

5. Language Models and Recurrent Neural Networks

LING-581-Natural Language Processing 1

Instructor: Hakyung Sung

September 23, 2025

*Acknowledgment: These course slides are based on materials from CS224N @ Stanford University; Dr. Kilho Shin @ Kyocera

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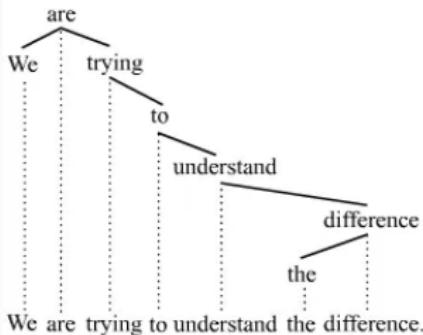
1. Lesson plan
2. Language modeling
3. n-gram language models
4. Window-based neural language models
5. RNNs
6. Wrap-up

Review

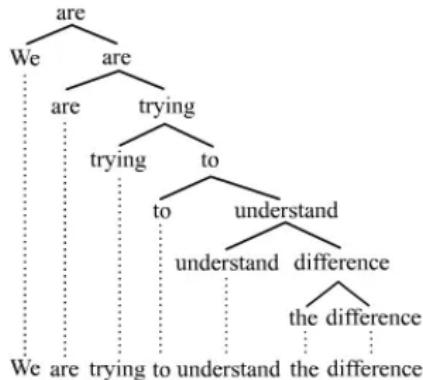
Review

- Syntactic structure: Consistency and dependency
- Dependency grammar and treebanks
- Dependency parsing
- Transition-based dependency parsing
- Neural dependency parsing

Review: Dependency grammar vs. Constituency parsing

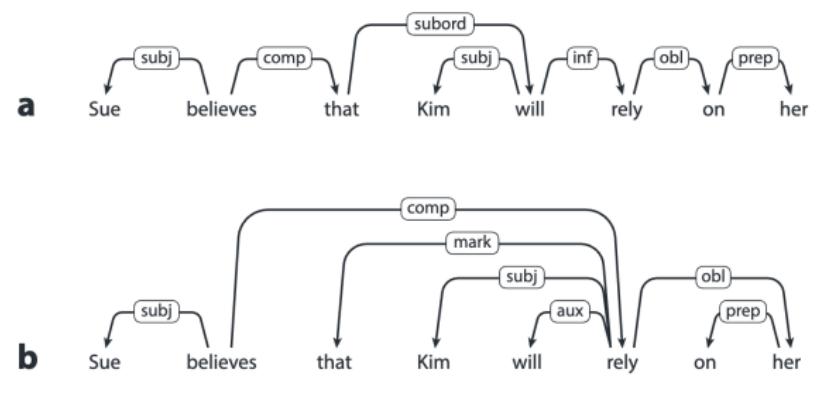


Dependency



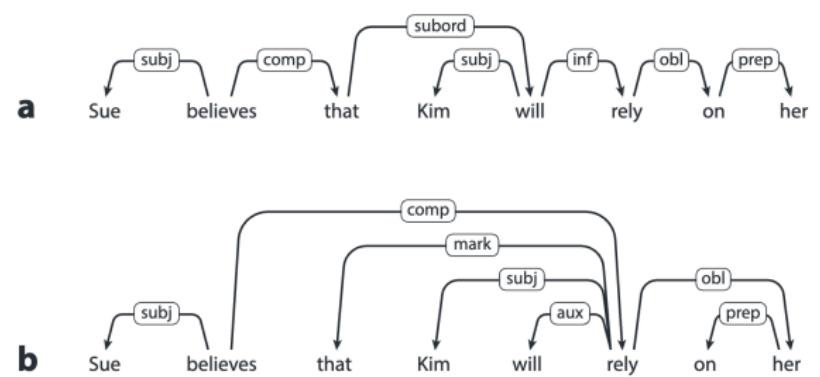
Constituency (BPS)

Review: Universal dependency grammar



Sourced from De Marneffe, M. C., & Nivre, J. (2019). Dependency grammar. Annual Review of Linguistics, 5(1), 197-218. Figure 4

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"UD gives priority to dependency relations between **content words**, while function words are attached to the content word."

Review: Universal dependency grammar

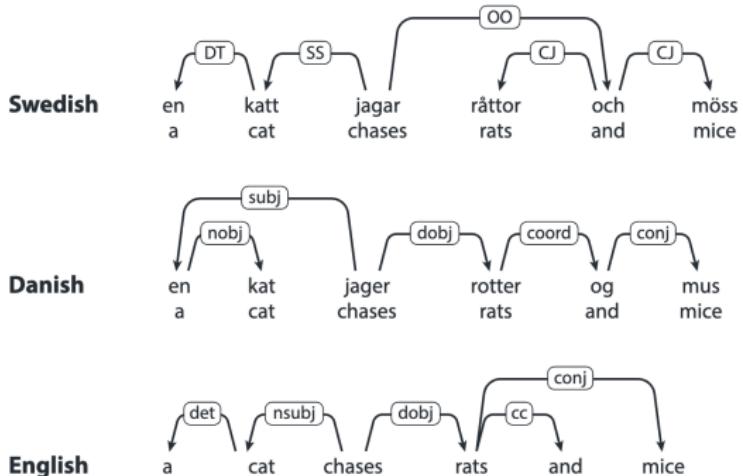


Figure 6

Dependency trees for parallel sentences in Swedish, Danish, and English.

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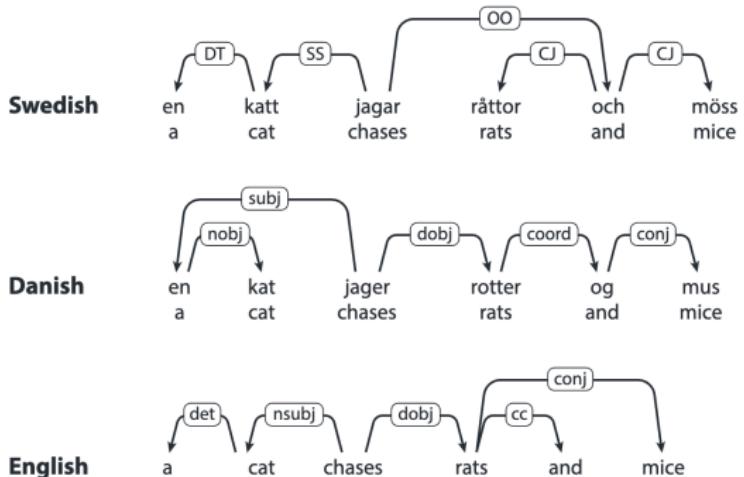


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“The goal is to support **multilingual research** in NLP and linguistics by enabling sound comparative evaluation across languages.”

Review: Terminology

- treebank
- UAS vs. LAS

Greedy transition-based parsing: Example

Sentence: *I saw him*

Initial State: Stack = [ROOT], Buffer = [I, saw, him], Arcs = {}

Step	Stack	Buffer	Transition	New Arc
1	[ROOT]	[I, saw, him]	SHIFT	—
2	[ROOT, I]	[saw, him]	SHIFT	—
3	[ROOT, I, saw]	[him]	LEFT-ARC	saw → I (subj)
4	[ROOT, saw]	[him]	SHIFT	—
5	[ROOT, saw, him]	[]	RIGHT-ARC	saw → him (obj)
6	[ROOT, saw]	[]	RIGHT-ARC	ROOT → saw (root)

Choosing the next parsing action

How should we decide the next parsing action?

- Parsing choice depends on the parsing algorithm:

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 - Current SOTA: Pre-trained transformers + graph-based biaffine decoders?

Thursday Lab

We'll continue working on building/training a dependency parser;
I've updated the dataset.

Lesson plan

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- Language modeling
- n-gram language models
- Window-based neural language models
- RNNs

Language modeling

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$$P(x_{t+1} \mid x_1, x_2, \dots, x_t).$$

- Here each x_i (and the predicted x_{t+1}) is drawn from a vocabulary

$$\mathcal{V} = \{w_1, w_2, \dots, w_{|\mathcal{V}|}\}.$$

The symbol w_j denotes the j -th word in \mathcal{V} .

Language modeling

- A language model can also be viewed as a system that **assigns a probability to an entire sequence of tokens**.
- For a text x_1, \dots, x_T , the joint probability is

$$\begin{aligned} P(x_1, \dots, x_T) &= P(x_1) P(x_2 | x_1) \cdots P(x_T | x_1, \dots, x_{T-1}) \\ &= \prod_{i=1}^T P(x_i | x_1, \dots, x_{i-1}) \end{aligned}$$

- This decomposition follows directly from the chain rule of probability.

Example: Sequence probability

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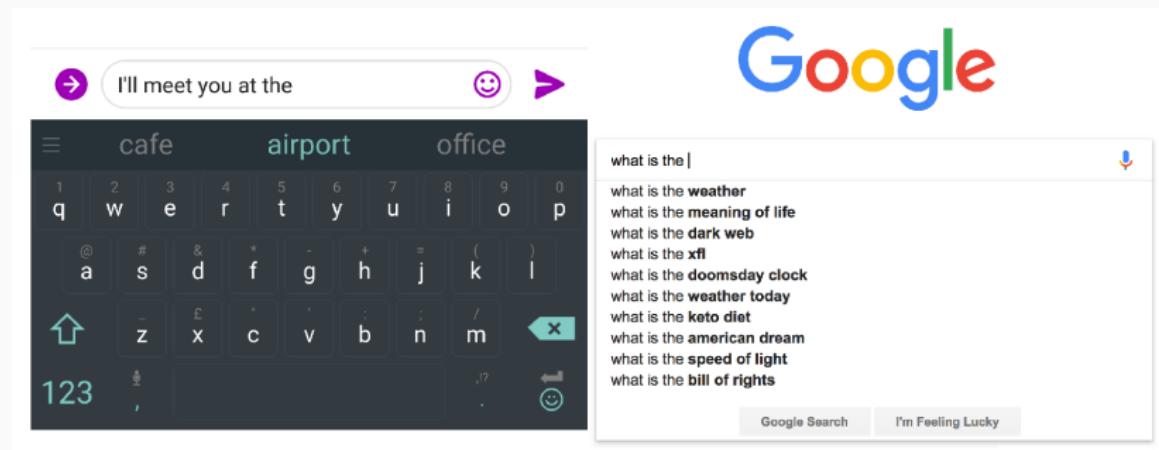
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- Multiplying these gives the overall probability of the sentence:

$$P(\text{"I like apples"}) = P(\text{"I"}) \cdot P(\text{"like"} \mid \text{"I"}) \cdot P(\text{"apples"} \mid \text{"I like"})$$

You use language models every day!

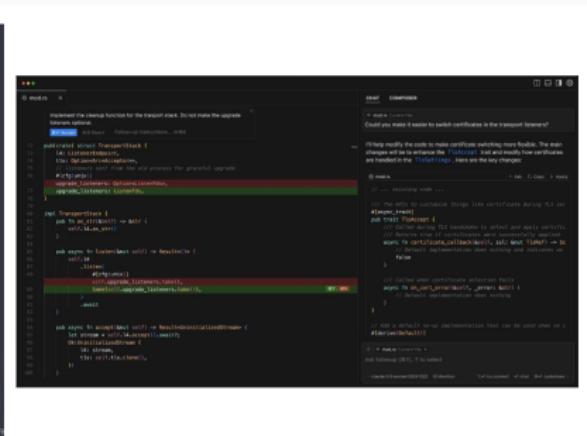


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ChatGPT

Examples	Capabilities	Limitations
"Explain quantum computing in simple terms"	Remembers what user said earlier in the conversation	May occasionally generate incorrect information
"Got any creative ideas for a 10 year old's birthday?"	Allows user to provide follow-up corrections	May occasionally produce harmful instructions or biased content
"How do I make an HTTP request in Javascript?"	Trained to decline inappropriate requests	Limited knowledge of world and events after 2021

ChatGPT is optimized for dialogue. Our goal is to make AI systems more natural to interact with, and your feedback will help us improve our system.



The screenshot shows a code editor window with Java code. The code is part of a class named `TransportTest` and includes methods for testing certificate handling. The code uses annotations like `@Test` and `@Before`. It imports various classes from `org.junit`, `java.util`, and `com.example`. The code is annotated with comments explaining its purpose, such as "Implement the cleanup function for the transport stack. Do not make the upgrade" and "Help modify the code to make certificate validation more flexible". There are also sections for "Initial code" and "Changes made".

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 - summarization
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 - etc.
- Everything else in NLP has been rebuilt upon language modeling: ChatGPT is an LM!

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- This leads us to **n-gram language models**.

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 - Unigrams: the, students, opened, their
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 - Trigrams:
 - 4-grams:
- **Idea:** Collect statistics about how often n-grams occur and use them to predict the next word.

n-gram language models: 1. Markov assumption

$$P(x_{t+1} \mid x_t, \dots, x_1) \approx P(x_{t+1} \mid x_t, \dots, x_{t-n+2})$$

- t : position of the current token in the sequence
- n : size of the n -gram (the model looks back $n - 1$ tokens)

Only the last $(n - 1)$ words matter.

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- This reduces the problem from “consider the whole history” to “just a short context window.”
- **Analogy:** Predicting the next word is like continuing a conversation. (i.e., You don’t need to remember everything said 5 minutes ago, just the last few words.)

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- e.g., the cat is cute

n-gram language models: 3. Example (4-gram)

As the proctor started the clock, the students opened their _____

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Problems with n-gram language models

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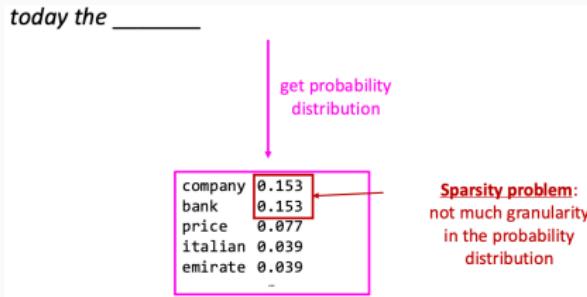
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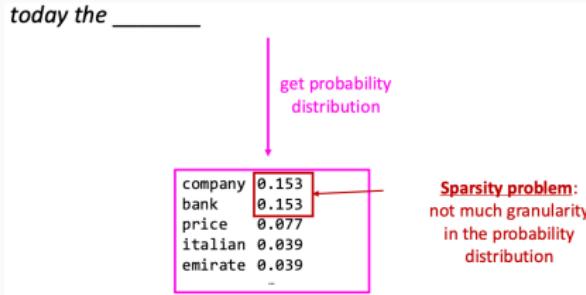
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 - Sparsity worsens as n increases (rarely $n > 5$ in practice).

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Window-based neural language models

Fixed-window neural language models

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3. Combine them through a feed-forward network
4. Output a probability distribution for the next word



Fixed-window neural language models

output distribution

$$\hat{y} = \text{softmax}(Uh + b_2) \in \mathbb{R}^{|V|}$$

hidden layer

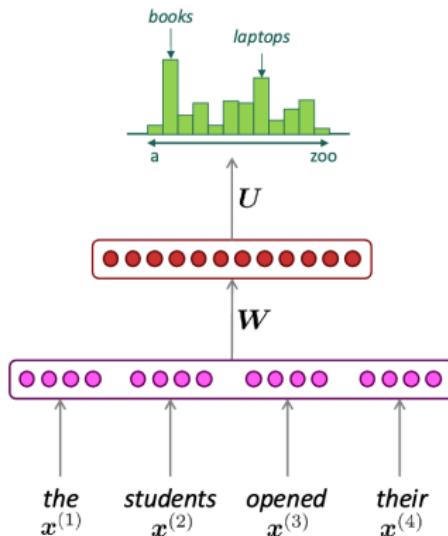
$$h = f(We + b_1)$$

concatenated word embeddings

$$e = [e^{(1)}; e^{(2)}; e^{(3)}; e^{(4)}]$$

words / one-hot vectors

$$x^{(1)}, x^{(2)}, x^{(3)}, x^{(4)}$$



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 - Enlarging the window makes the weight matrix W grow huge.

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 3. capture sequential order and proximity.

RNNs

Overview

RNNs are widely used to process continuous data such as time series.

They work by **retaining past information** while processing new input.

For example, changes in stock prices.



Or sequences like words in a sentence.



Or sequences like words in a sentence.



Or sequences like words in a sentence.



This is an awesome

Or sequences like words in a sentence.



This is an awesome

Or sequences like words in a sentence.



This is an awesome
sentence

Or sequences like words in a sentence.



This is an awesome
sentence that

Or sequences like words in a sentence.



This is an awesome
sentence that was

Or sequences like words in a sentence.



This is an awesome
sentence that was
written

RNNs can effectively handle data where order matters.



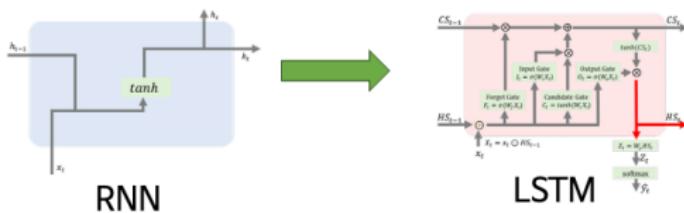
This is an awesome
sentence that was
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Think of RNNs as learning by extracting temporal features from time-series data.

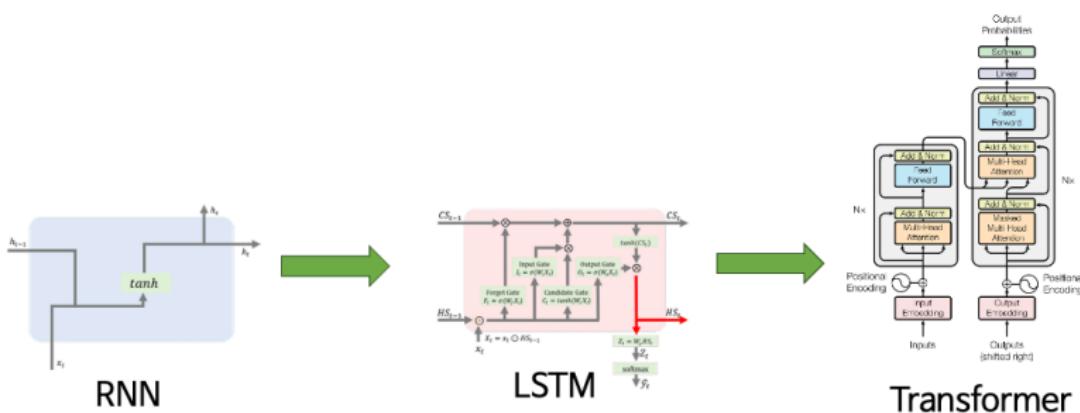


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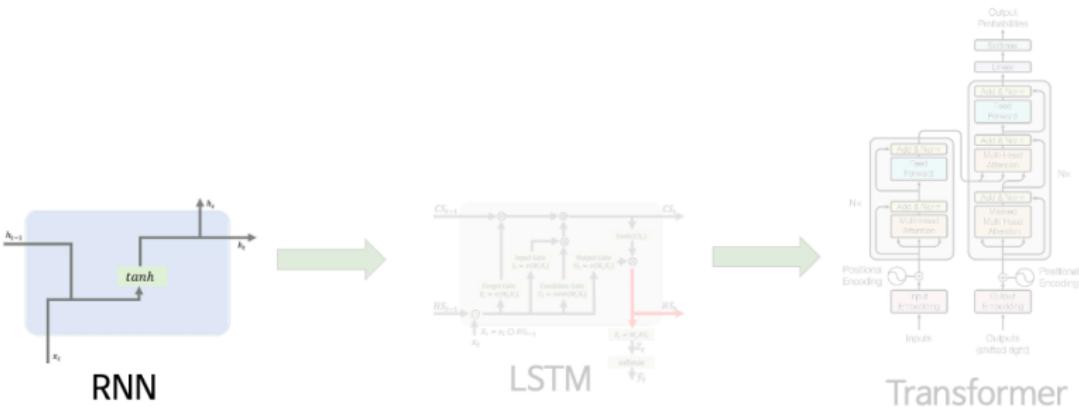
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Moreover, RNNs evolved into LSTMs and eventually into Transformers.



We will discuss (1) the structure, (2) the algorithms for learning sequential information, and (3) the uses of RNNs.

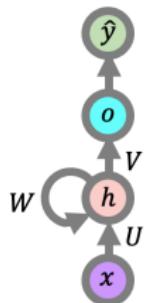


1. Structure

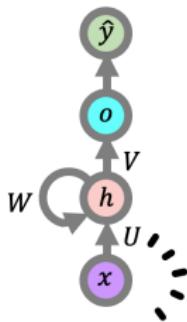
The structure of an RNN is simpler than you might think.



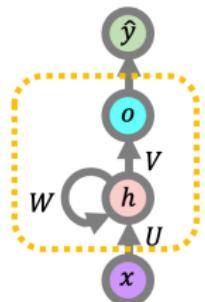
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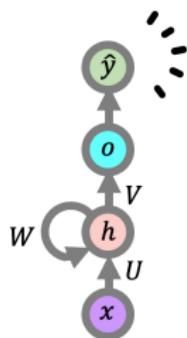
What an RNN does is to take an **input vector** x ,



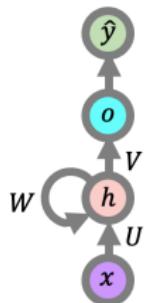
perform internal computations,



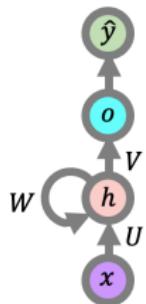
and produce an **output vector** \hat{y} .



This is the **feedforward process** of the RNN.

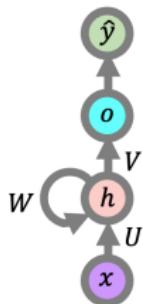


The **types** of input vectors x and output vectors \hat{y} can vary widely.



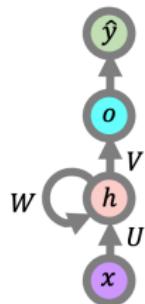
Time-series data (e.g., characters, stock price graphs, musical notes), as long as it can be represented sequentially, can be used as input.

'this' 'is' 'an' 'awesome' 'sentence' 'that' 'was' 'written'



So, what is the benefit of processing sequential data?

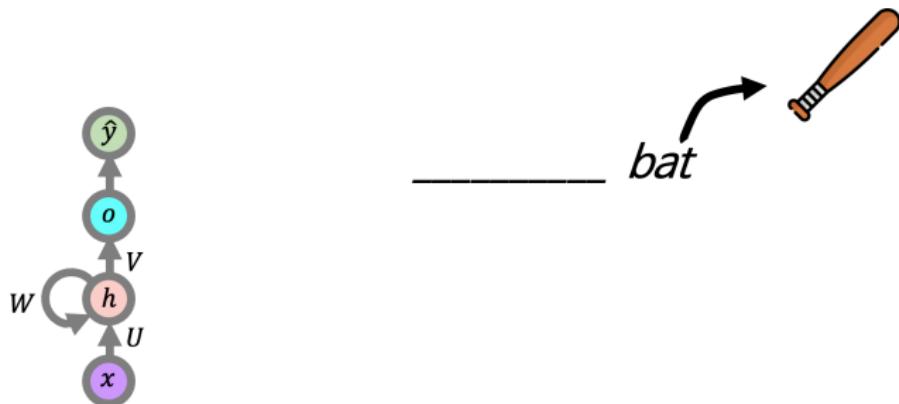
For example, let's assume this RNN is a model that translates *English* into *Spanish*.



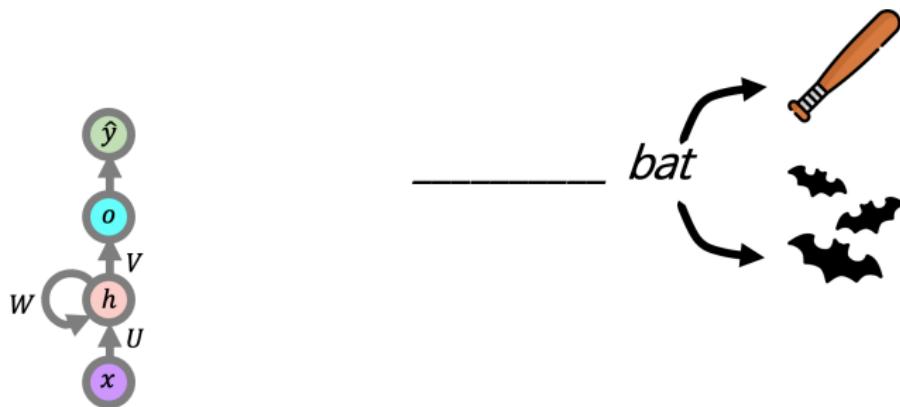
Suppose it encounters the word *bat* in English.



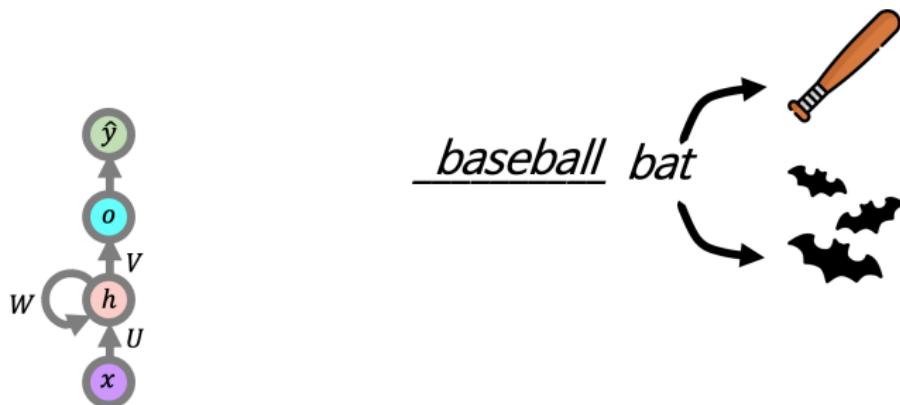
It has two possible meanings: a **baseball bat** or a flying bat.



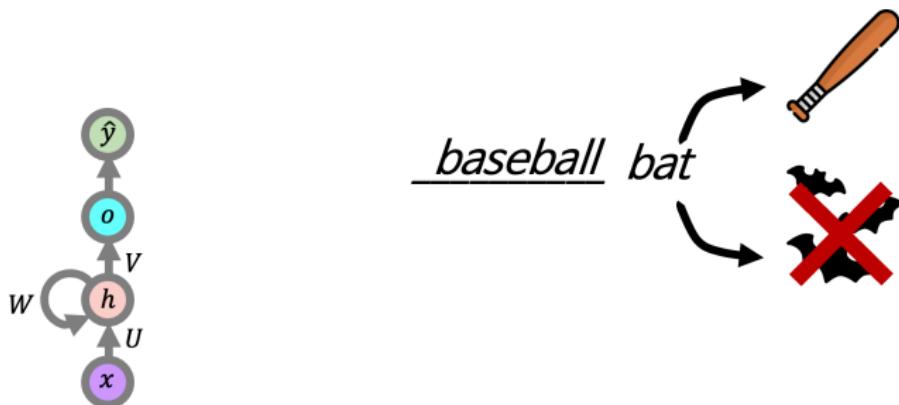
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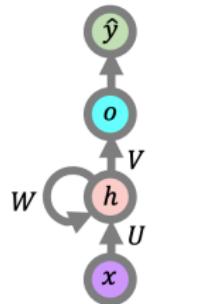
However, if the previous word is *baseball*,



then with high probability, *bat* will be translated as *bate* in Spanish.



When translating *baseball*,



baseball

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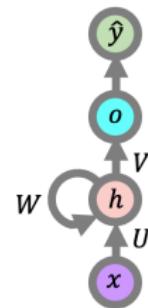
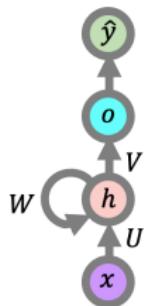


the internal state h is set with the processed representation of *baseball*.

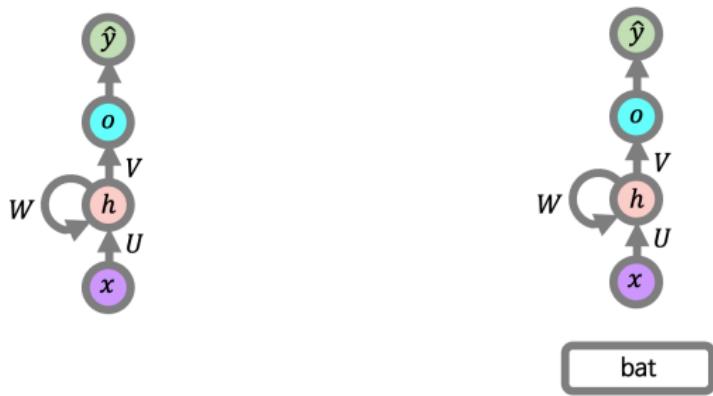


Through this internal computation, *baseball* is translated into *béisbol*,

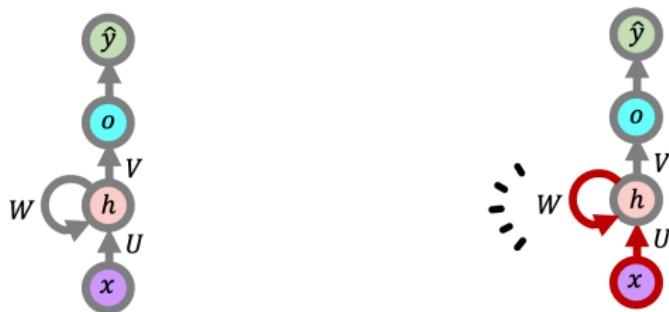
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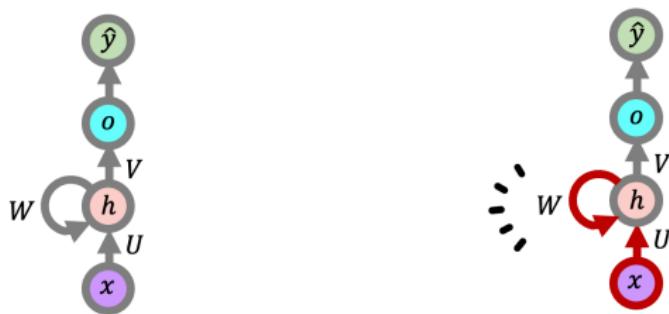
and when the model later encounters *bat*,



the hidden state created while translating *baseball*



influences how *bat* is translated – this is the core idea of an RNN.



The hidden state h gets updated to combine the context of both *baseball* and *bat*,



and thus, the probability of translating *bat* as *bate* (bat for baseball) becomes much higher than translating it as *murciélagos* (the animal).



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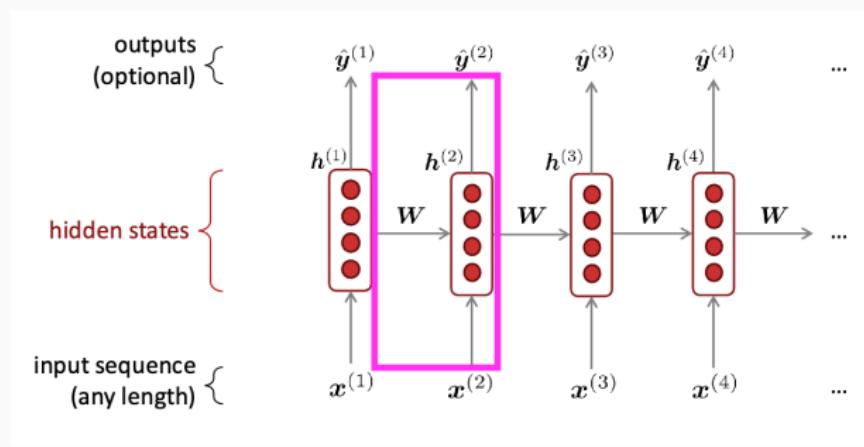


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2. Algorithm/Training

- Idea: Repeatedly apply the same weight matrix W at each time step
- Maintain a hidden state over time, feeding it back into the network to capture temporal dependencies



2-1. The Simple RNN language model

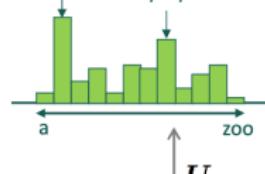
output distribution

$$\hat{y}^{(t)} = \text{softmax}(\mathbf{U}h^{(t)} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

$$\hat{y}^{(4)} = P(\mathbf{x}^{(5)} | \text{the students opened their})$$

books

laptops



hidden states

$$h^{(t)} = \sigma(\mathbf{W}_h h^{(t-1)} + \mathbf{W}_e e^{(t)} + \mathbf{b}_1)$$

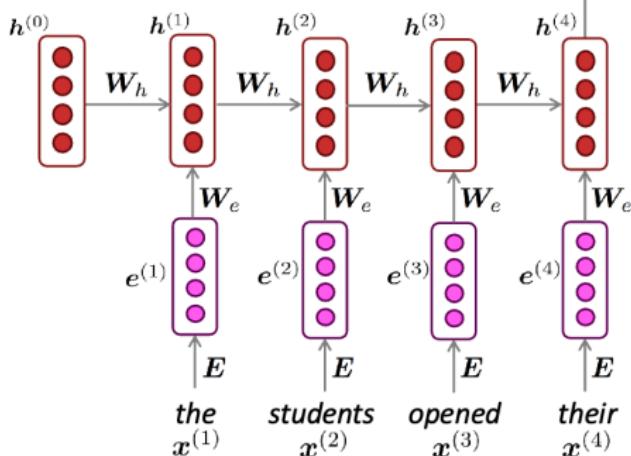
$h^{(0)}$ is the initial hidden state

word embeddings

$$e^{(t)} = \mathbf{E}x^{(t)}$$

words / one-hot vectors

$$\mathbf{x}^{(t)} \in \mathbb{R}^{|V|}$$



Note: this input sequence could be much longer now!

2-2. Training an RNN Language Model

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- *autoregressive, causal LM generation*

- **Loss** at step t :

$$\mathcal{J}^{(t)} = - \sum_{i=1}^{|V|} y_i^{(t)} \log \hat{y}_i^{(t)} = - \log \hat{y}_{w_{t+1}}^{(t)},$$

where:

- $y^{(t)}$: one-hot vector for the true next word w_{t+1} .
- $\hat{y}^{(t)}$: predicted probability distribution over the vocabulary from the softmax layer.
- This is the **cross-entropy loss** between the predicted distribution and the true label.

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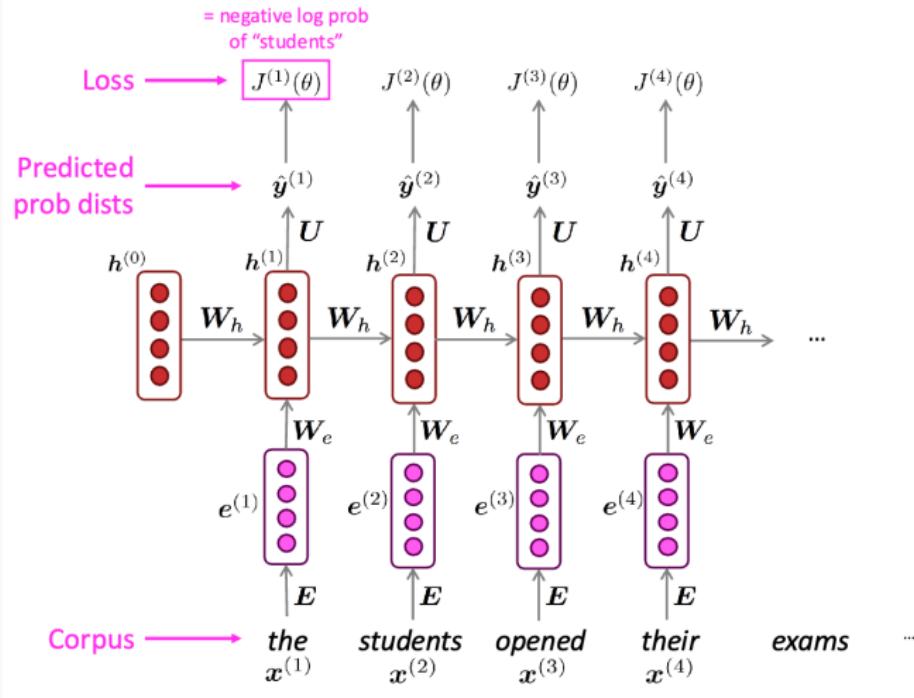
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- Compare: Higher probability for the correct word \Rightarrow lower loss.
- Example: if $\hat{y}_{\text{apple}} = 0.9$, then loss = $-\log 0.9 \approx 0.105$.



- Overall (average) loss over the sequence:

$$\mathcal{J}(\theta) = \frac{1}{T} \sum_{t=1}^T \mathcal{J}^{(t)}(\theta)$$

- Sum the losses across all time steps.
- Divide by the sequence length T to normalize for varying sequence lengths.
- This gives the **average negative log-likelihood per word**, the main training objective of the RNN language model.

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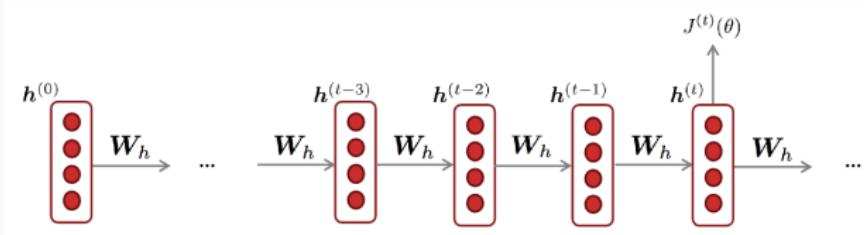
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 4. and repeat.

2-4. Training: Backpropagation

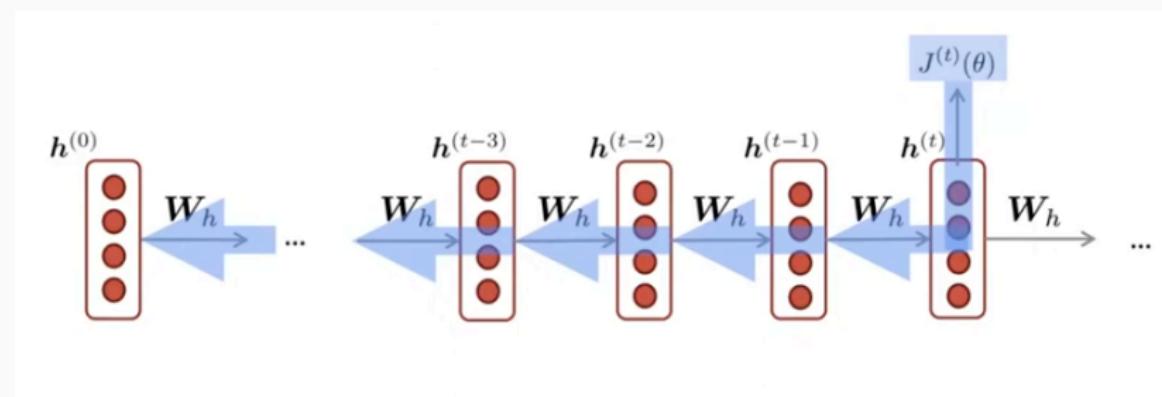


- The loss $J^{(t)}(\theta)$ depends on the shared weight matrix W_h at every time step.
- Therefore,

$$\frac{\partial J^{(t)}}{\partial W_h} = \sum_{k=1}^t \frac{\partial J^{(t)}}{\partial W_h^{(k)}}.$$

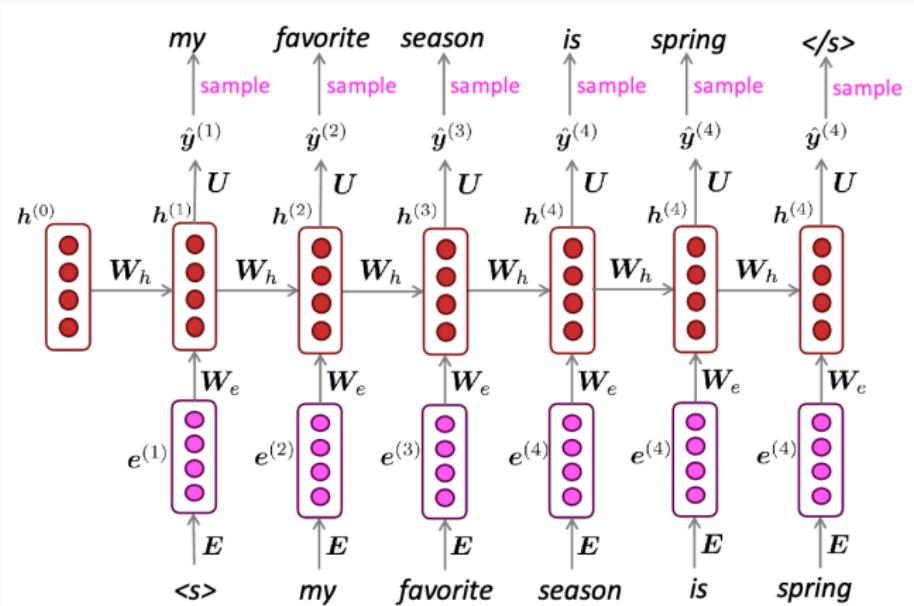
- In other words, the gradient w.r.t. the repeated parameter is the sum of its contributions over all time steps.
- **Why?** Because the RNN “unrolls” in time but reuses the same W_h at each step (parameter sharing).

2-4. Training: Backpropagation



2-5. Application: Generating text

Just like an n-gram language model, you can use an RNN model to generate text by **repeated sampling**. The sampled output becomes the next step's input.



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Example

- Start token: $\langle S \rangle$
 - Step 1: Model predicts distribution, sample *my*
 - Step 2: Input = *my*, sample *favorite*
 - Step 3: Input = *favorite*, sample *season*
 - Step 4: Input = *season*, sample *is*
 - Step 5: Input = *is*, sample *spring*
- ⇒ “my favorite season is spring”

RNN-LM trained on *Harry Potter*:



“Sorry,” Harry shouted, panicking—“I’ll leave those brooms in London, are they?”

“No idea,” said Nearly Headless Nick, casting low close by Cedric, carrying the last bit of treacle Charms, from Harry’s shoulder, and to answer him the common room perched upon it, four arms held a shining knob from when the spider hadn’t felt it seemed. He reached the teams too.

Source: <https://medium.com/deep-writing/harry-potter-written-by-artificial-intelligence-8a9431803da6>

2-6. Evaluation: Perplexity

- The most common evaluation metric for language models is **perplexity**.

$$\text{Perplexity}(w_{1:T}) = P(w_{1:T})^{-\frac{1}{T}} = \exp\left(-\frac{1}{T} \sum_{t=1}^T \log P(w_t | w_{1:t-1})\right)$$

<https://github.com/asahi417/lmppl>

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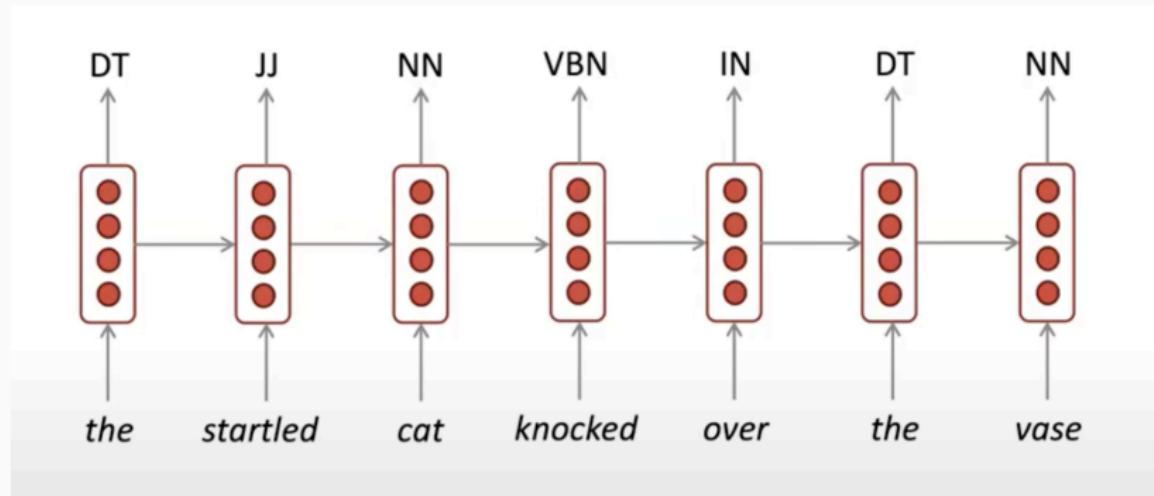
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 - Equivalent to the model’s effective average **branching factor** (i.e., how many plausible next words it considers at each step).
- Interpretation:** Lower perplexity \Rightarrow model is less “perplexed” and makes more accurate predictions.

<https://github.com/asahi417/lmppl>

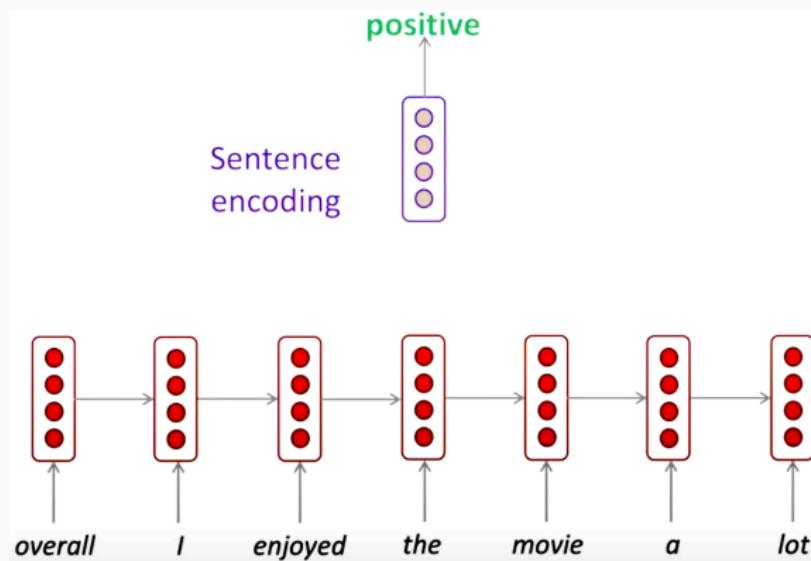
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2-7. Other uses: Sequence tagging

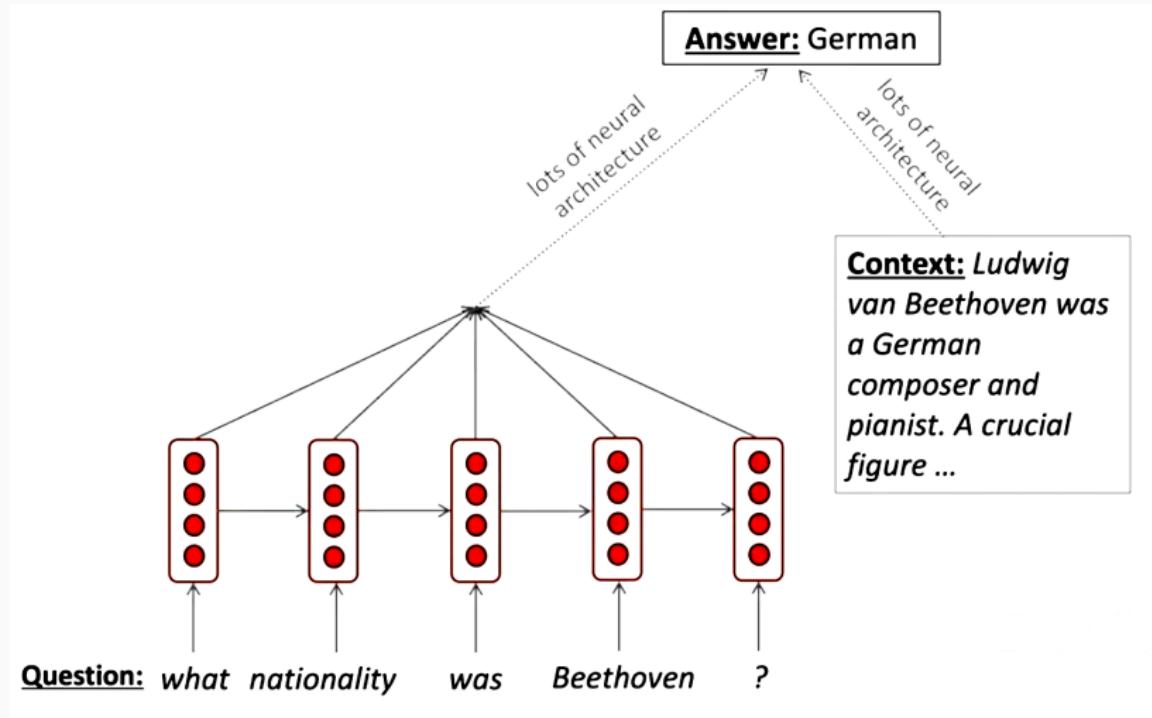
e.g., part-of-speech tagging, named-entity recognition



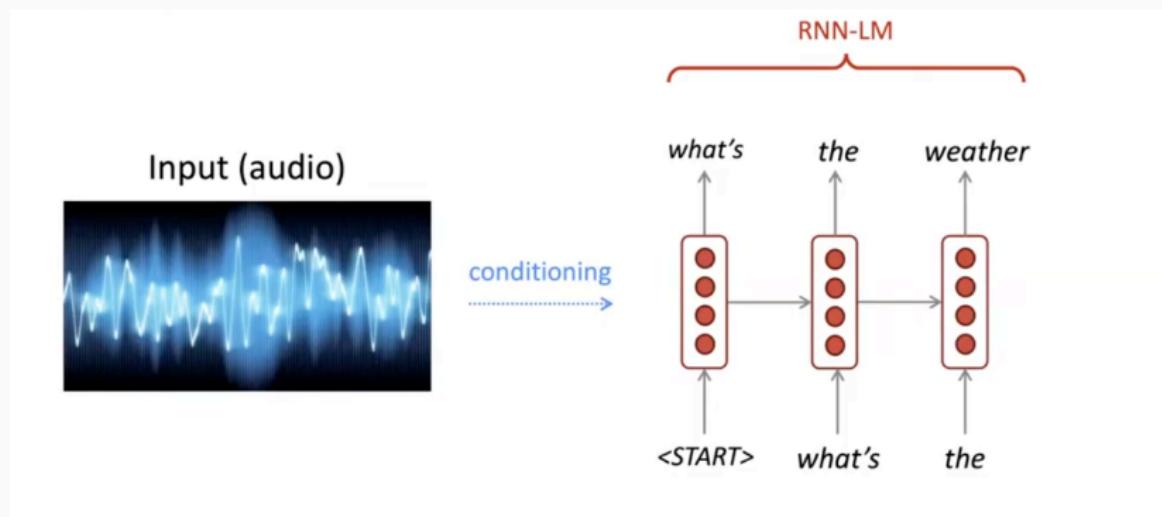
e.g., sentiment classification



e.g., question answering, machine translation



e.g., speech recognition, machine translation, summarization



Wrap-up

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