Question/Claim Prove that for any natural number n,

$$\sum_{i=1}^{n} 2^i = 2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2 \tag{1}$$

Proof By mathematical induction.

Initial Step For $n = 1$, the identity (1) reduces to:

$$2 = 2^{1+1} - 2 = 4 - 2 = 2$$

which is true since both sides are equal to 2.

Inductive Step Assume identity (1) is true for n:

$$\sum_{i=1}^{n} 2^i = 2^{n+1} - 2 \tag{2}$$

Add 2^{n+1} to both sides of (2):

$$2^{n+1} + \sum_{i=1}^{n} 2^i = 2^{n+1} + 2^{n+1} - 2 \tag{3}$$

By definition of summation and algebra, (3) reduces to:

$$\sum_{i=1}^{n+1} 2^i = 2 * 2^{n+1} - 2 = 2^{(n+1)+1} - 2$$

Which is identity (1) with $n + 1$ in place of n. Hence by the principle of mathematical induction, the identity holds for all $n \in \mathcal{N}$.