Question/Claim Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

Proof By Division Theorem and elimination of invalid forms.

By the Division Theorem, any integer can be expressed in one of the following forms:

$$4n, 4n + 1, 4n + 2, 4n + 3, n \in \mathcal{Z}. \tag{1}$$

Since natural numbers are subset of integers, each of them can also be expressed in one of the forms listed in (1).

Among those listed in (1),

$$4n = 2(2n)$$

which is clearly even since $2|4n$ is true by definition of divisibility. Similarly,

$$4n + 2 = 2(2n + 1)$$

which is also even since $2|(4n + 2)$ is true by definition of divisibility.

Since both $4n, 4n + 2$ are even, odd numbers cannot be expressed as such. Hence, every odd natural number is of one of the remaining forms $4n + 1$ or $4n + 3, n \in \mathcal{Z}$.