Question/Claim Prove that for any integer n, at least one of the integers $n, n+2, n+4$ is divisible by 3.

Proof By Division Theorem and definition of divisibility.

Let arbitrary $n \in \mathcal{Z}$ be given. By the Division Theorem, n can be expressed in one of the forms:

$$3m, 3m+1, 3m+2, m \in \mathcal{Z}. \tag{1}$$

We shall prove the claim by examining each possible form of n in (1).

Case 1 Assume $n = 3m$. Hence, n is divisible by 3 since $a = m$ satisfies the following divisibility definition:

$$(\exists a \in \mathcal{Z})(3a = n)$$

Case 2 Assume $n = 3m + 1$. This implies:

$$n + 2 = 3m + 1 + 2 = 3(m + 1)$$

Hence, $n + 2$ is divisible by 3 since $a = m + 1$ satisfies the following divisibility definition:

$$(\exists a \in \mathcal{Z})(3a = n + 2)$$

Case 3 Assume $n = 3m + 2$. This implies:

$$n + 4 = 3m + 2 + 4 = 3(m + 2)$$

Hence, $n + 4$ is divisible by 3 since $a = m + 2$ satisfies the following divisibility definition:

$$(\exists a \in \mathcal{Z})(3a = n + 4)$$

From the three cases above, for any integer n, at least one of the integers $n, n+2, n+4$ is divisible by 3.