Question Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).

Claim The sum of any five consecutive integers is divisible by 5.

Proof By algebra and definition of divisibility.

For any arbitrary consecutive 5 integers, let $n \in \mathcal{Z}$ be the smallest integer among them. In other words, the 5 consecutive integers are:

$$n, n+1, n+2, n+3, n+4.$$

We shall show that $5 | \sum_{i=n}^{n+4} i$ is true or

$$(\exists m \in \mathcal{Z})(5m = \sum_{i=n}^{n+4} i) \tag{1}$$

$$\begin{aligned} \sum_{i=n}^{n+4} i &= n + n + 1 + n + 2 + n + 3 + n + 4 \\ &= 5n + 10 = 5(n+2) \end{aligned}$$

Let $m = n + 2$. Clearly, m is an integer and satisfies (1). Hence by definition of divisibility, $(\forall n \in \mathcal{Z})(5 | \sum_{i=n}^{n+4} i)$. In other words, the sum of any five consecutive integers is divisible by 5.