Geometry Processing Lab 2012
Anisotropic Filtering of Non-Linear Surface Features

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1 Introduction

Nowadays, geometric data acquired through imaging or scanning devices has grown rapidly due to advances in technology, making it affordable in many aspects of our lives. However, when dealing with real data we always have to cope with measurement error which brings high frequency noise to our geometric models. Many researches have been conducted in order to remove noise from a scanned model while trying to preserve the underlying sampled surface. One of the seminal results was the work of Taubin et al. [Tau95] in which they use a signal processing approach to derive the Laplacian operator acting as a low-pass filter on the geometric signal. Even though the Laplacian operator is a powerful tool to remove high frequency noise, it isotropic behaviour makes it unable to preserve sharp features. Hildebrandt and Polthier [HP04] have developed an anisotropic method which can preserve high curvature features in a certain direction while suppressing unwanted curvature peaks in the other directions. This method makes it possible to denoise arbitrary surface meshes whereas non-linear geometric features e.g. curved surface regions and feature lines are preserved. This lab report explores and elaborates theory and practice needed to implement the prescribed mean curvature flow proposed by Hildebrandt and Polthier [HP04].

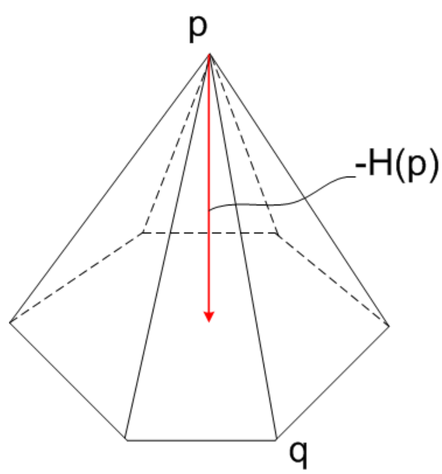

Figure 1: smooth a vertex p by moving it in the direction of the mean curvature vector $\vec{H}(p)$

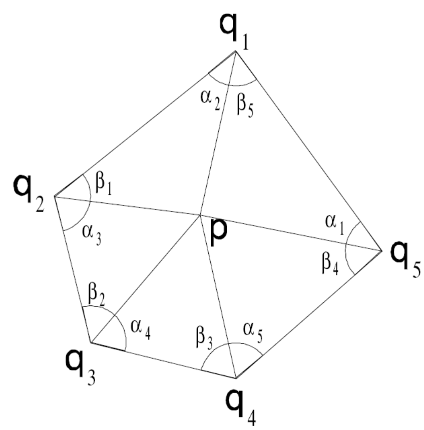

Figure 2: Cotangent weights

2 Smoothing Principle

A very intuitive smoothing operator one can think of is to move a vertex p to the center of gravity (c.o.g) of its one-ring neighbors $N_1(p)$:

$$p \leftarrow \frac{1}{\mid N_1(p) \mid} \sum_{q \in N_1(p)} q = p - \underbrace{\frac{1}{\mid N_1(p) \mid} \sum_{q \in N_1(p)} (p-q)}_{\triangle p} \tag{1}$$

Equation 1 reveals the update form of the smoothing operator in which the old vertex is moved by an amount of the update vector $\triangle p$ to the new position. The update vector $\triangle p$ can be generalized to have arbitrary weights over the 1-ring $\sum\limits_{q \in N_1(p)} w_q(p-q)$ other than uniform weights as in the equation 1. In fact, we can choose the weights such that the update vector points in the direction normal to the mesh surface. Such an update formula was first proposed by Pinkall and Polthier [PP93] known as cotangent weights:

$$\bigtriangledown_p \ area = \vec{H}(p) = 1/2 \sum_{q \in N_1(p)} (cot\alpha_q + cot\beta_q)(p-q) \tag{2}$$

Where $\vec{H}(p)$ is the mean curvature vector at a vertex p and equal the gradient of the area functional $\bigtriangledown_p \ area$ at that vertex. Figure 1 and 2 show

how the mean curvature vector is calculated.

3 Anisotropic Mean Curvature

The first step towards deriving an anisotropic mean curvature formula is to express the vertex mean curvature vector 2 in terms of an edge based mean curvature vector:

$$\vec{H}(e) = H_e \vec{N}_e \tag{3}$$

where $N_e = \frac{N_1+N_2}{\|N_1+N_2\|}$ is the edge normal vector as shown in figure 3 and $H_e = 2 \mid e \mid \cos\frac{\theta_e}{2}$ is the edge mean curvature which depends on the dihedral angle θ_e as illustrated in figure 4. Note that the smaller the dihedral angle, the sharper the edge resulting in the higher the mean curvature H_e. It can be shown in the work of Pothier [Pol02] that both mean curvature vectors 2 and 3 are related by the equation:

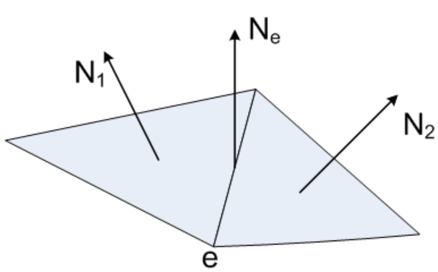

Figure 3: Edge normal vector N_e

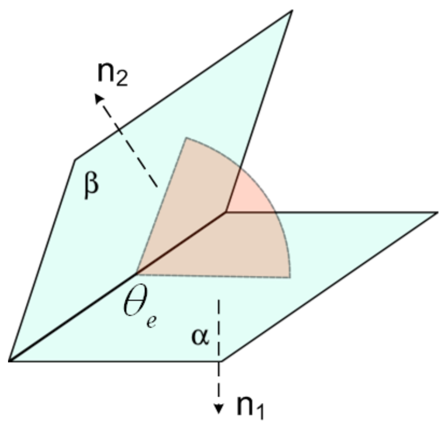

Figure 4: Dihedral angle θ_e

4 Prescribed Mean Curvature

Explain volume gradient in detail.

5 Implementation and Results

Matrix form: Ha, Mass (non-diagonal converges better =¿ larger time step), Taylor for implicit attempt.

6 Conclusion and Future Work

References

[HP04] Klaus Hildebrandt and Konrad Polthier. Anisotropic filtering of non-linear surface features. *Computer Graphics Forum*, 23:391–400, 2004.

[Pol02] Konrad Polthier. Polyhedral surfaces of constant mean curvature. Technical report, TU Berlin, 2002.

[PP93] Ulrich Pinkall and Konrad Polthier. Computing discrete minimal surfaces and their conjugates. *Experimental Mathematics*, 2:15–36, 1993.

[Tau95] Gabriel Taubin. A signal processing approach to fair surface design. In *Proceedings of the 22nd annual conference on Computer graphics and interactive techniques*, SIGGRAPH '95, pages 351–358, New York, NY, USA, 1995. ACM.