1654.737	2	117.364	0.000
Stirred	69.888	1	9.914	0.003
Residuals	303.133	43	NA	NA

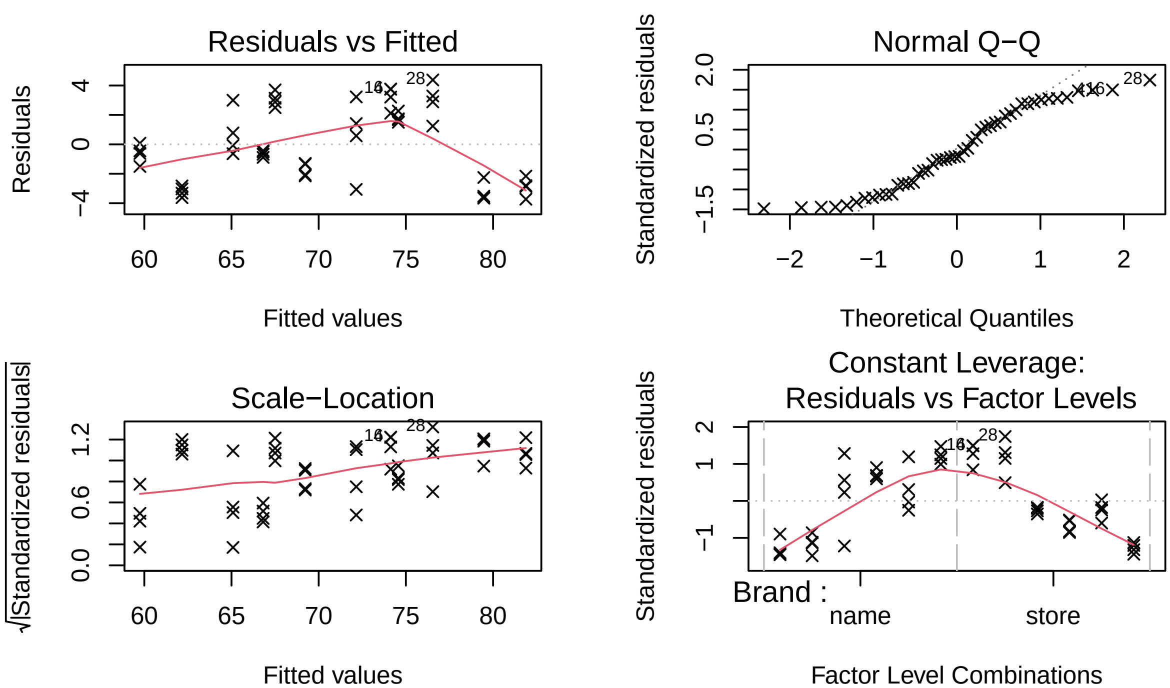


Table 13: Mixed Effects Models

Source	Df	Sum Sq	Mean Sq	Error Term	Error Df	F Score	Pr>F
Brand	1	342.0072	342.007154	MSAB	2	2.950221	0.2280
Temp	2	1654.7366	827.368276	MSAB	2	7.137041	0.1229
Brand*Temp	2	231.8519	115.925956	MSE	42	34.491507	0.0000
Residual	42	141.1685	3.361154	NA	NA	NA	NA

Table 14: Model4: Brand is Fixed and Temperature is Random

Brand	Mean	Effect
name	73.443	0.000
store	68.104	-5.339

Cov Parm	Estimate
Temp	44.465
Brand*Temp	14.071
Residual	3.361

Table 16: Model5: Brand is Random and Temperature is Fixed

Temp	Mean	Effect
6	78.006	0.000
23	70.690	-7.315
40	63.624	-14.381

Cov Parm	Estimate
Brand	9.42
Brand*Temp	14.071
Residual	3.361

Table 18: Model6: Brand and Temperature are Random

Cov Parm	Estimate
Brand	9.42
Temp	44.465
Brand*Temp	14.071
Residual	3.361

Table 19: Model 7: ANOVA Table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Brand	1	342.0071543	342.0071543	201.4523573	0.0000000
Temp	2	1654.7365514	827.3682757	487.3444529	0.0000000
Stirred	1	69.8878657	69.8878657	41.1660257	0.0000001
Order	1	0.9059095	0.9059095	0.5336076	0.4693512
Brand:Temp	2	234.3183134	117.1591567	69.0102180	0.0000000
Residuals	40	67.9082953	1.6977074	NA	NA

Table 20: Model 7: Type III ANOVA Table

	Sum Sq	Df	F value	Pr(>F)
(Intercept)	22049.754	1	12987.959	0.000
Brand	3.577	1	2.107	0.154
Temp	335.887	2	98.924	0.000
Stirred	6.156	1	3.626	0.064
Order	3.372	1	1.986	0.166
Brand:Temp	234.318	2	69.010	0.000
Residuals	67.908	40	NA	NA

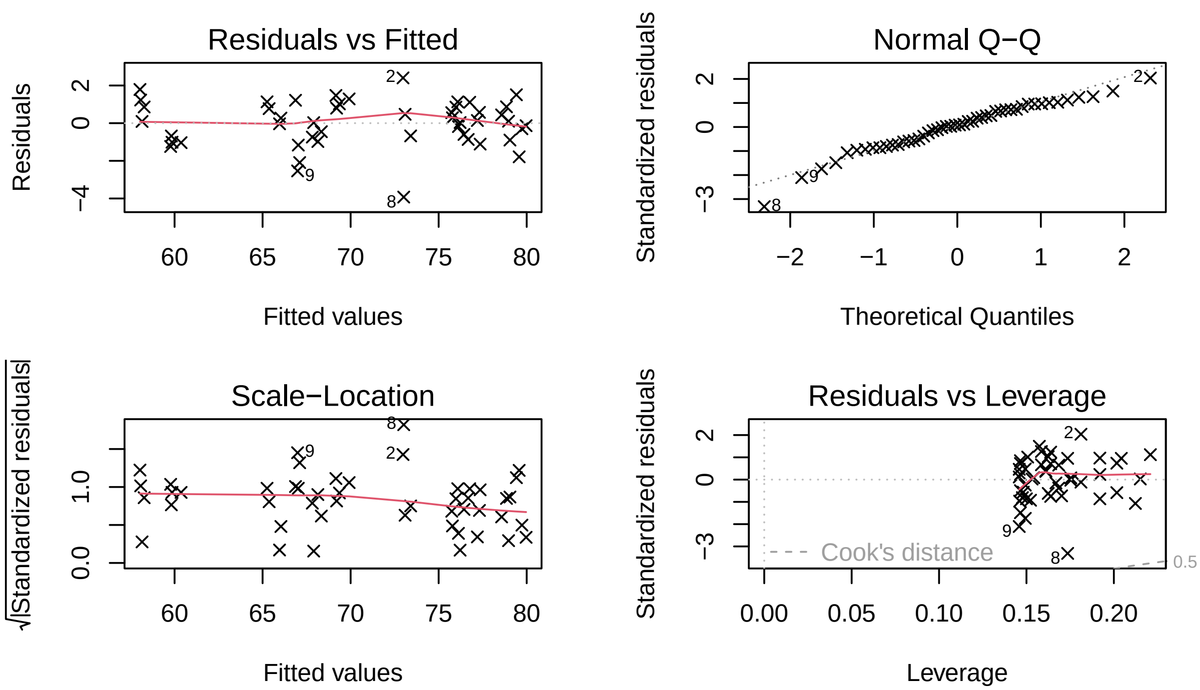
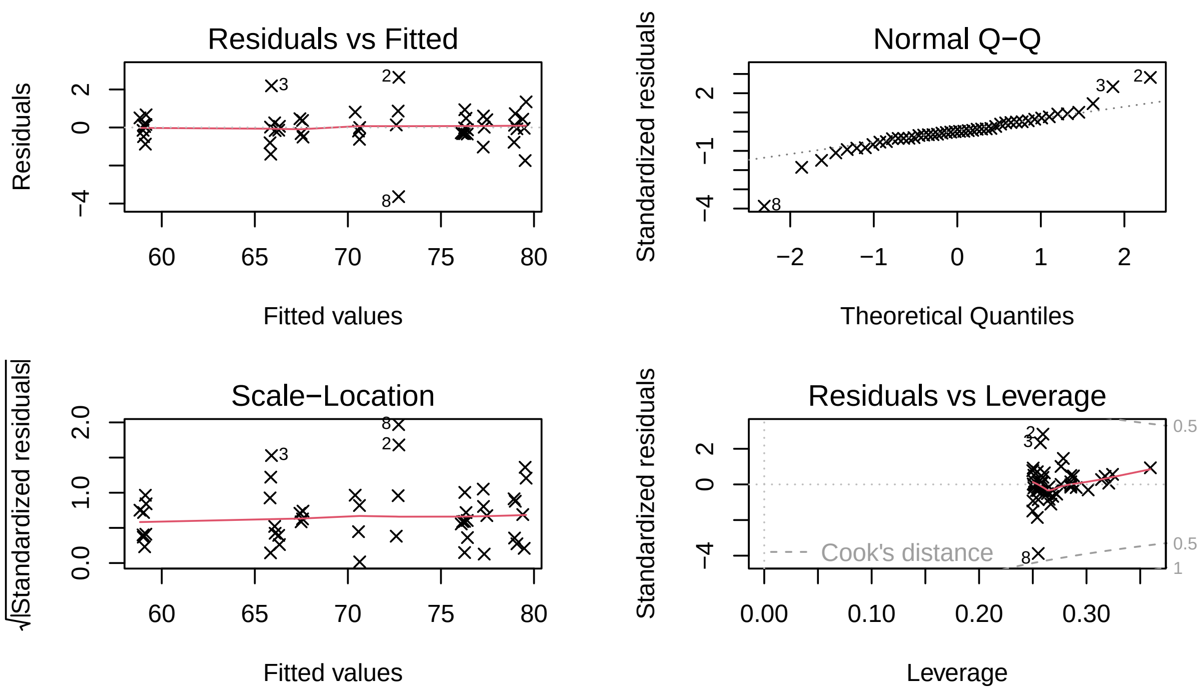


Table 21: Model 8: ANOVA Table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Brand	1	342.0071543	342.0071543	289.5117212	0.0000000
Temp	2	1654.7365514	827.3682757	700.3736925	0.0000000
Stirred	1	69.8878657	69.8878657	59.1606229	0.0000000
Order	1	0.9059095	0.9059095	0.7668595	0.3871609
Brand:Temp	2	234.3183134	117.1591567	99.1761391	0.0000000
Brand:Stirred	1	17.2952414	17.2952414	14.6405566	0.0005144
Temp:Stirred	2	0.0420436	0.0210218	0.0177951	0.9823712
Brand:Temp:Stirred	2	9.2246692	4.6123346	3.9043772	0.0294693
Residuals	35	41.3463412	1.1813240	NA	NA

Table 22: Model 8: Type III ANOVA Table

	Sum Sq	Df	F value	Pr(>F)
(Intercept)	15086.643	1	12770.961	0.000
Brand	2.356	1	1.994	0.167
Temp	220.855	2	93.478	0.000
Stirred	9.911	1	8.390	0.006
Order	0.243	1	0.206	0.653
Brand:Temp	69.452	2	29.396	0.000
Brand:Stirred	0.409	1	0.346	0.560
Temp:Stirred	3.500	2	1.482	0.241
Brand:Temp:Stirred	9.225	2	3.904	0.029
Residuals	41.346	35	NA	NA



Appendix: Code

```
library(tidyverse)
library(emmeans)
library(lme4)
library(lmerTest)
library(olsrr)
library(car)
library(cowplot)
df_eff <- read_csv('effervescence.csv', col_types = 'fffnnn')
df_stats <-
df_eff %>% group_by(Brand, Temp, Stirred) %>%
summarise('Min' = min(Time),
          '25%' = quantile(Time, probs = 0.25),
          'Mean' = mean(Time),
          'Median' = median(Time),
          '75%' = quantile(Time, probs = 0.75),
          'Max' = max(Time),
          'Range' = Max - Min,
          'Var' = var(Time),
          'n' = n())

#knitr::kable(df_stats)
df_eff %>% ggplot() + geom_boxplot(aes(fill = Brand, y = Time, x = Temp)) +
  facet_grid(cols = vars(Stirred)) + labs(title = "Stirred") + theme(
  plot.title = element_text(hjust = 0.5)
)
##3 factor interaction plot based on HW7 code
par(mfrow=c(1,2), mar = c(3.5,3.5,2,2))
with(df_eff%>%filter(Stirred=="yes"),interaction.plot(Temp,Brand,Time,
            type="b", pch=19, col=c(2,4), ylab="", xlab = "",
            main="Mean Time vs. Temp: Stirred = Yes",
            cex.main = 0.75, legend = FALSE))
legend("topright",
       title = "Brand",
       c("Name", "Store"),
       cex = 0.7,
       col = c("#DF536B","#2297E6"),
       pch = c(19,19), lty = c(2,1))
title(xlab = "Temperature", ylab = "Mean Dissolving Time (Minutes)", line = 2.25, cex.lab = 0.9)
#```

#```{r, echo=FALSE, message=FALSE, error=FALSE, fig.dim=c(6,3), dpi=250}
with(df_eff%>%filter(Stirred=="no"),interaction.plot(Temp,Brand,Time,
          type="b", pch=19, col=c(2,4), ylab="", xlab = "",
          main="Mean Time vs. Temp: Stirred = No",
          cex.main = 0.75, legend = FALSE))
legend("topright",
       title = "Brand",
       c("Name", "Store"),
       cex = 0.7,
       col = c("#DF536B","#2297E6"),
       pch = c(19,19), lty = c(2,1))
```

```
title(xlab = 'Temperature', ylab = "Mean Dissolving Time (Minutes)", line = 2.25, cex.lab = 0.9)
#model1
aov_eff <- aov(lm_eff <- lm(Time ~ Brand * Temp * Stirred, data = df_eff))


cooksD_values <- cooks.distance(lm_eff)

CD_plot <- ggplot() +
  geom_col(aes(y = cooksD_values, x = 1:length(cooksD_values)),
  width = 0.025, col = 'red')  +
  geom_point(aes(y = cooksD_values, x = 1:length(cooksD_values))) +
  xlab('Sample Points') + ylab("Cook's Distance") +
  geom_hline(yintercept = 0.25, lty = 2) +
  labs(title = "Cook's Distance")

#CD_plot <- ols_plot_cooksd_chart(lm_eff)
qqplot1 <- ggplot(df_eff, aes(sample = Time)) +
  stat_qq(shape = 20) +
  stat_qq_line(linetype = "dashed", col = 'red') +
  labs(x = "Theoretical Quantiles",
       y = "Sample Quantiles",
       title = "Normal Q-Q Plot")

plot_grid(CD_plot, qqplot1)
#qqnorm(df_eff$Time, pch = 20)
#qqline(df_eff$Time, col = "maroon", lwd = 2)
means_eff <- emmeans(aov_eff, specs = c('Brand', 'Temp', 'Stirred'))
#summary(means_eff)
cont_str_brd <-
contrast(means_eff, list(stirred = c(1/6, 1/6, 1/6, 1/6, 1/6, 1/6, -1/6, -1/6, -1/6, -1/6, -1/6, -1/6),
                         branding = rep(c(1/6,-1/6), 6)
                         )
         )

cont_strbrd <-

contrast(means_eff, list(stirredbrand = c(1/3, 0,  1/3, 0, 1/3, 0, -1/3, 0, -1/3, 0, -1/3, 0),
                         stirredstore = c(0, 1/3, 0,  1/3, 0, 1/3, 0, -1/3, 0, -1/3, 0, -1/3)
                         )
         )
cont_temp <-
contrast(means_eff, list(temp6_23 = c(1/4, 1/4, -1/4, -1/4, 0, 0, 1/4, 1/4, -1/4, -1/4, 0, 0),
                         temp6_40 = c(1/4, 1/4, 0, 0, -1/4, -1/4, 1/4, 1/4, 0, 0, -1/4, -1/4),
                         temp23_40 = c(0, 0, 1/4, 1/4, -1/4, -1/4, 0, 0, 1/4, 1/4, -1/4, -1/4),
                         temp6_rest = c(1/4, 1/4, -1/8, -1/8, -1/8, -1/8, 1/4, 1/4, -1/8, -1/8, -1/8, -
                         temp23_rest = c(-1/8, -1/8, 1/4, 1/4, -1/8, -1/8, -1/8, -1/8, 1/4, 1/4, -1/8, -
                         temp40_rest = c(-1/8, -1/8, -1/8, -1/8, 1/4, 1/4, -1/8, -1/8, -1/8, -1/8, 1/4,
                         ), options=list(adjust="bonferroni")
         )
#knitr::kable(confint(cont_str_brd))
#knitr::kable(confint(cont_strbrd))
#knitr::kable(confint(cont_temp))
#par(mfrow=c(2,2), mar = c(5,5,2,2))
```

```r
#plot(aov_eff)
#knitr::kable(summary(aov_eff)[[1]], 'simple', caption = 'Model 1 ANOVA Results')
#model with stirred as block effect without interaction
#model2
aov_block_eff <- aov(lm_block_eff <- lm(Time ~ Brand * Temp + Stirred, data = df_eff))
#model3
aov_block_eff_noint <- aov(lm_block_eff <- lm(Time ~ Brand + Temp + Stirred, data = df_eff))
#summary(lm_block_eff)
#knitr::kable(summary(aov_block_eff)[[1]], caption = "Model 2: ANOVA Table")
#par(mfrow=c(2,2), mar = c(5,5,2,2))
#plot(aov_block_eff)
#ols_plot_cooksd_chart(lm_block_eff)
#added covariate Order model with stirred as block effect without interaction
#model7
aov_block_order_eff <- aov(lm_block_order_eff <- lm(Time ~ Brand * Temp + Stirred + Order, data = df_ef
#summary(lm_block_order_eff)
#knitr::kable(summary(aov_block_order_eff)[[1]], caption = "Model 3: ANOVA Table")
#Anova(aov_block_order_eff, type=3) # type 3 SS
#par(mfrow=c(2,2), mar = c(5,5,2,2))
#plot(lm_block_order_eff)
#ols_plot_cooksd_chart(lm_block_order_eff)
#added covariate Order to model with 3 factor interaction
#model8
aov_three_order_eff <- aov(lm_three_order_eff <- lm(Time ~ Brand * Temp * Stirred + Order, data = df_ef
#summary(lm_three_order_eff)
#knitr::kable(summary(aov_three_order_eff)[[1]], 'simple', caption = "Model 7: ANOVA Table")
#Anova(aov_three_order_eff, type=3) # type 3 SS
#par(mfrow=c(2,2), mar = c(5,5,2,2))
#plot(lm_three_order_eff)
RMSE_function <- function(df_aov){

    r_mse <- sqrt(sum(df_aov$residuals^2)/df_aov$df)

    r_s <- 1 - tail(summary(df_aov)[1][[1]][[2]], n = 1) / sum(summary(df_aov)[1][[1]][2])

    a_r_s <- 1 - (1 - r_s)*(nrow(df_aov$model) - 1)/(df_aov$df)

    aic_ <- AIC(df_aov)

    bic_ <- BIC(df_aov)

    output_stats <- c(
        r_mse,
        r_s,
        a_r_s,
        aic_,
        bic_
    )
    return(output_stats)
}


model1 <- RMSE_function(aov_eff)
```

```r
model2 <- RMSE_function(aov_block_eff)
model3 <- RMSE_function(aov_block_eff_noint)
model7 <- RMSE_function(aov_block_order_eff)
model8 <- RMSE_function(aov_three_order_eff)
knitr::kable(df_stats, caption = "Data Summary Table")
knitr::kable(summary(means_eff), caption = "Least Squares Means")
knitr::kable(confint(cont_str_brd), caption = "Contrast Stirred and Brand")
knitr::kable(confint(cont_strbrd), caption = "Contrast Stirred versus Brand")
knitr::kable(confint(cont_temp), caption = "Contrast Temperatures")
knitr::kable(summary(aov_eff)[[1]], 'simple', digits = 3, caption = 'Model 1: ANOVA Table')
knitr::kable(Anova(aov_eff, type=3), 'simple', digits = 3, caption = 'Model 1: Type III ANOVA Table') #
par(mfrow=c(2,2), mar = c(5,5,2,2))
plot(aov_eff, pch = 4)
knitr::kable(summary(aov_block_eff)[[1]],
             'simple' , caption = "Model 2: ANOVA Table")
knitr::kable(Anova(aov_block_eff, type=3),
             'simple', digits = 3, caption = 'Model 2: Type III ANOVA Table')
par(mfrow=c(2,2), mar = c(5,5,2,2))
plot(aov_block_eff, pch = 4)
knitr::kable(summary(aov_block_eff_noint)[[1]],
             'simple', caption = "Model 3: ANOVA Table")
knitr::kable(Anova(aov_block_eff_noint, type=3),
             'simple', digits = 3, caption = 'Model 3: Type III ANOVA Table')
par(mfrow=c(2,2), mar = c(5,5,2,2))
plot(aov_block_eff_noint, pch = 4)

lm_eff_me <- lm(Time ~ Brand * Temp, data = df_eff)
aov_eff_me <- aov(lm_eff_me)
anova_eff_me <- anova(lm_eff_me)

me_table <- as_tibble(summary(aov_eff_me)[[1]][,1:3])
me_table <- cbind('Source' = c('Brand', 'Temp', 'Brand*Temp', 'Residual'),
                  me_table,
                  'Error Term' = c("MSAB", "MSAB", "MSE", "NA"))

MSA <-  me_table[1, 4]
MSB <-  me_table[2, 4]
MSAB <- me_table[3, 4]
MSE <-  round(me_table[4, 4],3)

a <- length(levels(df_eff$Brand))
b <- length(levels(df_eff$Temp))
n <- nrow(df_eff)/(a*b)

sigma_ab <- round((MSAB - MSE) / n,3)
sigma_a <- round((MSA - MSAB) / (b * n), 3)
sigma_b <- round((MSB - MSAB) / (a*n), 3)

f_scores <- c(MSA/MSAB,MSB/MSAB,MSAB/MSE, NA)

error_dof <- c(rep(tail(me_table$Df, n = 2)[1], 2), tail(me_table$Df, n = 1)[1], NA)

me_table['Error Df'] <- error_dof
```

```
me_table["F Score"] <- f_scores
f_test <- round(1 - pf(me_table['F Score'][[1]],
                       me_table['Df'][[1]],
                       me_table['Error Df'][[1]]),
                4)

me_table['Pr>F'] <- f_test



name_mean <- df_eff %>% filter(Brand == 'name') %>% select(Time) %>% unlist() %>% mean()
brand_fe <- df_eff %>% group_by(Brand) %>% summarise('Mean' = mean(Time), 'Effect' = Mean - name_mean)

name_mean <- df_eff %>% filter(Temp == '6') %>% select(Time) %>% unlist() %>% mean()
temp_fe <- df_eff %>% group_by(Temp) %>% summarise('Mean' = mean(Time), 'Effect' = Mean - name_mean)
knitr::kable(me_table, caption = 'Mixed Effects Models')

knitr::kable(brand_fe, digits = 3,
             caption = 'Model4: Brand is Fixed and Temperature is Random')

knitr::kable(cbind(c('Temp', 'Brand*Temp', 'Residual'),
                   c(sigma_b, sigma_ab, MSE)), col.names = c('Cov Parm', 'Estimate'), caption = "")
knitr::kable(temp_fe, caption = 'Model5: Brand is Random and Temperature is Fixed',
             digits = 3)
knitr::kable(cbind(c('Brand', 'Brand*Temp', 'Residual'),
                   c(sigma_a, sigma_ab, MSE)), col.names = c('Cov Parm', 'Estimate'),
             caption = "")
knitr::kable(cbind(c('Brand', 'Temp' , 'Brand*Temp', 'Residual'),
                   c(sigma_a, sigma_b,sigma_ab, MSE)), col.names = c('Cov Parm', 'Estimate'),
             caption = 'Model6: Brand and Temperature are Random')
knitr::kable(summary(aov_block_order_eff)[[1]],
             'simple', caption = "Model 7: ANOVA Table")
knitr::kable(Anova(aov_block_order_eff, type=3), 'simple',
             digits = 3, caption = 'Model 7: Type III ANOVA Table')
par(mfrow=c(2,2), mar = c(5,5,2,2))
plot(aov_block_order_eff, pch = 4)
knitr::kable(summary(aov_three_order_eff)[[1]],
             'simple', caption = "Model 8: ANOVA Table")
knitr::kable(Anova(aov_three_order_eff, type=3),
             'simple', digits = 3, caption = 'Model 8: Type III ANOVA Table')
par(mfrow=c(2,2), mar = c(5,5,2,2))
plot(lm_three_order_eff, pch = 4)
```