In the name of Allah, the Beneficent, the Merciful.

Assignment No.4

Artificial Neural Networks

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Q. No.1 (a):

The weight matrix for a Hopfield network with 5 nodes will be a 5x5 matrix, with weights W_{ij} in column i and row j having all diagonal value 0 because the diagonal elements W_{ii} as self-connection are not allowed in Hopfield networks.

The threshold vector for this network would be a 5-dimentional vector, with θ_i representing the bias of neuron i.

So, the weight matrix and threshold vector for the given network would be as follows:

$$W = \begin{bmatrix} 0 & -2 & 1 & 5 & 11 \\ -2 & 0 & -2 & -6 & -10 \\ 1 & -2 & 0 & 3 & 5 \\ 5 & -6 & 3 & 0 & -5 \\ 11 & -10 & 5 & -5 & 0 \end{bmatrix}, \qquad \theta = \begin{bmatrix} 2 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Q. No.1 (b): Asynchronous Update:

Update the nodes by using following equation:

$$Updated\ node = \sum W_{ij} * z_0 + \theta$$

First update Node 1

$$= 0 - 1 + 0 + 0 - 4 - 2 - 5 = -12$$

As the -12 < 0, so the state of node 1 remains -1.

Similarly, we can calculate the remaining nodes and their states. So, for the first iteration the updated 6 nodes and their states will be as follows:

Iteration 1 = $(-12, 0, 5, 4, 4, -1)^T$ with $z_1 = (-1, -1, 1, -1, 1, 1)^T$

Similarly, for we repeat the process and till the network converges.

Iteration 2 = $(-12, 0, 5, 4, 4, -1)^T$ with $z_2 = (-1, -1, 1, -1, -1, 1)^T$

Iteration 3 = $(-4, -16, 17, -2, -8, 7)^T$ with $z_3 = (-1, -1, 1, -1, -1, 1)^T = z_2$

So, algorithm converges here.

Q. No.1 (c): Synchronous Update:

Update the nodes by using following equation:

$$Updated\ node = \sum W_{ij} * z_0 + \theta$$

Iteration 1 = $(-12, 0, 5, 4, 4, -1)^T$ with $z_1 = (-1, -1, 1, 1, 1, -1)^T$

Iteration 2 = $(-8, 6, -5, -16, -4, -1)^T$ with $z_2 = (-1, 1, -1, -1, -1, 1)^T$

Iteration 3 = $(-2, -8, 9, 18, 12, 3)^T$ with $z_3 = (-1, -1, 1, 1, 1, 1)^T$

Iteration 4 = $(-12, -2, -9, -10, -16, 1)^T$ with $z_4 = (-1, -1, -1, -1, -1, 1))^T$

Iteration 5 = $(-2, -8, 9, 18, 12, 3)^T$ with $z_5 = (-1, -1, 1, 1, 1, 1)^T = z_3$

The network does not converge but switches between states z_3 and z_4.

Q. No.2 (a):

We can calculate the weight matrix for a Hopfield Network with two vectors from the following equation.

$$W_{ij} = \frac{1}{m}\sum_{k=1}^{m} x_i^k x_j^k$$

When i = j, $W_{ij} = 0$

For vector x_1:

$$W = \frac{1}{2}\begin{pmatrix} 0 & -1 & 1 & -1 & 1 & 1 \\ -1 & 0 & -1 & 1 & -1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ -1 & 1 & -1 & 0 & -1 & -1 \\ 1 & -1 & 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & -1 & 1 & 0 \end{pmatrix}$$

For vector x_2:

$$W = \frac{1}{2}\begin{pmatrix} 0 & 1 & 1 & -1 & -1 & -1 \\ 1 & 0 & 1 & -1 & -1 & -1 \\ 1 & 1 & 0 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 & 0 \end{pmatrix}$$

Now just add the weights for vector 1 and vector 2 to get final weight for the network.

$$W = \frac{1}{2}\begin{pmatrix} 0 & -1 & 1 & -1 & 1 & 1 \\ -1 & 0 & -1 & 1 & -1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ -1 & 1 & -1 & 0 & -1 & -1 \\ 1 & -1 & 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & -1 & 1 & 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 1 & 1 & -1 & -1 & -1 \\ 1 & 0 & 1 & -1 & -1 & -1 \\ 1 & 1 & 0 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 & 0 \end{pmatrix}$$

$$W = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Q. No.2 (b):

To confirm that the vectors $x_1 = (1, -1, 1, -1, 1, 1)^T$ and $x_2 = (1, 1, 1, -1, -1, -1)^T$ are stable states of the network, we can compute the output of the network when these vectors are input. If the output of the network is the same as the input, then the vector is a stable state of the network.

In a Hopfield network, the output of the network is given by the equation

$$\mathrm{y} = \mathrm{f}(\mathrm{w} * \mathrm{x} + \mathrm{b})$$

In a Hopfield network, the activation function is typically chosen to be the sign function, which maps each element of its input to 1, 0, or -1 depending on whether the element is positive, zero, or negative, respectively. The bias vector is typically set to zero.

$$y = \mathrm{f}(\mathrm{w} * (1,-1,1,-1,1,1)^T + 0)$$

$$y = (2,-2,2,-2,2,2)^T = (1,-1,1,-1,1,1)^T = x_1$$

Since the output of the network is the same as the input, x_1 is a stable state of the network.

Similarly, the output of the network when x_2 is input would be:

$$y = \mathrm{f}(\mathrm{w} * (1,1,1,-1,-1,-1)^T + 0)$$

$$y = (2,2,2,-2,-2,-2)^T = (1,1,1,-1,-1,-1)^T = x_2$$

Since the output of the network is the same as the input, x_2 is also a stable state of the network.

Q. No. 3 & 4:

As per the requirements the code is implemented in the notebooks. Please see the notebooks.