

# Mesh Smoothing based on Anisotropic Mean Curvature Flow

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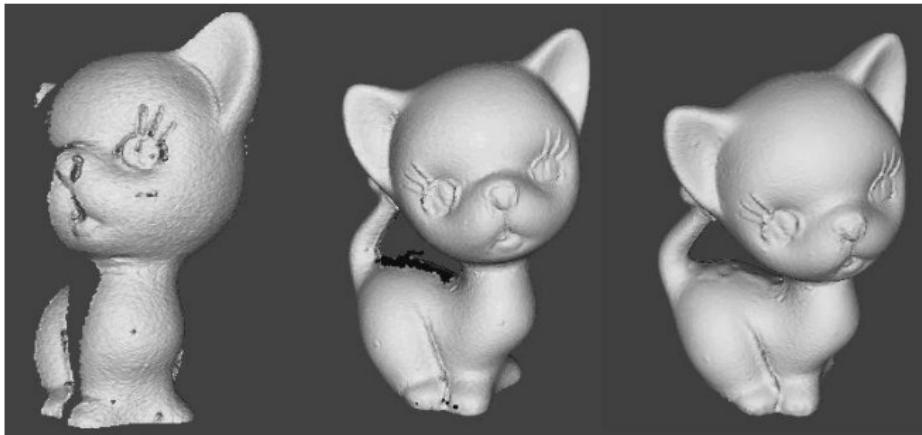
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# Outline

- 1 Motivation
- 2 Anisotropic Prescribed Mean Curvature
- 3 Results
- 4 Summary and Future work

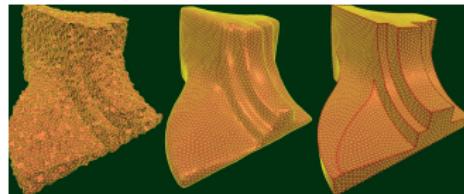
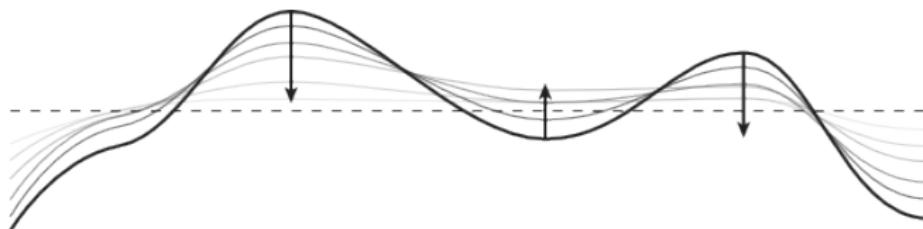
# Geometry pipeline



- Many range image patches are acquired by a 3D scanner
- Patches are aligned and merged
- Holes are filled and the mesh is **smoothed**

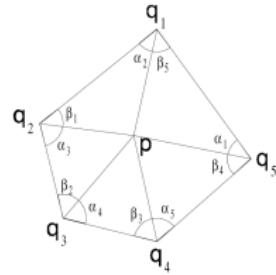
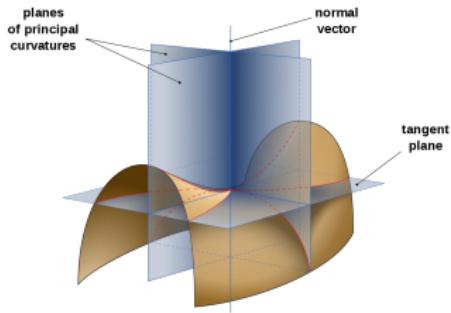
# Remove noise from 3D scanners

- Idea: Diffuse high curvature region by averaging over its neighborhood.



- Isotropic e.g. Laplacian: sharp features not preserved
- Anisotropic mean curvature: preserves sharp features

# Mean Curvature



- Curvature: measure deviation from flat
- Mean Curvature: average between min and max curvatures
- Smooth: reduce curvature (equivalent to area gradient)

$$\nabla_p \text{area } M = \vec{H}(p) = 1/2 \sum_{q_i \in \text{link } p} (\cot\alpha_i + \cot\beta_i)(p - q_i)$$

# Mean Curvature

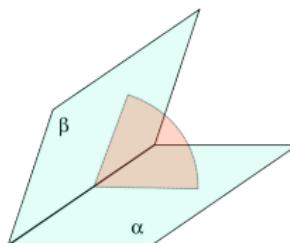


Figure : Dihedral angle  $\theta_e$

- Let  $\vec{H}(e) = H_e \vec{N}_e$  be an edge mean curvature vector, where  $\vec{N}_e$  is edge normal and  $H_e = 2|e|\cos(\theta_e/2)$  then:

$$\vec{H}(p) = 1/2 \sum_{e=(p,q), q \in \text{link } p} \vec{H}(e)$$

# Anisotropic Mean Curvature

- Weight less for feature vertices to avoid smoothing sharp features

$$\vec{H}_A(p) = 1/2 \sum_{e=pq, q \in \text{link } p} w(H_e) H_e \vec{N}(e)$$

$$w_{\lambda,r}(a) = \begin{cases} 1 & \text{for } |a| \leq \lambda \\ \frac{\lambda^2}{r(\lambda - |a|)^2 + \lambda^2} & \text{for } |a| > \lambda \end{cases}$$

# Anisotropic mean curvature flow

- Explicit iteration step of the anisotropic mean curvature flow

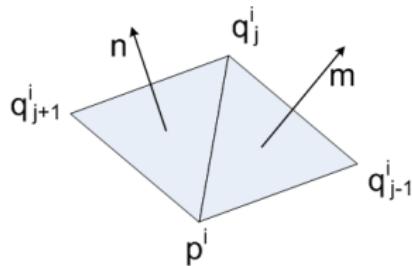
$$p^{j+1} = p^j - \frac{3s}{\text{area(star } p^j)} \vec{H}_A(p^j)$$

- s controls the speed of an integration step
- Matrix form:

$$\mathcal{P}^{j+1} = \mathcal{P}^j - sM^{-1}\vec{H}_A(\mathcal{P}^j)$$

$$M_{pq} = \begin{cases} \frac{1}{6} \text{area(star } p), & \text{if } p = q \\ \frac{1}{12} \text{area(star } e), & \text{if there is an edge } e = (p, q) \\ 0, & \text{otherwise} \end{cases}$$

# Anisotropic mean curvature - Matrix form



$$\begin{aligned}
 \vec{H}_A(p^i) &= 1/2 \sum_{q_j^i \in N(p^i)} w(H_{e_j}) H_{e_j} \frac{1}{\|\mathbf{n} + \mathbf{m}\|} (\mathbf{n} + \mathbf{m}) \\
 &= \sum_{q_j^i \in N(p^i)} w_j \left[ (q_j^i - p^i) \times (q_{j+1}^i - p^i) + (q_{j-1}^i - p^i) \times (q_j^i - p^i) \right] \\
 &= \sum_{q_j^i \in N(p^i)} w_j \left[ (q_j^i - p^i) \times (q_{j+1}^i - q_{j-1}^i) \right] \\
 &= \sum_{q_j^i \in N(p^i)} w_j \left[ A_\times \cdot q_{j+1}^i - A_\times \cdot q_{j-1}^i \right]
 \end{aligned}$$

# Put them all together

- sparse global matrix containing blocks of  $3 \times 3$  cross matrices
- row index corresponds to each one-ring center  $p^i$
- column index corresponds to a one-ring neighbor  $q_j^i$

$$\vec{H}_A(\mathcal{P}^i) = \begin{bmatrix} \cdots & \cdots & \cdots \\ \sum_{q_j^i \in N(p^i)} w_j A_x & \cdots & - \sum_{q_j^i \in N(p^i)} w_j A_x \\ \cdots & \cdots & \cdots \end{bmatrix}_{3n \times 3n} \cdot \begin{bmatrix} q_{j+1}^i \\ \vdots \\ q_{j-1}^i \end{bmatrix}_{3n}$$

# Drawback of Anisotropic mean curvature

- slow down the smoothing process in regions with high curvature.
- cause deformations of the surface

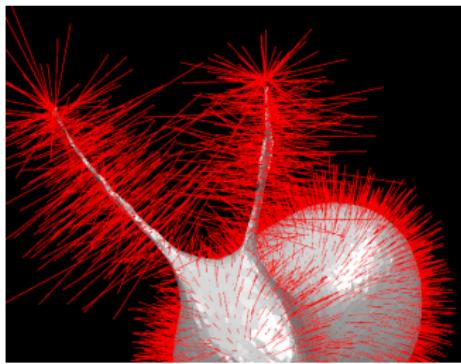


Figure : anisotropic mean curvature after 100 iterations

# Prescribed Mean Curvature

- Energy minimization. AMC:  $\vec{H}_A(p) \rightarrow 0$ , APMC:  $\vec{H}_A(p) \rightarrow H\vec{V}_A(p)$ , where  $H$  is some constant mean curvature
- compute mean curvature ( $H$ ), smooth this scalar field
- evolve the surface towards a surface having this smoothed mean curvature

$$p^{j+1} = p^j - \frac{3s}{\text{area(star } p^j)} (\vec{H}_A(p^j) - f(p^j) \vec{V}_A(p^j))$$

- $f$  is a function, that prescribes the anisotropic mean curvature
- $\vec{V}_A$  is an anisotropic volume gradient.

# Results

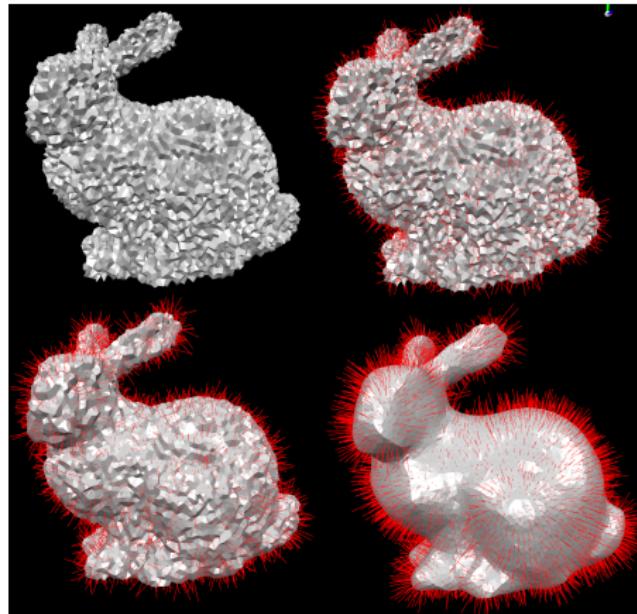


Figure : bunny after 0, 5, 10, 30 iterations

# Results

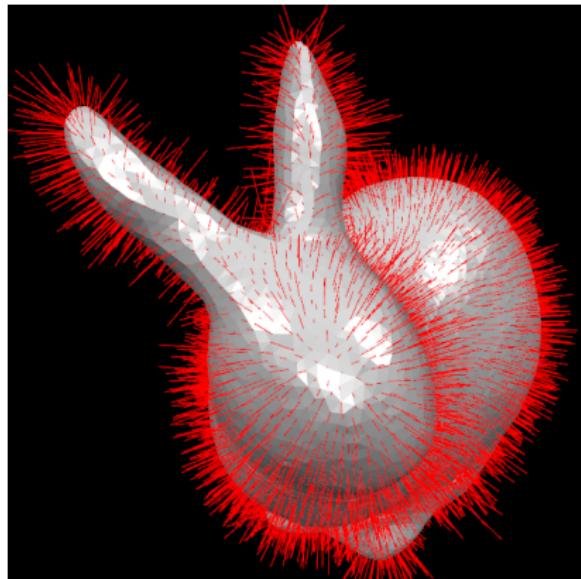


Figure : after 100 iterations

# Results

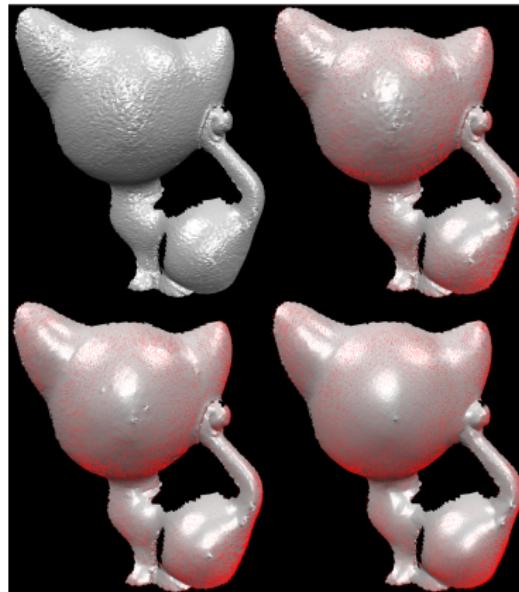


Figure : after 0, 5, 10, 100 iterations

# Results

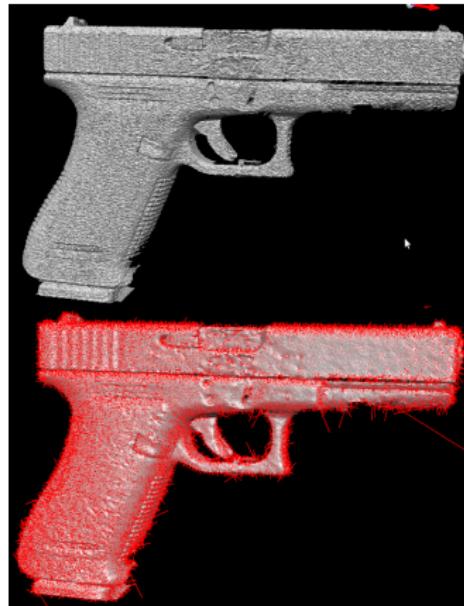


Figure : after 0, 20 iterations

# Summary

- Anisotropic mean curvature: reduce smoothing in high curvature regions
  - Preserve linear features but leads to deformation of curves
- Anisotropic prescribed mean curvature: evolve the surface to the precomputed, smoothed mean curvature
- Future work
  - Implement implicit integration methods
  - Extend to process boundaries