Shape Inflation With an Adapted Laplacian Operator For Hybrid Quad/Triangle Meshes

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Outline

Motivation

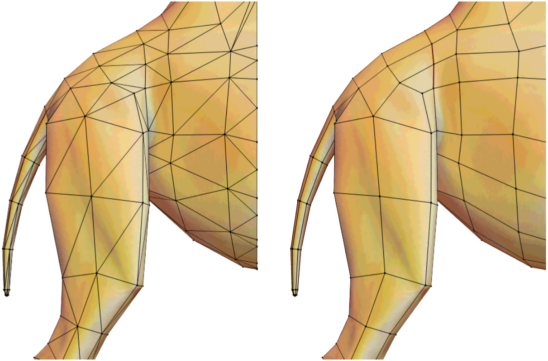


Figure: Triangle vs Quads

Related Work

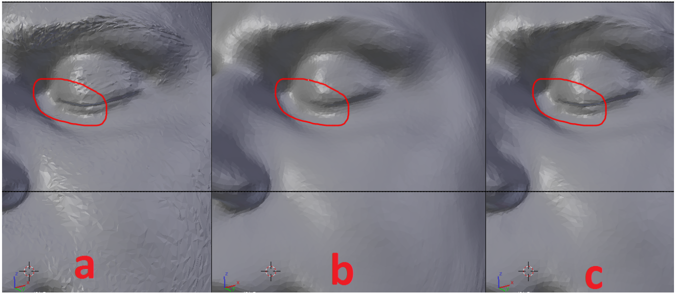
- Tools based on the Laplacian operator [8, 9, 10].
- Offset methods for polygonal meshing [1, 13].
- Shape edition [3, 6].
- Digital sculpting [11, 4].

Laplacian Smoothing

Area integral of the surface S.

$$E(S) = \int_S \kappa_1^2 + \kappa_2^2 dS$$

Where κ_1 and κ_2 are the two principal curvatures of the surface S.



Gradient of Voronoi Area

The area change produced by the movement of v_i is called the gradient of *Voronoi region* [7, 2, 5]

$$\nabla A = \frac{1}{2}\sum_{j} \left(\cot \alpha_j + \cot \beta_j\right)\left(v_i - v_j\right)$$

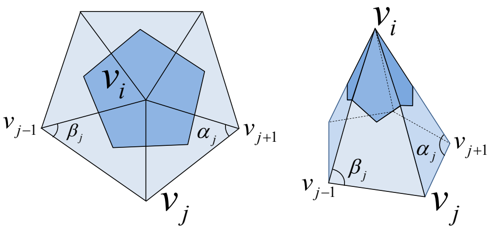


Figure: Area of the Voronoi region around v_i in dark blue. v_j belong to the first neighborhood around v_i. α_j and β_j opposite angles to edge $\overrightarrow{v_j - v_i}$.

Laplace Beltrami Operator

Laplace Beltrami Operator for Hybrid Quad/Triangle Meshes
TQLBO

$$\triangle_S(v_i) = 2\kappa\mathbf{n} = \frac{\nabla A}{A} = \frac{1}{2A}\sum_{v_j \in N_1(v_i)} w_{ij}(v_j - v_i)$$

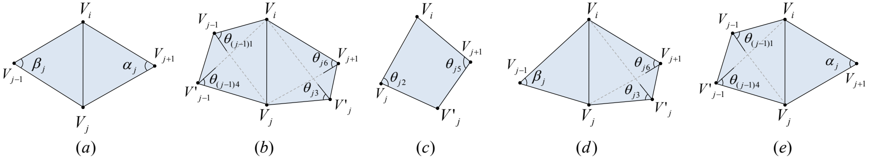


Figure: The 5 basic triangle-quad cases with common vertex V_i and the relationship with V_j and V'_j. (a) Two triangles [2]. (b) (c) Two quads and one quad [12]. (d) (e) Triangles and quads (TQLBO) Our Contribution.

LBO as a matrix equation

We define a TQLBO as a matrix equation

$$L(i,j) = \begin{cases} -\frac{1}{2A_i} w_{ij} & \text{if } j \in N(v_i) \\ \frac{1}{2A_i} \sum\limits_{k \in N(v_i)} w_{ik} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Where L is a $n \times n$ matrix, n is the number of vertices of a given mesh M, $N(v_i)$ is the 1-ring neighborhood with shared face to v_i, A_i is the ring area around v_i, and w_{ij} is.

$$w_{ij} = \begin{cases} (\cot \alpha_j + \cot \beta_j) & \text{case } a. \\ \frac{1}{2}\left(\cot \theta_{(j-1)1} + \cot \theta_{(j-1)4} + \cot \theta_{j3} + \cot \theta_{j6}\right) & \text{case } b. \\ (\cot \theta_{j2} + \cot \theta_{j5}) & \text{case } c. \\ \frac{1}{2}(\cot \theta_{j3} + \cot \theta_{j6}) + \cot \beta_j & \text{case } d. \\ \frac{1}{2}\left(\cot \theta_{(j-1)1} + \cot \theta_{(j-1)4}\right) + \cot \alpha_j & \text{case } e. \end{cases}$$

The Shape Inflation

A standard diffusion process is used.

$$\frac{\partial V}{\partial t} = \lambda L(V)$$

To solve this equation, implicit integration is used as well as a normalized version of TQLBO matrix

$$(I - |\lambda dt|\, W_p L)\, V' = V^t$$

$$V^{t+1} = V^t + \operatorname{sign}(\lambda)\,(V' - V^t)$$

where L is the TQLBO, V' are the smoothing vertices, V^t are the actual vertices positions, W_p is a diagonal matrix with vertex weights, and λdt is the inflate factor.

Sculpting
Inflate Brush

Real-time brushes require the Laplacian matrix is constructed with the vertices that are within the sphere radius defined by the user, reducing the matrix to be processed.

$$L(i,j) = \begin{cases} -\frac{w_{ij}}{\sum\limits_{j \in N(v_i)} w_{ij}} & \text{if } \|v_i - u\| < r \wedge \|v_j - u\| < r \\ 0 & \text{if } \|v_i - u\| < r \wedge \|v_j - u\| \geq r \\ \delta_{ij} & \text{otherwise} \end{cases}$$

Where $v_j \in N(v_i)$, u is the sphere center of radius r. The matrices should remove rows and columns of vertices that are not within the radius.

Test LBO in Triangles and Quads

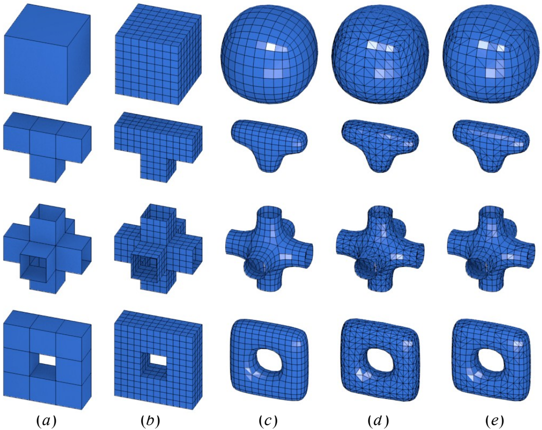


Figure: (a) Original Model. (b) Simple subdivision. (c), (d) (e) Laplacian smoothing with $\lambda = 7$ and 2 iterations: (c) for quads, (d) for triangles, (e) for triangles and quads random chosen.

Shape Inflation

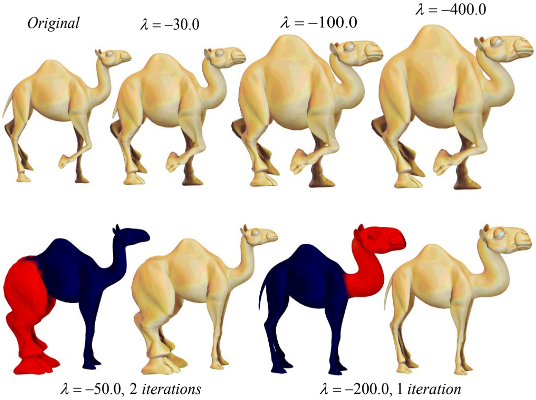


Figure: Top row: Original camel model in left. Shape inflation with $\lambda = -30.0$, $\lambda = -100.0$, $\lambda = -400.0$. Bottom row: Shape inflation with weight vertex group, $\lambda = -50.0$ and 2 iterations for the legs, $\lambda = -200.0$ and 1 iteration for the head and neck.

Shape inflation Brush

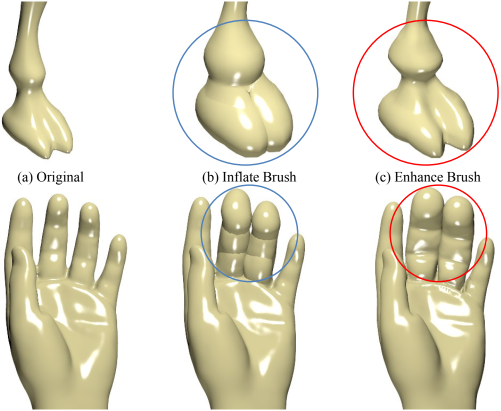


Figure: Top row: (a) Leg Camel, (b) Traditional inflate brush for leg into blue circle, (c) Shape inflation brush for leg into red circle. Bottom row: (a) Hand, (b) Traditional inflate brush for fingers into blue circle, (c) Shape inflation brush for fingers in red circle.

Shape inflation Brush

Performance

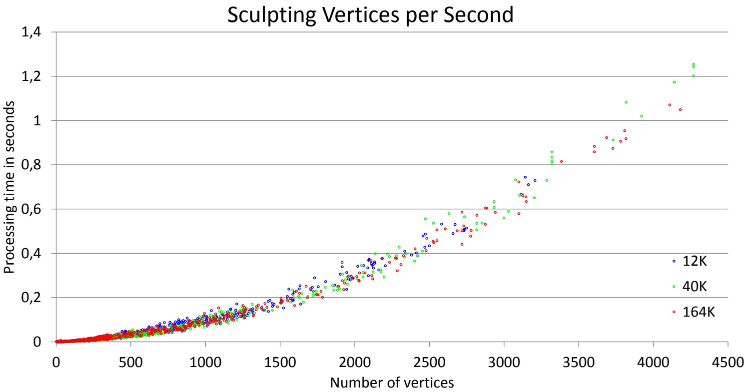


Figure: Performance of our dynamic shape inflation brush in terms of the sculpted vertices per second. Three models with 12K, 40K, 164K vertices used for sculpting in real time.

Invariant Under Isometric Transformations

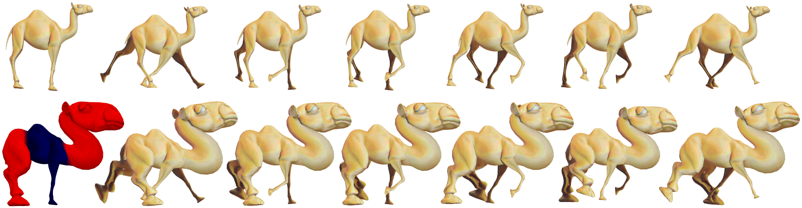


Figure: The method is pose insensitive. The inflation for the different poses are similar in terms of shape. Top row: Original walk cycle camel model. Bottom row: Shape inflation with weight vertex group, $\lambda = -400$ and 2 iterations.

Conclusion

- This work presented an extension of the Laplace Beltrami operator for hybrid quad/triangle meshes.
- This paper has introduced a new way to change silhouettes in a mesh for sculpting.
- The method works properly with isometric transformations, opening the possibility of introducing it on the process of animation.

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