5. Language Models and Recurrent Neural Networks

LING-581-Natural Language Processing 1

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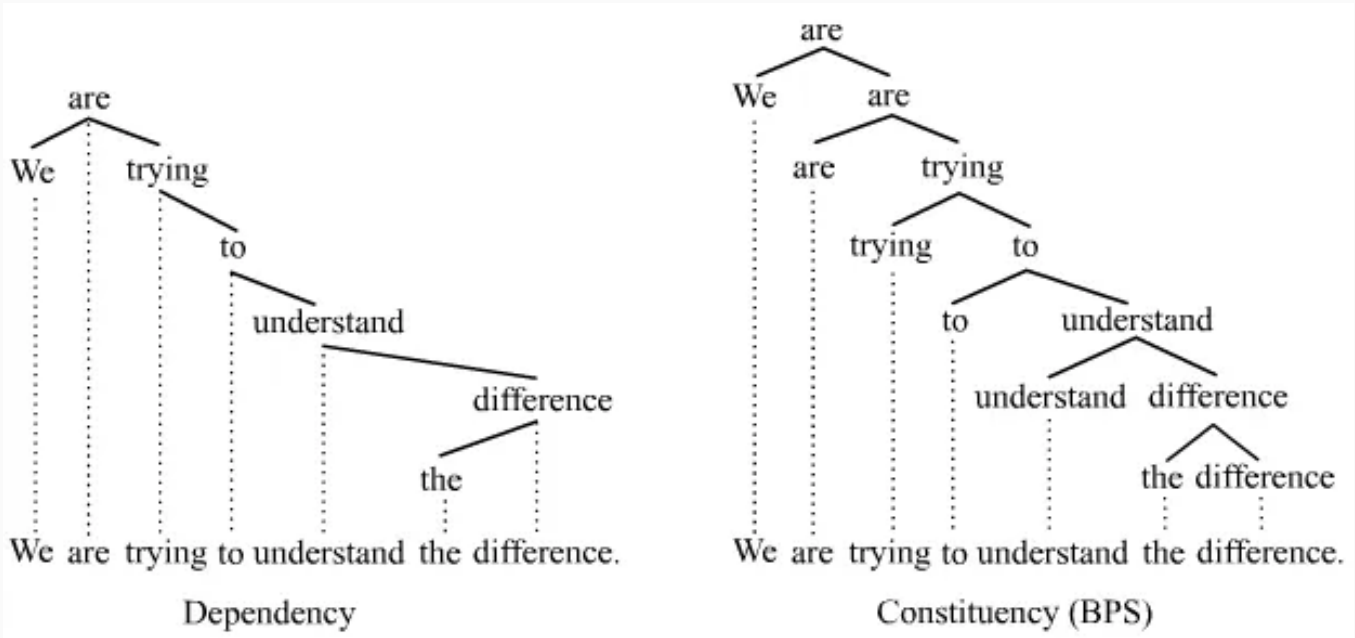
Table of contents

Review

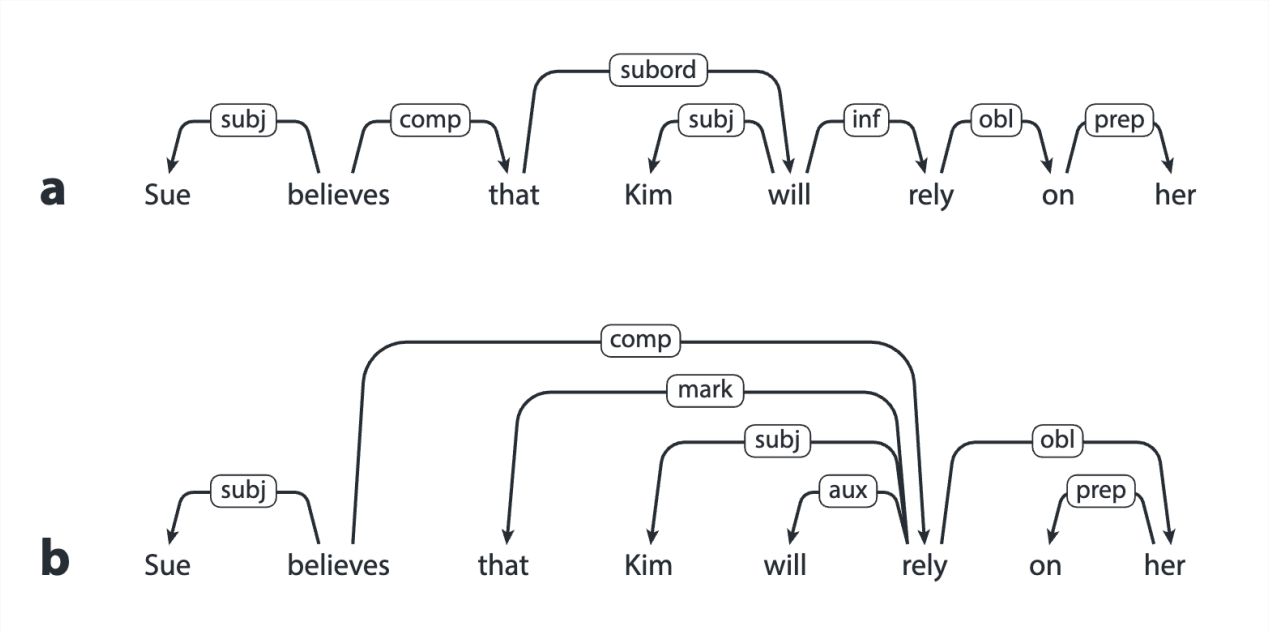
Review

- Syntactic structure: Consistency and dependency
- Dependency grammar and treebanks
- Dependency parsing
- Transition-based dependency parsing
- Neural dependency parsing

Review: Dependency grammar vs. Constituency parsing



Review: Universal dependency grammar



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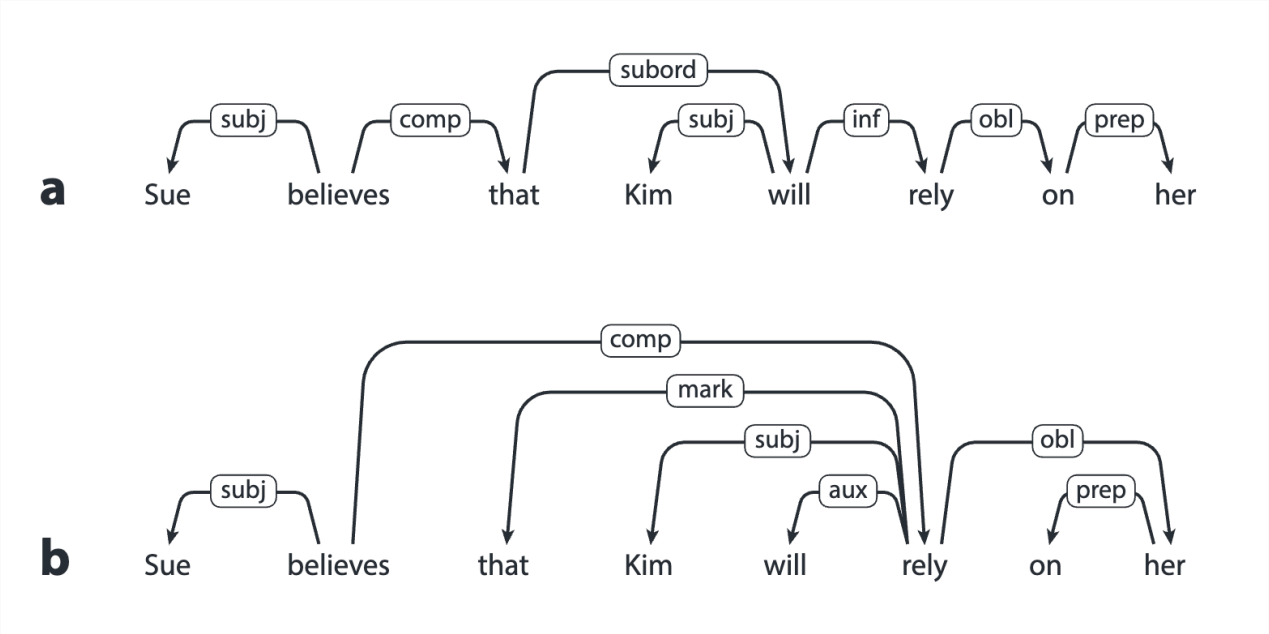

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"UD gives priority to dependency relations between **content words**, while function words are attached to the content word."

Review: Universal dependency grammar

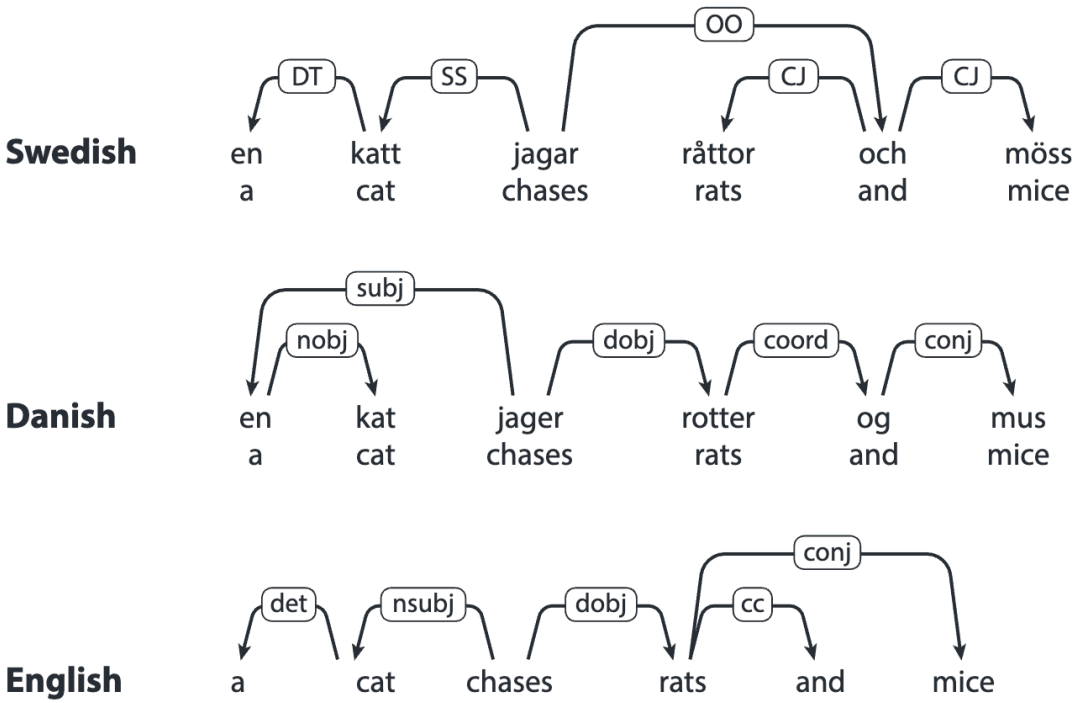

Figure 6

Dependency trees for parallel sentences in Swedish, Danish, and English.

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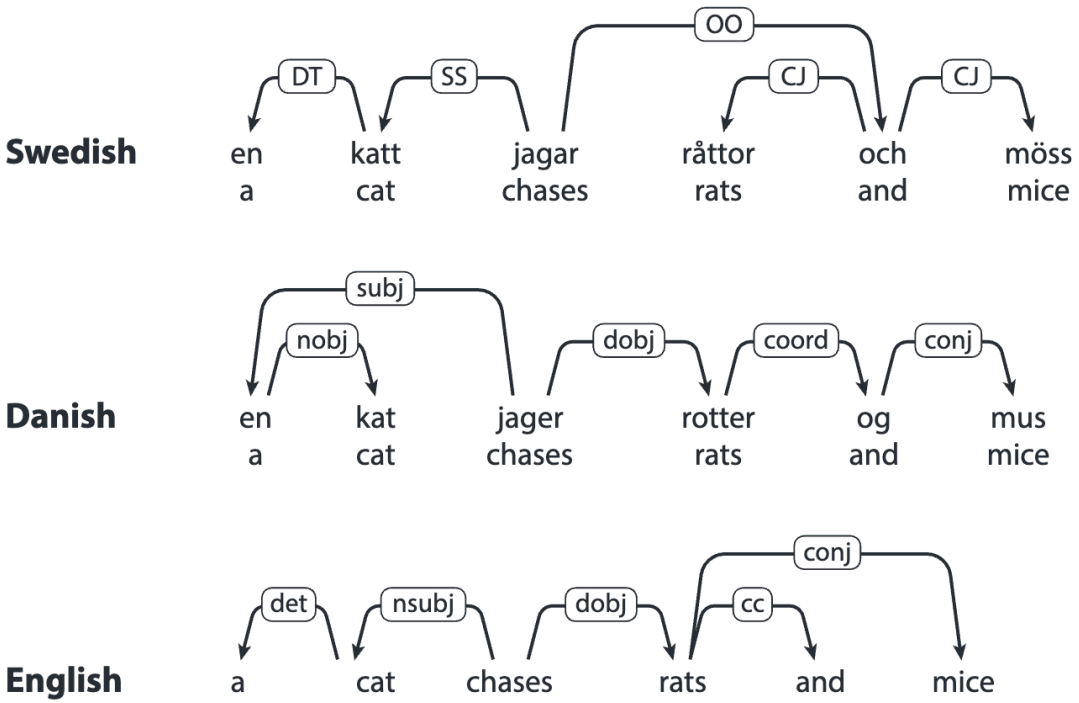

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"The goal is to support **multilingual research** in NLP and linguistics by enabling sound comparative evaluation across languages."

Review: Terminology

- treebank
- UAS vs. LAS

Greedy transition-based parsing: Example

Sentence: *I saw him*

Initial State: Stack = [ROOT], Buffer = [I, saw, him], Arcs = {}

Step	Stack	Buffer	Transition	New Arc
1	[ROOT]	[I, saw, him]	SHIFT	—
2	[ROOT, I]	[saw, him]	SHIFT	—
3	[ROOT, I, saw]	[him]	LEFT-ARC	saw $\rightarrow$ I (subj)
4	[ROOT, saw]	[him]	SHIFT	—
5	[ROOT, saw, him]	[ ]	RIGHT-ARC	saw $\rightarrow$ him (obj)
6	[ROOT, saw]	[ ]	RIGHT-ARC	ROOT $\rightarrow$ saw (root)

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How should we decide the next parsing action?

- Parsing hoice depends on the parsing algorithm:

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  - Current SOTA: Pre-trained transformers + graph-based biaffine decoders?

Lesson plan

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- Language modeling
- n-gram language models
- Window-based neural language models
- RNNs

Language modeling

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$$P(x_{t+1} \mid x_1, x_2, \dots, x_t).$$

- Here each x_i (and the predicted x_{t+1}) is drawn from a vocabulary

$$\mathcal{V} = \{ w_1, w_2, \dots, w_{|\mathcal{V}|} \}.$$

  The symbol w_j denotes the j-th word in $\mathcal{V}$.

Language modeling

- A language model can also be viewed as a system that assigns a probability to an entire sequence of tokens.
- For a text $x_1, \dots, x_T$, the joint probability is

$$
\begin{aligned}
&P(x_1, \dots, x_T) \\
&= P(x_1)\, P(x_2 \mid x_1) \cdots P(x_T \mid x_1, \dots, x_{T-1}) \\
&= \prod_{i=1}^{T} P(x_i \mid x_1, \dots, x_{i-1})
\end{aligned}
$$

- This decomposition follows directly from the chain rule of probability.

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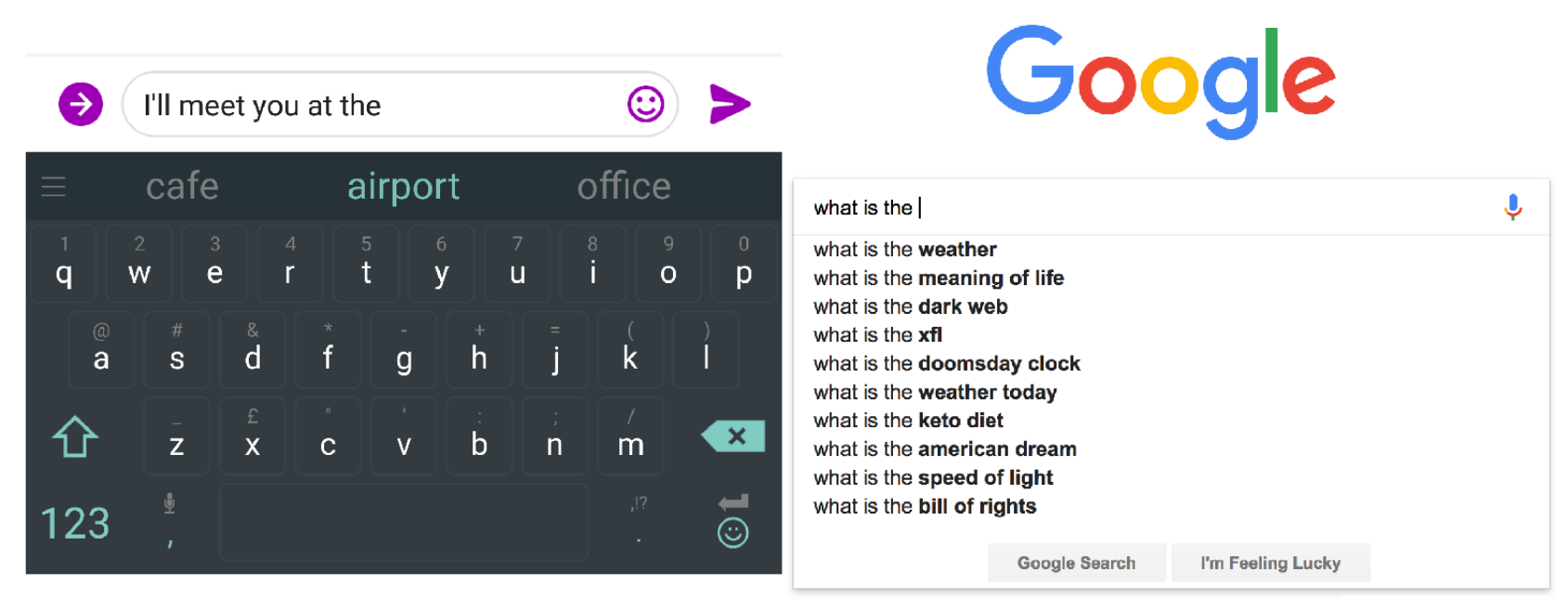
- Multiplying these gives the overall probability of the sentence:

$$P(\text{“I like apples”}) = P(\text{“I”}) \cdot P(\text{“like”} \mid \text{“I”}) \cdot P(\text{“apples”} \mid \text{“I like”})$$

You use language models every day!



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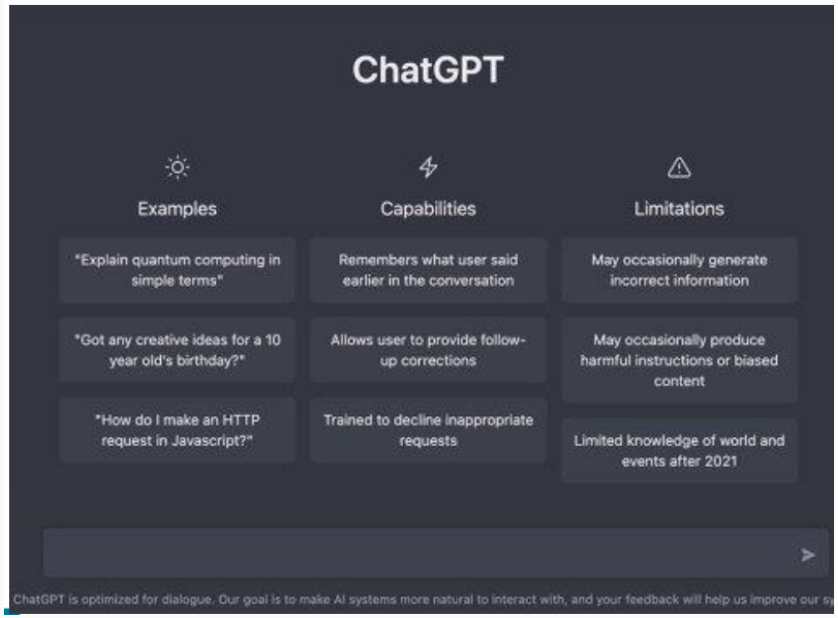

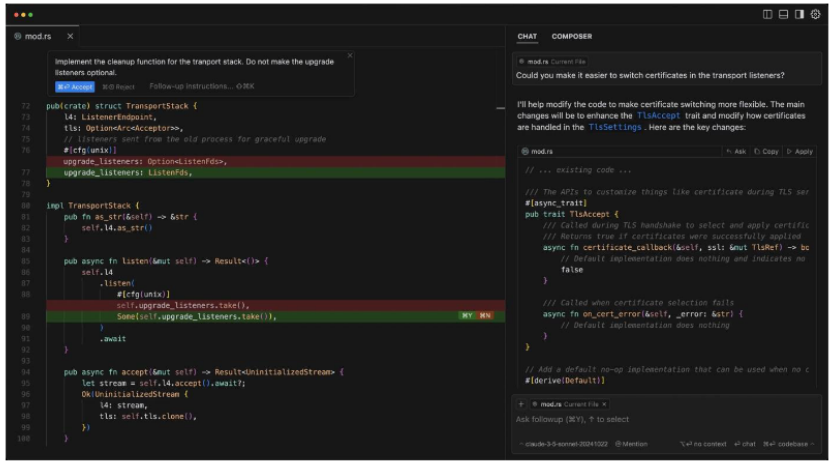

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  - etc.
- Everything else in NLP has been rebuilt upon language modeling: ChatGPT is an LM!

How do we build a language model?

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- This leads us to n-gram language models.

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  - 4-grams:
- **Idea:** Collect statistics about how often n-grams occur and use them to predict the next word.

n-gram language models: 1. Markov assumption

$$P(x_{t+1} \mid x_t, \dots, x_1) \approx P(x_{t+1} \mid x_t, \dots, x_{t-n+2})$$

- t: position of the current token in the sequence
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Only the last $(n-1)$ words matter.

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- This reduces the problem from "consider the whole history" to "just a short context window."
- **Analogy:** Predicting the next word is like continuing a conversation. (i.e., You don't need to remember everything said 5 minutes ago, just the last few words.)

n-gram language models: 2. conditional probability

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- e.g., *the* ***cat is*** *cute*

n-gram language models: 3. Example (4-gram)

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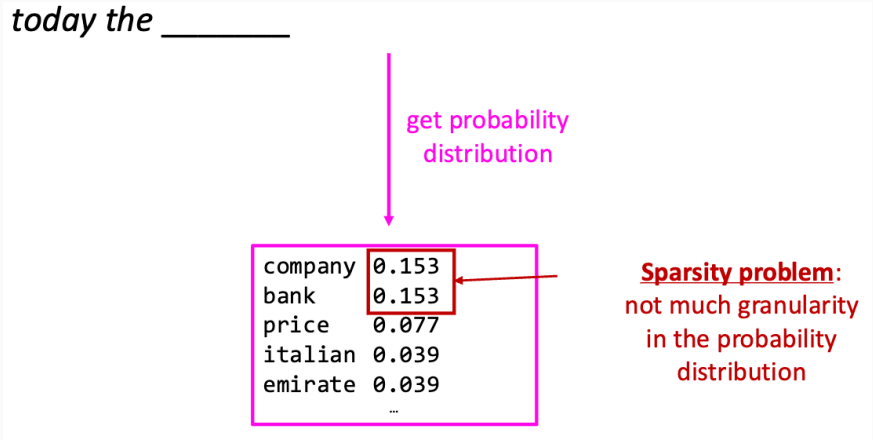

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- **Practical note:** Easy to build (e.g., a trigram LM on a million-word corpus in seconds), but results could be sparse and limited.
  - Sparsity worsens as n increases (rarely $n > 5$ in practice).

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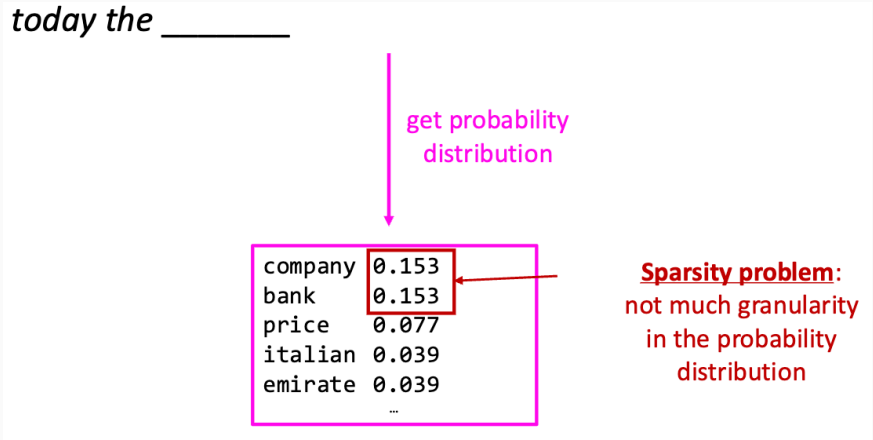

Window-based neural language models

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2. Map each word to an embedding
3. Combine them through a feed-forward network
4. Output a probability distribution for the next word

Fixed-window neural language models

output distribution

$$\hat{\boldsymbol{y}} = \text{softmax}(\boldsymbol{U}\boldsymbol{h} + \boldsymbol{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

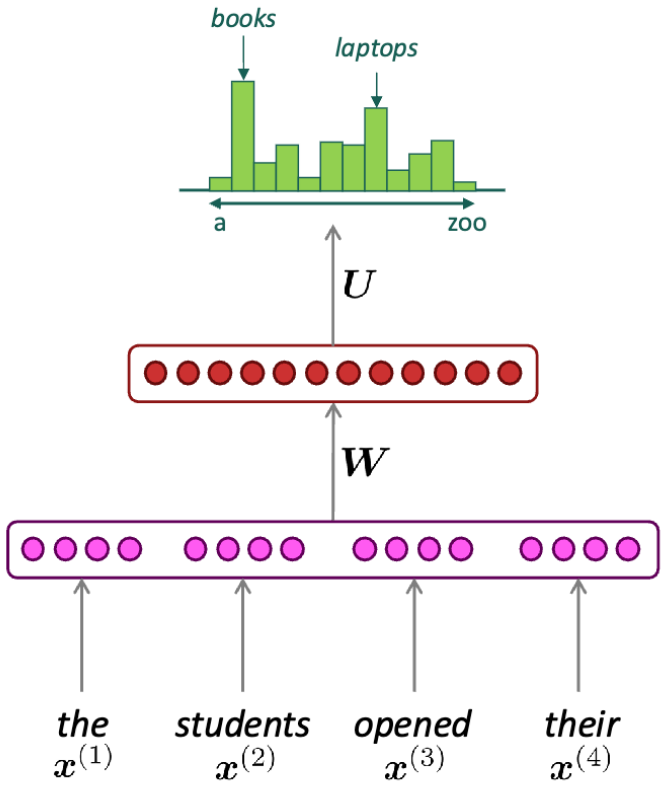
$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{e} + \boldsymbol{b}_1)$$

concatenated word embeddings

$$\boldsymbol{e} = [\boldsymbol{e}^{(1)}; \boldsymbol{e}^{(2)}; \boldsymbol{e}^{(3)}; \boldsymbol{e}^{(4)}]$$

words / one-hot vectors

$$\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}, \boldsymbol{x}^{(4)}$$



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  3. capture sequential order and proximity.

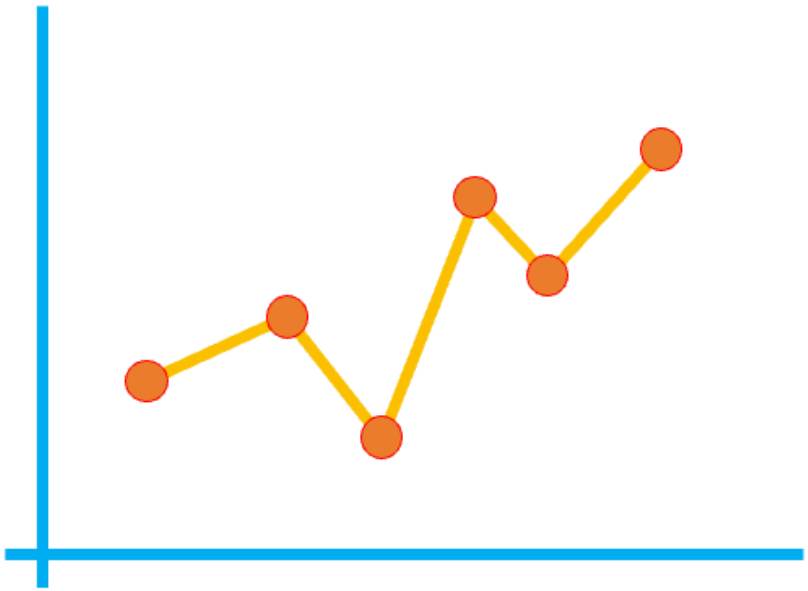
RNNs

Overview

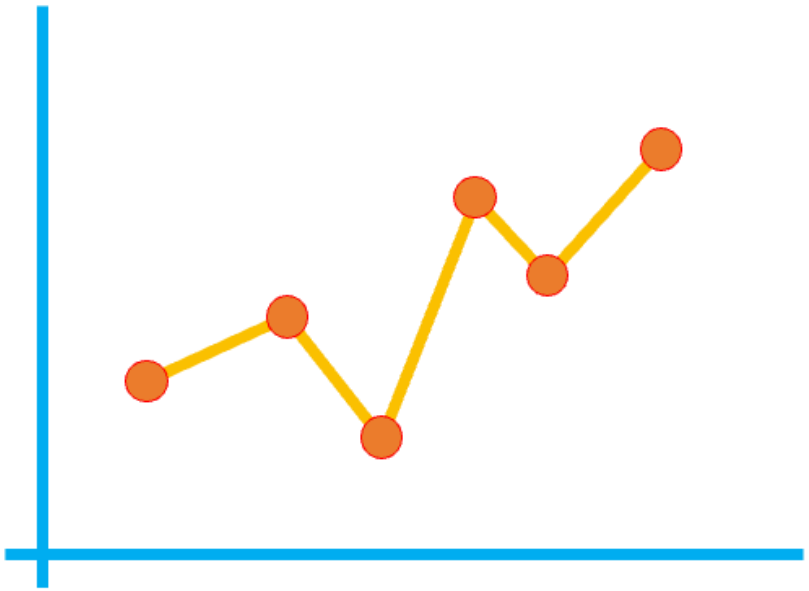
RNNs are widely used to process continuous data such as time series.

They work by **retaining past information** while processing new input.

For example, changes in stock prices.



Or sequences like words in a sentence.



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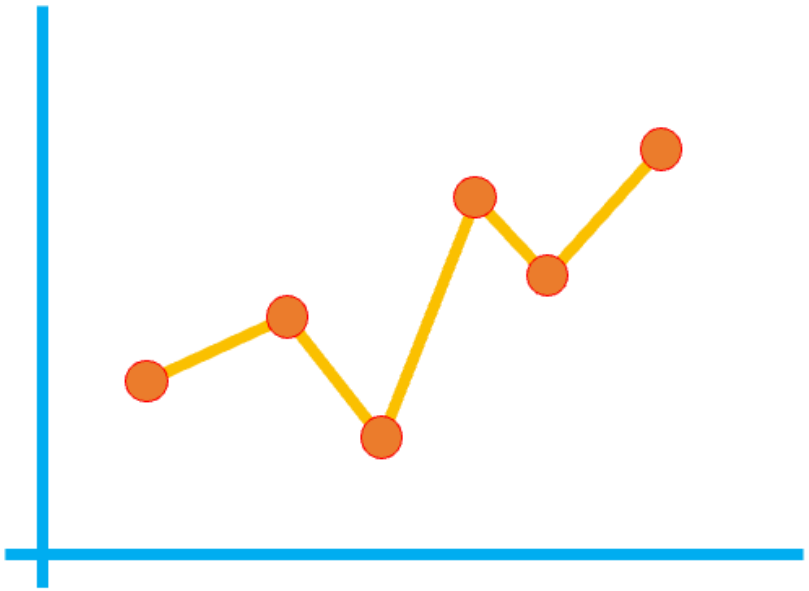

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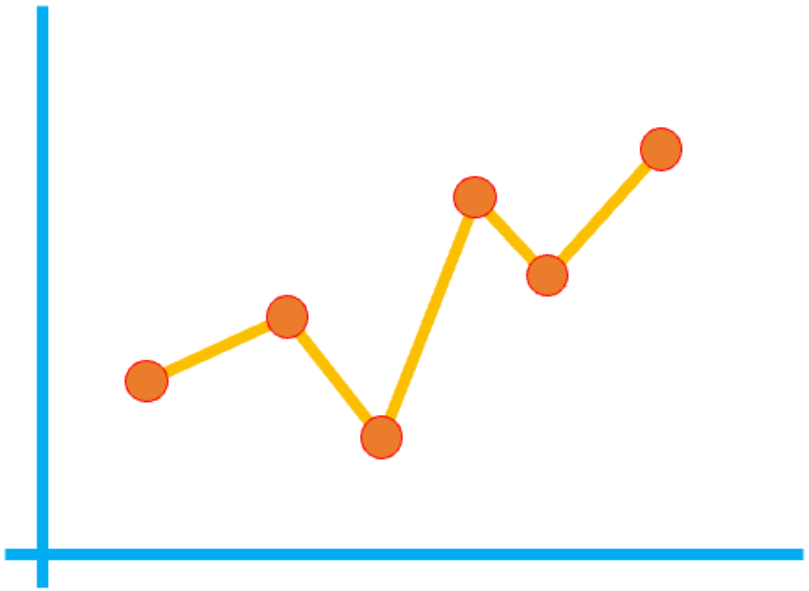

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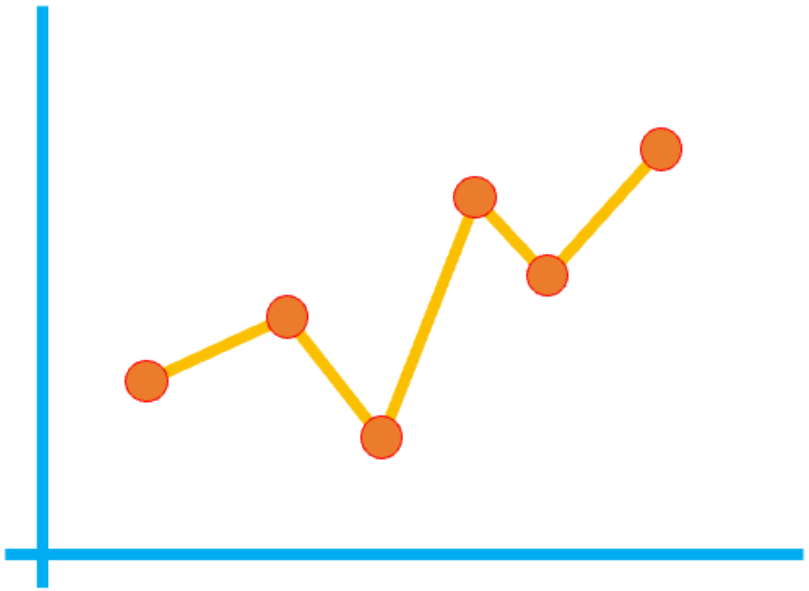

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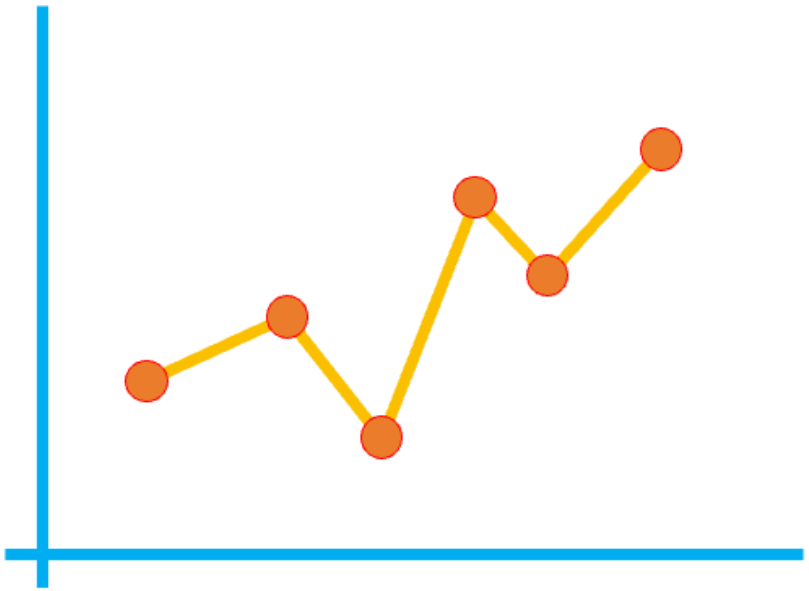

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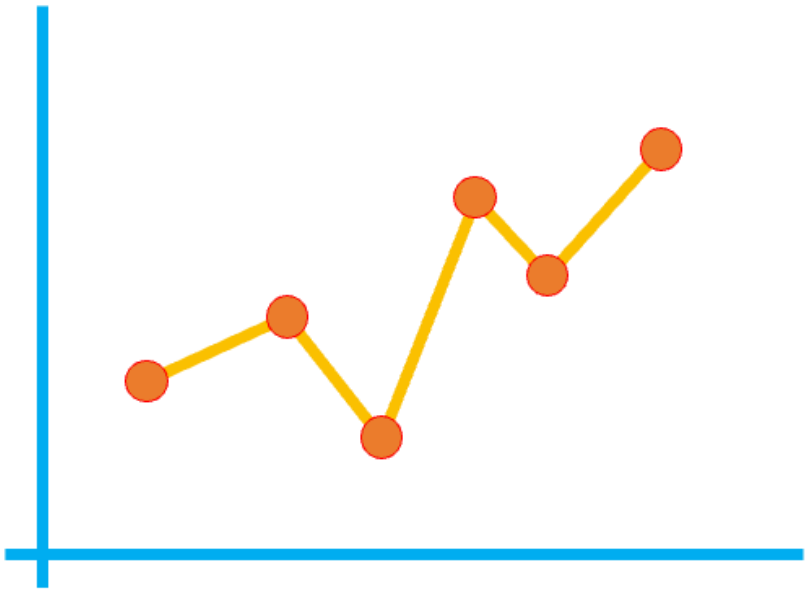

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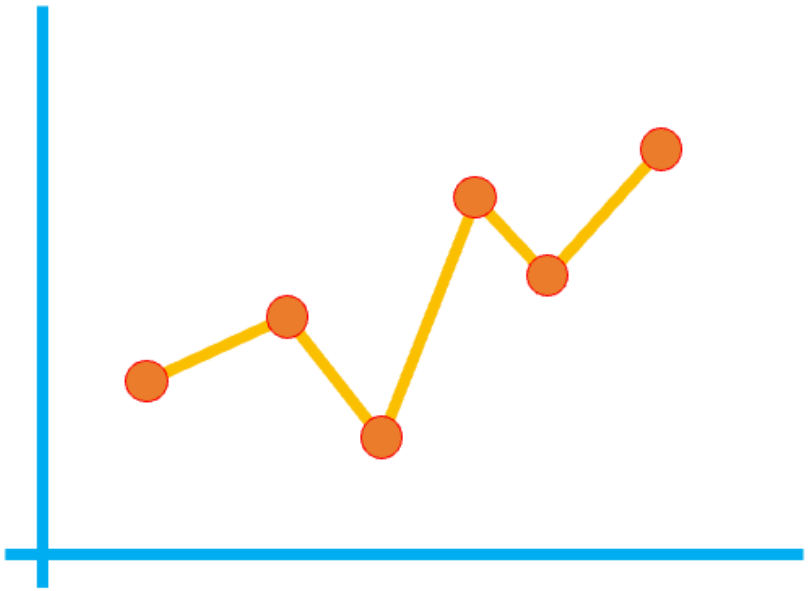

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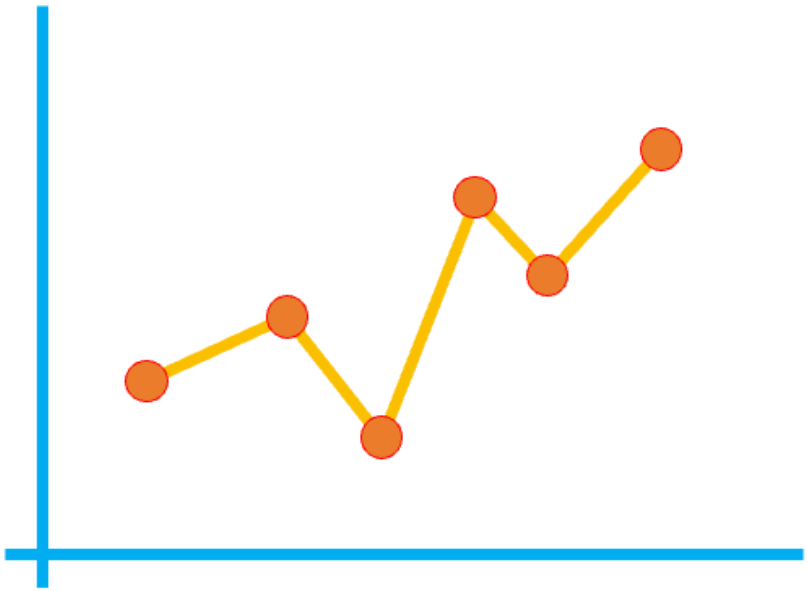
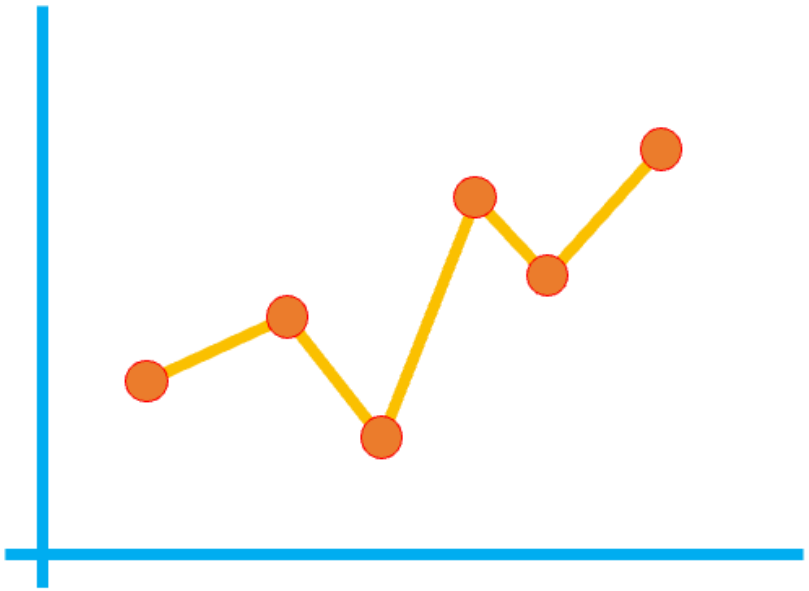
RNNs can effectively handle data where order matters.



This is an awesome sentence that was written by me

Think of RNNs as learning by extracting temporal features from time-series data.



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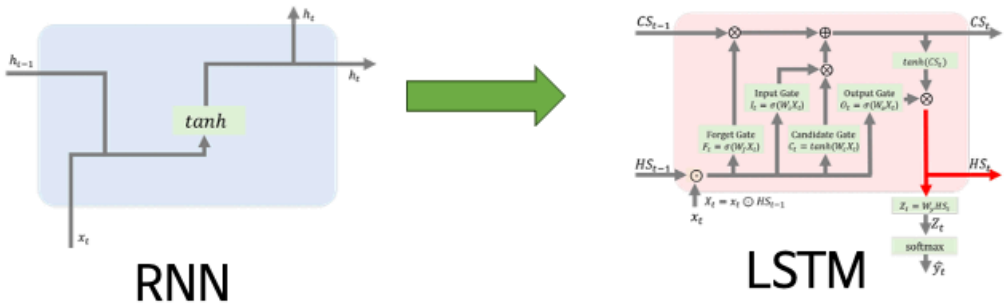
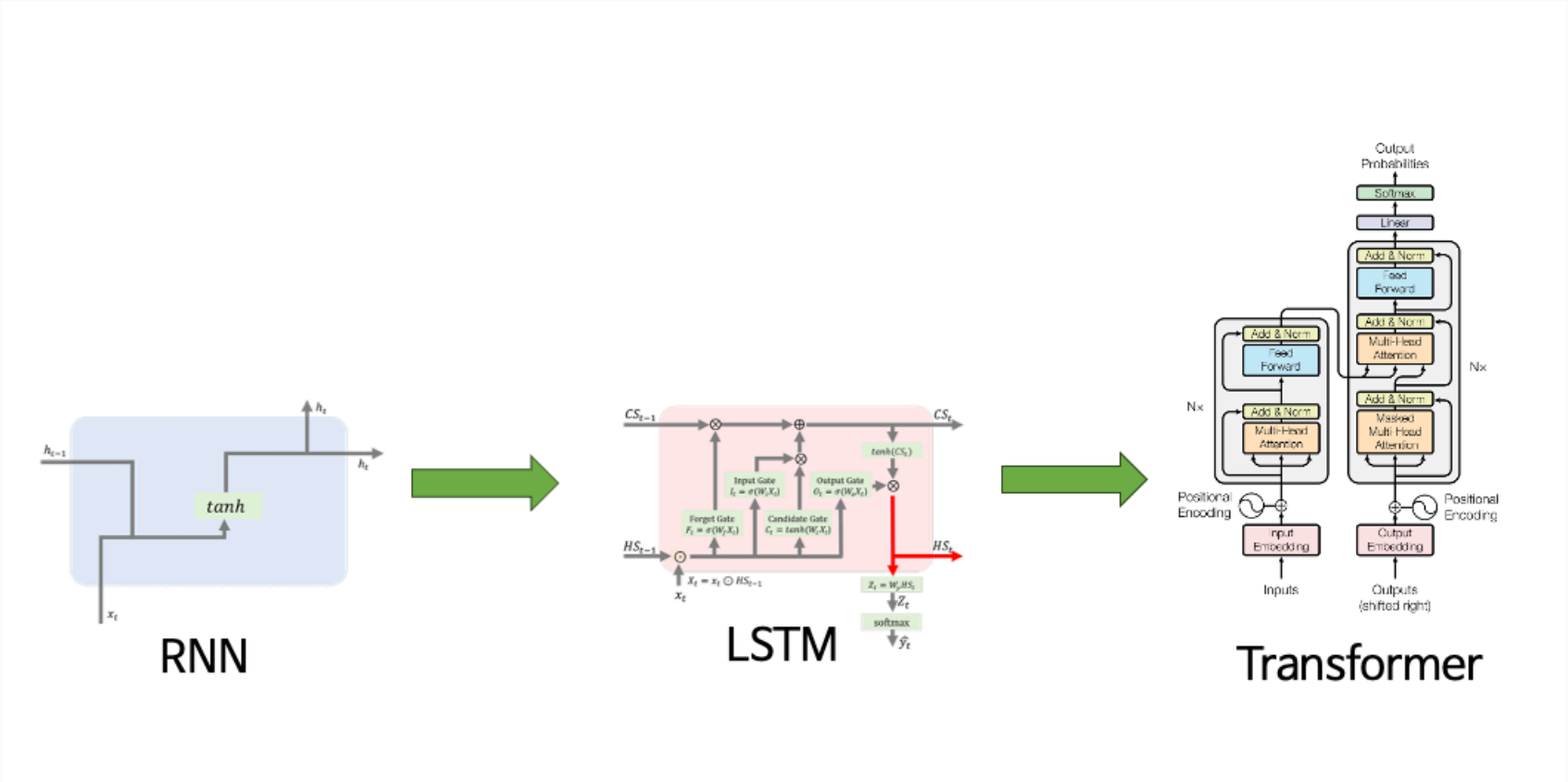
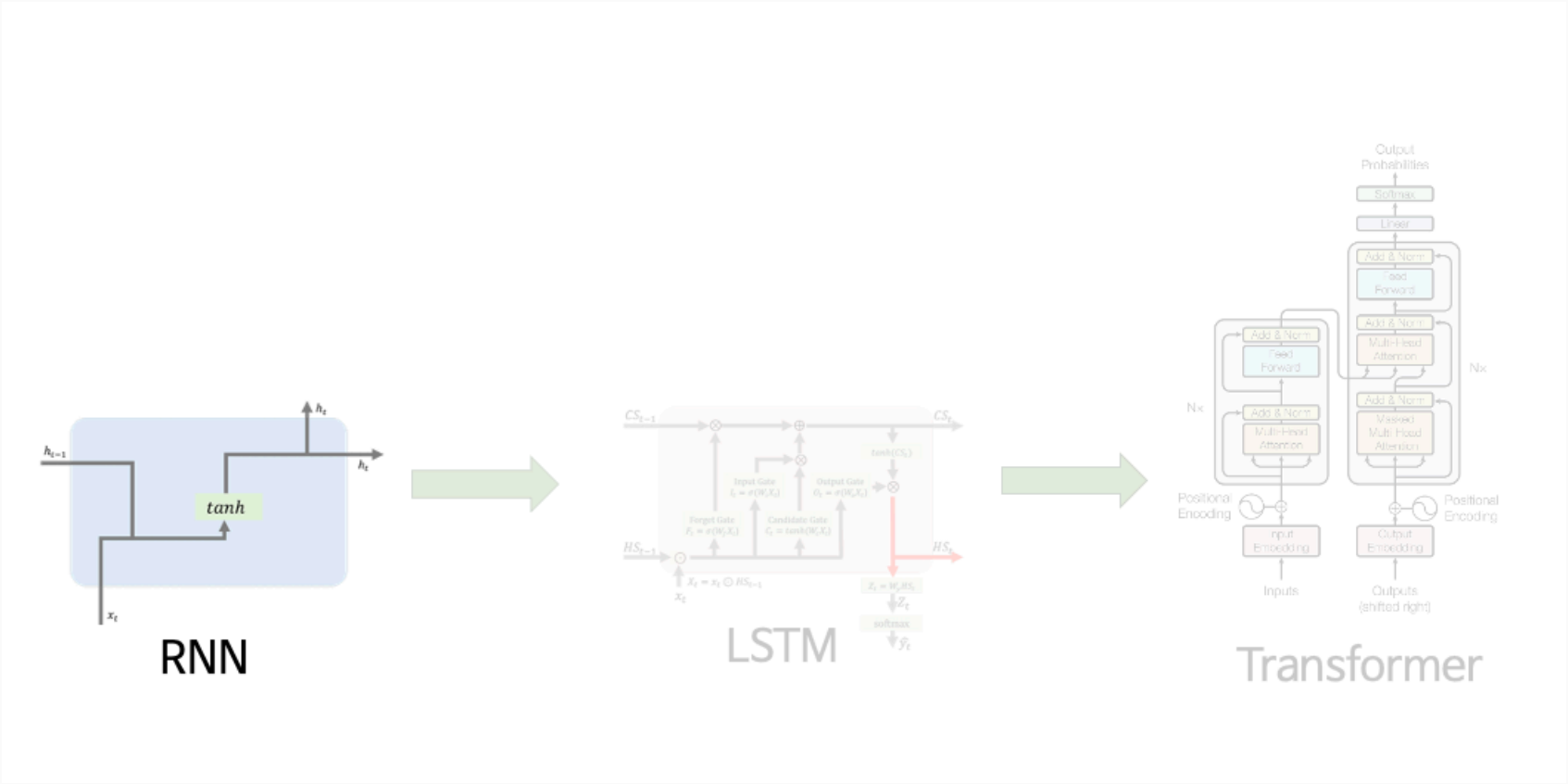
Moreover, RNNs evolved into LSTMs



Moreover, RNNs evolved into LSTMs and eventually into Transformers.

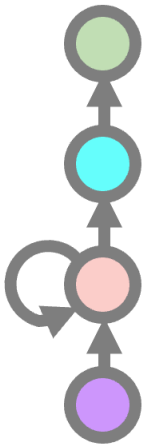


We will discuss (1) the structure, (2) the algorithms for learning sequential information, and (3) the uses of RNNs.



1. Structure

The structure of an RNN is simpler than you might think.



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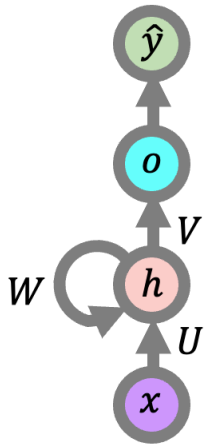
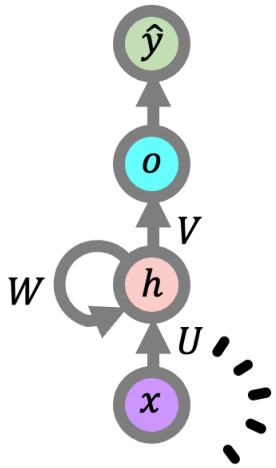
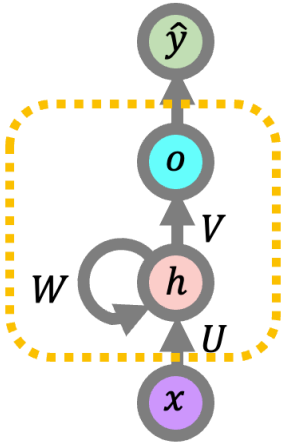


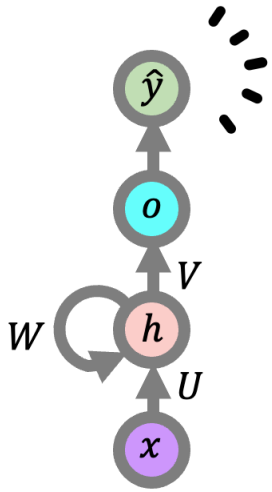
What an RNN does is to take an **input vector** x,



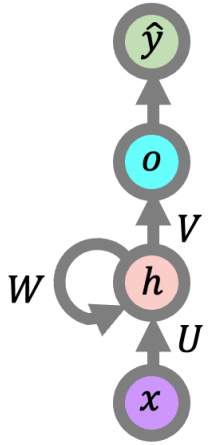
perform **internal computations**,



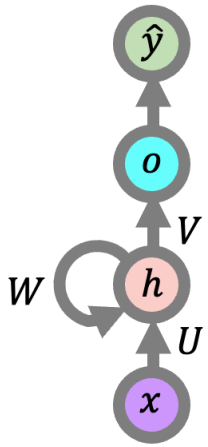
and produce an **output vector** $\hat{y}$.



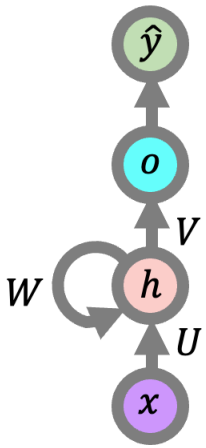
This is the **feedforward process** of the RNN.



The **types** of input vectors x and output vectors $\hat{y}$ can vary widely.



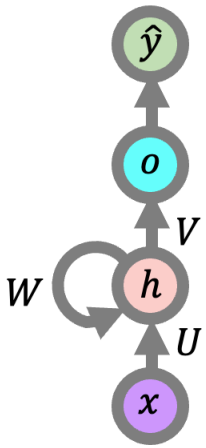
Time-series data (e.g., characters, stock price graphs, musical notes), as long as it can be represented sequentially, can be used as input.



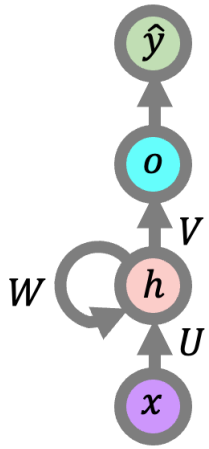
‘this’ ‘is’ ‘an’ ‘awesome’ ‘sentence’ ‘that’ ‘was’ ‘written’

So, what is the benefit of processing sequential data?

For example, let's assume this RNN is a model that translates *English* into *Spanish*.

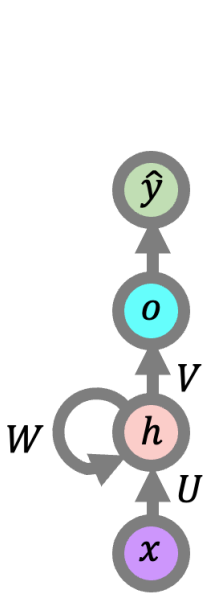


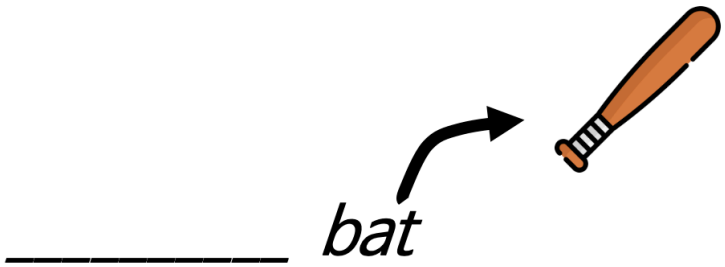
Suppose it encounters the word *bat* in English.



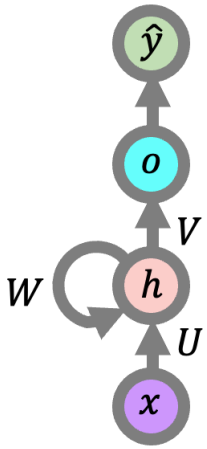
_____ *bat*

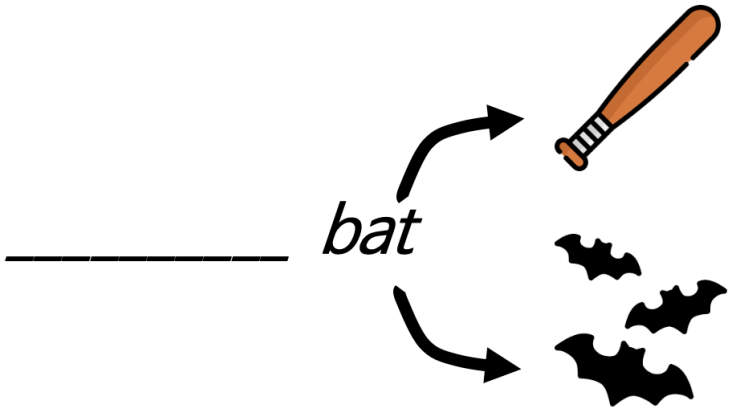
It has two possible meanings: a **baseball bat** or a flying bat.



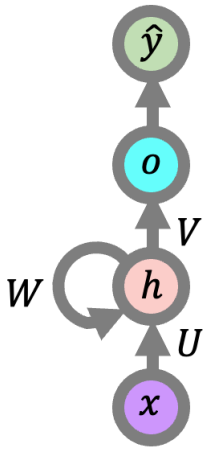


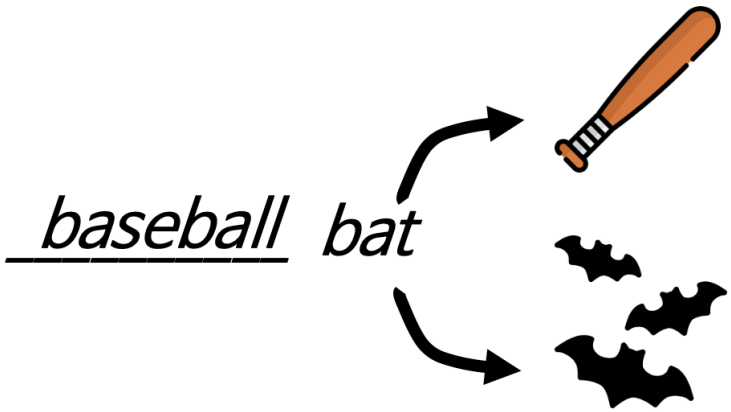
It has two possible meanings: a baseball bat or **a flying bat**.



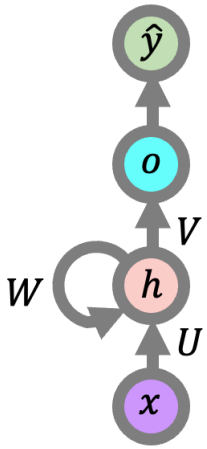


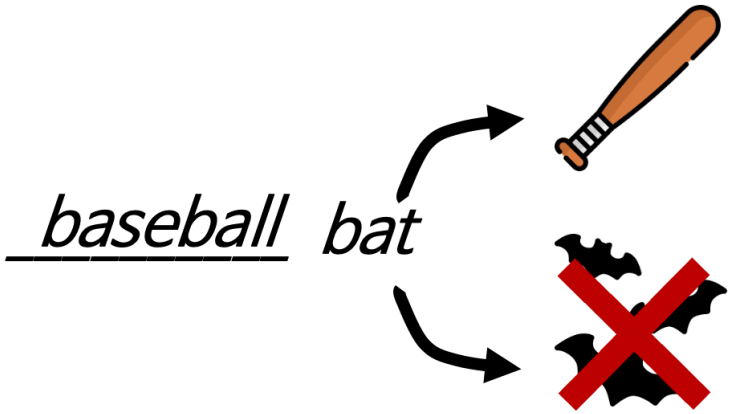
However, if the previous word is *baseball*,



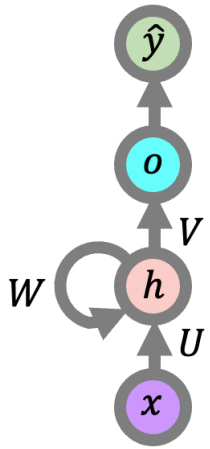


then with high probability, *bat* will be translated as *bate* in Spanish.





When translating *baseball*,



baseball

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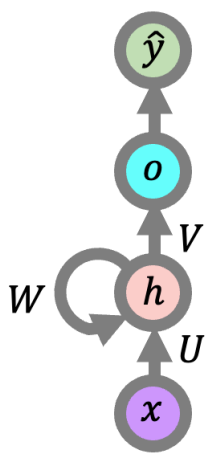
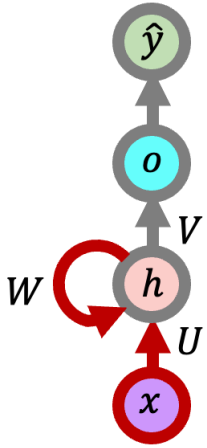
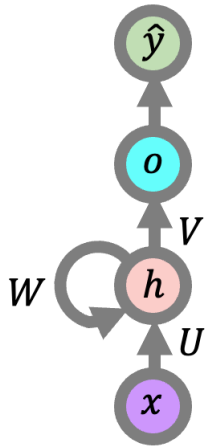
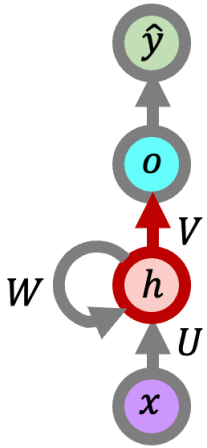


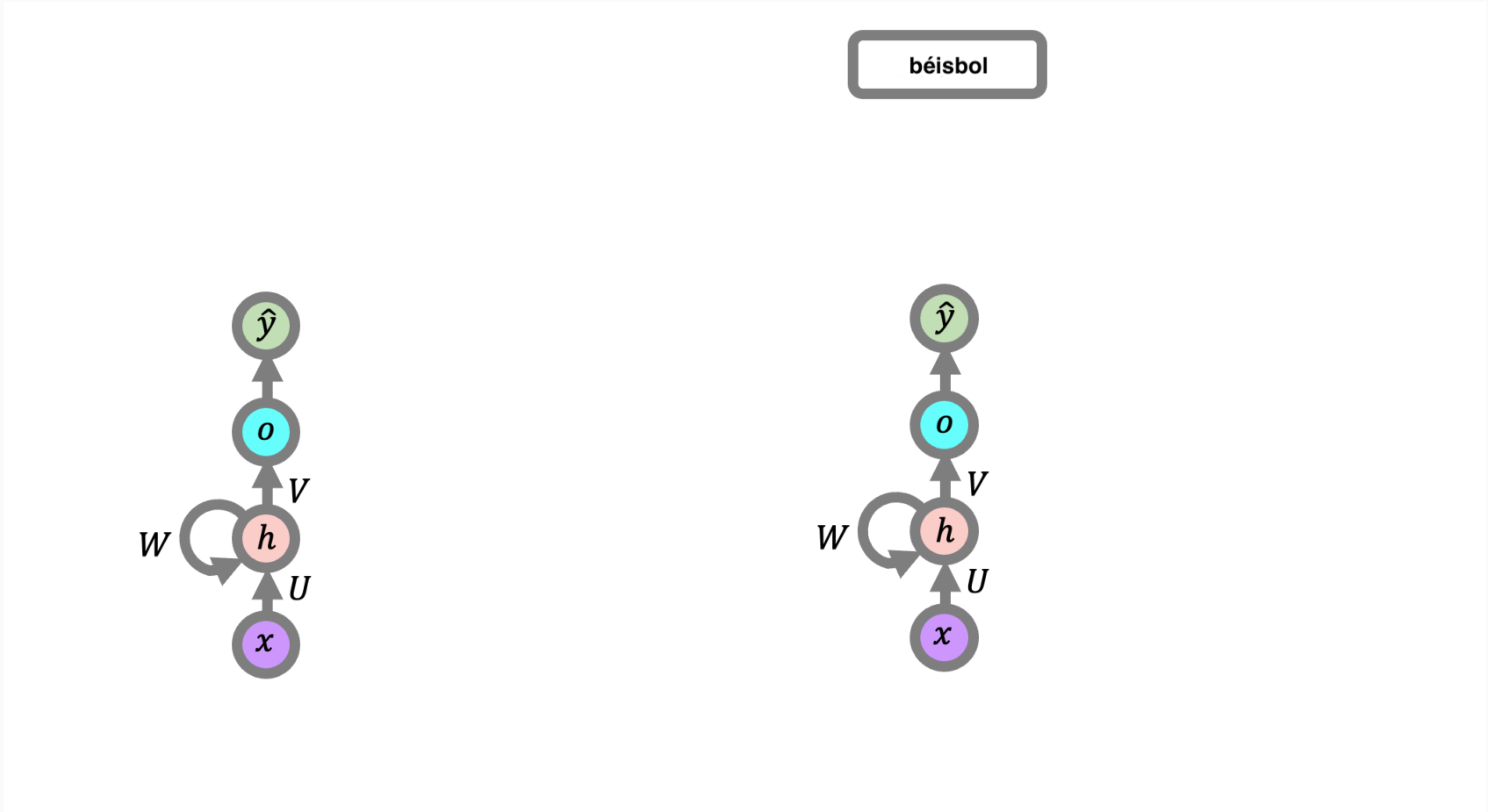


the internal state h is set with the processed representation of *baseball*.

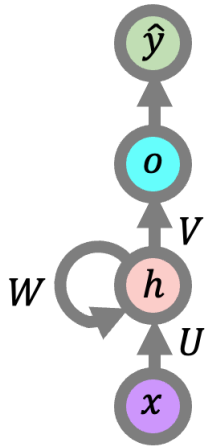


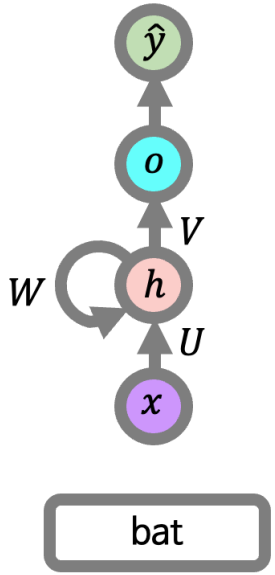


Through this internal computation, *baseball* is translated into *béisbol*,

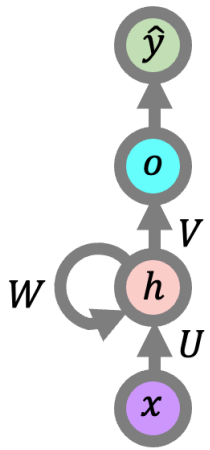


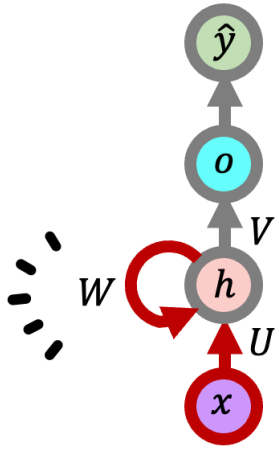
and when the model later encounters *bat*,



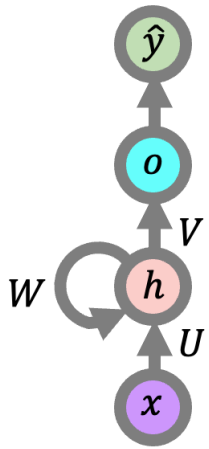


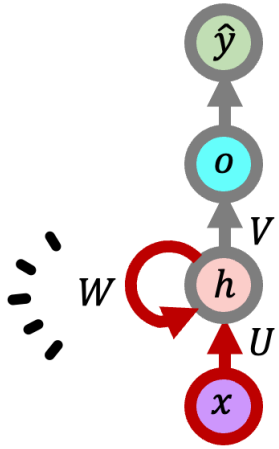
the hidden state created while translating *baseball*



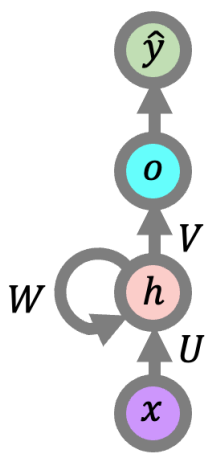


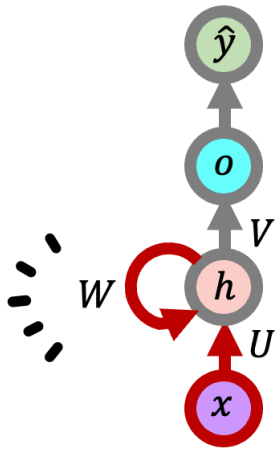
influences how *bat* is translated – this is the core idea of an RNN.





The hidden state h gets updated to combine the context of both *baseball* and *bat*,





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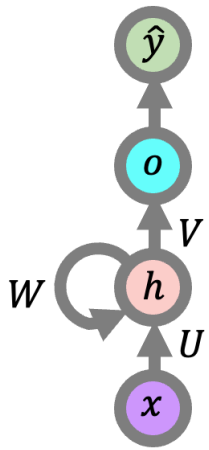

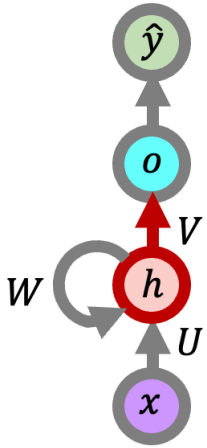

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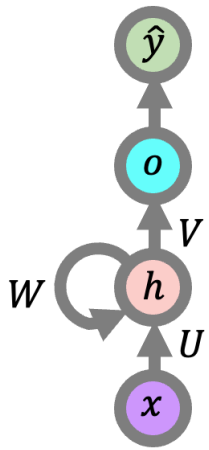

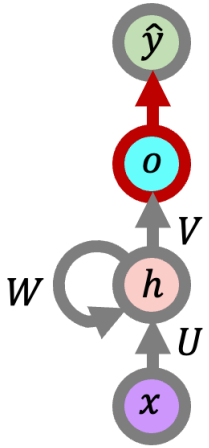

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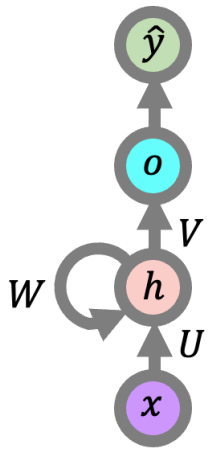

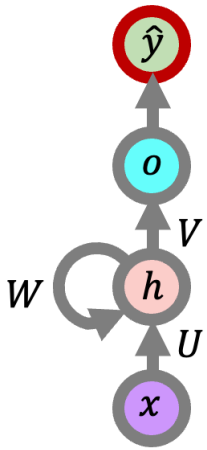

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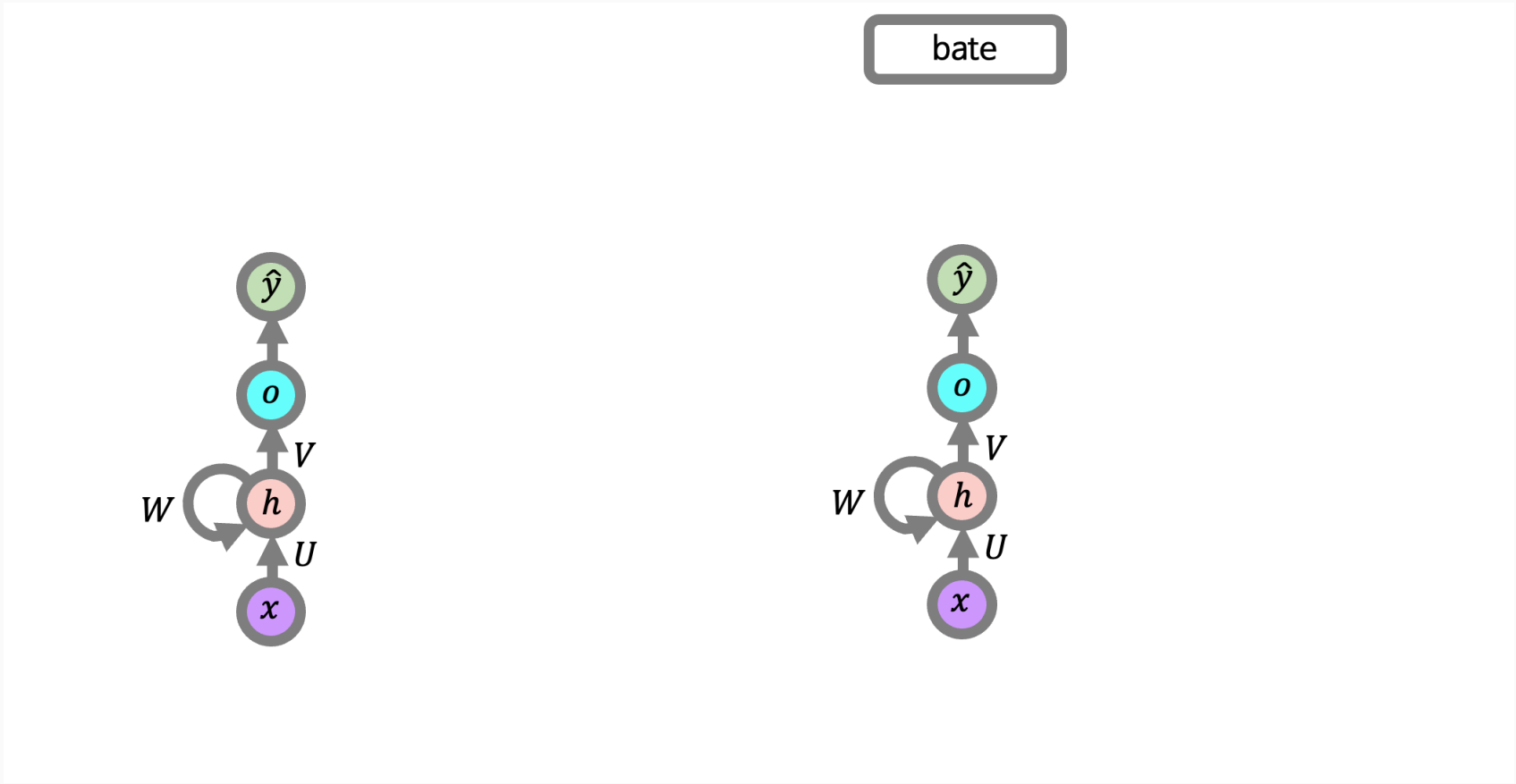
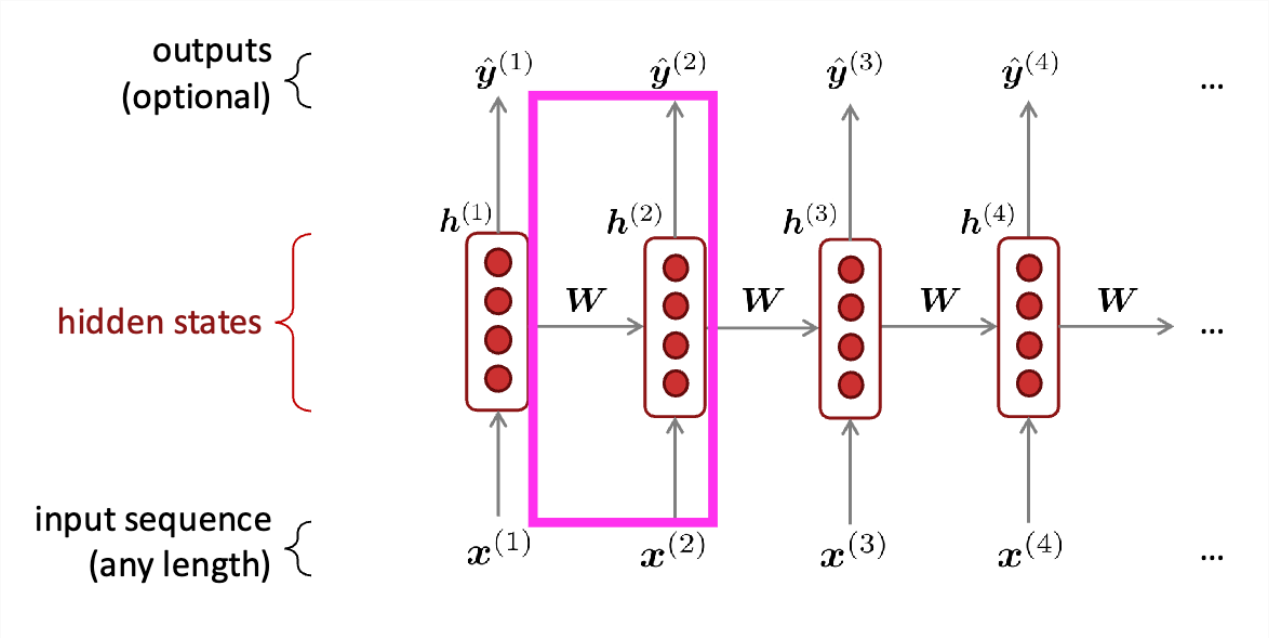


2. Algorithm/Training

- Idea: Repeatedly apply the same weight matrix W at each time step
- Maintain a hidden state over time, feeding it back into the network to capture temporal dependencies



2-1. The Simple RNN language model

output distribution

$$\hat{\boldsymbol{y}}^{(t)} = \text{softmax}\left(\boldsymbol{U}\boldsymbol{h}^{(t)} + \boldsymbol{b}_2\right) \in \mathbb{R}^{|V|}$$

hidden states

$$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h\boldsymbol{h}^{(t-1)} + \boldsymbol{W}_e\boldsymbol{e}^{(t)} + \boldsymbol{b}_1\right)$$

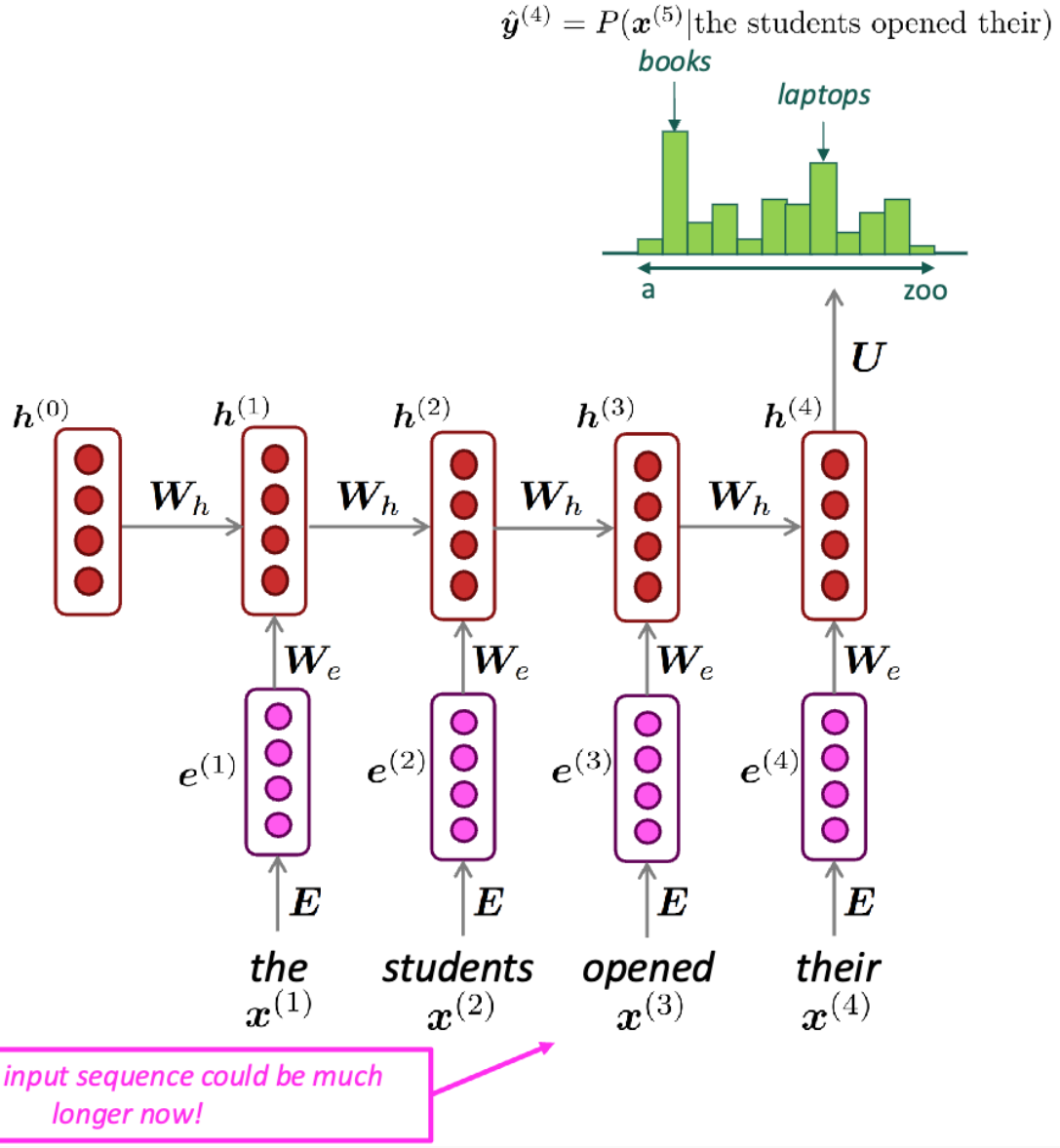
$\boldsymbol{h}^{(0)}$ is the initial hidden state

word embeddings

$$\boldsymbol{e}^{(t)} = \boldsymbol{E}\boldsymbol{x}^{(t)}$$

words / one-hot vectors

$$\boldsymbol{x}^{(t)} \in \mathbb{R}^{|V|}$$



***Note**: this input sequence could be much longer now!*

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1. Start with a large text corpus, represented as a sequence of words $w_1, \ldots, w_{T-1}, w_T$.

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   - *autoregressive, causal LM generation*

- **Loss** at step t:

$$\mathcal{J}^{(t)} \;=\; -\sum_{i=1}^{|V|} y_i^{(t)} \log \hat{y}_i^{(t)} \;=\; -\log \hat{y}_{w_{t+1}}^{(t)},$$

  where:

  - $y^{(t)}$: one-hot vector for the true next word w_{t+1}.
  - $\hat{y}^{(t)}$: predicted probability distribution over the vocabulary from the softmax layer.
  - This is the **cross-entropy loss** between the predicted distribution and the true label.

Example: Why one-hot vectors in the loss?

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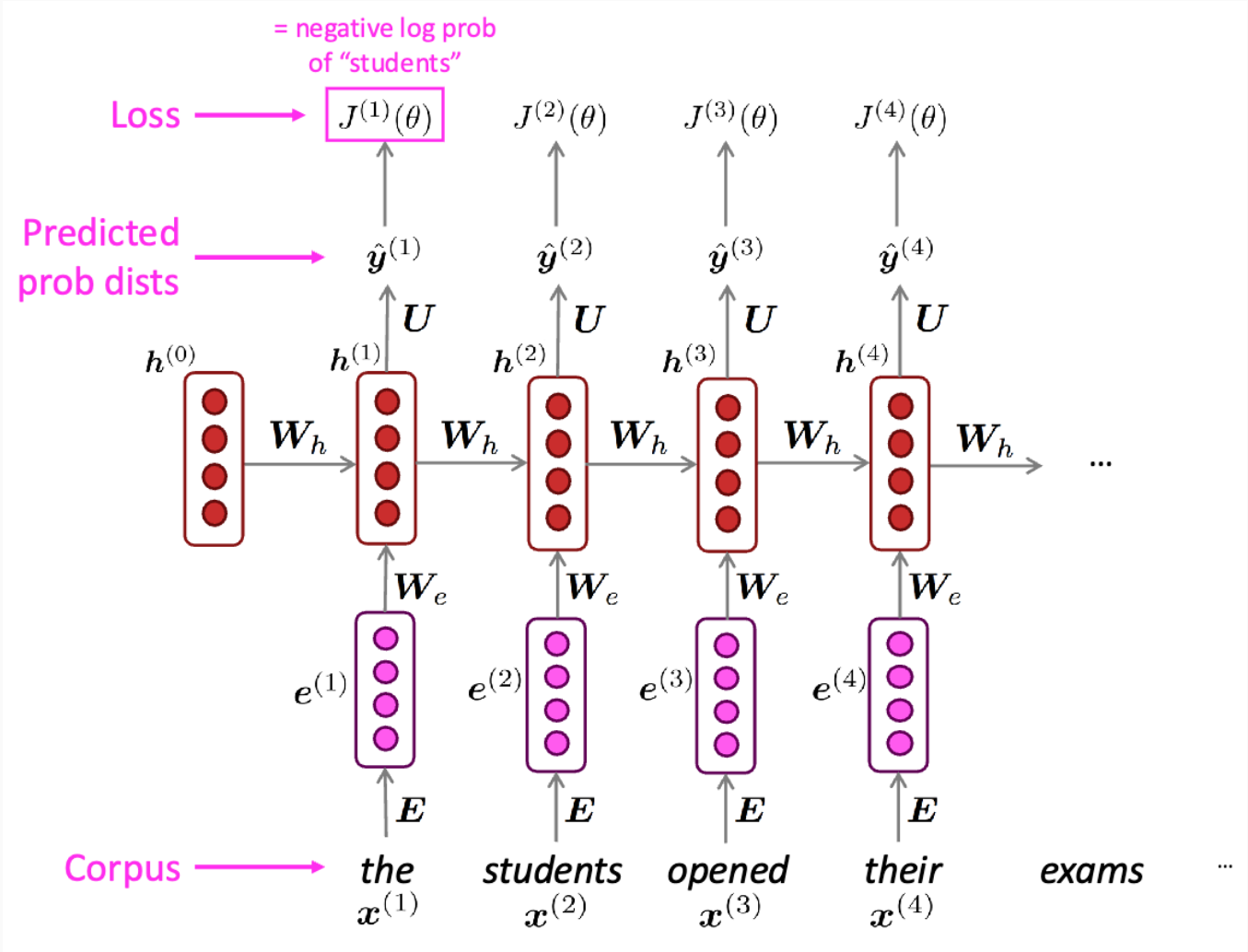
  - Example: if $\hat{y}_{\text{apple}} = 0.9$, then loss $= -\log 0.9 \approx 0.105$.


= negative log prob of "students"
Loss
Predicted prob dists
Corpus
$J^{(1)}(\theta)$
$J^{(2)}(\theta)$
$J^{(3)}(\theta)$
$J^{(4)}(\theta)$
$\hat{\boldsymbol{y}}^{(1)}$
$\hat{\boldsymbol{y}}^{(2)}$
$\hat{\boldsymbol{y}}^{(3)}$
$\hat{\boldsymbol{y}}^{(4)}$
$\boldsymbol{U}$
$\boldsymbol{h}^{(0)}$
$\boldsymbol{h}^{(1)}$
$\boldsymbol{h}^{(2)}$
$\boldsymbol{h}^{(3)}$
$\boldsymbol{h}^{(4)}$
$\boldsymbol{W}_h$
$\boldsymbol{W}_e$
$\boldsymbol{e}^{(1)}$
$\boldsymbol{e}^{(2)}$
$\boldsymbol{e}^{(3)}$
$\boldsymbol{e}^{(4)}$
$\boldsymbol{E}$
the
$\boldsymbol{x}^{(1)}$
students
$\boldsymbol{x}^{(2)}$
opened
$\boldsymbol{x}^{(3)}$
their
$\boldsymbol{x}^{(4)}$
exams
...

- Overall (average) loss over the sequence:

$$\mathcal{J}(\theta) \;=\; \frac{1}{T}\sum_{t=1}^{T}\mathcal{J}^{(t)}(\theta)$$

  - Sum the losses across all time steps.
  - Divide by the sequence length T to normalize for varying sequence lengths.
  - This gives the **average negative log-likelihood per word**, the main training objective of the RNN language model.

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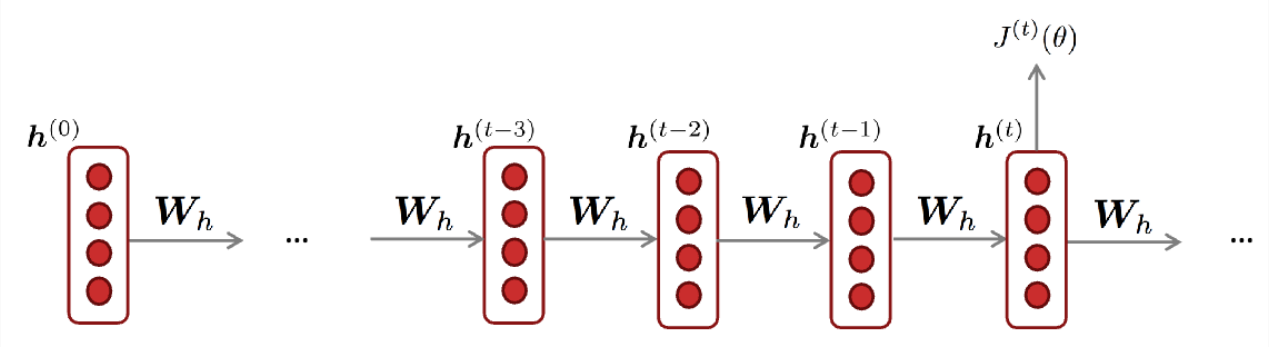
  4. and repeat.

2-4. Training: Backpropagation

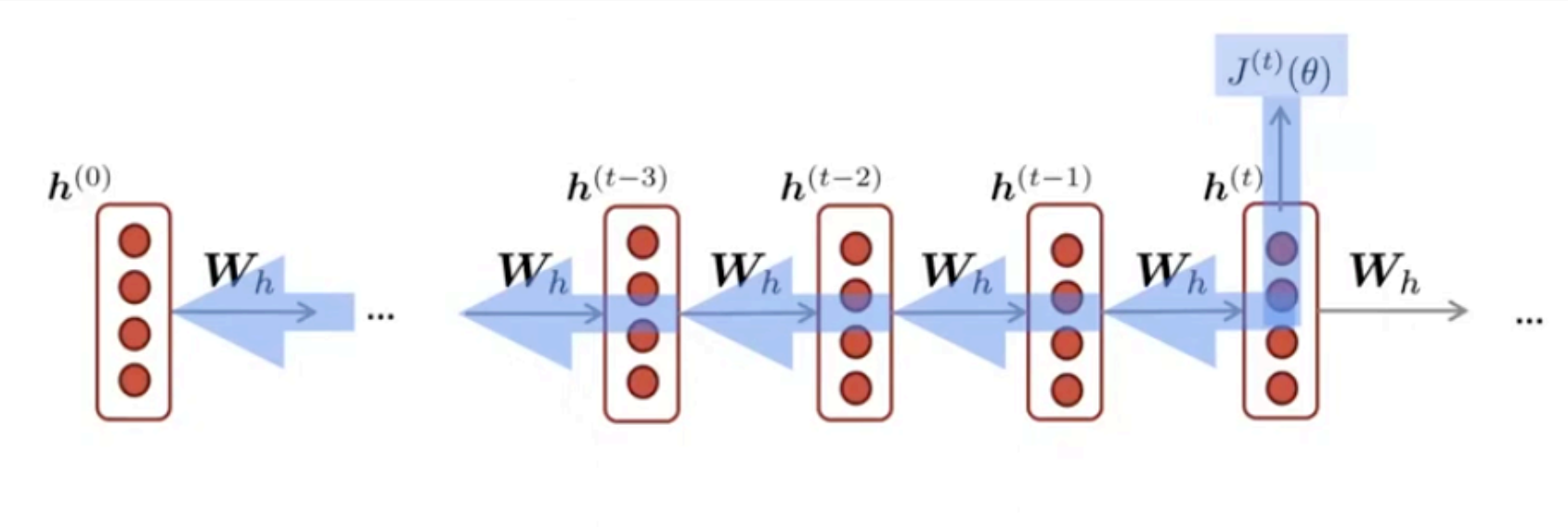


- The loss $J^{(t)}(\theta)$ depends on the shared weight matrix W_h at every time step.
- Therefore,

$$\frac{\partial J^{(t)}}{\partial W_h} = \sum_{k=1}^{t} \frac{\partial J^{(t)}}{\partial W_h^{(k)}}.$$

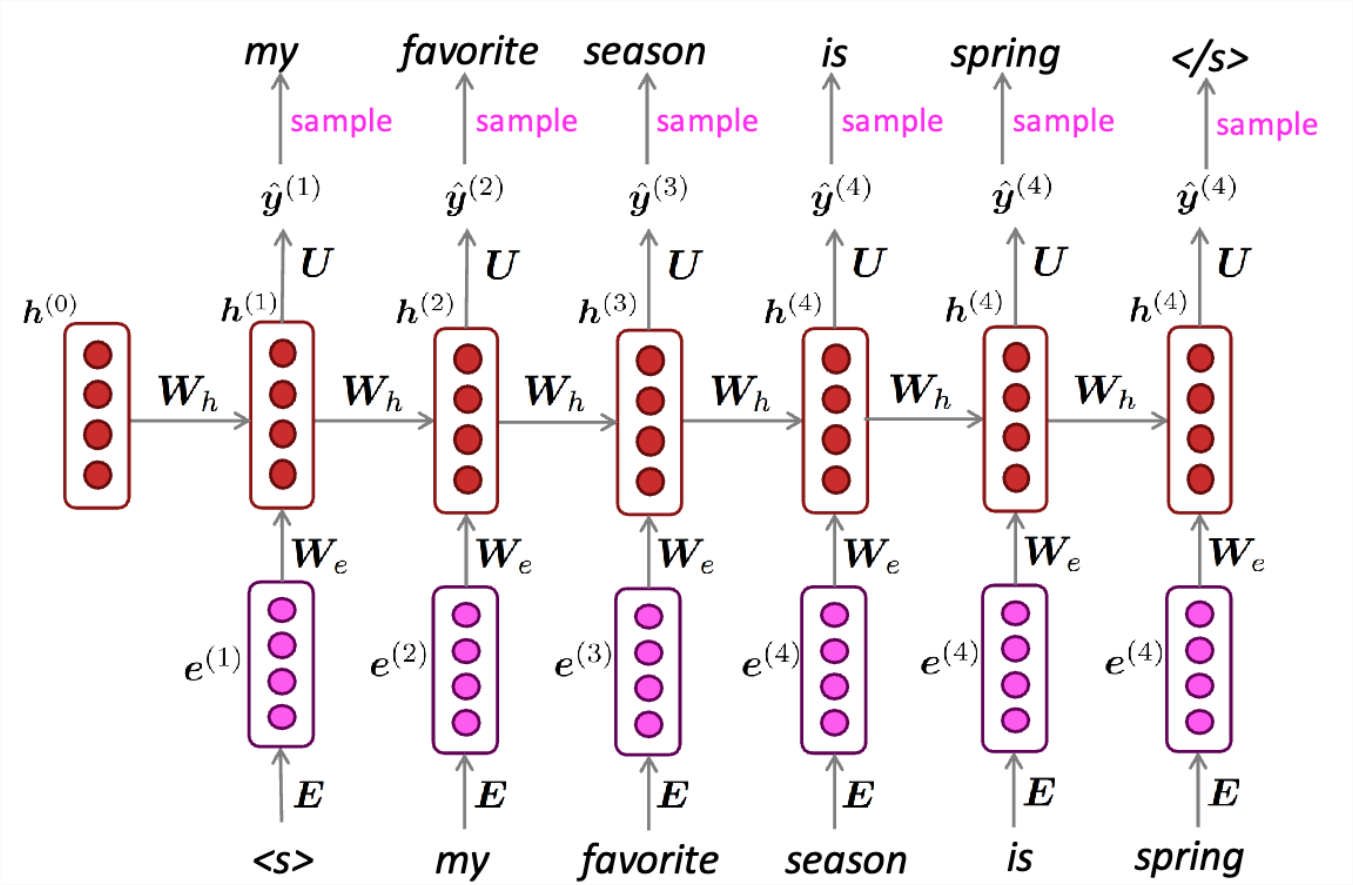
- In other words, the gradient w.r.t. the repeated parameter is the sum of its contributions over all time steps.
- **Why?** Because the RNN "unrolls" in time but reuses the same W_h at each step (parameter sharing).

2-4. Training: Backpropagation



2-5. Application: Generating text

Just like an n-gram language model, you can use an RNN model to generate text by **repeated sampling**. The sampled output becomes the next step's input.



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Example

- Start token: *<S>*
- Step 1: Model predicts distribution, sample *my*
- Step 2: Input = *my*, sample *favorite*
- Step 3: Input = *favorite*, sample *season*
- Step 4: Input = *season*, sample *is*
- Step 5: Input = *is*, sample *spring*

$\Rightarrow$ "my favorite season is spring"

RNN-LM trained on *Harry Potter*:

"Sorry," Harry shouted, panicking—"I'll leave those brooms in London, are they?"

"No idea," said Nearly Headless Nick, casting low close by Cedric, carrying the last bit of treacle Charms, from Harry's shoulder, and to answer him the common room perched upon it, four arms held a shining knob from when the spider hadn't felt it seemed. He reached the teams too.

Source: https://medium.com/deep-writing/harry-potter-written-by-artificial-intelligence-8a9431803da6

2-6. Evaluation: Perplexity

- The most common evaluation metric for language models is perplexity.

$$\text{Perplexity}(w_{1:T}) = P(w_{1:T})^{-\frac{1}{T}} = \exp\left(-\frac{1}{T}\sum_{t=1}^{T}\log P(w_t \mid w_{1:t-1})\right)$$

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2-6. Evaluation: Perplexity

- The most common evaluation metric for language models is perplexity.

$$\text{Perplexity}(w_{1:T}) = P(w_{1:T})^{-\frac{1}{T}} = \exp\left(-\frac{1}{T}\sum_{t=1}^{T}\log P(w_t \mid w_{1:t-1})\right)$$

- **Definition:** Exponential of the average **cross-entropy loss**.
- **Intuition:**
  - Measures how "surprised" the model is when predicting the test data.

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- **Interpretation:** Lower perplexity $\Rightarrow$ model is less "perplexed" and makes more accurate predictions.

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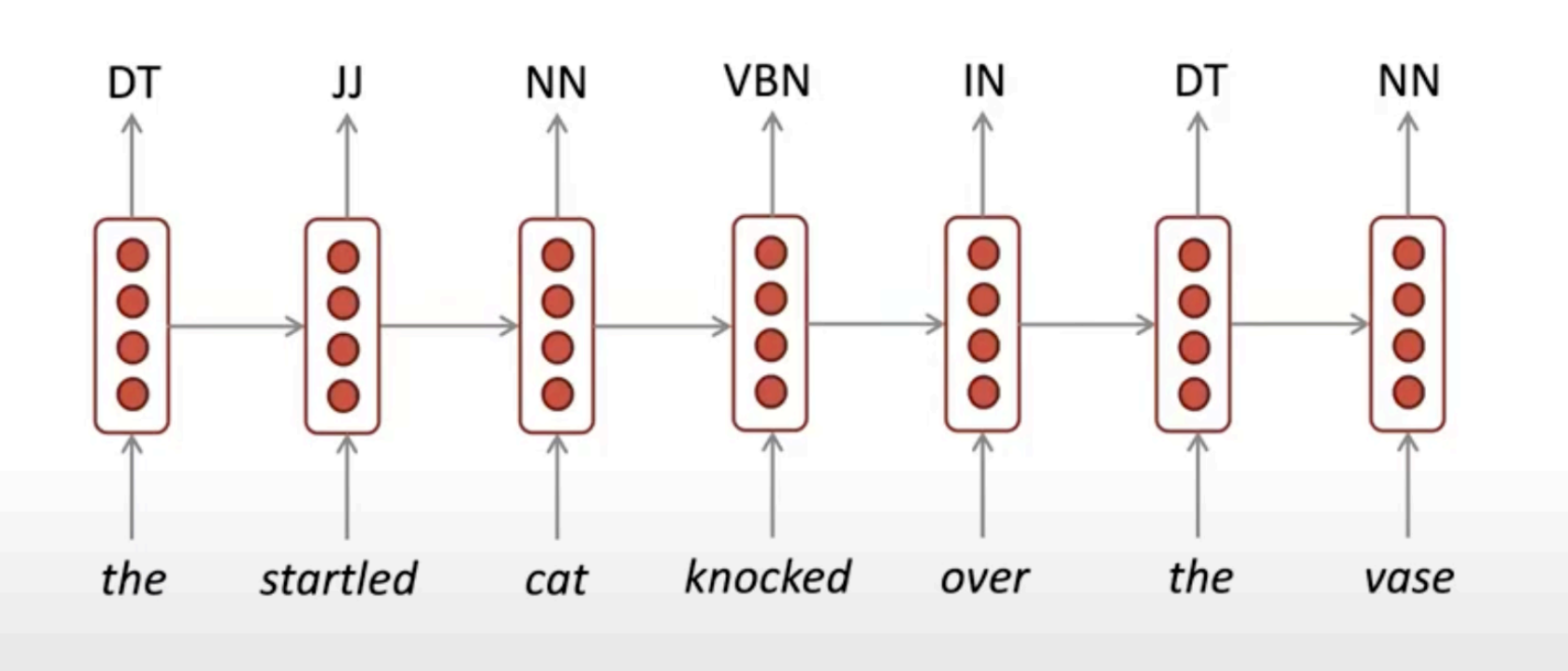
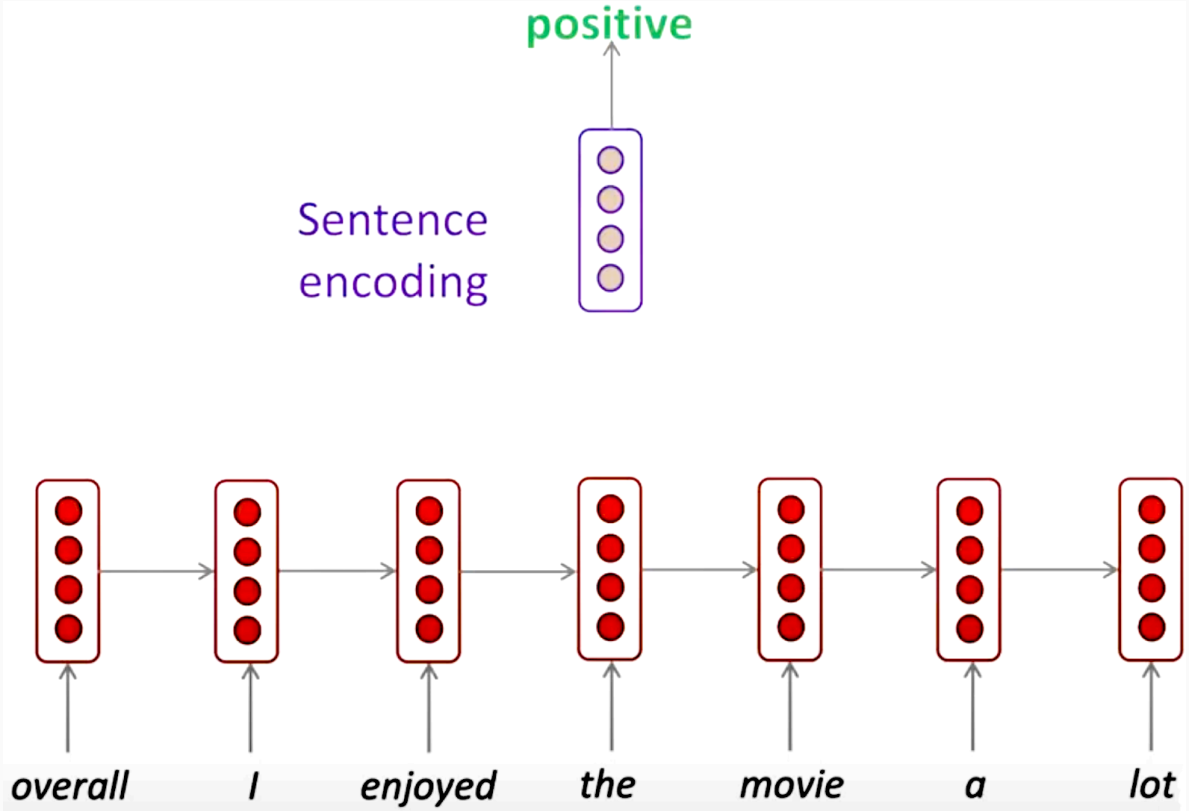
2-7. Other uses: Sequence tagging

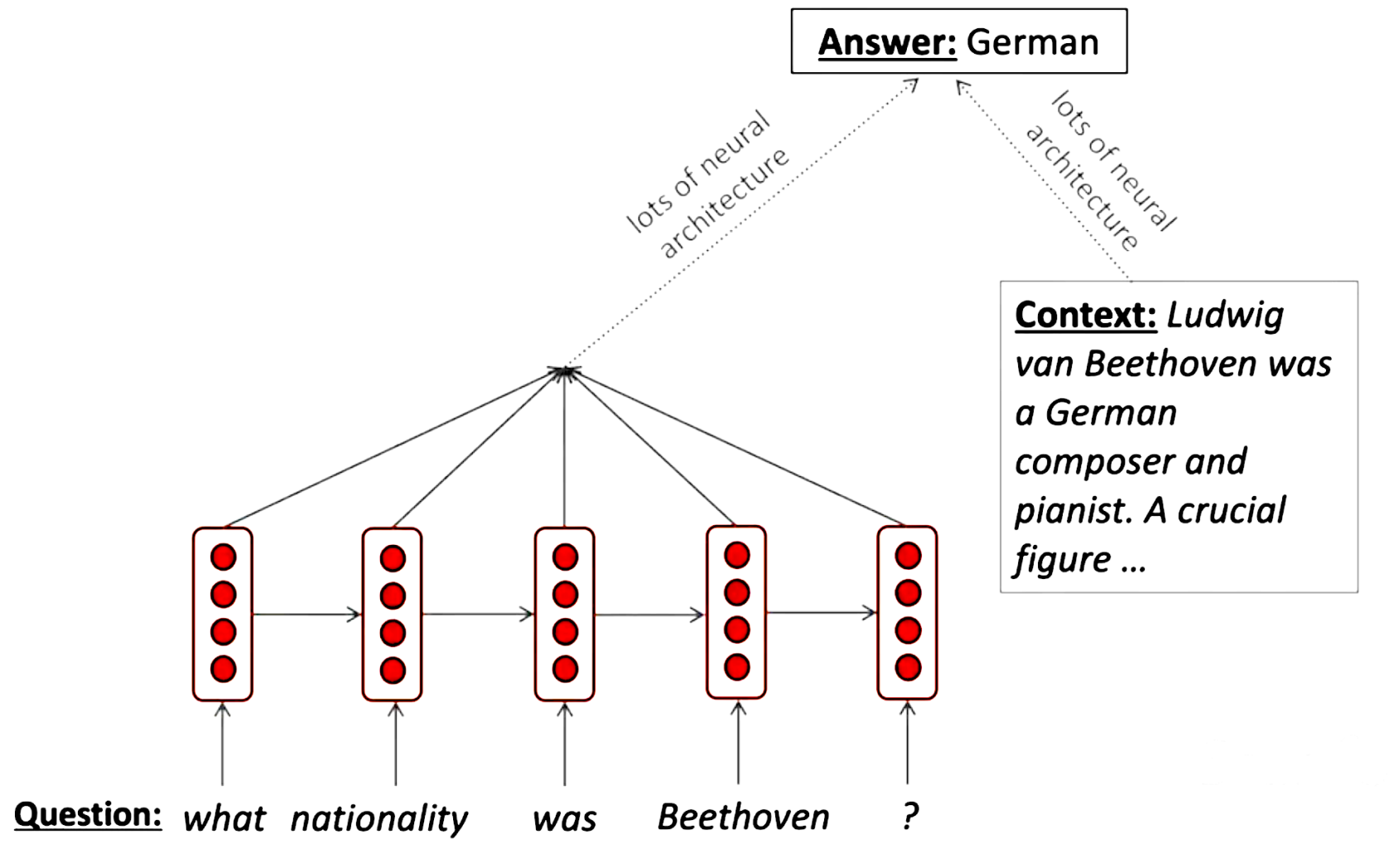
e.g., part-of-speech tagging, named-entity recognition



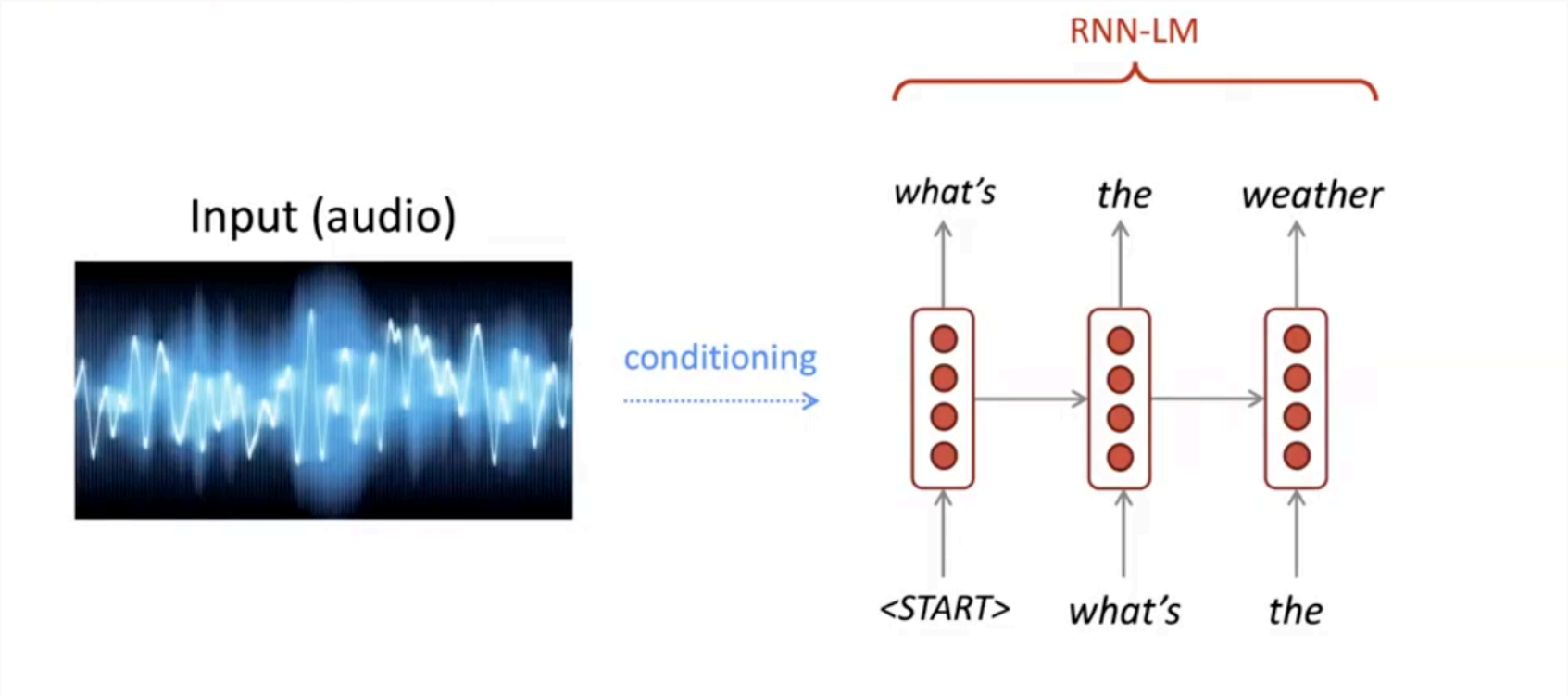
e.g., sentiment classification



e.g., question answering, machine translation



e.g., speech recognition, machine translation, summarization



Wrap-up

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- n-gram language models

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- RNNs
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  - structure
  - loss function

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  - applications