236861 Numerical Geometry of Images

Tutorial 11

Active Contours

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Active Contours - introduction

- *Active contours* is a catch-all name for finding the curve that best segments an image.
- This is known as *segmentation*.
- Segmentation is highly related to tracking.
- In $3D$ - *active surfaces*.

Region-based vs. Edge-based functionals

Many segmentation functionals can be divided into two groups

- Edge-based - the object to be segmented should have its boundary visible in the image, as some sort of prominent edge.
- Region-based - the region of the object in the image should have a different statistic in some feature space, compared to its surroundings.

Often, these functionals are combined with information on the shape of the region being extracted.

Examples for functionals

- Edge-based segmentation - Snakes (Terzopolous-Kass-Witkin '87), Geometric active contours (Caselles-Catte-Coll-Dibos '93, Malladi-Sethian-Vemuri '95), Geodesic active contours (Caselles-Kimmel-Sapiro '98).
- Region-based - (Cohen-Bardinet-Ayache '93), Chan-Vese ('01), Bhattacharyya (Freedman-Zhang '04, Rathi-Michailovich-Tannenbaum '06).

Various *shape priors* are available:

- Geometry-based - Curvature based (Sethian '85, Osher-Sethian '88), affine curvature based (Angenent-Sapiro-Tannenbaum '98) and many more.
- Shape-Spaces - Leventon-Grimson-Faugeras '00, Cremers-Tischhäuser-Weickert-Schnörr '02, and many more.
- Projective geometry based - Damberville-Sandhu-Yezzi-Tannenbaum '08, Sandhu-Damberville-Yezzi-Tannenbaum '08.

Snakes

- The *snakes* model try to segment the image based on the following energy:

$$E_{snake} = E_{int} + E_{ext}$$

where

$$E_{int} = \int \alpha |c'|^2 + \beta |c''|^2 ds$$

and, as one simple example,

$$E_{ext} = -|\nabla I|^2$$

- Optimization is done using splines.

Geometric active contours

- *Geometric active contours* attempt to segment an object based on its edges, in a level-set framework.
- The initial contour is chosen to include the object.
- The contour evolves according to

$$c_t = g(I)\kappa N$$

- where $g(\cdots)$ is a function which should drop to zero at edges.
- The contour evolution tends to smooth the contour, if no other information is available.
- The contour according to this evolution will shrink to a point. Hence, a *balloon force* (Cohen '91) may be added

$$c_t = (g(I)\kappa - \beta)N$$

Geodesic active contours

- However the choice of a balloon force is arbitrary.
- It is not clear if we actually minimize some functional, and the global minimizer is not clear either.
- The *geodesic active contour* tries to remedy this by minimizing the following weighted length functional:

$$\int_0^{L(c)} g(I)ds$$

- $g(I)$ constitutes an (inverse) edge indicator. For example,

$$g(I) = \frac{1}{\sqrt{|\nabla I|^2 + \epsilon}}$$

Geodesic active contours (cont.)

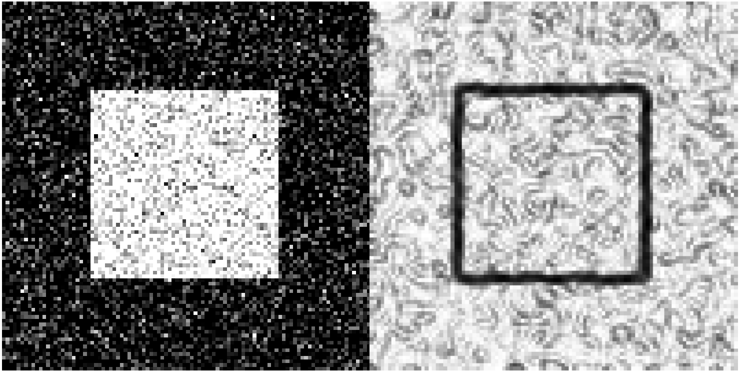


Figure: Left: an image. Right: An example g function

- The functional states that curves segmenting the object should try and surround it with a minimal weighted arclength.

Geodesic active contours (cont.)

- This can be given a physical interpretation: We are looking for the trajectory of a particle on a map, where the potential energy at each point is $-\lambda g(I)^2$, and we assume the particle's trajectory should form a closed simple curve.
- The potential energy of the particle is given by

$$\mathcal{U}(c) = -\lambda g(I)^2$$

- From physics, the *Hamiltonian* will be:

$$\mathcal{H}(c) = \frac{m}{2}|c'|^2 + \mathcal{U}(c)$$

- and the *Lagrangian*, or difference between kinetic and potential energy, is

$$\mathcal{L}(c) = \frac{m}{2}|c'|^2 - \mathcal{U}(c)$$

- This gives us the classical approach of *snakes*

Geodesic active contours (cont.)

- The E-L equation will simply state conservation of energy for this particle under the chosen trajectory. The trajectory chosen should corresponde to the Hamiltonian.
- *Maupertuis' principle:* Curves in Euclidean space which are extremal corresponding to the Hamiltonian, and have an energy level E_0, are geodesics with respect to the metric

$$g_{ij} = 2m\left(E_0 - \mathcal{U}(c)\right)\delta_{ij}$$

Geodesic active contours (cont.)

- The initial energy level E_0 is arbitrary.
- We now choose $E_0 = 0$ (for an ideal edge we want $E_{ext} = E_{int} = 0$), and obtain

$$min \int_0^1 \sqrt{\lambda 2mg(I)^2}|c'|dq = min \int_0^1 \sqrt{2m\lambda}g(I)|c'|dq = \sqrt{2m\lambda} min \int_0^{L(c)} g(I)ds$$

- Which is an intrinsic, Euclidean-invariant functional with no need for additional parameters.
- The resulting curve evolution is given by

$$c_t = (g(I)\kappa - \langle \nabla g, N \rangle) N$$

Geodesic active contours (cont.)

- Note the geodesic interpretation also works for parts of the curve (Cohen and Kimmel, '97).

Figure: Segmentation by geodesics. Left: The metric $\sqrt{g}$. Right: The resulting geodesic.

Geodesic active contours (example)

gac2.mp4

Geodesic active contours (implementation)

Two main techniques are available for efficiently implementing the geodesic active contours model

- *Narrow band* (Chopp '93, Adalsteinsson and Sethian '95) methods compute the levelset function on only part of the image, where relevant.
- Semi-implicit schemes allow us to take large time steps and converge faster.
- Multigrid methods may also be used (Kenigsberg et. al. '04, Papandreou and Maragos '07).

Narrow band and splitting schemes may be combined! (Goldenberg, Kimmel, Rivlin, Rudzsky '01)

The Mumford-Shah model

- Looking only locally at the gradient for segmentation is quite limited, in terms of basin of attraction and robustness to noise. Region statistics give us a lot more information.
- The Mumford-Shah model of image description ('89) partitions the image into:
  - Smooth parts, which we can approximate by smooth functions, and
  - A small amount of edges.
- The resulting functional is

$$\alpha \int_{\Omega \setminus \Gamma} |\nabla u|^2 d\Omega + \beta \int_{\Omega} |u - u_0|^2 d\Omega + \mathcal{H}^{N-1}(\Gamma)$$

- where the term $\mathcal{H}^{N-1}(\Gamma)$ denotes the measure of boundary curves, at which discontinuities are allowed.
- The functional is defined both in terms of a 2-dimensional function, and a set of 1-dimensional curves.
- The optimization is not trivial..

The Mumford-Shah model (Cont.)

- Among other schemes, a well-known scheme has been suggested by Ambrosio and Tortorelli ('92) which replaces the discontinuities with an indicator function v

$$\int_\Omega \left[\alpha(1-v)^2 \|\nabla u\|^2 + \beta |u - u_0|^2 + \rho \|\nabla v\|^2 + \frac{v^2}{2\rho} \right] d\Omega$$

- With optimization carried out on both u and v (EL's are given in their papers).
- One term states the measure of the discontinuity.
- Another term is a viscosity term, favoring smooth solutions.
- Taking $\rho \to 0$ converges to the original MS problem in a Γ-convergence process.
- Many other schemes are available..

Active Contours Without Edges

- Chan and Vese ('00) took the Mumford-Shah model and used it to create a region-statistics based segmentation algorithm.
- Similar to a more specific, single-region, approach presented by Cohen et. al. in '93.
  - The image is assumed to be made of an object and a background, replacing the $\mathcal{H}^{N-1}(\Gamma)$ term with the length of a closed curve.
  - Inside and outside the object, the image intensity is assumed to be a Gaussian around a certain mean.
- This suggests a simplified model of the image, where the object has color μ_1 and the background has color μ_2, and a separating contour is assumed to be closed and simple.
- The resulting functional is

$$\int_R (I - \mu_1)^2 \, d\Omega + \int_{\Omega \setminus R} (I - \mu_2)^2 \, d\Omega + \lambda \oint ds$$

Active Contours Without Edges (cont.)

- Optimization is done on both discrete parameters μ_1, μ_2, and on the levelset function.
- The levelset formulation involves a smoothed Heaviside approximation $H_\epsilon(\phi)$

$$\int_R (I - \mu_1)^2 H_\epsilon(\phi) + (I - \mu_2)^2 (1 - H_\epsilon(\phi)) + \lambda \delta_\epsilon(\phi) \|\nabla \phi\| d\Omega$$

Active Contours Without Edges (cont.)

- The resulting PDE is

$$\phi_t = \left[\lambda \nabla^T \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \left((I - \mu_1)^2 - (I - \mu_2)^2 \right) \right] \delta_\epsilon(\phi)$$

- Optimization for μ_1, μ_2 is trivial.
- Alternating minimization.

Shape regularization

- Obviously, some sort of regularization is needed for the shape of the contour.
  - For example, without the curve length term, the Chan-Vese model simplifies into the *k-means* algorithm.
- A natural choice of regularization for curves is to use some measure of the curve:
  - Often, the length of the contour is added to the functional, resulting in the addition of a curvature flow term.
  - Another possibility is affine curvature.
  - The geodesic active contours model contains its own shape regularization.
- However, the silhouette or boundary of most objects we try to segment should be based on more than Occam's razor..

Linear Shape Spaces

- Leventon, Grimson and Faugeras, 2000.
- Look at the linear space of signed distance maps of a given shape, based on several examples.
- Perform PCA for these SDM's (requires alignment).

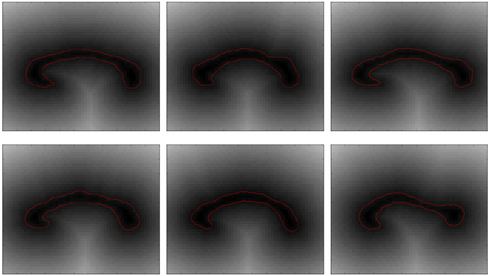


Figure: Signed distance maps of corpus callosum examples. Leventon et. al. '00

Linear Shape Spaces (cont.)

- Obtain a statistical representation of common variations in the object shape.

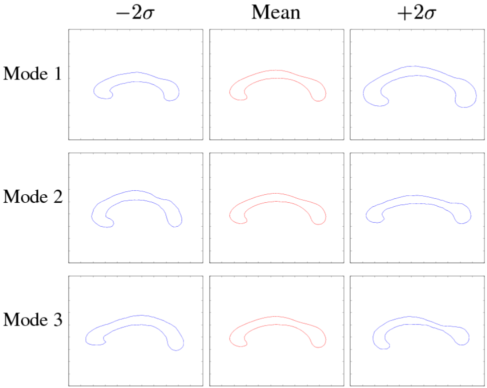


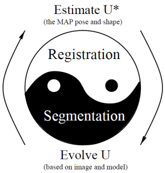
Figure: First 3 modes of corpus callosum examples. Leventon et. al. '00

Linear Shape Spaces (cont.)

- Given the current levelset, its representation in the linear shape space is given by
$$\alpha = U_k^T (u - \mu),$$
- where U_k is the matrix of vectors (read: SDMs) spanning the examples of SDMs, obtained by PCA.
- or, as an SDM,

$$\widetilde{u} = U_k \alpha$$

All of the above discussion assumes that the contour is registered using the pose parameter p.



Linear Shape Spaces (cont.)

- Given a current contour, its “correct” representation is given by a *maximum a-posteriori* (MAP) estimation.

$$P(\alpha, p|u, I) = \frac{P(u, I|\alpha, p)P(\alpha, p)}{P(u, I)} = \frac{P(u|\alpha, p)P(I|\alpha, p, u)P(\alpha)P(p)}{P(u, I)}.$$

- As often happens, α is assumed to be Gaussian..

$$\alpha \sim N(0, \Sigma_\alpha)$$

- where Σ_α is computed from the given examples.
- u is connected to α, p using $\widetilde{u}$.
- I is only statistically linked to u in the model,

$$P(I|\alpha, p, u) = P(I|u)$$

Non-Linear Shape Spaces

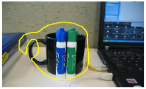
Later works have extended the model:

- Chen et. al. '01, Tsai et. al. '01, Rousson and Paragios '02,
- Cremers '02,
- Damberville '06, Yezzi et. al. '07,
- Riklin-Raviv et. al. '07, Etyngier et. al. '07, Randall et. al. '07, Thorstensen et. al. '08 - extended the work to nonlinear dimensionality reduction algorithms, including treatment of the pre-image problem.
- NLDR Algorithms in use: KPCA, LLE, Diffusion maps - and more..
- Extensions by different solutions to the pre-image problem, incorporation of dynamics, different assumption on the transformations allowed and more..

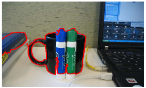
Projective Geometry Shape Priors

- Damberville, Sandhu '08.
- As always in computer vision - if your model is accurate, you can expect much better results
- Shape spaces work fine in medical images, where you really have complete information of the object (once reconstructed..)
- Doesn't work as well for $3D$ object and a single viewpoint.
- Different object poses are not approximated well by a linear space, or by close-to-linear spaces.
- Instead, why not parameterize the contour as an object silhouette?
- If the shape is variable, the variations should be in terms of the 3D object, not its projected silhouette.

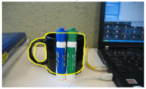
Projective Geometry Shape Priors (cont.)



(a_3)



(b_3)



(c_3)

Figure: Taken from Sandhu et. al. , '09 : Initial pose and shape, AC result, result using prior.