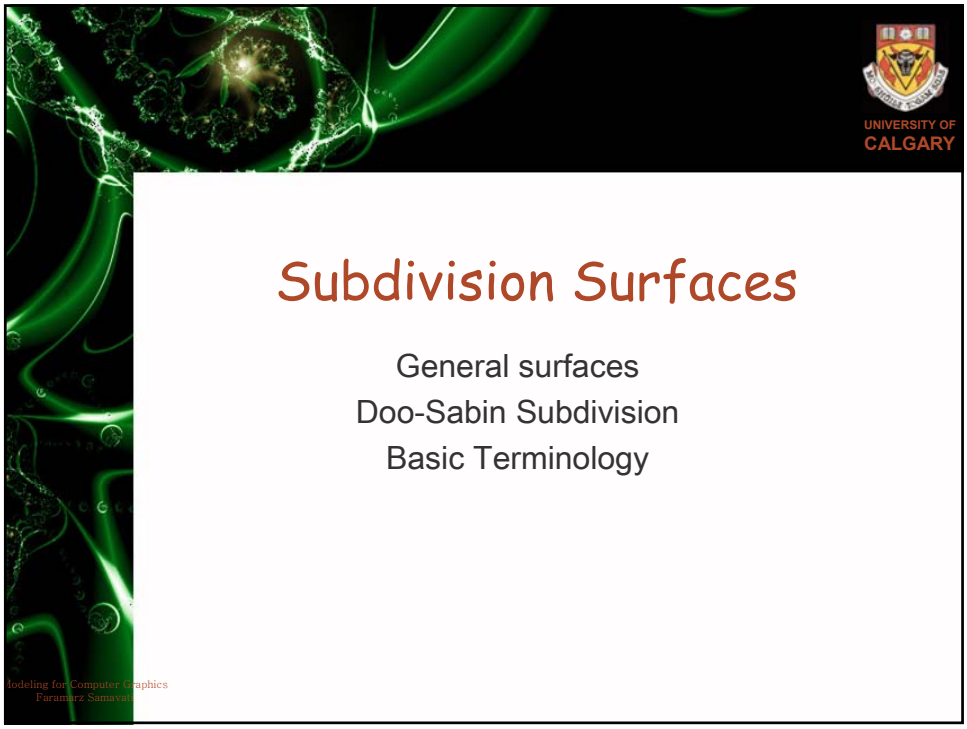
UNIVERSITY OF
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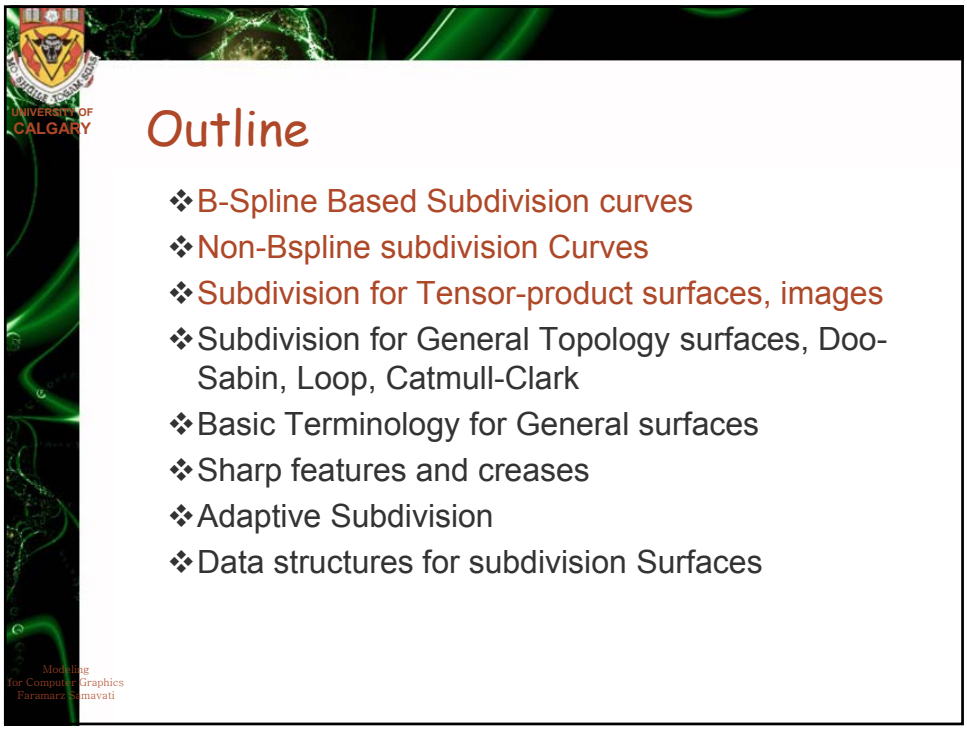
Subdivision Surfaces
General surfaces
Doo-Sabin Subdivision
Basic Terminology
Modeling for Computer Graphics
Faramarz Samavati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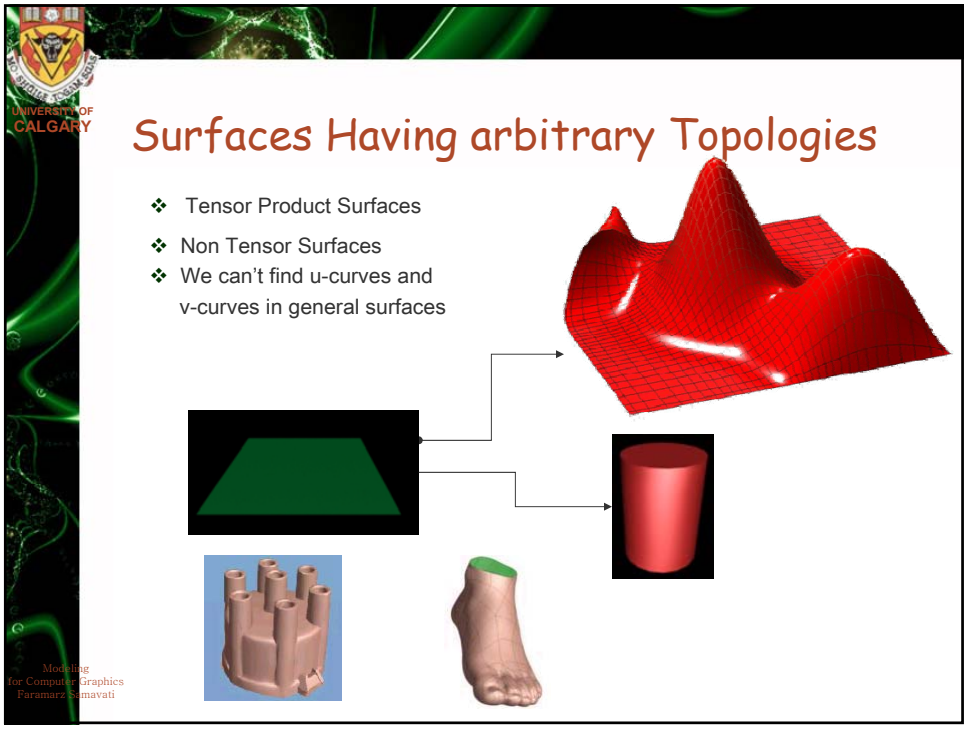
Outline
B-Spline Based Subdivision curves
Non-Bspline subdivision Curves
Subdivision for Tensor-product surfaces, images
Subdivision for General Topology surfaces, Doo-Sabin, Loop, Catmull-Clark
Basic Terminology for General surfaces
Sharp features and creases
Adaptive Subdivision
Data structures for subdivision Surfaces
Modeling
for Computer Graphics
Faramarz Samavati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Surfaces Having arbitrary Topologies
Tensor Product Surfaces
Non Tensor Surfaces
We can't find u-curves and v-curves in general surfaces
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General Subdivision
Coarse mesh Subdivision Fine mesh
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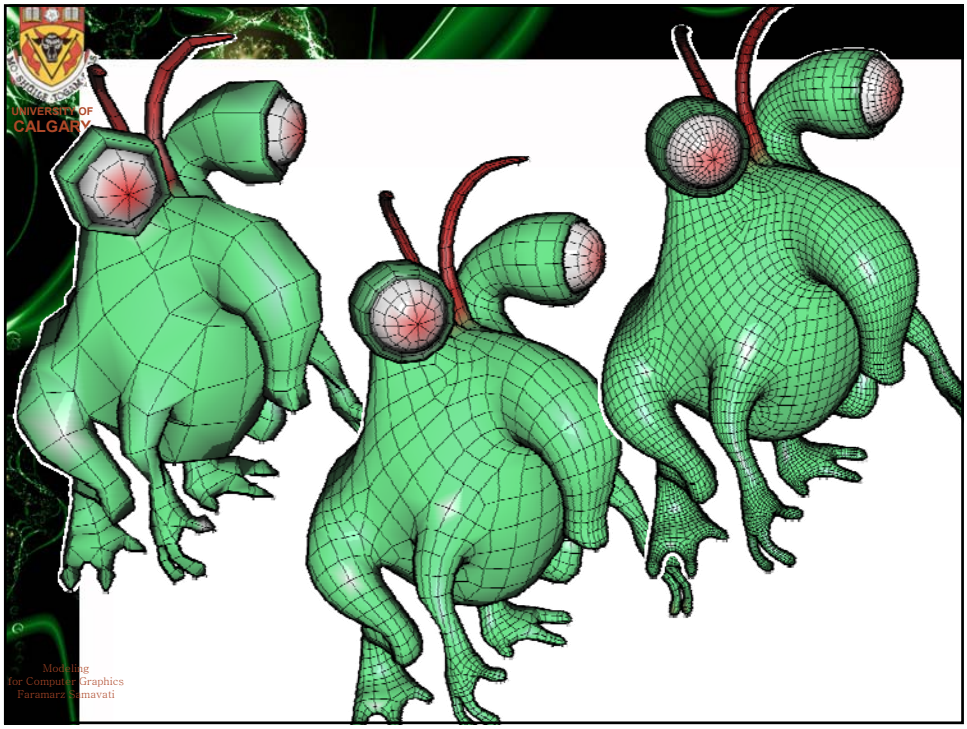
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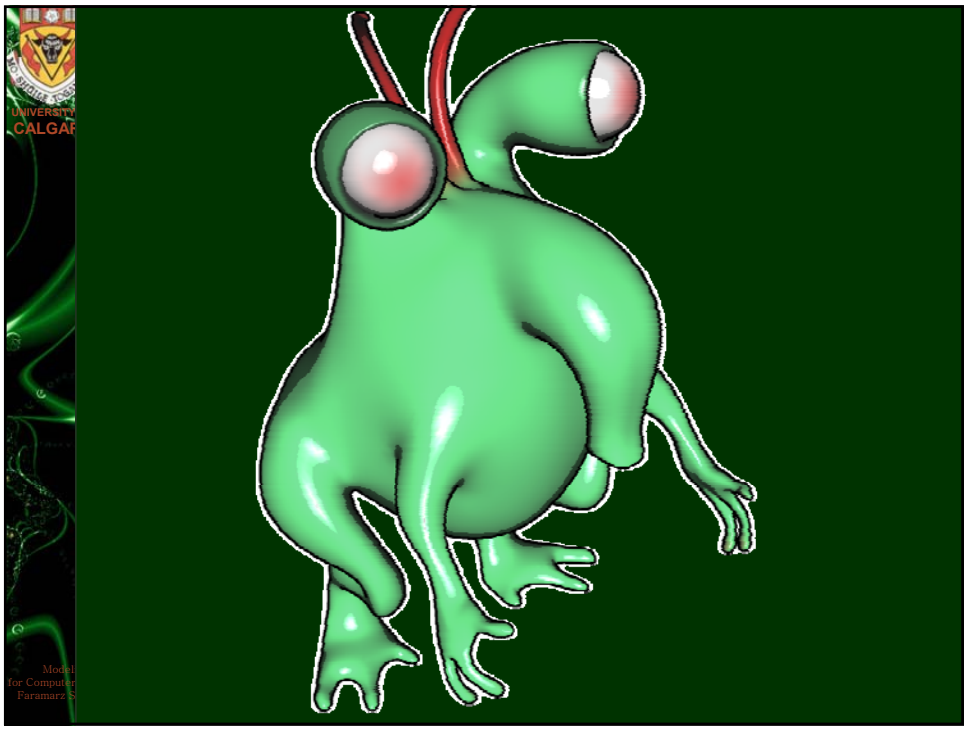
UNIVERSITY
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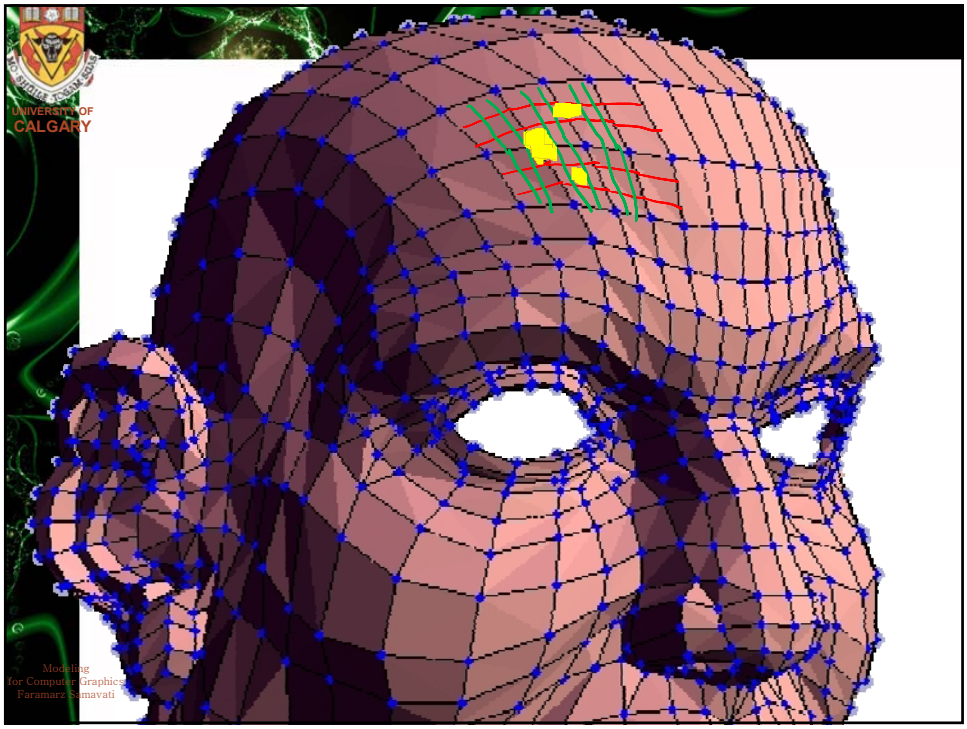
Model
for Computer
Faramarz S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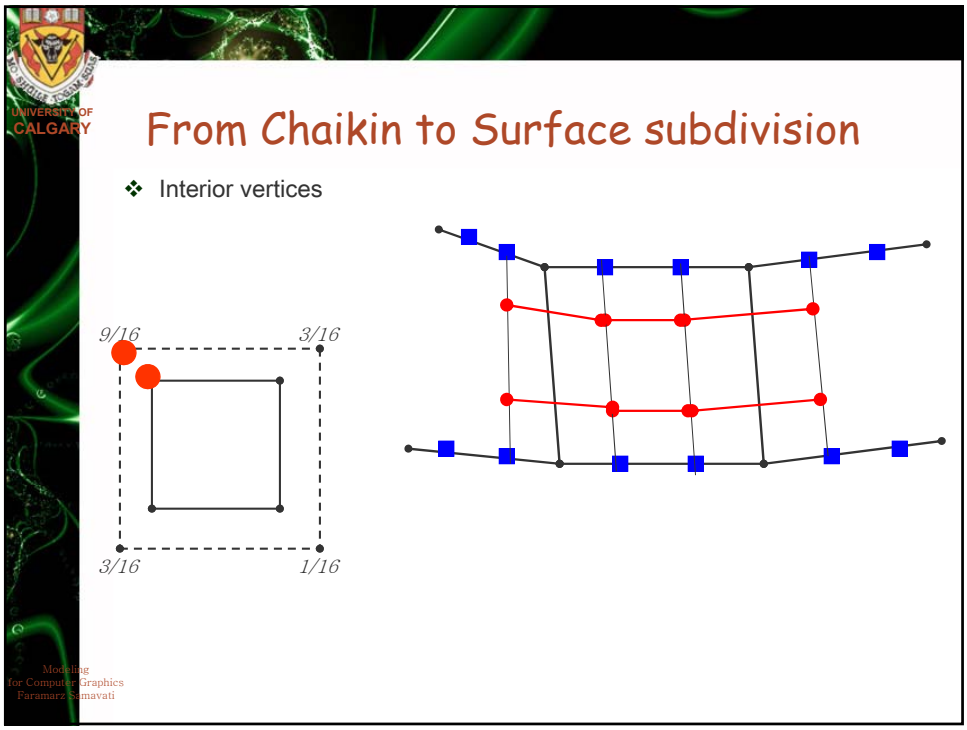
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From Chaikin to Surface subdivision
Interior vertices
9/16
3/16
3/16
1/16
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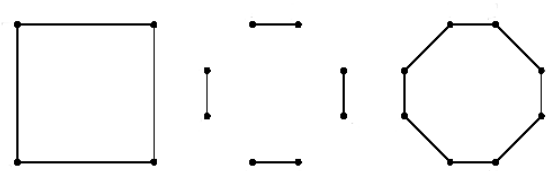
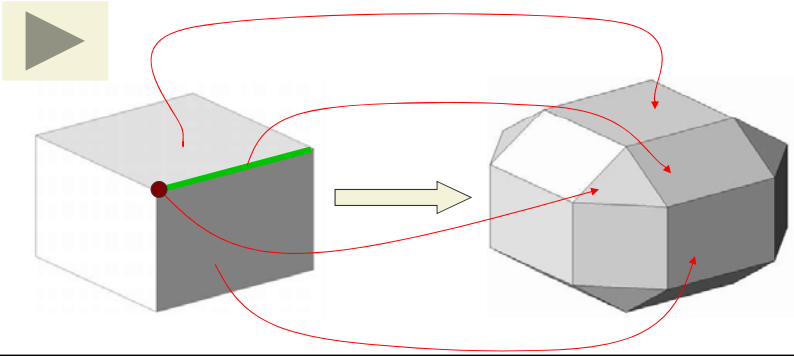
From Chaikin to Doo-Sabin Subdivision

- 3D corner cutting method
- Generalization of Chaikin
- Each step =2 stages
- Contracting and adjoining



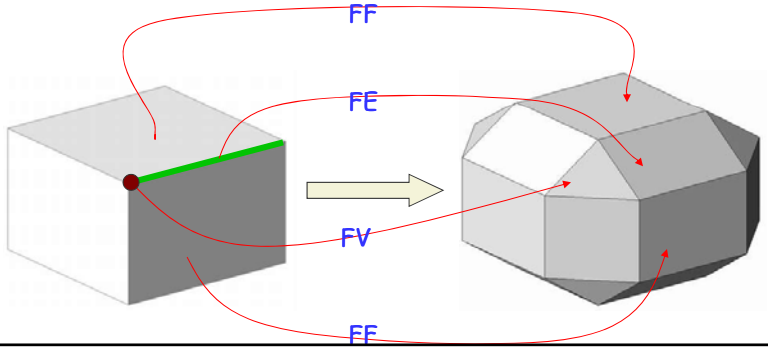
One Step of Doo-Sabin Subdivision

- One Step of Subdivision (F,V) ⟹ (NF,NV)
- Line ⟹ Face ⟹ Curve ⟹ Surface

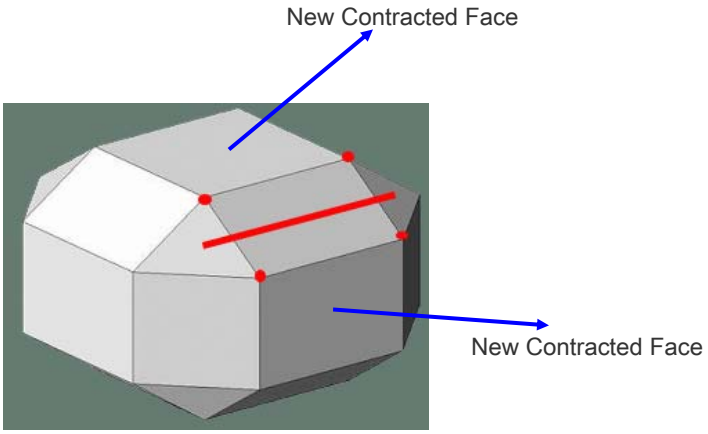


Types of Faces

- Three types of New Faces(NF) :
- FF : Face- to - Face
- FE : Face - to - Edge
- FV: Face - to - vertex

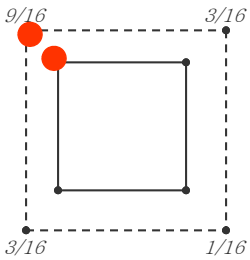


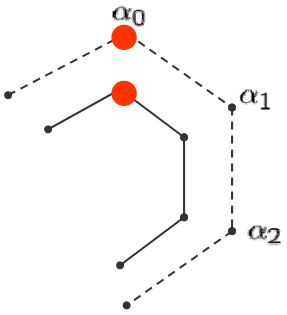
Edge-to-faces



Mask Details

- Interior vertices



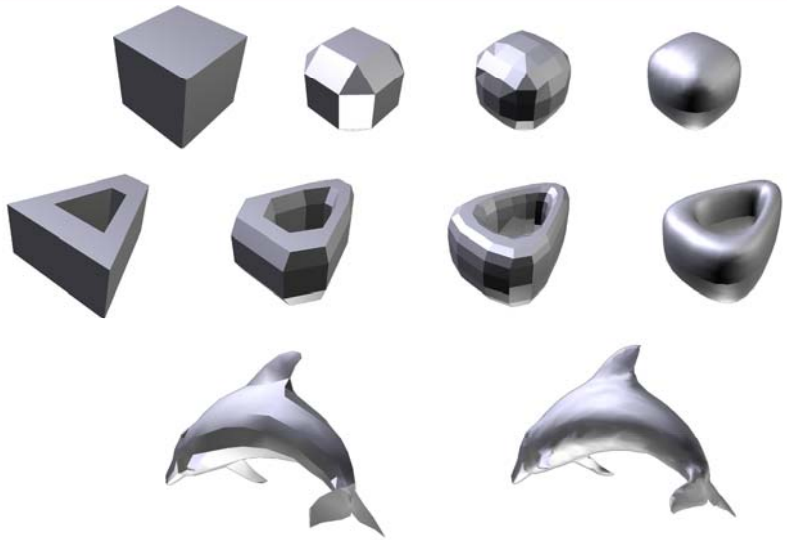


$$\alpha_0 = \frac{1}{4} + \frac{5}{4n}$$

$$\alpha_i = \frac{(3+2\cos(2i\pi/n))}{4n}$$

n= number of vertices on the face

Examples



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Example of Doo Subdivision

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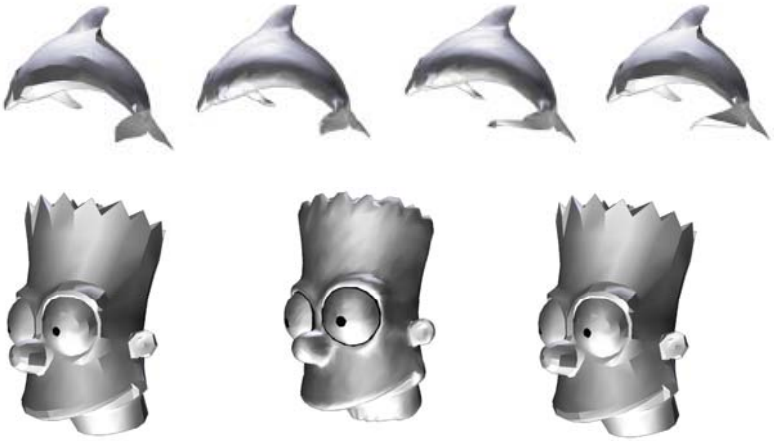
Multiresolution based on reverse of Doo-Sabin subdivision



Multiresolution Representation of Surface with Arbitrary Topology by Reversing Doo Subdivision, F. F. Samavati, N. Mahdavi-Amiri and R. H. Bartels, Computer Graphic Forum, Vol.21, No.2, pp.121-136, 2002.

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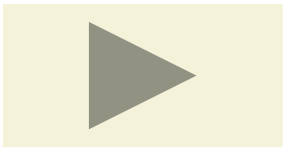
Cubic Tragedy



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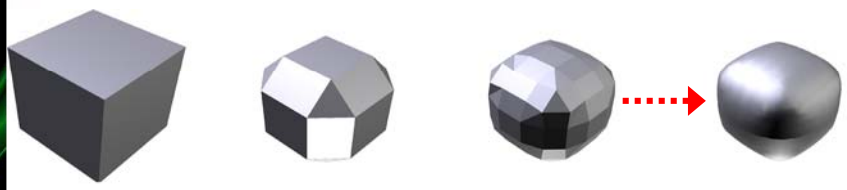
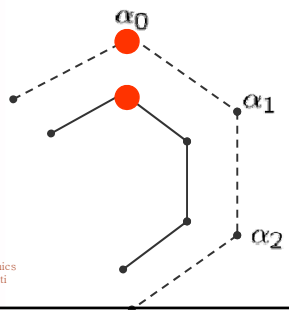
Summary



M^0 M^1 M^2 M^∞

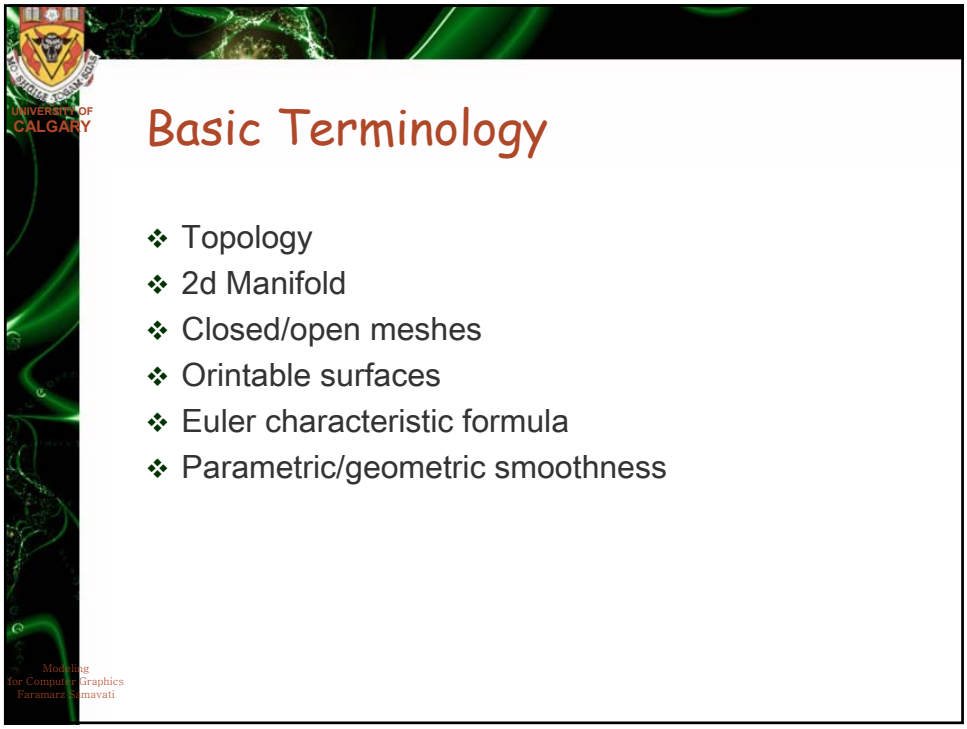


$$\alpha_0 = \frac{1}{4} + \frac{5}{4n}$$

$$\alpha_i = \frac{(3+2\cos(2i\pi/n))}{4n}$$

n= number of vertices on the face

mask as a weighted averaging!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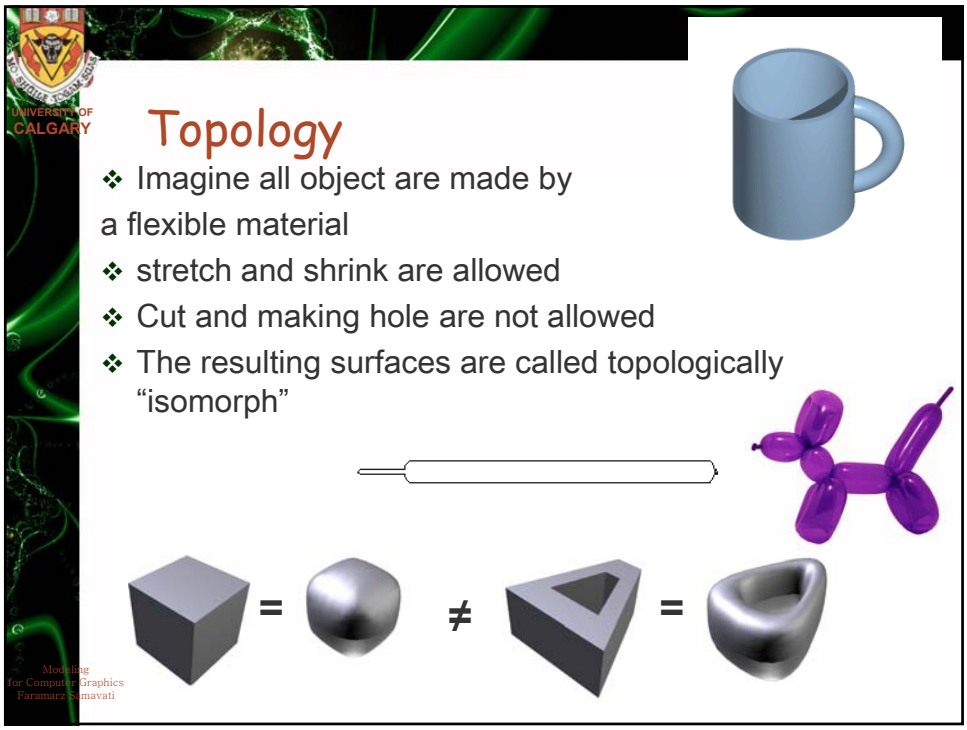
Basic Terminology
Topology
2d Manifold
Closed/open meshes
Orintable surfaces
Euler characteristic formula
Parametric/geometric smoothness
Modeling
for Computer Graphics
Faramarz Samavati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Topology
Imagine all object are made by
a flexible material
stretch and shrink are allowed
Cut and making hole are not allowed
The resulting surfaces are called topologically
"isomorph"
=
≠
=
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for Computer Graphics
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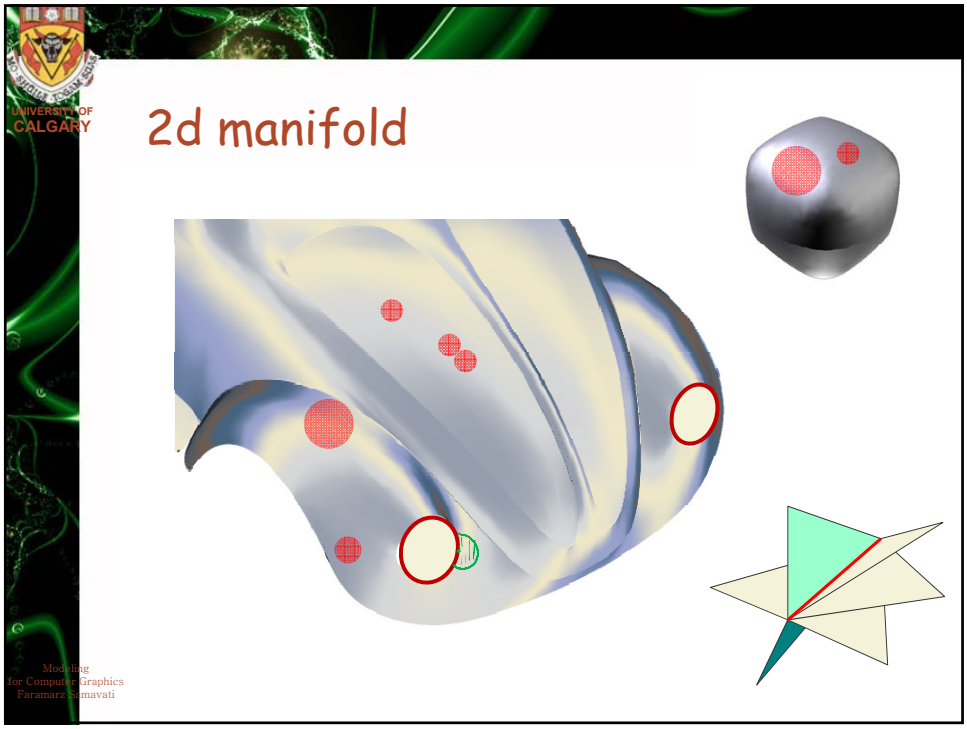
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2d manifold
Modeling for Computer Graphics
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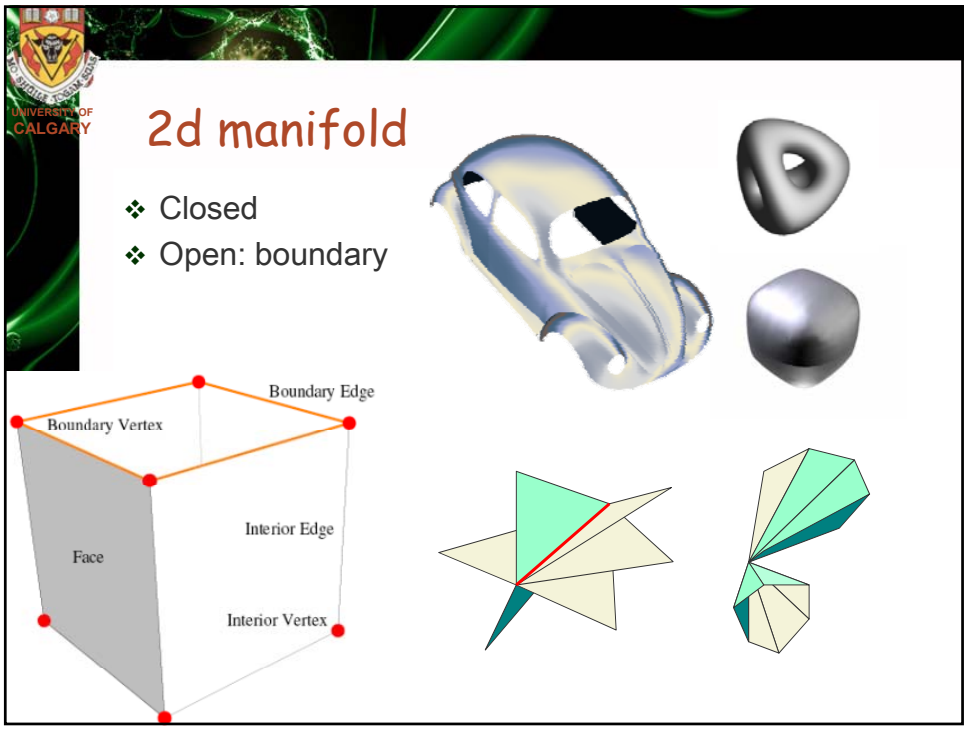
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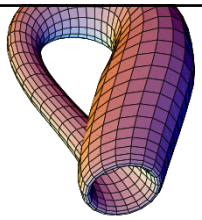
2d manifold
Closed
Open: boundary
Boundary Edge
Boundary Vertex
Interior Edge
Face
Interior Vertex

Orientation

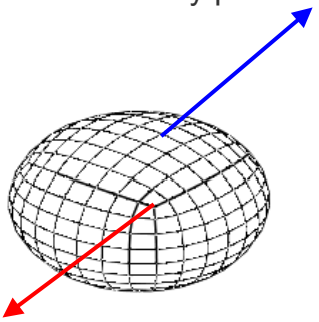


- Consistent orientation

For all points (or faces in mesh)

Parametric smoothness and geometric smoothness

Regular: inherited from u- and v-curves
Usually parametric smoothness



Ir-regular: requires eigenanalysis
just geometric smoothness can be checked

Euler characteristic formula

Name	Image	V (vertices)	E (edges)	F (faces)
Tetrahedron		4	6	4
Hexahedron or cube		8	12	6
Octahedron		6	12	8
Dodecahedron		20	30	12
Icosahedron		12	30	20

Simple closed meshes:
E=V+F-2

Euler-Poincare Formula

- E=V+F-2(1-G)
- G is the genus of the surface
- O(E)=O(V+F)

Geneus
0
1
4

Doo Surface Properties

- Convex hull property $(\sum \alpha_i = 1)$
- It produces mostly four sided faces
- Large region of the new polyhedron are covered with tensor product surface .
- Doo-Sabin surfaces are thus “mostly bi-quadratic B-splines”
- Limit surface has the first degree of parametric smoothness for regular vertices and first degree of geometric smoothness for ir-regular vertices

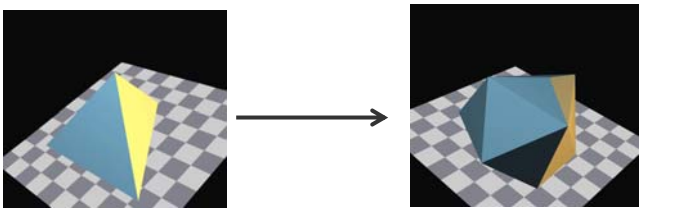
M^0 M^1 M^2 M^∞

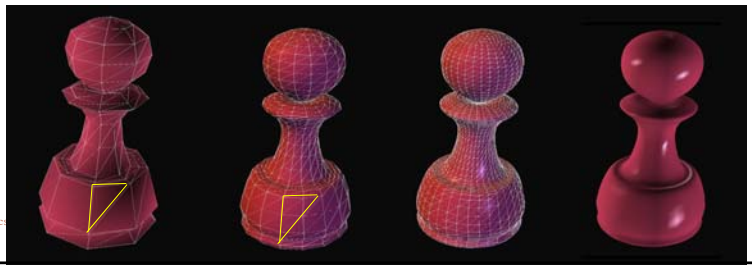
Subdivision Surfaces

Loop and Catmull-Clark Subdivision

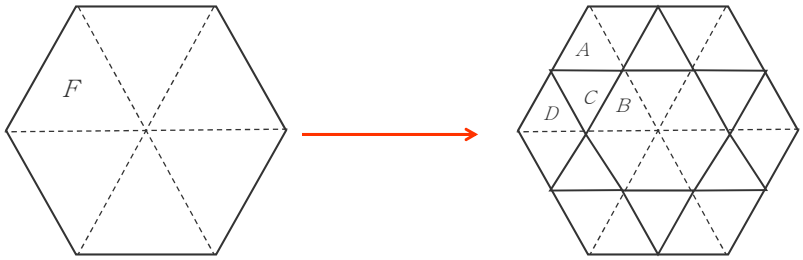
Loop Subdivision

- A triangle-based subdivision

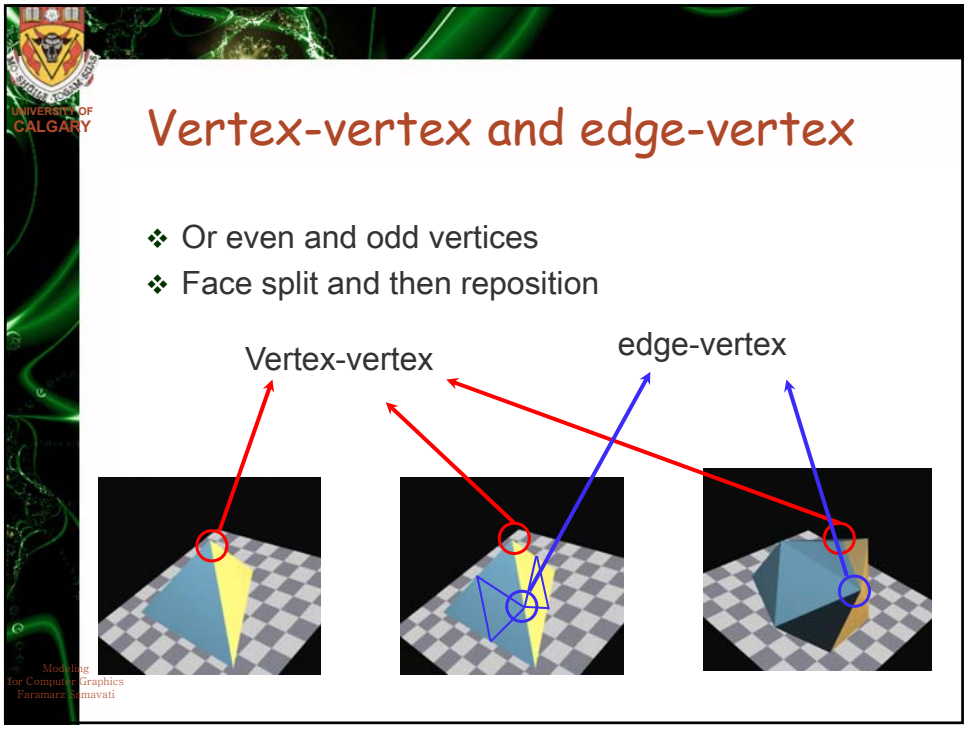




Face Splitting



- Every face splits to four new faces
  F ⟶ A,B,C,D
- Updating the face structure
- Or equivalently: insert midpoints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Vertex-vertex and edge-vertex
Or even and odd vertices
Face split and then reposition
Vertex-vertex
edge-vertex
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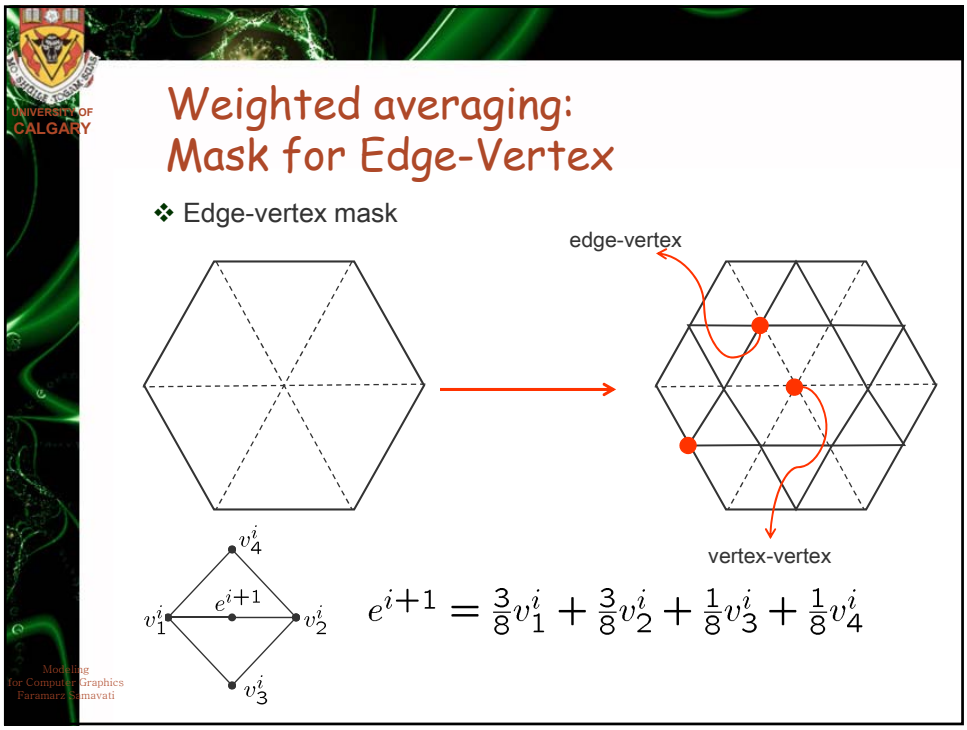
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Weighted averaging: Mask for Edge-Vertex
Edge-vertex mask
edge-vertex
vertex-vertex
v_4^i
e^{i+1}
v_1^i
v_2^i
v_3^i
$e^{i+1} = \frac{3}{8}v_1^i + \frac{3}{8}v_2^i + \frac{1}{8}v_3^i + \frac{1}{8}v_4^i$
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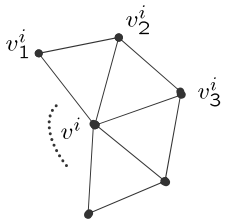
Mask for Vertex-Vertex

- n: number of adjacent vertices (valences)
- n > s



$$v^{i+1} = (1 - n\alpha)v^i + \alpha \sum v_j^i$$

$$\alpha = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

- v^i has much more influence on v^{i+1}
- n=3 , $\alpha = \frac{3}{16}$ or $v^{i+1} = \frac{7}{16}v^i + \frac{3}{16}(v_1^i + v_2^i + v_3^i)$

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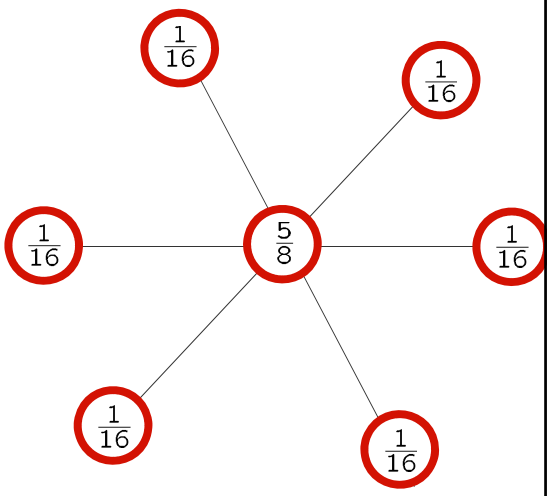
Example: regular mask (n=6)

- Displacement Mask



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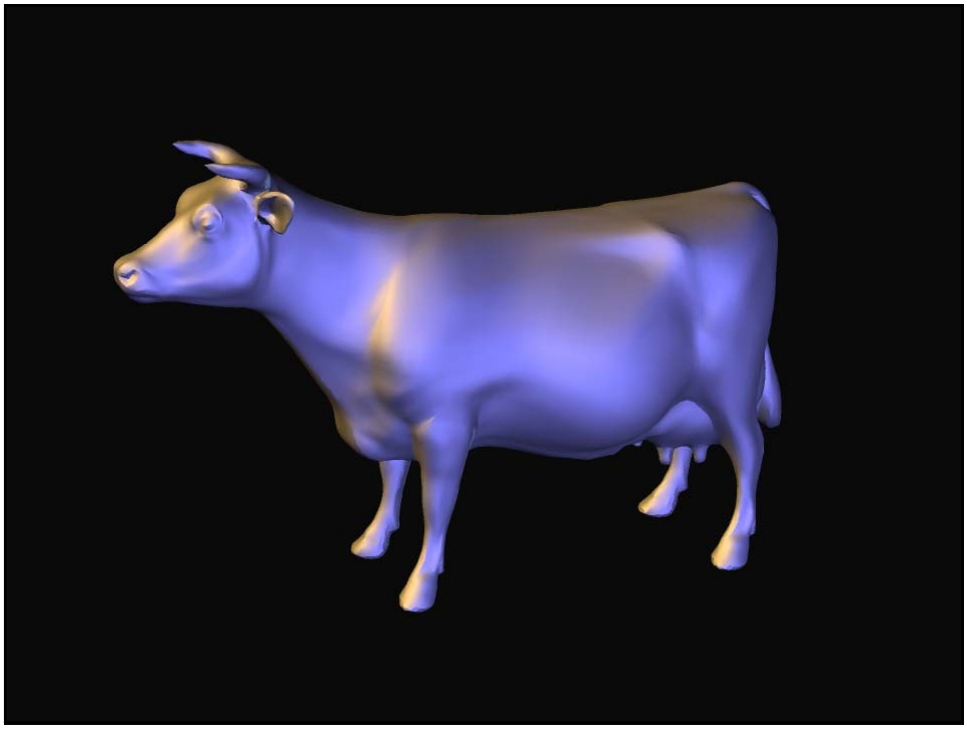

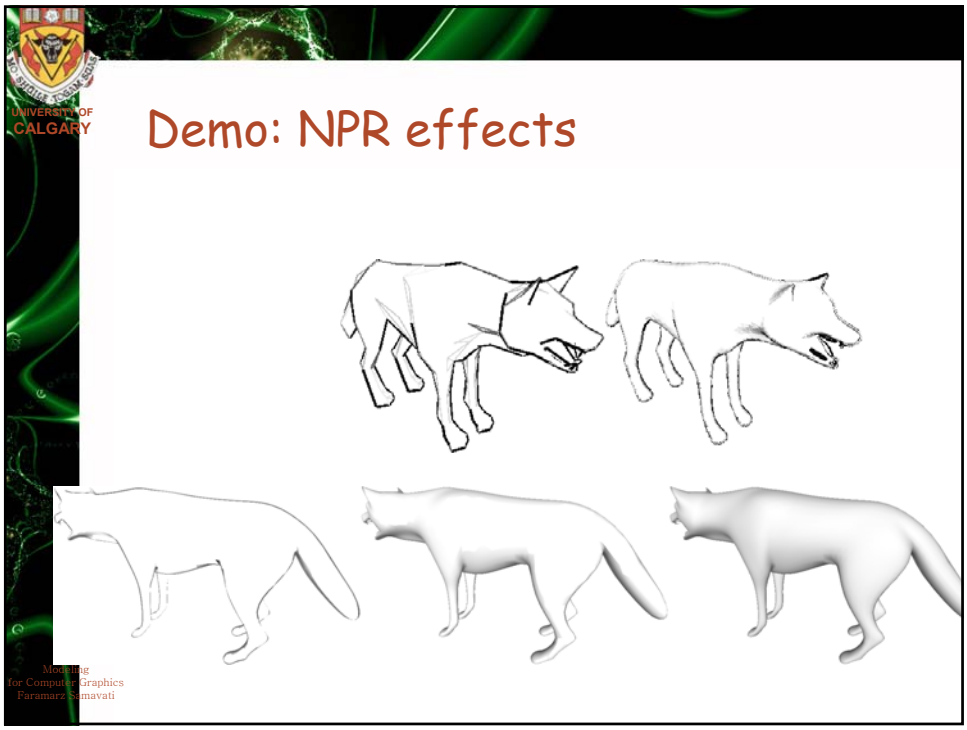
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Demo: NPR effects
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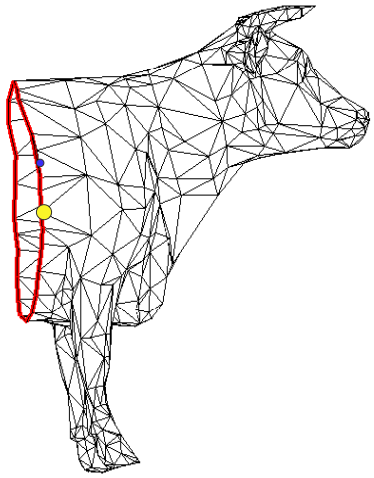
Boundary

- It is extension of the open curves
- No enough neighbours
- Using internal points move the boundary vertices!



- Demo with open cube

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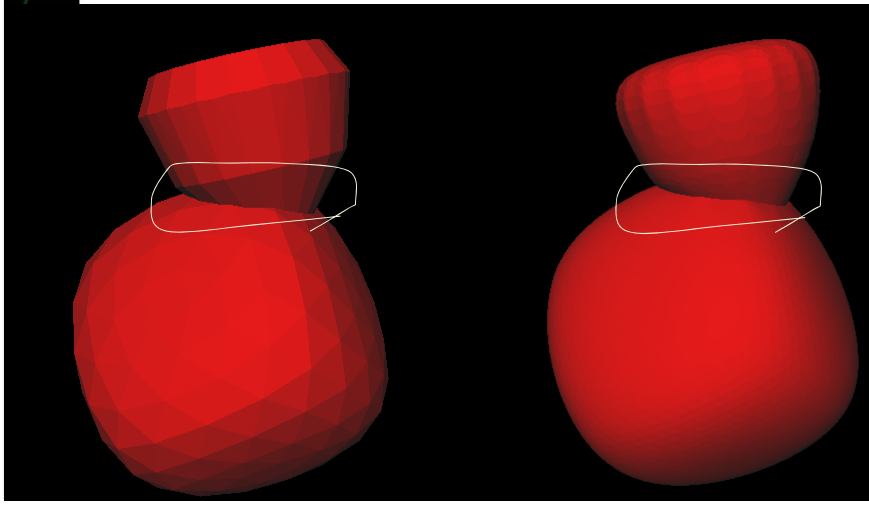
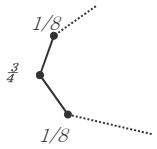
Matched boundaries for two disjoint meshes

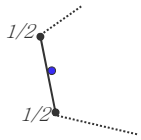


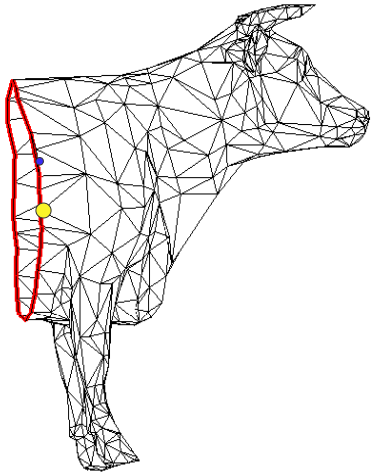
Boundary masks

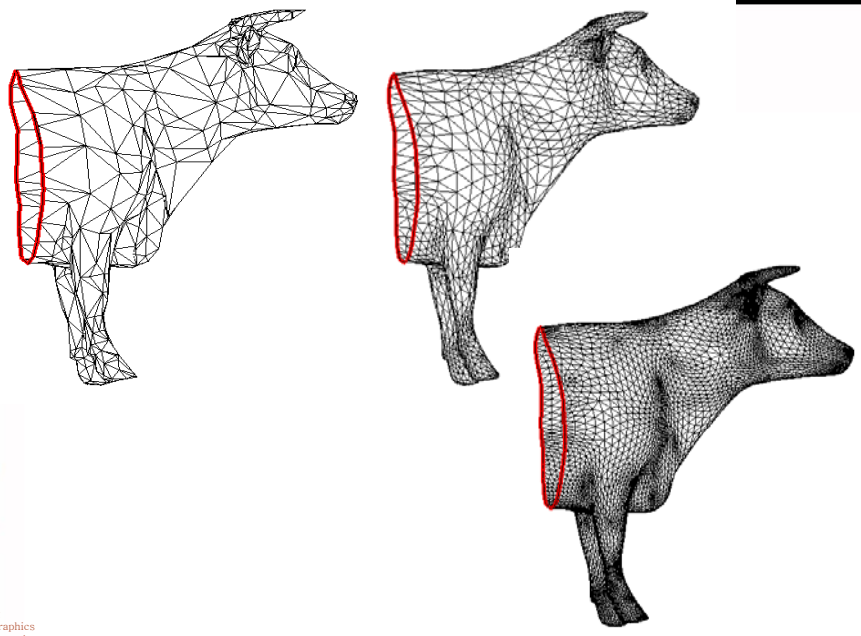
- Even Vertex



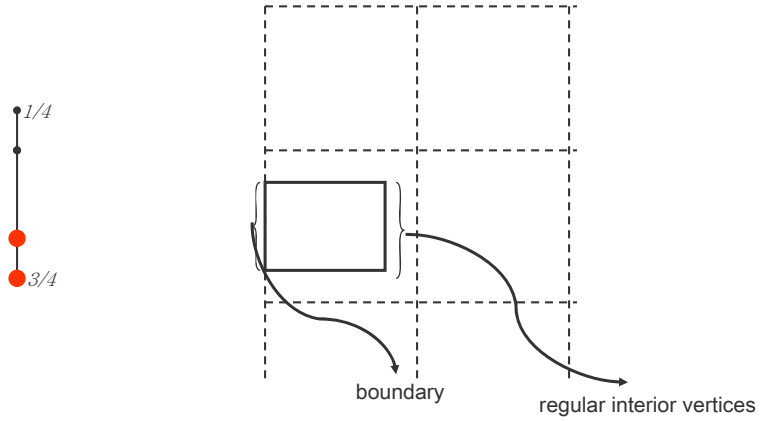
- Odd Vertex







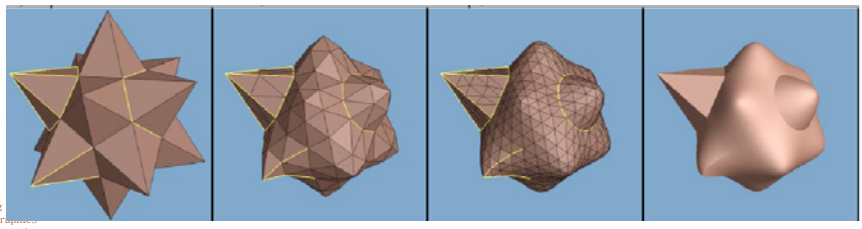
Boundary Mask for Doo-Sabin



❖Chaikin scheme for the boundary

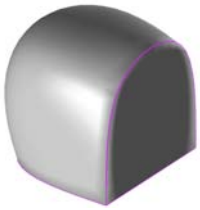
Creases and Corners

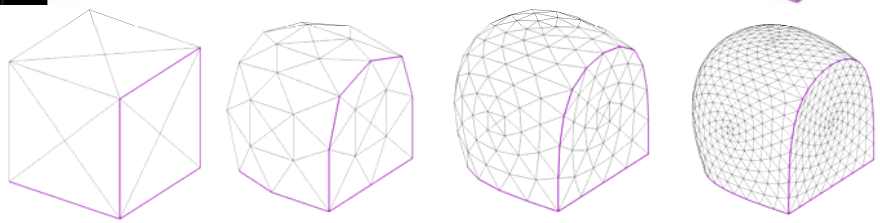
- Using sharp edges to create creases and corners
- Tangent planes continuity across sharp edges is relaxed
- Creases are considered as boundary
- Resulting edges of sharp edges are tagged as sharp



Corner points

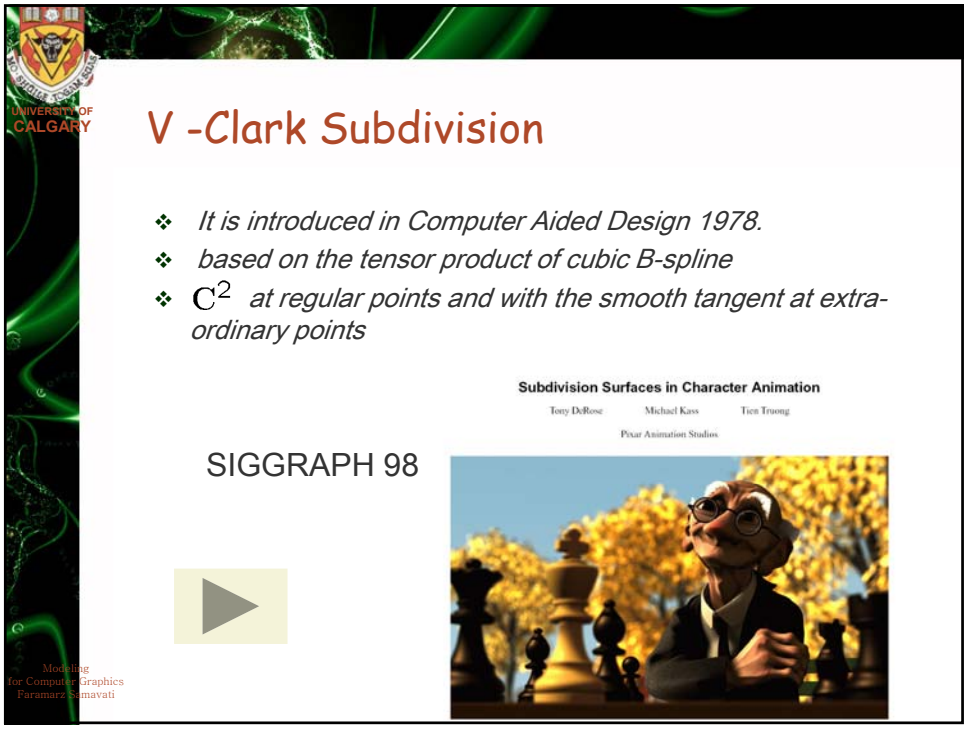
- $v^i = v^{i+1}$ (are unchanged by subdivision)
- Vertices with sharp valence more than two are usually considered as corner point





Properties

- Second degree of continuity for regular vertices
- First degree of geometric continuity for non-regular vertices
- Strong convex hull property
- *It produces mostly regular vertices*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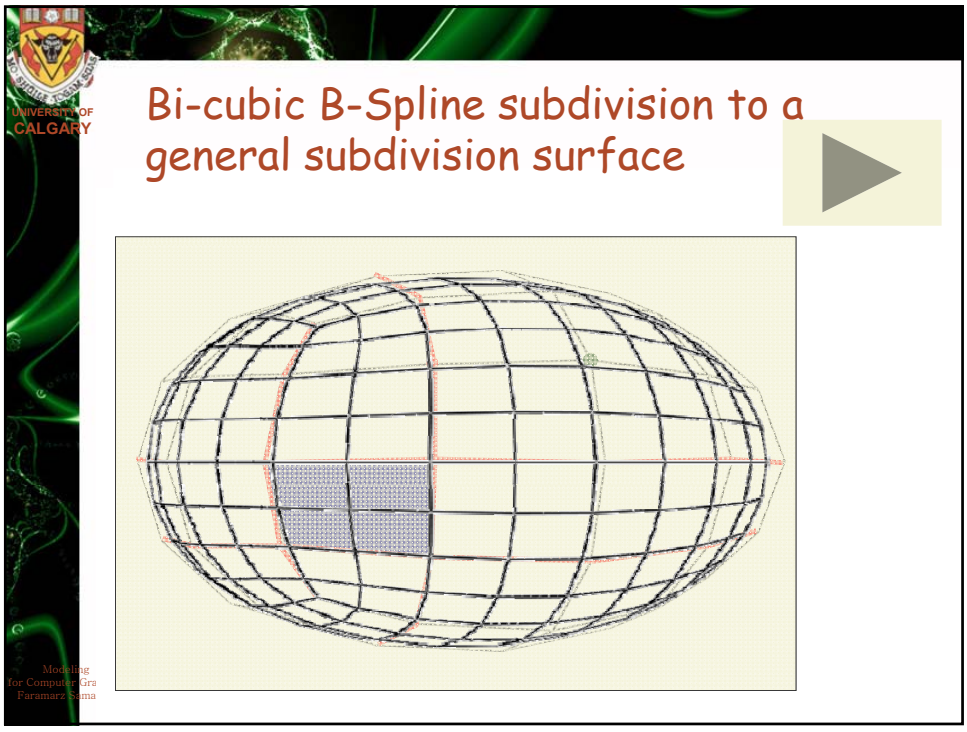
V -Clark Subdivision
It is introduced in Computer Aided Design 1978.
based on the tensor product of cubic B-spline
C^2 at regular points and with the smooth tangent at extra-ordinary points
SIGGRAPH 98
Subdivision Surfaces in Character Animation
Tony DeRose
Michael Kass
Tien Truong
Pixar Animation Studios
Modeling
for Computer Graphics
Faramarz Samavati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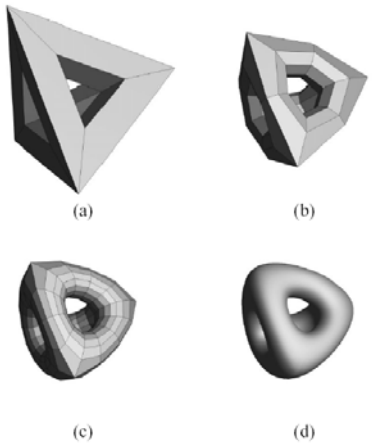
Bi-cubic B-Spline subdivision to a general subdivision surface
Modeling
for Computer Gra
Faramarz Sama

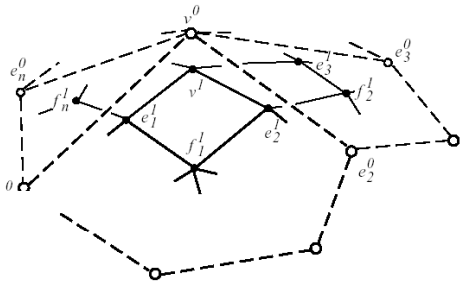
Catmull-Clark Subdivision

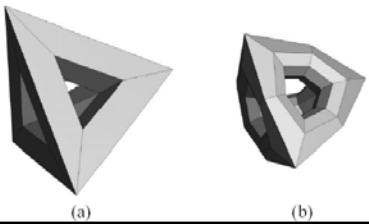
- Splitting each face into a collection of quadrilateral subfaces
- The new vertices are computed using certain weighted average



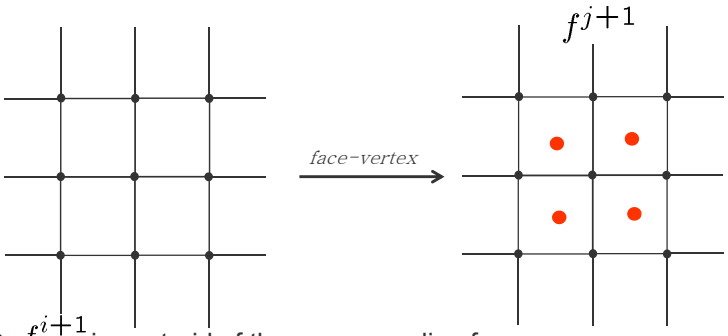
Weighted average

- Three different types of vertices
- Face point
- Edge point
- Vertex point



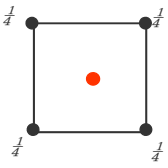


Face points

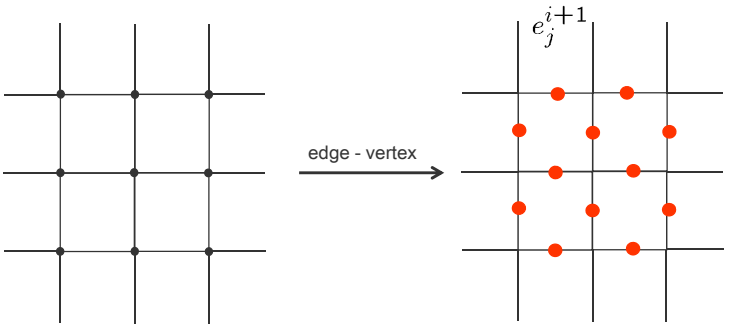


- f_j^{i+1} is centorid of the corresponding face or

$$f_j^{i+1} = \frac{1}{n}\sum_{k=1}^{n} v_k^i$$

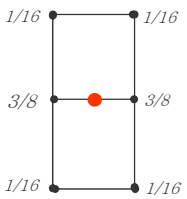


Edge-Vertex

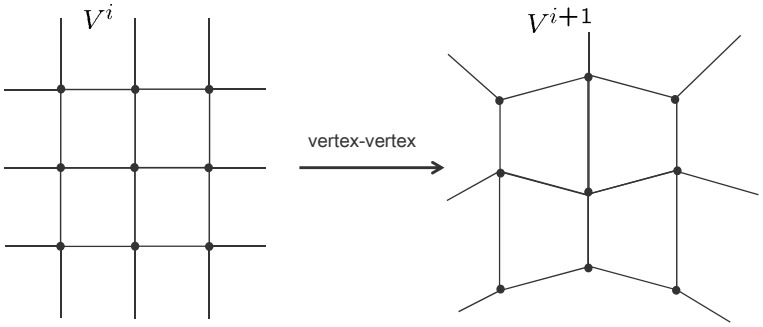


- e_j^{i+1} average the edge's end points and two face points on either side of the edge

$$e_j^{i+1} = \frac{v^i + e_j^i + f_{j-1}^{i+1} + f_j^{i+1}}{4}$$



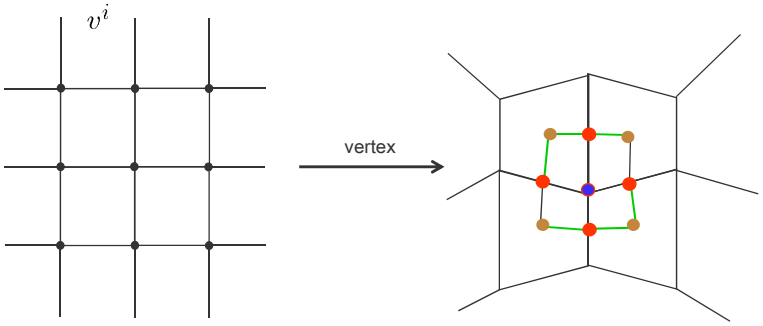
Vertex-Vertex



❖ New vertex point v^{i+1} : assuming there are n faces around v^i. It is computed by:

$$v^{i+1} = \frac{n-2}{n} v^i + \frac{1}{n^2} \sum_j e_j^i + \frac{1}{n^2} \sum_j f_j^{i+1}$$

New Faces



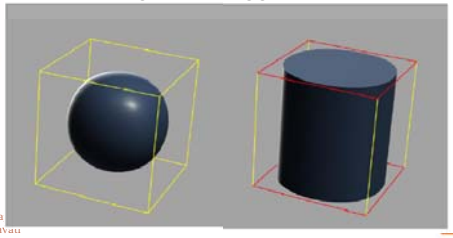
❖ Each new face consist of a loop:

$$v \rightarrow e_j \rightarrow f_j \rightarrow e_{j+1} \rightarrow v$$

Boundary, sharp edges, corners

- Cubic B-Spline curve masks for boundary and sharp edges
- Crease is a chain of sharp edges
- Corner points keep unchanged

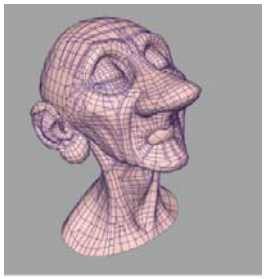
Red edges are tagged sharp



sharp creases are used between the skin and the finger nails

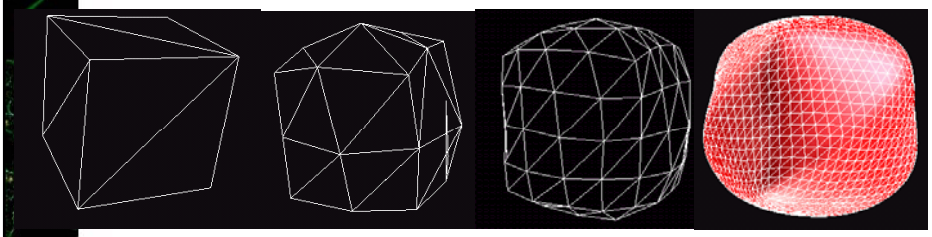
Example

- Geri's
  - Head
  - Hands
  - Clothing: Jacket, pants, shirt, tie and shoes

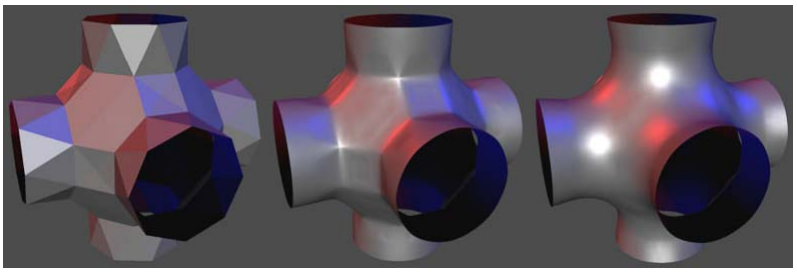


Butterfly Scheme

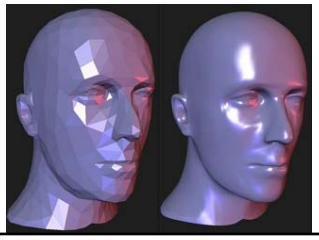
- It was proposed by Dyn, Levin and Gregory (TOG 1991)
- It is a triangular based subdivision
  - Face split step is the same as Loop
- Interpolating C^1 subdivision

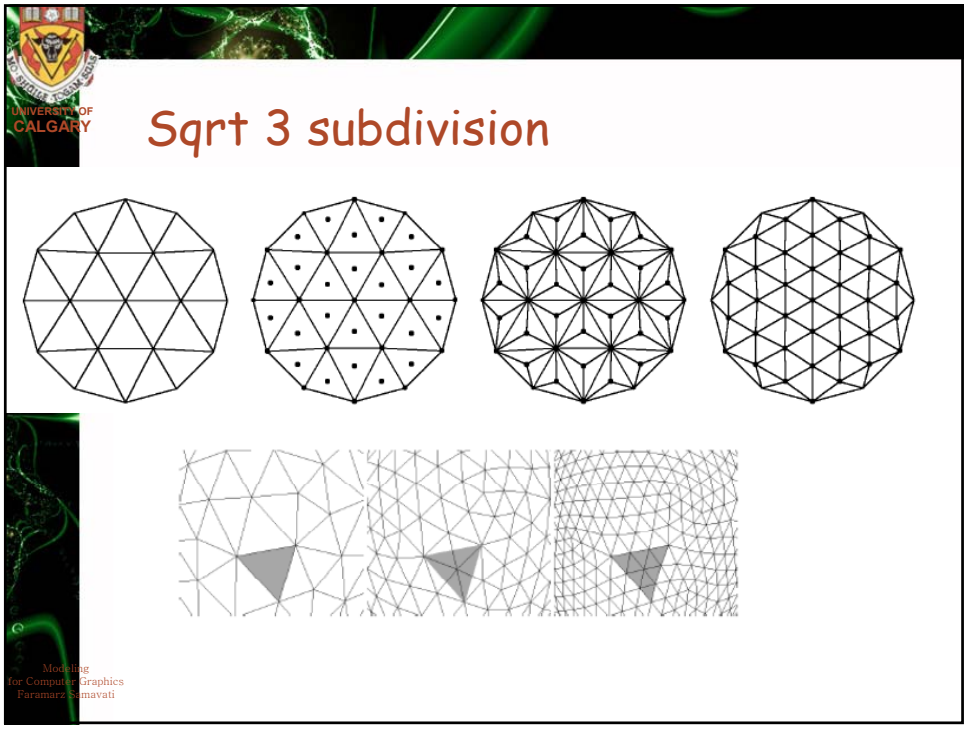


Modified Butterfly Subdivision



Control mesh original scheme Modified scheme

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Sqrt 3 subdivision
Modeling
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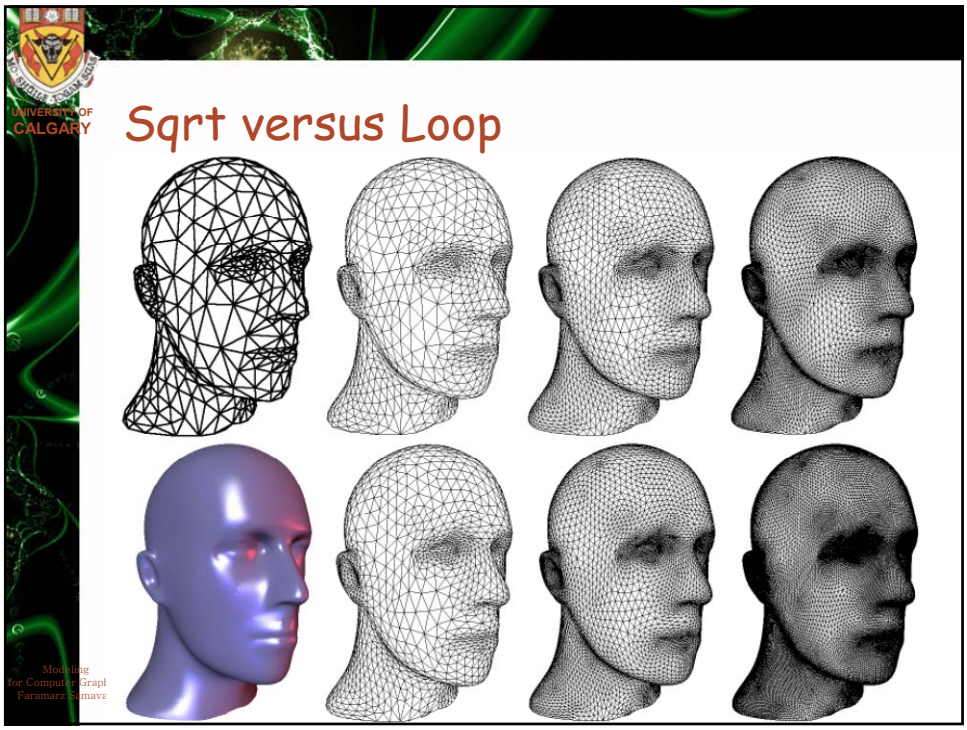
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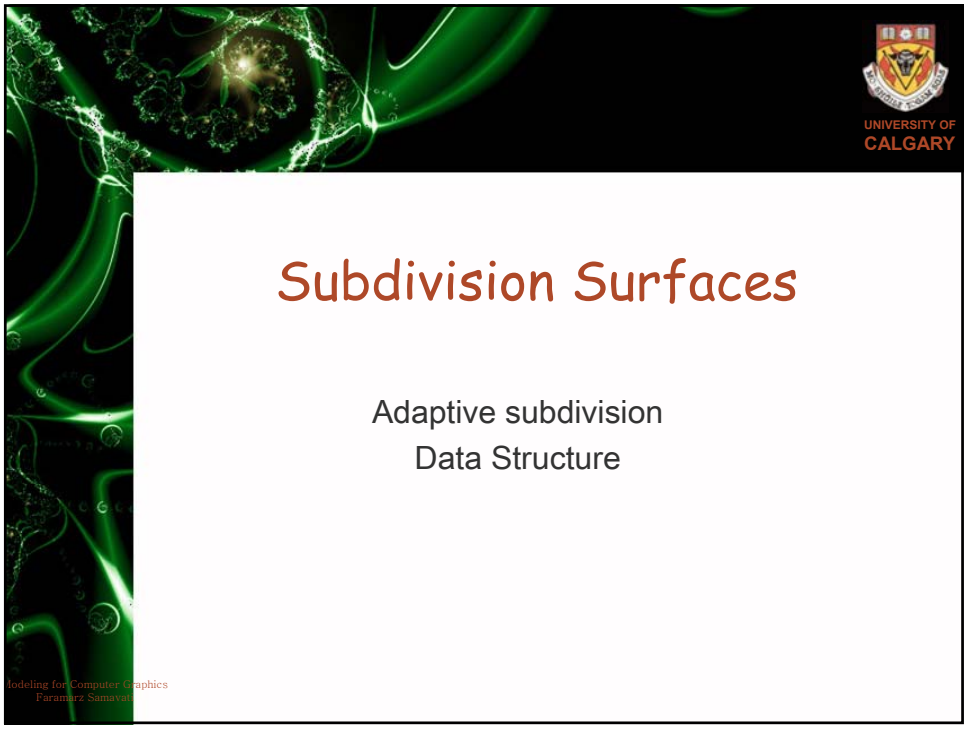
Sqrt versus Loop
Modeling
for Computer Grapl
Faramarz Samava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Subdivision Surfaces
Adaptive subdivision
Data Structure
Modeling for Computer Graphics
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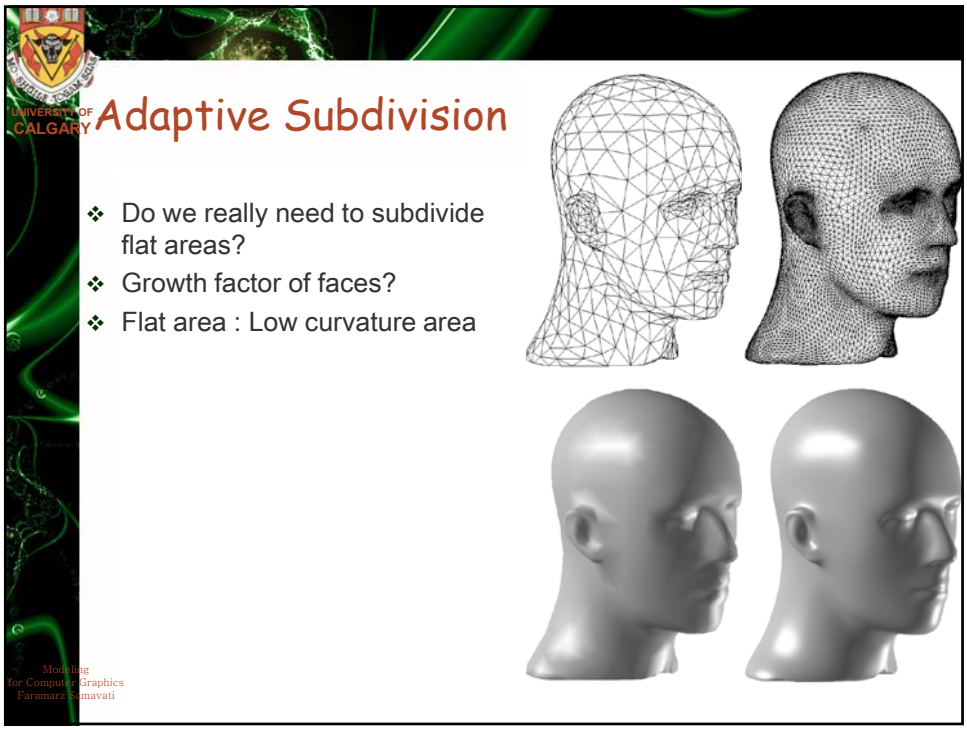
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Adaptive Subdivision
Do we really need to subdivide flat areas?
Growth factor of faces?
Flat area : Low curvature area
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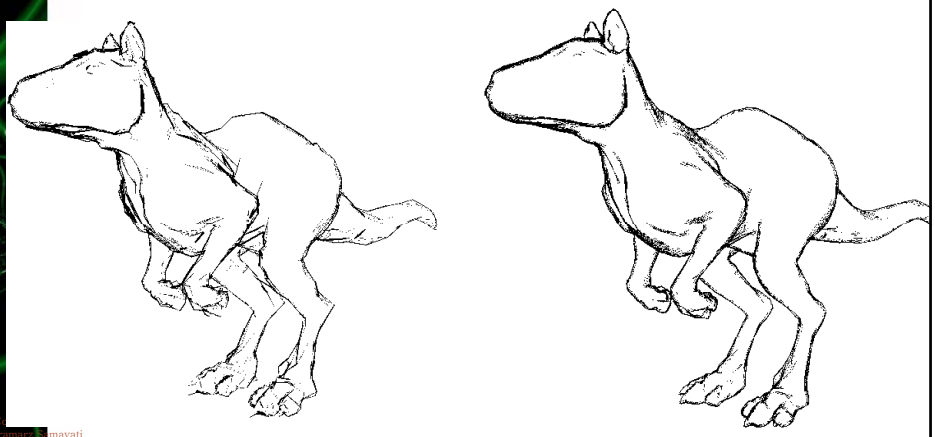
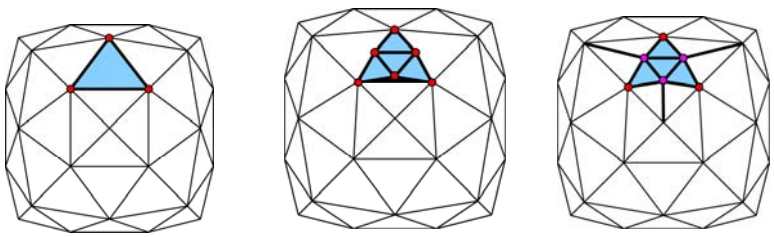
Interest based selected area

- Features: silhouette, high curvature, …

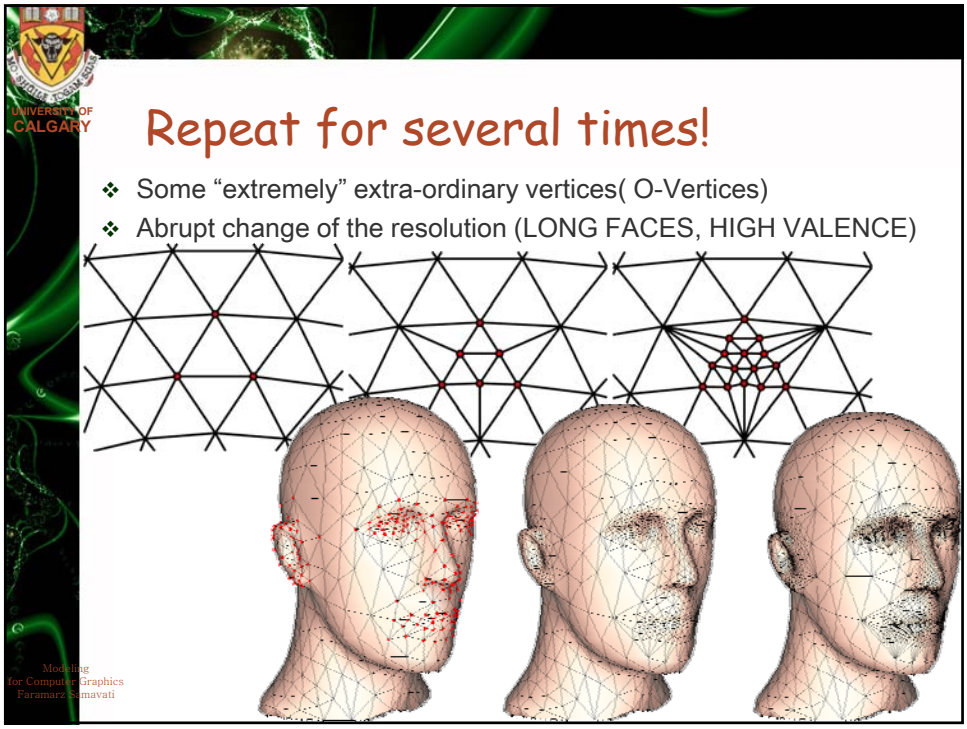


Adaptive subdivision (Loop)

- Just subdivide and split some triangles
- Cracks !
- Solution: insert new edges (T-junctions)



A. Amresh, G. Farin, and A. Razdan. Adaptive subdivision schemes for triangular meshes,2003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Repeat for several times!
Some “extremely” extra-ordinary vertices(O-Vertices)
Abrupt change of the resolution (LONG FACES, HIGH VALENCE)
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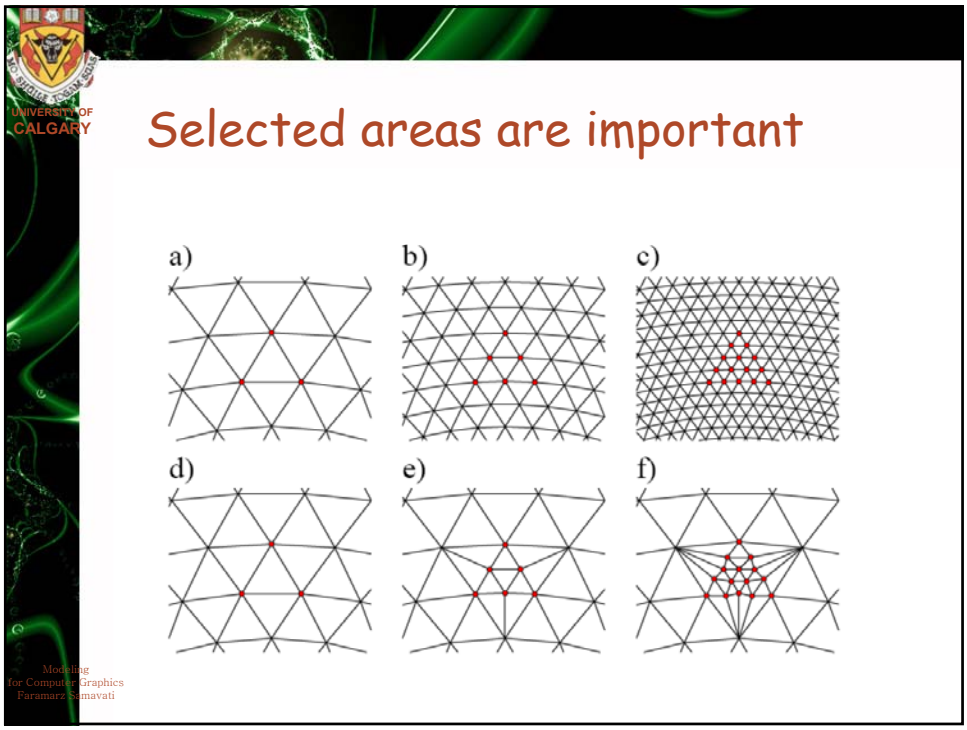
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Selected areas are important
a)
b)
c)
d)
e)
f)
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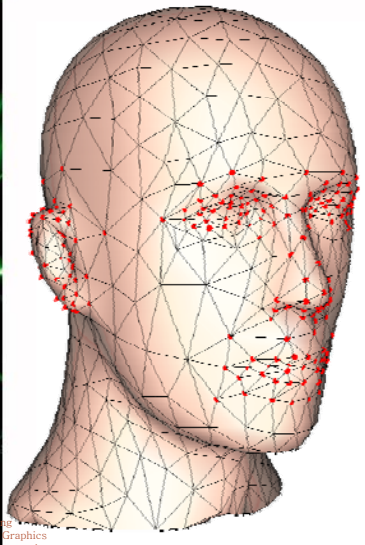
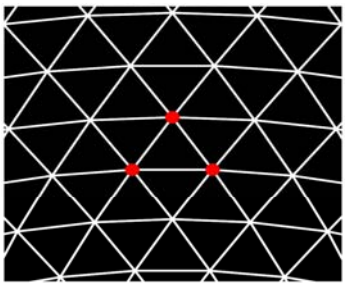
Red-Green Rules





Bank. **Refinement algorithms and data structures for regular local mesh refinement**. 1983

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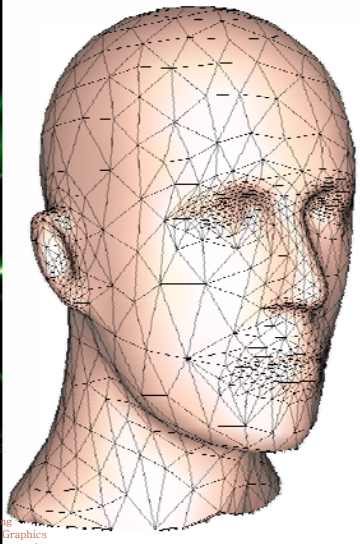
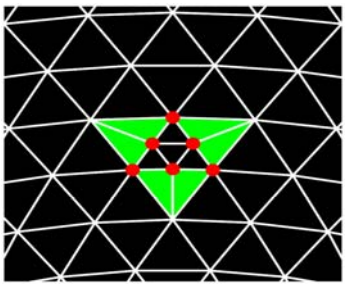
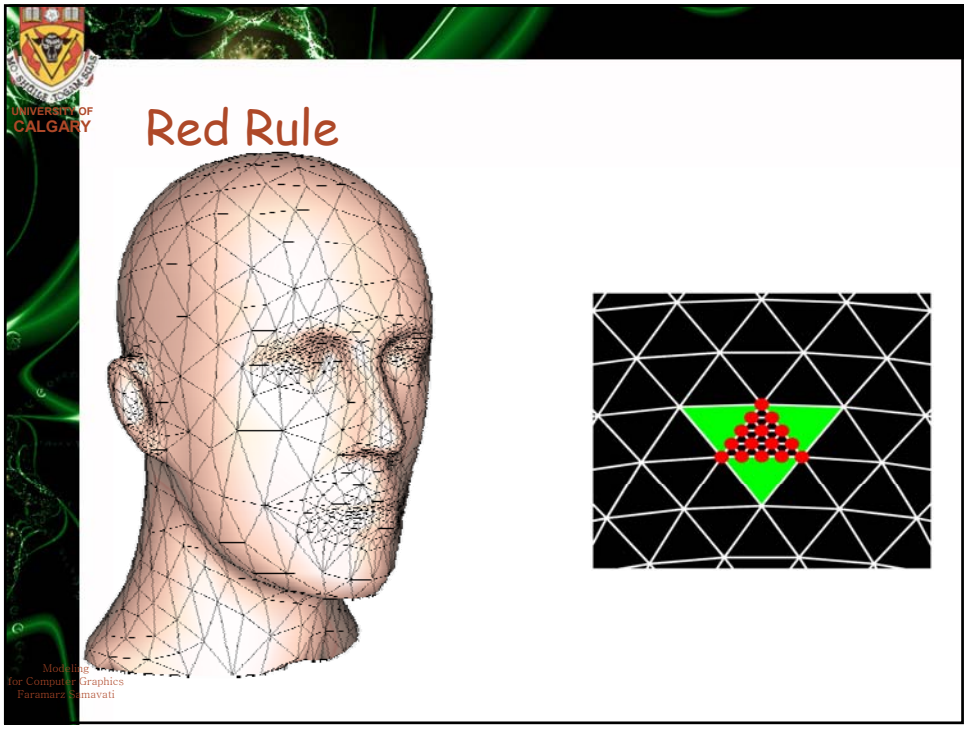
Green Rule



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Red Rule
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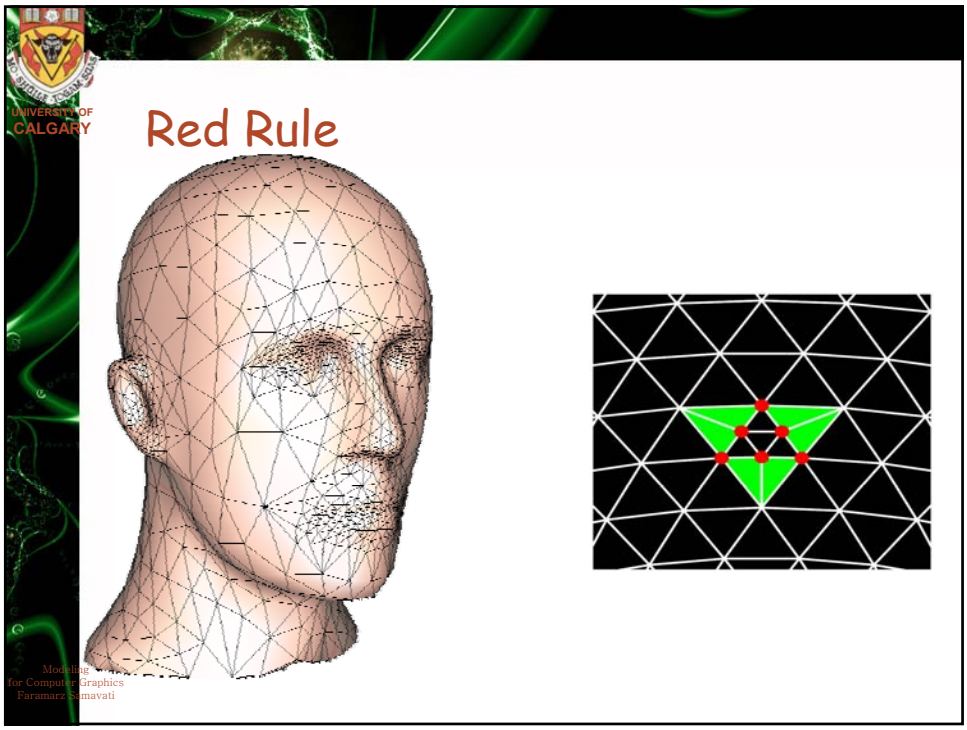
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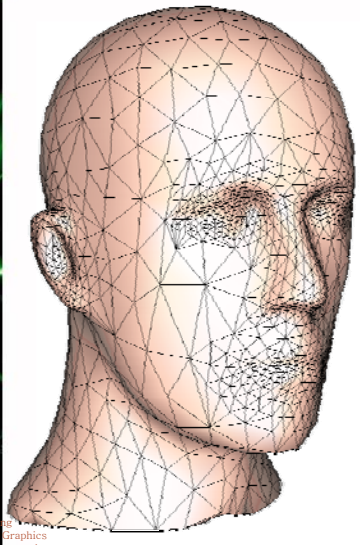
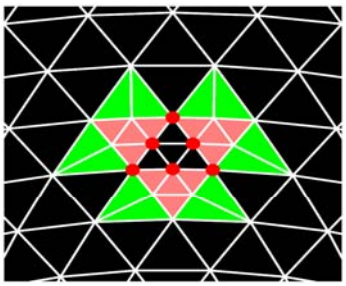
Red Rule





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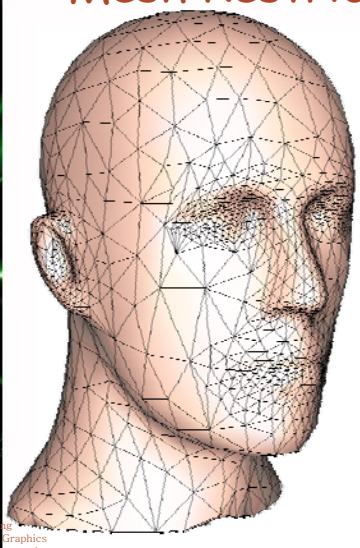
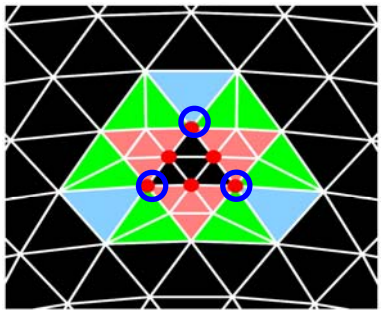
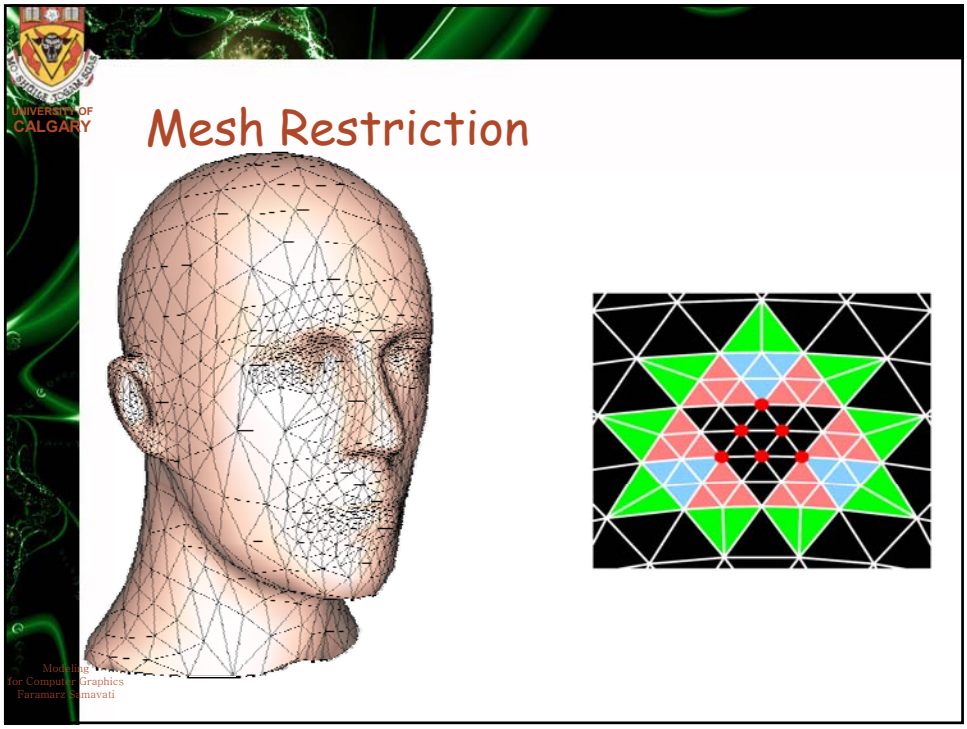
Mesh Restriction





Zorin, Schröder, Sweldens. **Interactive multiresolution mesh editing**. 1997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Mesh Restriction
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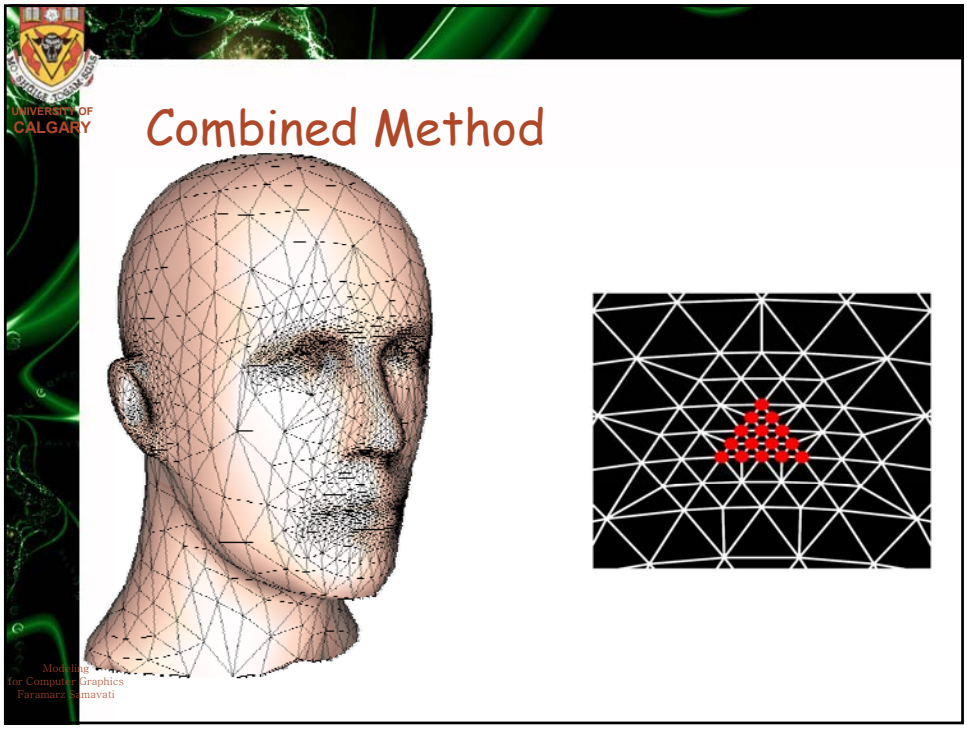
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Combined Method
Modeling
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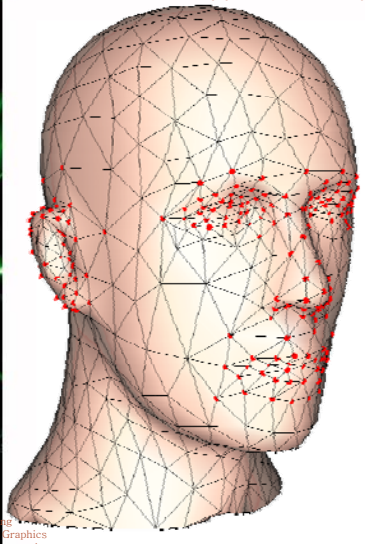
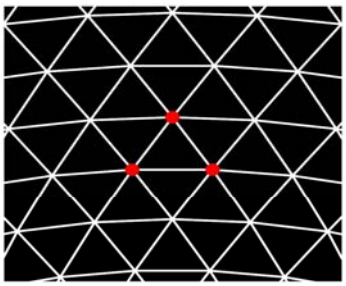
Incremental Method





Pakdel, Samavati. **Incremental adaptive Loop subdivision**. 2004
Pakdel, Samavati. **Incremental Catmull-Clark subdivision**. 2005

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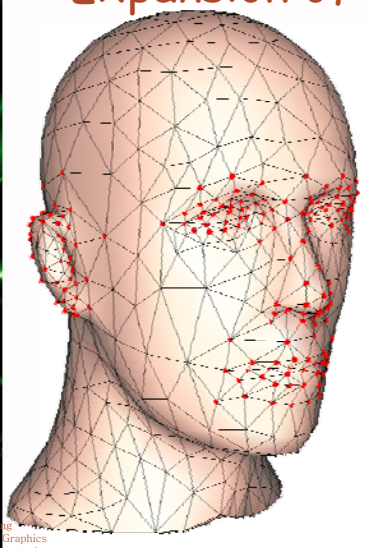
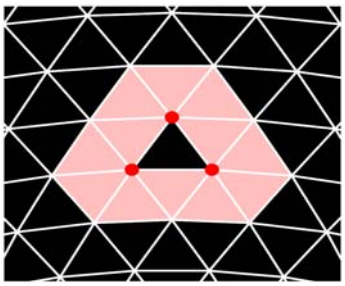
Expansion of Selected Area





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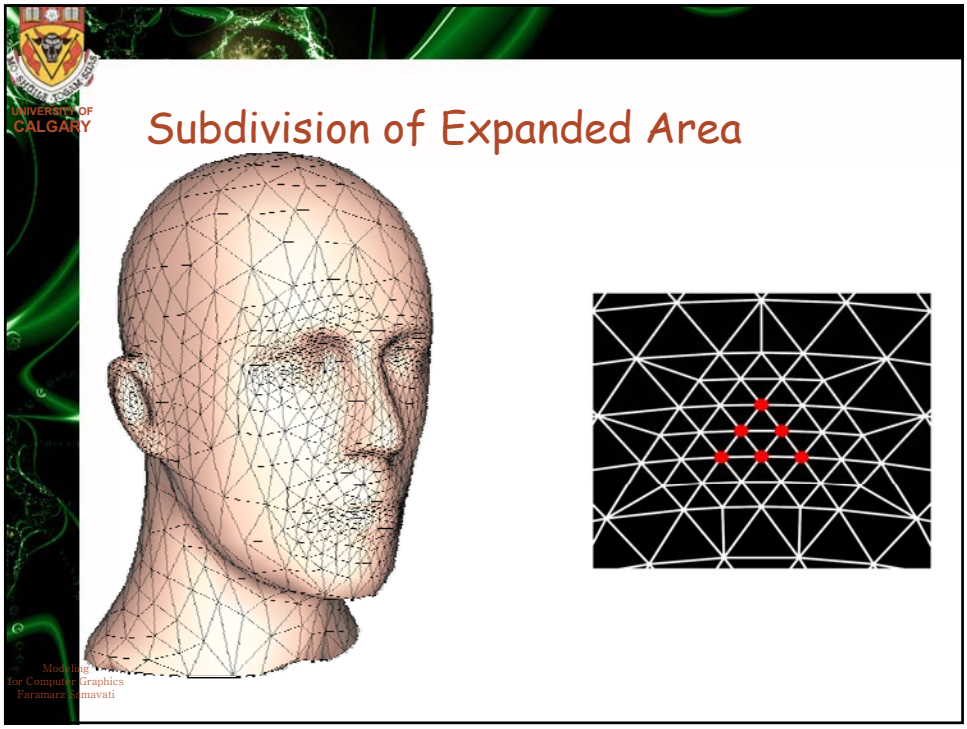
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Subdivision of Expanded Area
Modeling
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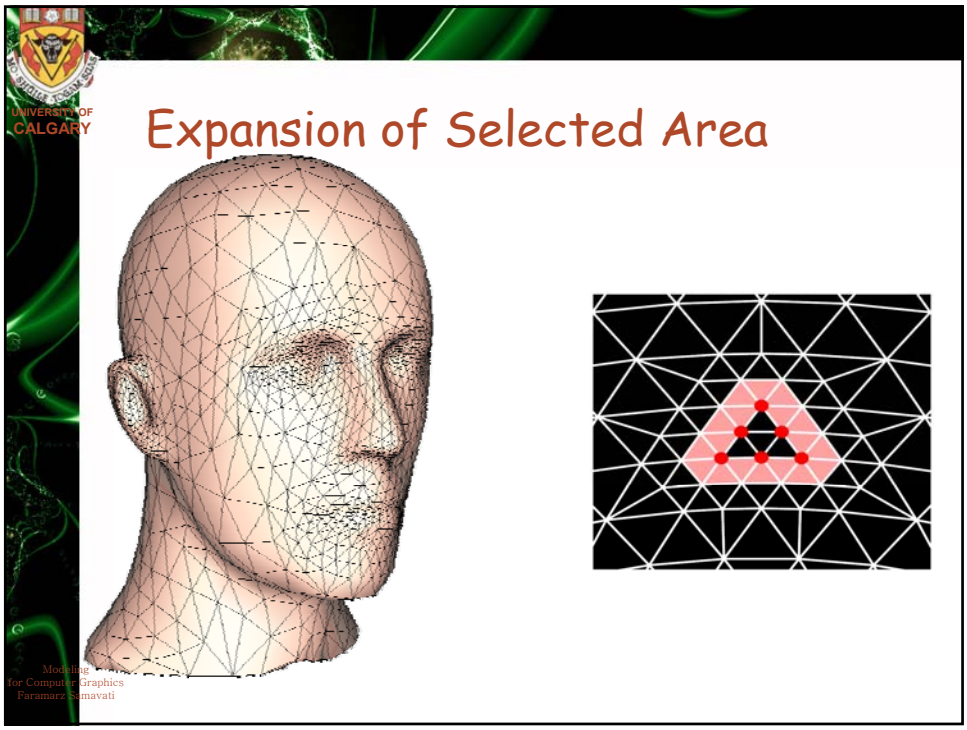
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Expansion of Selected Area
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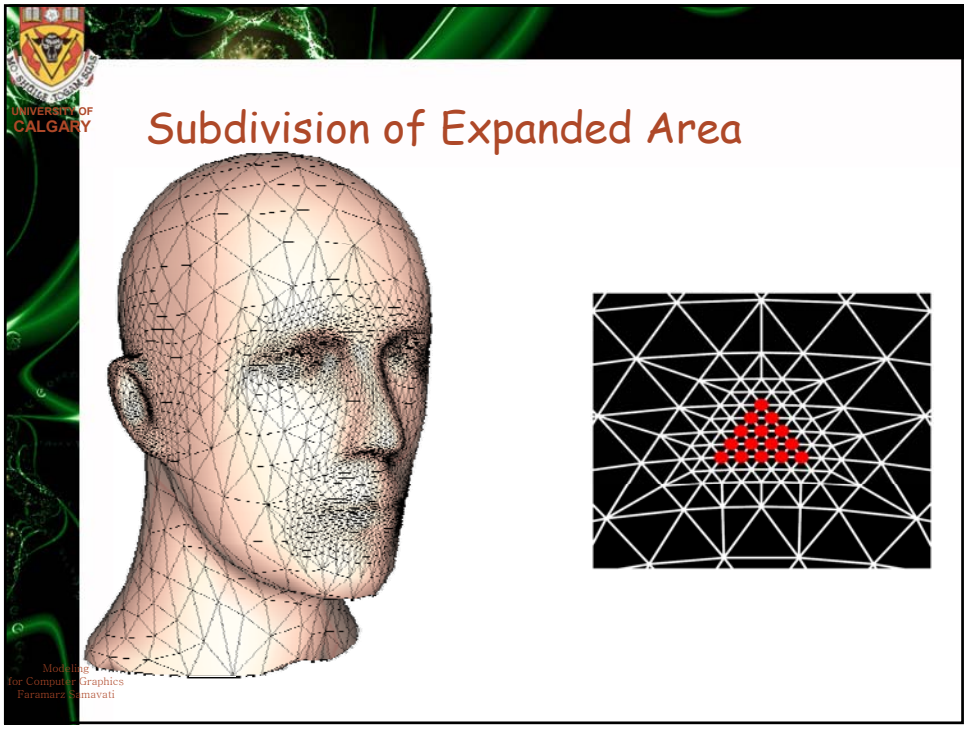
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Subdivision of Expanded Area
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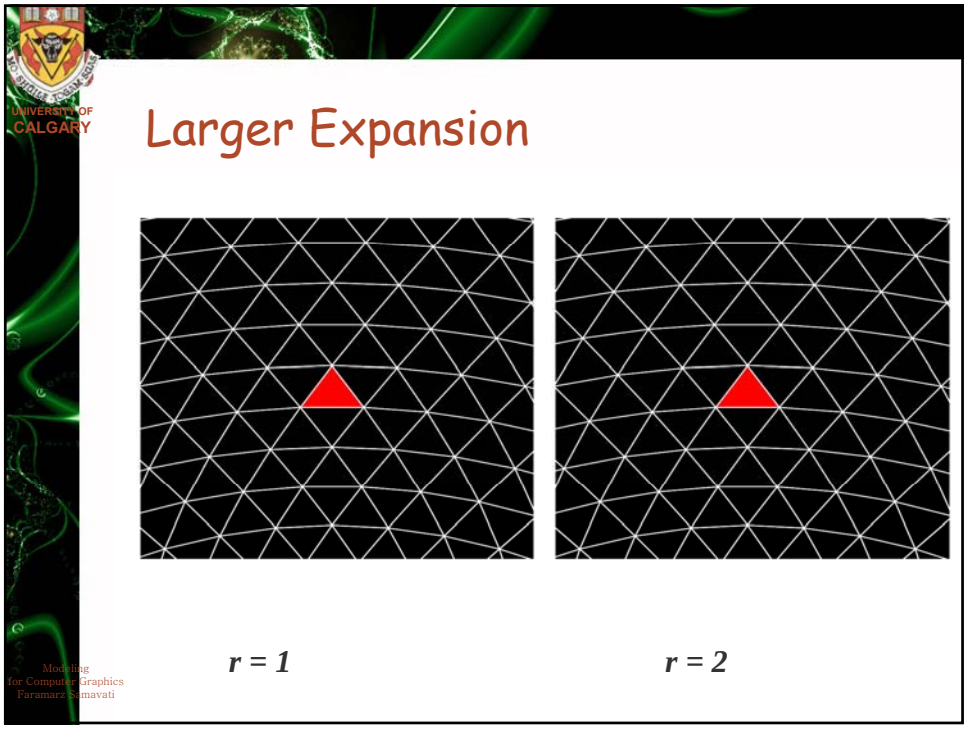
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Larger Expansion
r = 1
r = 2
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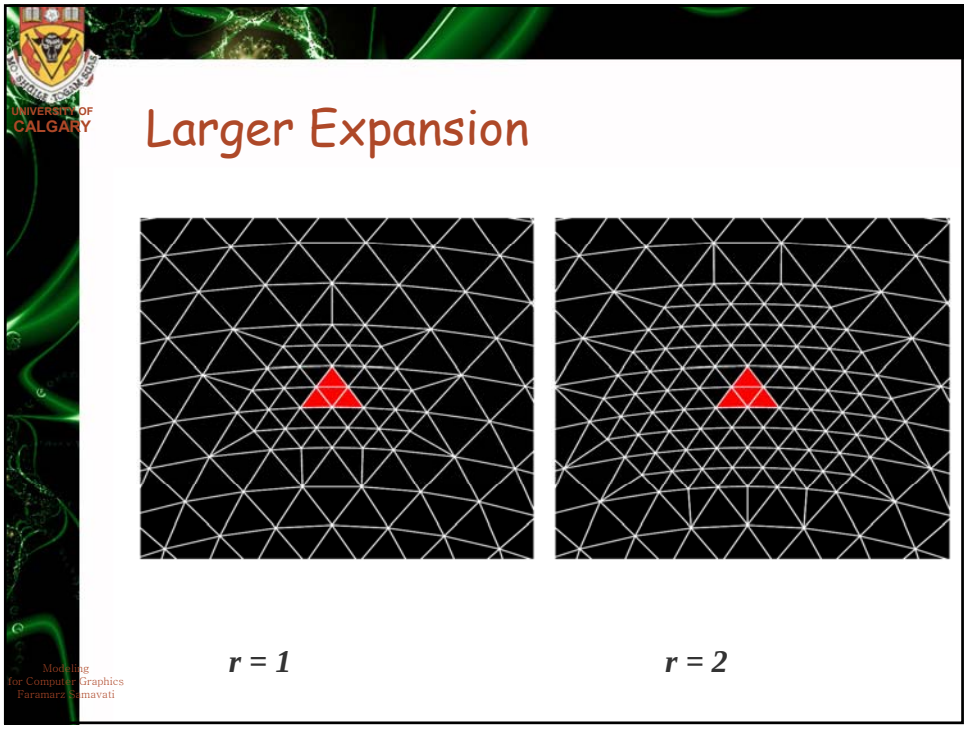
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Larger Expansion
r = 1
r = 2
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for Computer Graphics
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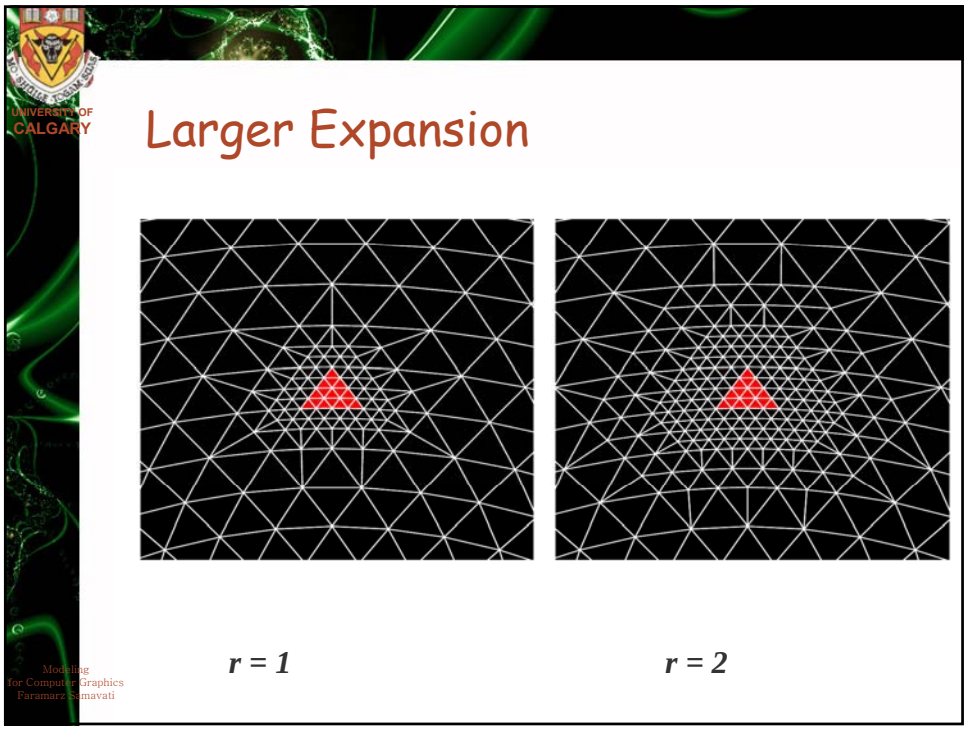
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Larger Expansion
r = 1
r = 2
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for Computer Graphics
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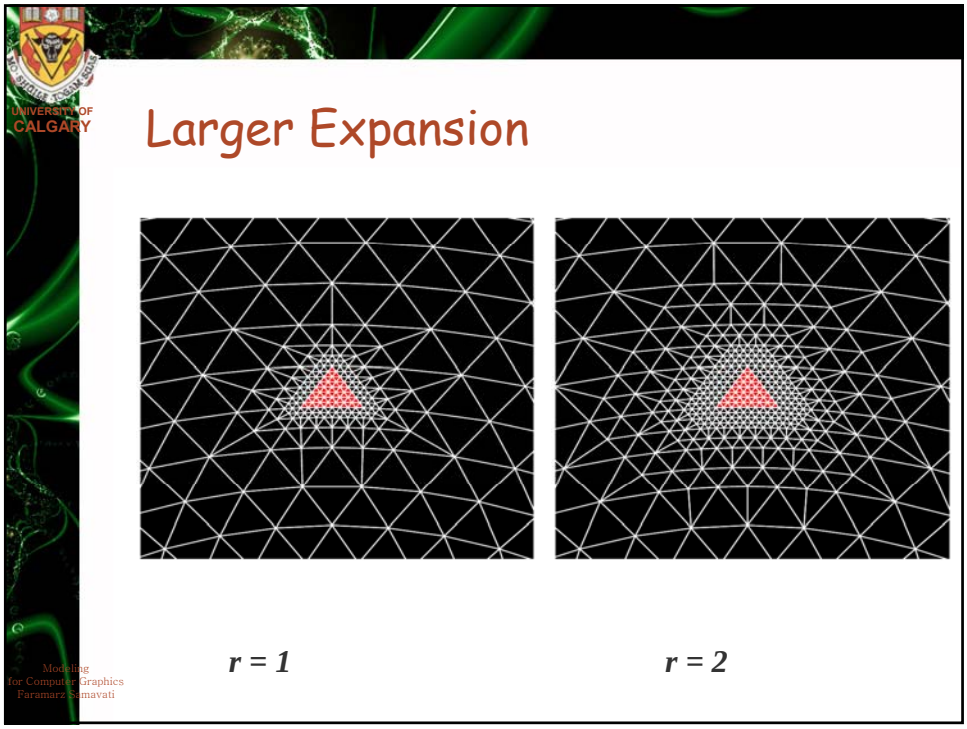
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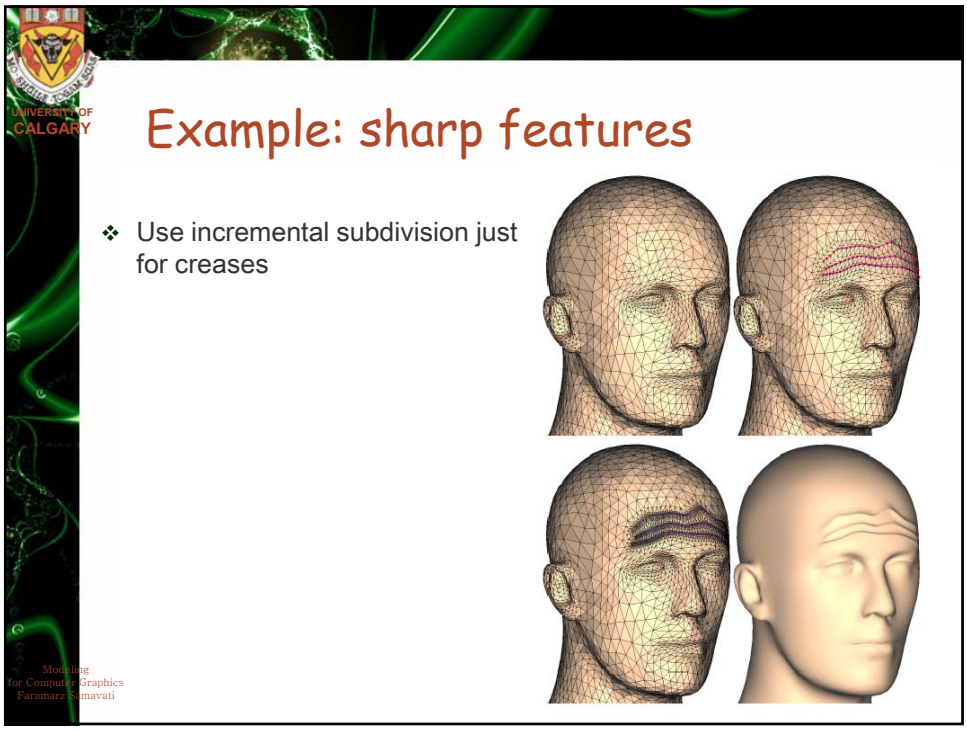
Larger Expansion
r = 1
r = 2
Modeling
for Computer Graphics
Faramarz Samavati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Example: sharp features
Use incremental subdivision just for creases
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for Computer Graphics
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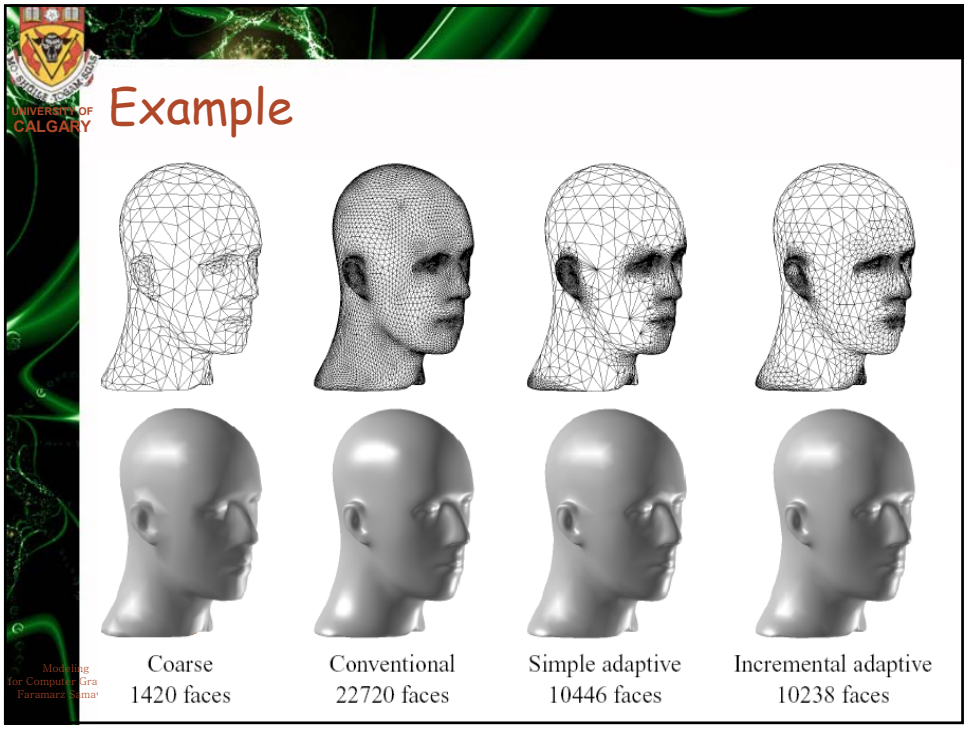
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Example
Coarse
1420 faces
Conventional
22720 faces
Simple adaptive
10446 faces
Incremental adaptive
10238 faces
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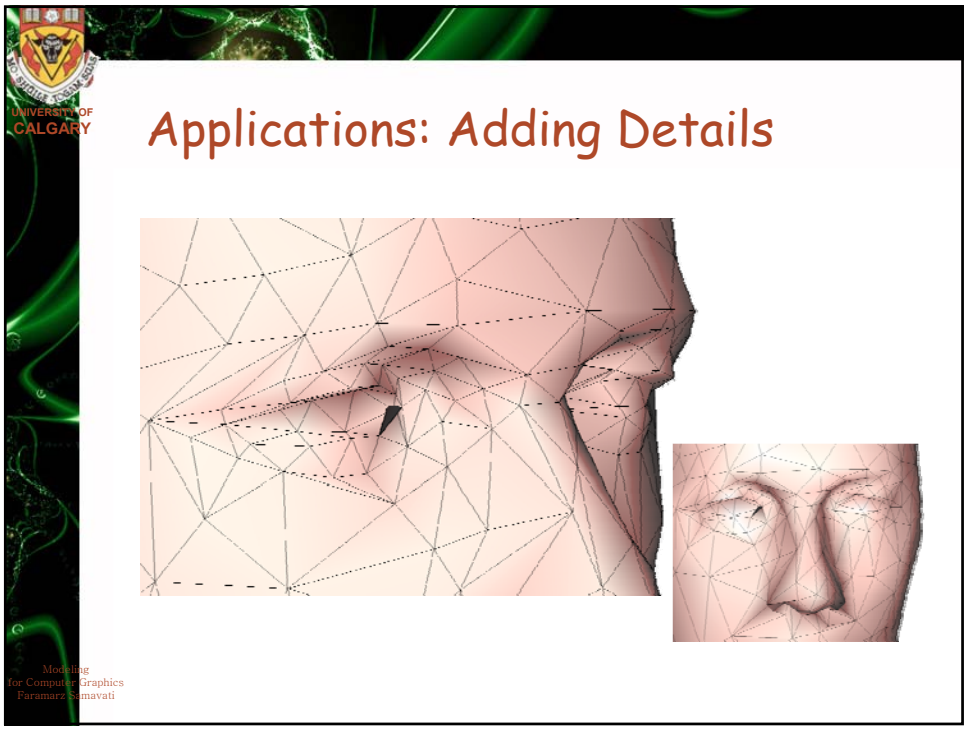
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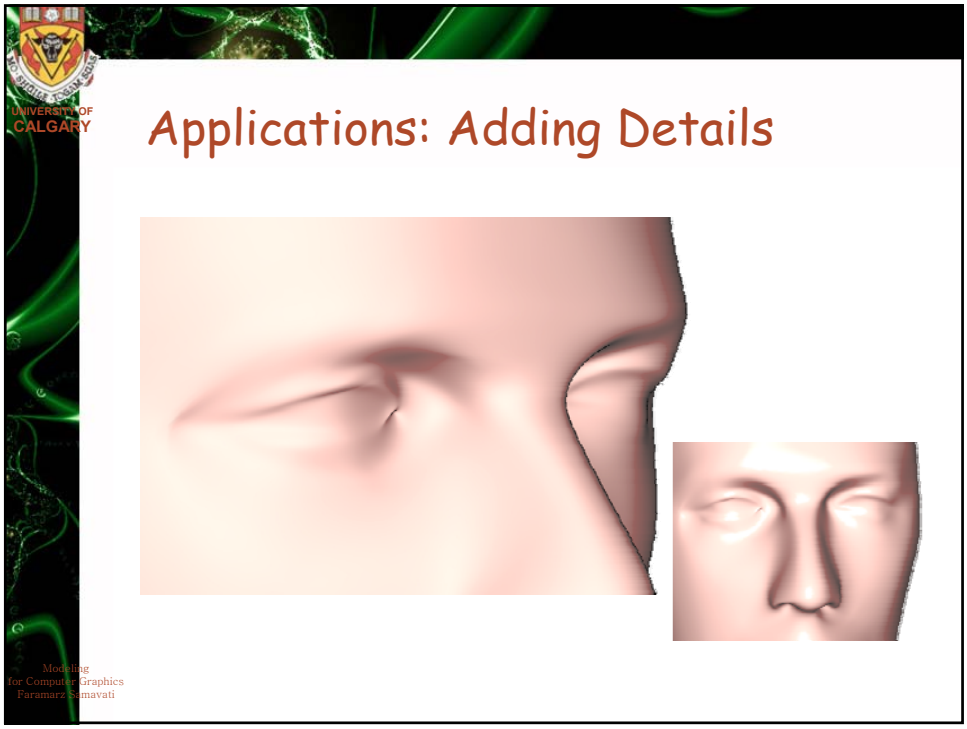
Applications: Adding Details
Modeling
for Computer Graphics
Faramarz Samavati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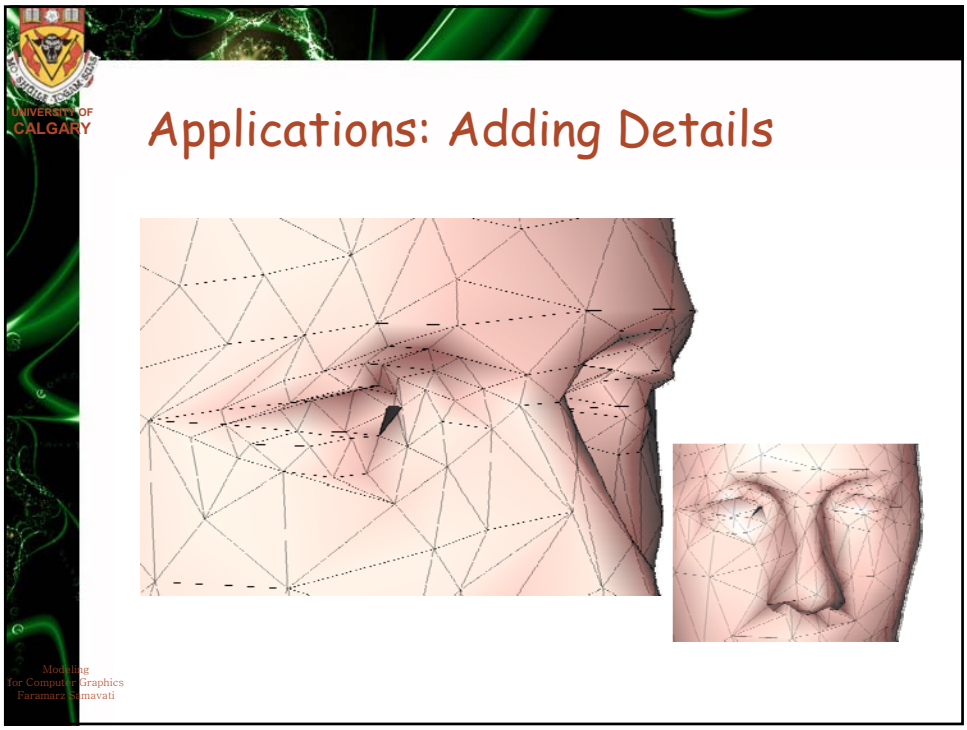
Applications: Adding Details
Modeling
for Computer Graphics
Faramarz Samavati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Applications: Adding Details
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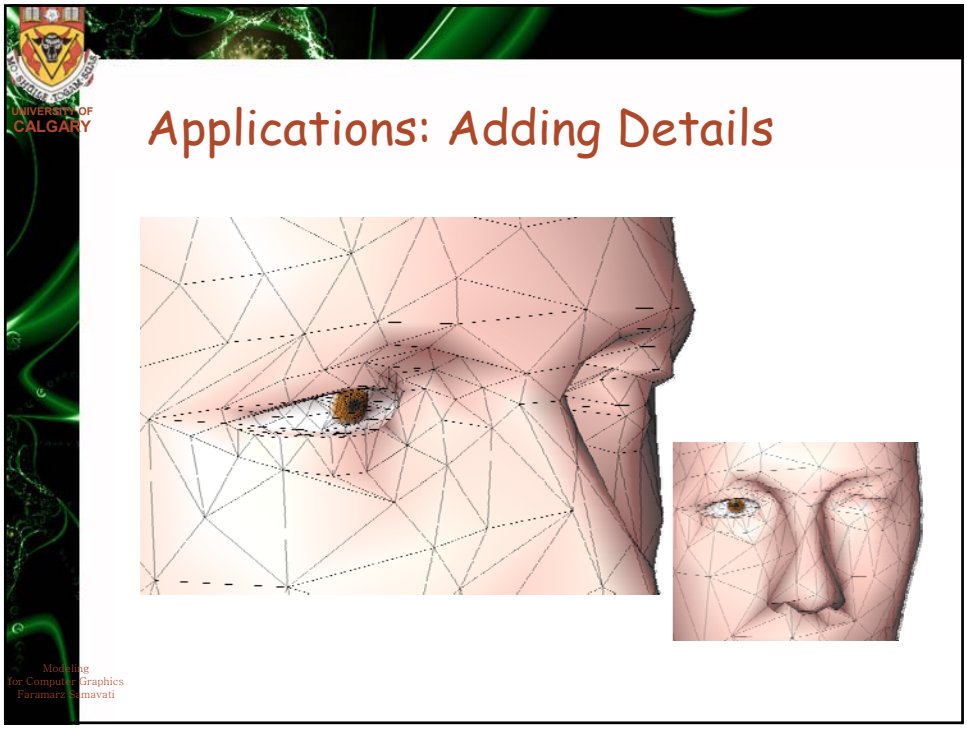
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Applications: Adding Details
Modeling
for Computer Graphics
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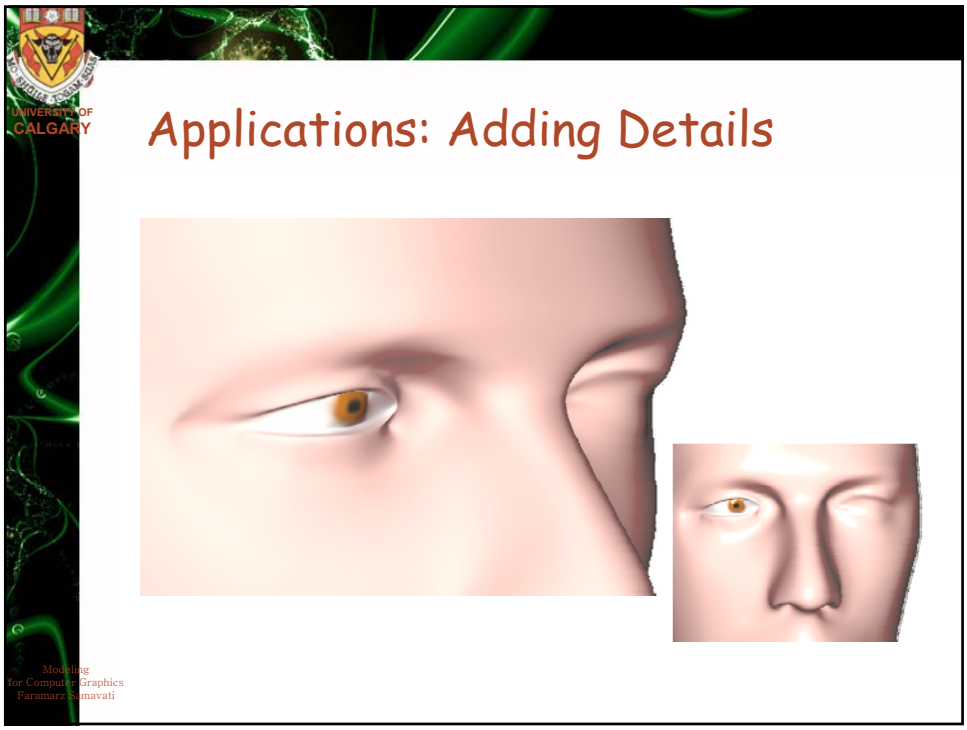
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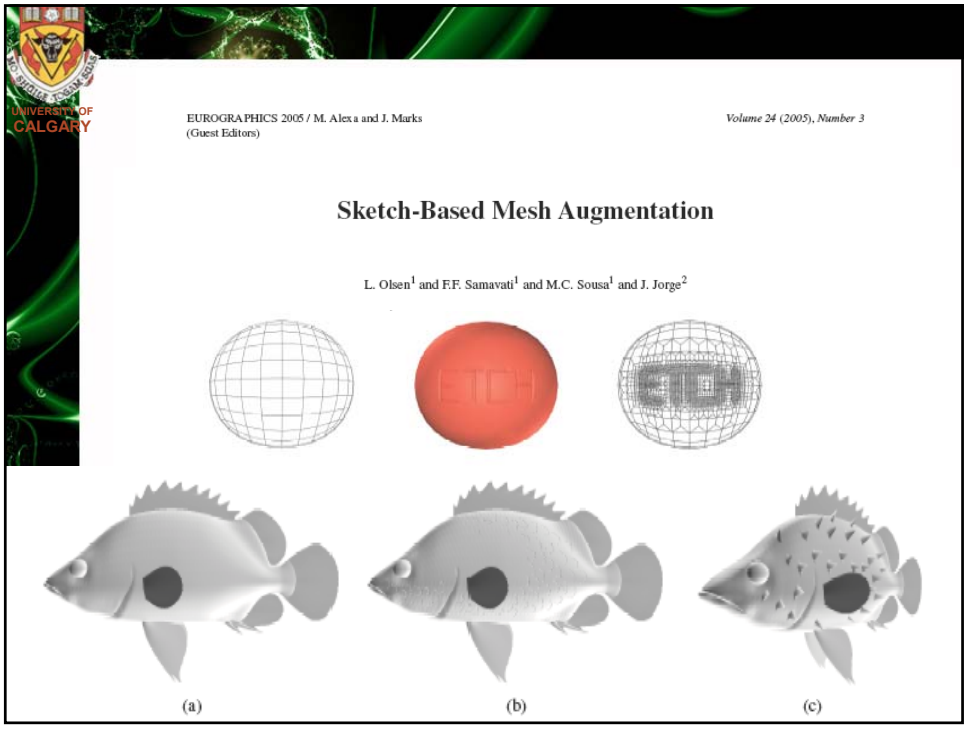
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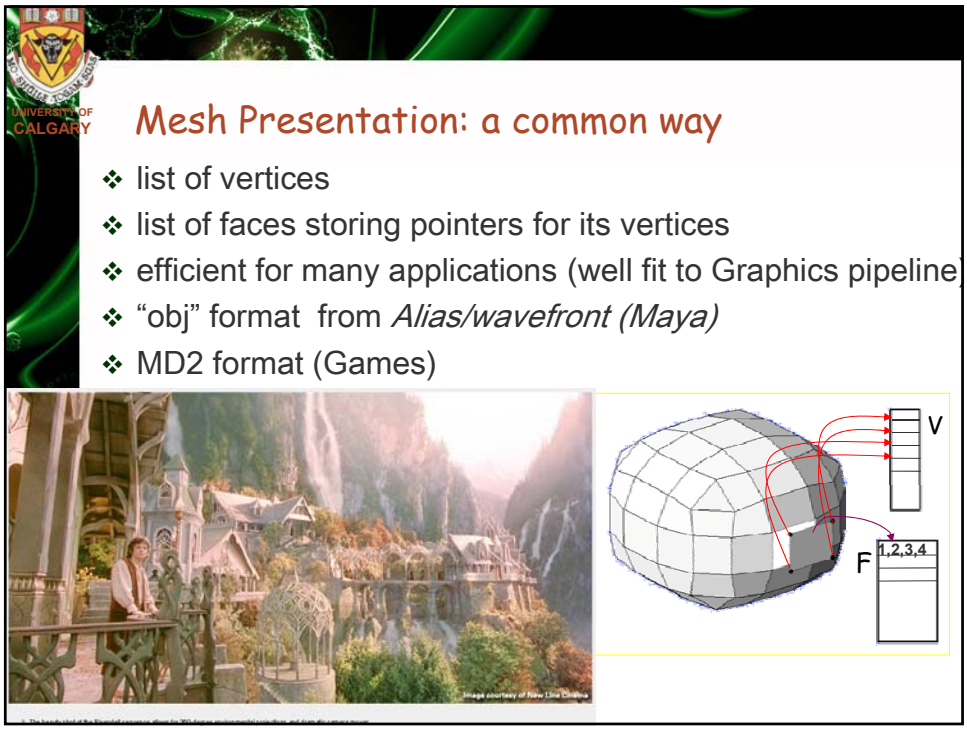
EUROGRAPHICS 2005 / M. Alexa and J. Marks
(Guest Editors)
Volume 24 (2005), Number 3
Sketch-Based Mesh Augmentation
L. Olsen[1] and F.F. Samavati[1] and M.C. Sousa[1] and J. Jorge[2]
(a)
(b)
(c)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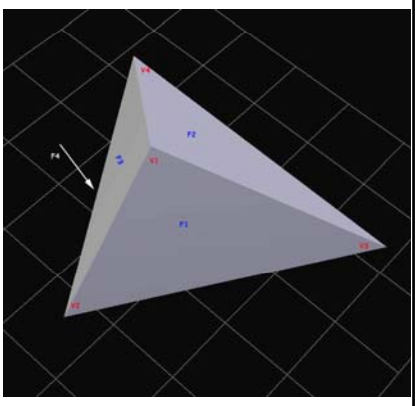
Mesh Presentation: a common way
list of vertices
list of faces storing pointers for its vertices
efficient for many applications (well fit to Graphics pipeline)
"obj" format from Alias/wavefront (Maya)
MD2 format (Games)
V
F
1,2,3,4

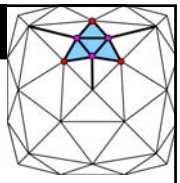
Obj Example

- Tetrahedron

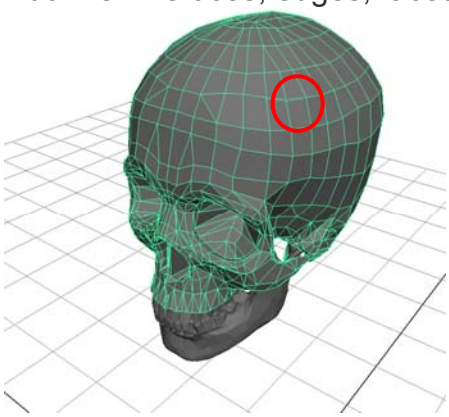
```
# Obj file format with ext .obj
v 1.0 1.0 1.0
v 2.0 0.0 0.0
v 0.0 0.0 0.0
v 1.0 1.6 0.0
f 1 3 2
f 1 2 4
f 1 4 3
f 2 3 4
```

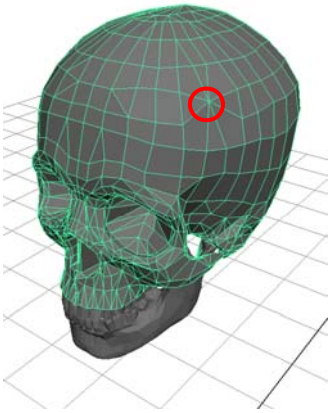


Mesh processing



Collapse: edge, vertex, face
Split: edge, vertex, face
Add: new vertices, edges, faces





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Queries

- Given a vertex, Which edges are share it?
- Given a vertex, Which vertices are adjacent to it?
- Given a vertex, which faces are adjacent to it?
- …

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Inefficiencies for mesh processing

- Hard to add a new point/vertex/face
- find the vertex neighborhood which is important for any mesh operation is very hard
- Better methods ??

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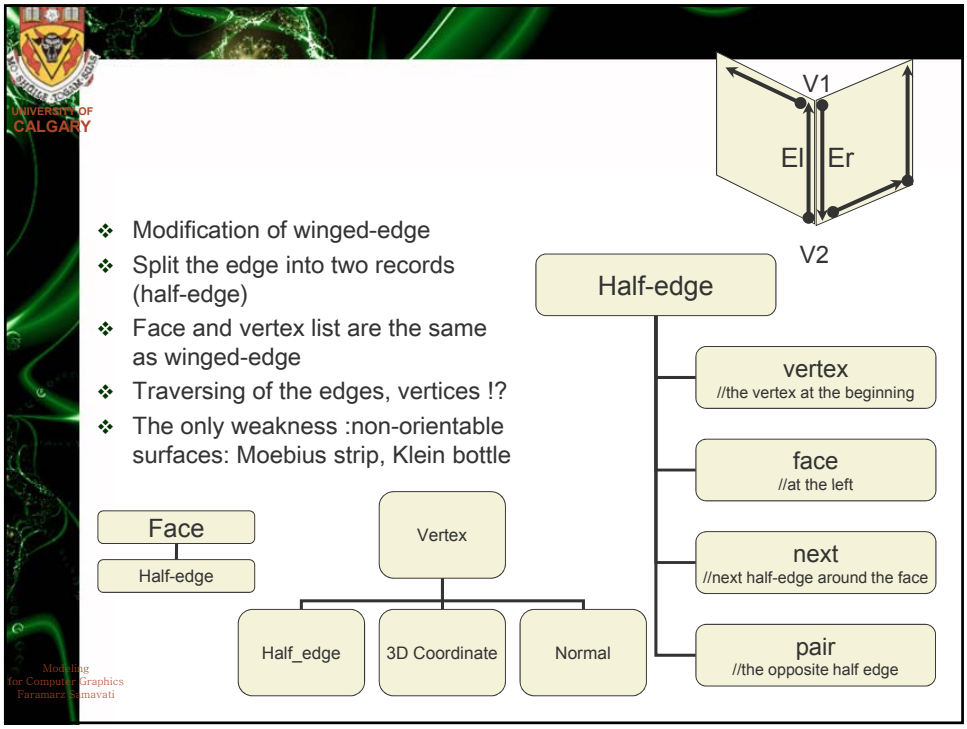
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V1
El
Er
V2
Modification of winged-edge
Split the edge into two records (half-edge)
Face and vertex list are the same as winged-edge
Traversing of the edges, vertices !?
The only weakness :non-orientable surfaces: Moebius strip, Klein bottle
Half-edge
vertex
//the vertex at the beginning
face
//at the left
next
//next half-edge around the face
pair
//the opposite half edge
Face
Half-edge
Vertex
Half_edge
3D Coordinate
Normal
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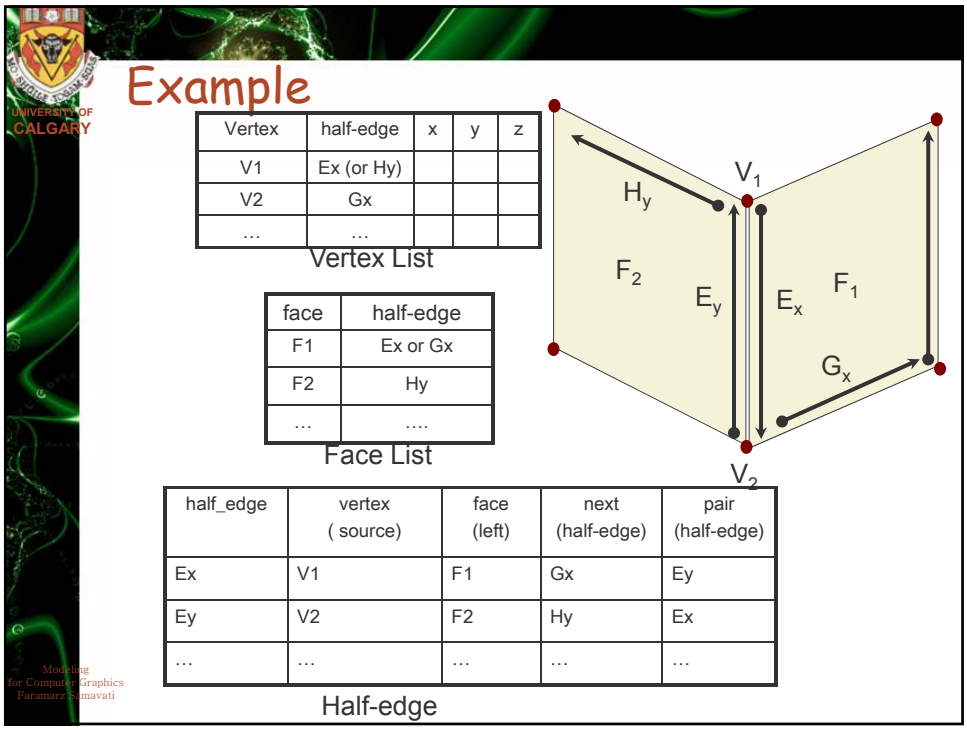
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Example
Vertex
half-edge
x
y
z
V1
Ex (or Hy)
V2
Gx
…
…
Vertex List
face
half-edge
F1
Ex or Gx
F2
Hy
…
….
Face List
half_edge
vertex (source)
face (left)
next (half-edge)
pair (half-edge)
Ex
V1
F1
Gx
Ey
Ey
V2
F2
Hy
Ex
…
…
…
…
…
Half-edge
V_1
H_y
F_2
E_y
E_x
F_1
G_x
V_2
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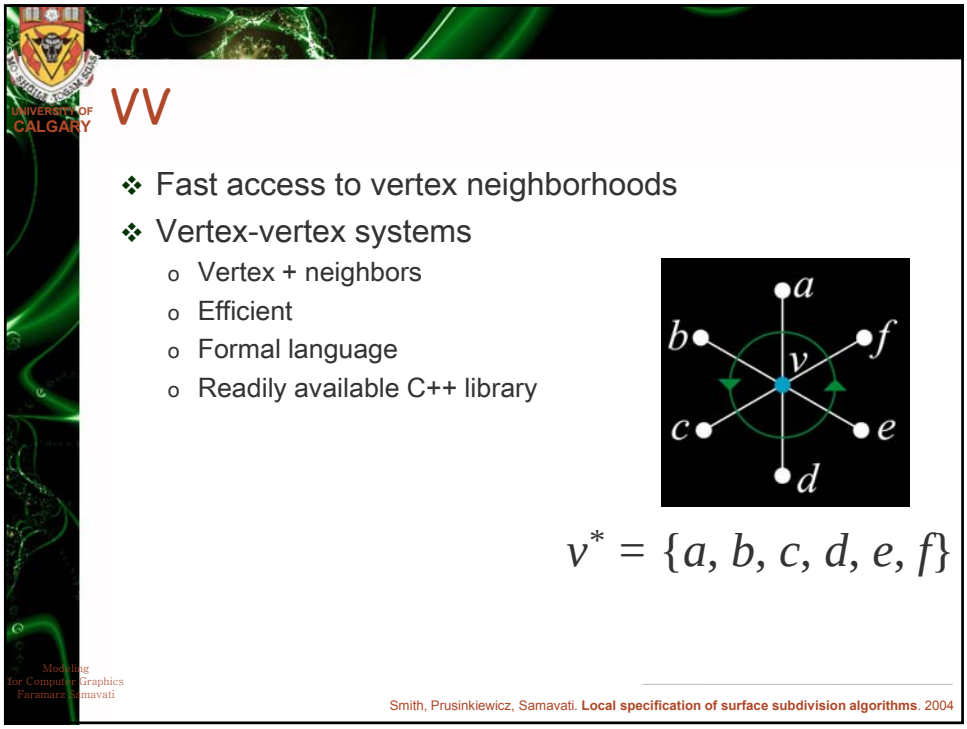
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VV
Fast access to vertex neighborhoods
Vertex-vertex systems
Vertex + neighbors
Efficient
Formal language
Readily available C++ library
a
b
f
v
c
e
d
$v^* = \{a, b, c, d, e, f\}$
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Smith, Prusinkiewicz, Samavati. Local specification of surface subdivision algorithms. 2004