Question Say whether the following is true or false and support your answer by a proof. $(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12)$

Claim $(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12)$ is false.

Proof By contradiction.

Assume the following claim is true:

$$(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12) \tag{1}$$

Observe that, since natural numbers are positive:

$$3m > 0, 5n > 0 \tag{2}$$

Also note that $3m < 12$. Otherwise, $5n = 12 - 3m \leq 0$, which contradicts (2). Hence, the possible values of m is:

$$\{m | m \in \mathcal{N}, 0 < 3m < 12\} = \{1, 2, 3\}$$

Case 1 Assume that $m = 1$ satisfies (1). The claim is equivalent to:

$$(\exists n \in \mathcal{N})(5n = 12 - 3(1) = 9)$$

This is not true since 9 is not divisible by 5. In light of this contradiction, $m = 1$ does not satisfy (1).

Case 2 Assume that $m = 2$ satisfies (1). The claim is equivalent to:

$$(\exists n \in \mathcal{N})(5n = 12 - 3(2) = 6)$$

This is also not true since 6 is not divisible by 5. In light of this contradiction, $m = 2$ does not satisfy (1).

Case 3 Assume that $m = 3$ satisfies (1). The claim is equivalent to:

$$(\exists n \in \mathcal{N})(5n = 12 - 3(3) = 3)$$

This is also not true since 3 is not divisible by 5. In light of this contradiction, $m = 3$ does not satisfy (1).

From all cases above, there is no pair of natural numbers m, n that satisfies (1). In light of this contradiction, the following statement must be false:

$$(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12)$$