

Spike spectra for recurrences

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Abstract: A novel kind of power spectrum is constructed, the *inter spike spectrum*, which transforms any signal into its spike-frequency domain. This method clearly shows the apparent cycles in the data and overcomes the problems for spike-train-like signals when using the obvious idea of Fourier-transforming it. We invent this instructive approach with the idea of transforming the τ -recurrence rate of a recurrence plot (RP), which often has a spiky appearance. The τ -recurrence rate is the density of recurrence points along diagonals of the RP, which are parallel to the main diagonal with a distance of τ . In this context the inter spike spectrum can be interpreted as a nonlinear power spectrum of a potentially high dimensional system which constitutes the RP. The proposed measure is robust to noise and is able to detect and analyze bifurcations.

Keywords: Decomposition; Frequency Analysis; Recurrence Analysis; Bifurcations

PACS: 05.45.Tp, 05.90.+m, 89.90.+n, 02.70.Uu, 05.10.Ln, 05.45.-a, 05.45.Ac

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Text.

Text.

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**Figure 2.** This is a wide figure.

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MDPI Multidisciplinary Digital Publishing Institute
DOAJ Directory of open access journals
TLA Three letter acronym
LD Linear dichroism

Appendix A

Appendix A.1

The appendix is an optional section that can contain details and data supplemental to the main text—for example, explanations of experimental details that would disrupt the flow of the main text but nonetheless remain crucial to understanding and reproducing the research shown; figures of replicates for experiments of which representative data are shown in the main text can be added here if brief, or as Supplementary Data. Mathematical proofs of results not central to the paper can be added as an appendix.

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Entry 1	Data	Data
Entry 2	Data	Data

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References

- Author 1, T. The title of the cited article. *Journal Abbreviation* **2008**, *10*, 142–149.
- Author 2, L. The title of the cited contribution. In *The Book Title*; Editor 1, F., Editor 2, A., Eds.; Publishing House: City, Country, 2007; pp. 32–58.
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