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Transit route and frequency design: Bi-level modeling and hybrid artificial bee colony algorithm approach

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ABSTRACT

This paper proposes a bi-level transit network design problem where the transit routes and frequency settings are determined simultaneously. The upper-level problem is formulated as a mixed integer non-linear program with the objective of minimizing the number of passenger transfers, and the lower-level problem is the transit assignment problem with capacity constraints. A hybrid artificial bee colony (ABC) algorithm is developed to solve the bi-level problem. This algorithm relies on the ABC algorithm to design route structures and a proposed descent direction search method to determine an optimal frequency setting for a given route structure. The descent direction search method is developed by analyzing the optimality conditions of the lower-level problem and using the relationship between the lower- and upper-level objective functions. The step size for updating the frequency setting is determined by solving a linear integer program. To efficiently repair route structures, a node insertion and deletion strategy is proposed based on the average passenger demand for the direct services concerned. To increase the computation speed, a lower bound of the objective value for each route design solution is derived and used in the fitness evaluation of the proposed algorithm. Various experiments are set up to demonstrate the performance of our proposed algorithm and the properties of the problem.

1. Introduction

Transit network design has received considerable attention over the last two decades due to its practical importance. For example, in Hong Kong, over 90% of the 11 million daily trips that people make involve public transport. Hence, a well-designed transit network is important for meeting passenger demand. Guihaire and Hao (2008) and Kepaptsoglou and Karlaftis (2009) provided comprehensive reviews in this area. Previous works on this topic focus on route design (e.g., Mandl, 1980; Murray, 2003; Wan and Lo, 2003; Li et al., 2011, 2012), frequency setting (e.g., Furth and Wilson, 1982; LeBlanc, 1988; Hadas and Shnaiderman, 2012), timetabling (e.g., Wong et al., 2008; Fleurent et al., 2004), vehicle scheduling (e.g., Bunte et al., 2006), crew scheduling (e.g., Wren and Rousseau, 1993), fare structure (e.g., Li et al., 2009), fleet size determination (e.g., Li et al., 2008), and a combination of the above (e.g., Ceder and Wilson, 1986; Lee and Vuchic, 2005; Szeto and Wu, 2010).

The majority of previous studies have considered the optimization of transit route structures and service frequencies separately. For example, Fernandez and Marcotte (1992), Constantin and Florian (1995), Zubieta (1998), Gao et al. (2004), Uchida et al. (2005, 2007), and Leiva et al. (2010) proposed models for optimizing frequencies to achieve different objectives

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within an existing transit network, whereas Laporte et al. (2010) and Yu et al. (2012) focused exclusively on designing route structures. Both transit route structure and frequency setting determine the level of service (e.g., in terms of in-vehicle congestion and waiting time at bus stops); more importantly they determine whether the service has sufficient capacity to meet passenger demand. Therefore, it is important to simultaneously optimize the transit route structure and the frequency setting.

In transit network design, it is essential to consider the in-vehicle congestion issue. In-vehicle congestion leads to increased waiting and travel times, along with the comfort problem prompted by a lack of seats for passengers. This comfort problem can be particularly serious if the trip time is long or demand is high. Generally, there are two approaches to addressing the congestion issue: capacity constraint and the congestion cost function. The capacity constraint approach (e.g., Kurauchi et al., 2003; Lei and Chen, 2004; Lam et al., 1999, 2002; Cepeda et al., 2006; Sumalee et al., 2009, 2011; Schmöcker et al., 2008, 2011; Szeto et al., 2013; Cortés et al., 2013) incorporates capacity constraints in transit assignment models that disallow flows on transit vehicles to be greater than the corresponding capacity. The congestion cost function approach (e.g., Spiess and Florian, 1989; de Cea and Fernández, 1993; Lo et al., 2003; Li et al., 2008, 2009, 2011; Sun and Gao, 2007; Teklu, 2008; Szeto et al., 2011a; Szeto and Jiang, 2014) adopts an unbounded increasing convex function to model the effect of in-vehicle congestion on waiting time. Although both approaches have been used in the literature, practically speaking, the former is more realistic because the latter can result in an unacceptable line flow that is far greater than the corresponding capacity.

In addition to the congestion issue, it is important to consider passenger transfers between transit vehicles, as they can generate passenger inconvenience. The number of passenger transfers is an important network performance indicator, especially in Hong Kong, for the following reasons. First, the total number of passenger transfers reflects the number of passengers without direct services to their destinations, which can indicate inconvenience. Second, passengers always complain when there are no direct services to their destinations (Szeto and Jiang, 2012). The total number of passenger transfers also indirectly reflects the number of complaints regarding lack of direct services. Optimizing the number of passenger transfers can reduce the number of complaints implicitly. However, very few studies have considered this number. Baaj and Mahmassani (1990) embedded the transfer concept into their route generation procedures, such that a route with more than two transfers was abandoned. Similarly, the number of passenger transfers was modeled implicitly in Zhao et al. (2005). The travel cost calculated in the objective function excluded the travel costs of routes with more than two transfers, yet they did not optimize the total number of passengers needing to transfer between transit vehicles. Guan et al. (2006) used the total number of passenger transfers as a surrogate of transfer and waiting times in passenger line assignment, which is the lower level problem of their transit network design problem. Jara-Díaz et al. (2012) considered the total number of passenger transfers to investigate the condition under which a transit network design with transfers is preferable. Most of the existing studies have used the total passenger travel time as the objective function. However, there is no guarantee that minimizing the total number of passenger transfers also minimizes the total passenger travel time. In some cases, there can be a tradeoff between the total number of passenger transfers and total passenger travel time (Szeto and Wu, 2010). It is essential to explicitly capture the total number of passenger transfers in the objective function.

This paper proposes a bi-level model for designing transit routes and their frequencies that explicitly minimizes the total number of passenger transfers in the objective function of the upper-level problem and incorporates strict capacity constraints to address the in-vehicle congestion in the lower-level problem. This bi-level model is formulated as a mixed integer non-linear program that is NP-hard and considers the route choice behavior of passengers through the lower-level user-equilibrium problem. The model also considers the stop location choice of each route within each zone of the study area. This model differs from the bi-level models proposed by Constantin and Florian (1995), Gao et al. (2004) and Uchida et al. (2005, 2007) in the sense that they only considered frequency setting, whereas our model further considers route design and stop location choice.

To solve transit network design problems, exact methods (e.g., Wan and Lo, 2003) and metaheuristics such as genetic algorithms (GAs) (e.g., van Nes et al., 1988; Bielli et al., 2002; Chakroborty and Dwivedi, 2002; Tom and Mohan, 2003; Ngamchai and Lovell, 2003; Shih et al., 1998; Fan and Machemehl, 2006a; Mazloumi et al., 2012) and simulated annealing (e.g., Fan and Machemehl, 2006b; Zhao and Zeng, 2006) have been used. A hybrid artificial bee colony (ABC) algorithm—a matheuristic that combines a metaheuristic and an exact algorithm—is developed for the transit network design problem as an improvement to the original ABC algorithm, a metaheuristic proposed by Karaboga (2005) and motivated by the foraging behavior of honey bees.

Compared with existing evolutionary algorithms such as GAs, the ABC algorithm has a better local search mechanism that improves the solution quality. More recently, the ABC algorithm has been applied to solve complex engineering optimization problems. For example, Kang et al. (2009) successfully applied an ABC algorithm to the parameter identification of concrete dam-foundation systems. Karaboga (2009) proposed an ABC algorithm to solve a digital filter design problem and obtained good results. Karaboga and Ozturk (2009) used an ABC algorithm to train neural networks for pattern classification, and their results on benchmark instances showed that such use was efficient. Szeto et al. (2011b) improved the ABC algorithm to solve a capacitated vehicle routing problem. Szeto and Jiang (2012) enhanced the ABC algorithm to solve a single-level transit network design problem without considering the in-vehicle congestion effect. Long et al. (2014) improved the ABC algorithm to solve a turn restriction design problem. Szeto and Jiang (2012) and Long et al. (2014) showed that their proposed ABC algorithms are better than the GA for solving their problems, but it has not yet been improved to solve bi-level transit network design problems that consider in-vehicle congestion. This study enhances the ABC algorithm to solve this problem.

The proposed algorithm relies on the ABC algorithm to design route structures and a proposed descent direction search method to determine an optimal frequency setting for a given route structure. A node insertion and deletion strategy for

repairing the route structures is developed based on average-direct-demand, which is defined as the average passenger demand on the direct services concerned. The descent direction search method is developed by analyzing the optimality conditions of the lower-level problem and using the relationship between the lower- and upper-level objective functions. The step size for updating the frequency setting is determined by solving a linear integer program formed by the derivative obtained by the Lagrange function of the lower-level problem. The Simplex method is used to solve the lower-level problem. To increase the computation speed, a lower bound of the objective value for each route design solution is derived and used in the fitness evaluation for the hybrid ABC algorithm.

Various experiments are conducted to demonstrate the effectiveness of our proposed algorithm. They illustrate the effects of various node insertion and deletion strategies and the effects of different parameter values and forms of fitness functions on the performance of the hybrid ABC algorithm. A realistic case study is conducted to show that under demand uncertainty, the optimal solution obtained from the hybrid ABC algorithm is better than the existing bus network design in terms of the average number of passenger transfers, and is more robust in terms of handling passenger demand. We also use the Winnipeg network to demonstrate that the performance of our proposed method is better than that of a GA to solve our problem. The experiments illustrate the effects of different design parameters such as minimum frequency, maximum fleet size, and the maximum numbers of routes and intermediate stops on the objective value. The results show that a higher minimum frequency can lead to a higher number of passenger transfers, and multiple design solutions are possible.

The main contributions of this study are as follows.

(1) Proposing a bi-level model to simultaneously solve the transit route design and frequency setting problems while considering the candidate transit stop location available in each zone in the study area and two inconvenience factors: transfers between transit vehicles and in-vehicle congestion.
(2) Developing a new matheuristic—the hybrid ABC algorithm—to solve the model.
(3) Examining the properties of the bi-level problem and the performance of the algorithm.
(4) Demonstrating the applicability of the proposed model and algorithm in realistic situations.

The remainder of this paper is organized as follows. Section 2 introduces the bi-level model. The proposed hybrid ABC method is described in Section 3, and numerical examples are presented in Section 4. Finally, the conclusions and future research directions are given in Section 5.

2. Bi-level formulation of the problem

Consider a study area with a connected (bus) transit network represented by a directed graph G with N nodes, E links (or arcs), and one dummy node (node 0) introduced for the ease of formulating the problem. The study area is separated into many zones, each of which is represented by a centroid. The centroid is the origin node aggregating the travel demand within the zone. Each centroid is connected to all of the candidate transit stops and terminals in that zone, in which a transit terminal for a bus service can be a candidate stop for another bus service. Each centroid also generates N' types of travel demand, each of which is designated to one centroid (or destination node) outside the study area. Each of the N' centroids is connected to bus terminals or bus stops in their individual zone. Both the bus terminals and centroids in each of these zones are connected to the transit network in the study area. The following notations are used in this paper.

Sets	
Z_U	a set of nodes in the upper-level network, excluding the depot
G_s	a set of centroids within the study area
H_m	a set of candidate stops connecting to centroid m
U	a set of starting bus terminals inside the study area
V	a set of ending terminals outside the study area
G_d	a set of centroids/destinations outside the study area
C	a set of bus terminals and candidate stops within the study area
Z_L	a set of nodes (including centroids) in the lower-level network
T^R	a set of transfer links or arcs in the lower-level network
A	a set of transit links in the lower-level network
A_i^+	a set of transit links coming out from node i; and
A_i^-	a set of transit links going into node i
Indices	
i, j, m	indices of nodes
e	the index of a centroid/destination outside the study area
e'	the index of an ending bus terminal outside the study area; and
r	the route index

(*continued on next page*)

Parameters	
c_{ij}	the in-vehicle travel time on the shortest path between nodes i and j
c_a	the in-vehicle travel time on link a
s_t	the average time for stopping at a node
d_m^e	the travel demand from node m to centroid e
W	the maximum bus fleet size allowed for the network
k_{cap}	the capacity of a bus
R_{max}	the maximum number of routes in the bus network
f_{min}	the minimum frequency of a route
S_{max}	the maximum number of stops (including the bus terminal) within the study area on a route
T_{max}	the maximum route travel time within the study area; and
p	a very large value used in the sub-tour elimination constraint
Decision Variables	
Lower-level decisions	
v_t^e	the number of passenger transfers on transit link t to destination e
v_a^e	the flow on link a to destination e
ω_i^e	the total waiting time at node i for all flows to destination e
$\mathbf{v}$	$[v_a^e]$
$\mathbf{w}$	$[\omega_i^e]$
Upper-level decisions	
q_{ir}	the node potential at node i, which is needed in the sub-tour elimination constraint for bus route r
X_{ijr}	1 if route r (r = 1 to R_{max}) passes through node j immediately after node i, and 0 otherwise
X_{0jr}	1 if route r starts at node j, and 0 otherwise
X_{i0r}	1 if route r ends at node i, and 0 otherwise
X_{00r}	1 if route r is not available, and 0 otherwise
f_r	the frequency of route r
$\mathbf{X}$	$[X_{ijr}]$; and
$\mathbf{f}$	$[f_r]$
Functions of decision variables	
T_r	the trip time of route r from the starting terminal to the ending terminal
δ_r^e	1 if route r connects the terminal that links to centroid e, and 0 otherwise
f_a	the frequency of link a; and
$d_i^{e'}$	the travel demand from node i to bus terminal e'

2.1. Upper-level problem

The upper-level problem is to determine the frequency of and a route structure for each transit line within the study area. The number of transfers within the study area is unlimited and their possible locations must remain within the study area. The upper-level problem is formulated as follows.

$$\min_{\mathbf{x},\mathbf{f}} z_1 = \sum_{t\in T^R}\sum_{e\in G_d} v_t^e, \tag{1}$$

subject to

$$\sum_{j\in U\cup\{0\}} X_{0jr} = 1 \quad \text{for} \quad r = 1 \text{ to } R_{max}, \tag{2}$$

$$\sum_{i\in V\cup\{0\}} X_{i0r} = 1 \quad \text{for} \quad r = 1 \text{ to } R_{max}, \tag{3}$$

$$\sum_{\substack{i\in Z_U\cup\{0\}\\ i\neq j}} X_{ijr} - \sum_{\substack{i\in Z_U\cup\{0\}\\ i\neq j}} X_{jir} = 0 \quad \text{for} \quad j\in Z_U,\ r = 1 \text{ to } R_{max}, \tag{4}$$

$$\sum_{\substack{i\in Z_U\cup\{0\},\\ i\neq j}} X_{ijr} \leqslant 1 \quad \text{for} \quad j \in Z_U,\ r = 1 \text{ to } R_{max}, \tag{5}$$

$$\sum_{\substack{j\in Z_U\cup\{0\},\\ j\neq i}} X_{ijr} \leqslant 1 \quad \text{for} \quad j \in Z_U,\ r = 1 \text{ to } R_{max}, \tag{6}$$

$$X_{jjr} = 0 \quad \text{for} \quad j \in Z_U,\ r = 1 \text{ to } R_{max}, \tag{7}$$

$$T_r = \sum_{i\in Z_U} \sum_{j\in Z_U, j\neq i} X_{ijr}(c_{ij} + s_t) - s_t \quad \text{for} \quad r = 1 \text{ to } R_{max}, \tag{8}$$

$$\sum_{r=1}^{R_{max}} 2 f_r T_r (1 - X_{00r}) \leqslant W, \tag{9}$$

$$f_{min}(1 - X_{00r}) \leqslant f_r \quad \text{for} \quad r = 1 \text{ to } R_{max}, \tag{10}$$

$$\sum_{i\in C} \sum_{j\in C, j\neq i} X_{ijr} \leqslant S_{max} \quad \text{for} \quad r = 1 \text{ to } R_{max}, \tag{11}$$

$$\sum_{i\in C} \sum_{j\in C, j\neq i} X_{ijr}(c_{ij} + s_t) - s_t \leqslant T_{max} \quad \text{for} \quad r = 1 \text{ to } R_{max}, \tag{12}$$

$$\sum_{r=1}^{R_{max}} \sum_{i\in H_m} \sum_{j\in Z_U\cup\{0\}, j\neq i} X_{ijr} \geqslant 1 \quad \text{for} \quad m \in G_s, \tag{13}$$

$$\sum_{r=1}^{R_{max}} f_r k_{cap} \delta_r^e \geqslant \sum_{m\in G_s} d_m^e \quad \text{for} \quad e \in G_d, \tag{14}$$

$$\delta_r^e = \sum_{i\in Z_U\setminus V e'\in H_e} \sum X_{ie'r} \quad \text{for} \quad e \in G_d, \text{ and} \tag{15}$$

$$q_{ir} - q_{jr} + p X_{ijr} \leqslant p - 1 \quad \text{for} \quad i,j \in Z_U, \quad i \neq j, \quad r = 1 \text{ to } R_{max}. \tag{16}$$

Objective (1) is to minimize the sum of transfer passengers. Constraint (2) ensures that all of the bus service routes start from a bus terminal selected from the available locations inside the study area. Constraint (3) ensures that each of the service routes ends at a bus terminal selected from the available locations outside the study area. It should be noted that the rth route is not needed to provide bus services when $X_{00r} = 1$. Constraint (4) ensures that with the exception of dummy nodes, any node on a service route has one preceding and one following node. Constraints (5)–(7) ensure that each node can be visited by a particular route at most once. Constraint (8) calculates the in-vehicle travel time (including stop time) of a service route. Constraint (9) ensures that the fleet size used cannot exceed the available fleet size. Constraint (10) ensures that the frequency of each service route is not less than the minimum allowable frequency. Constraints (11) and (12) restrict the number of intermediate stops and the trip time within the study area, respectively. Constraint (13) is the zone covering constraint, and ensures that at least one of the candidate stops in each zone is served by at least one transit line. Constraint (14) is the capacity constraint, which ensures that there is enough line capacity to meet passenger demand heading to each destination/centroid outside of the study area. Constraint (15) determines whether route r ends at terminal e'. Constraint (16) is the sub-tour elimination constraint, which is extended from Miller et al. (1960).

In this formulation, the decisions are the route structures and frequencies. However, under the preceding setting and constraint (13), the route design automatically also considers the stop location choice in a zone because there is more than one candidate stop in each zone in general, and a route may not pass through all of them.

2.2. Lower-level problem

The lower-level problem requires another network representation to depict the passenger route choice behavior under a given set of transit routes defined by the upper-level problem. The network representation for the lower-level problem is extended from the one proposed by Nguyen and Pallottino (1988). The network is also represented by nodes and links (or arcs). However, a node may represent a bus stop in a transit line, a boarding node, an alighting node, or a centroid. A link is used to connect two adjacent nodes. Each link has three attributes: travel time, frequency, and capacity.

Fig. 1 is a graph representation of a centroid connecting one general transit stop served by n transit lines. Similar to the graphical representation proposed by Nguyen and Pallottino (1988), there is a pair of boarding and alighting arcs connecting the bus stop of each transit line, s_i, $i = 1,\ldots,n$, to the stop node (as represented by the node defined by the dashed line in Fig. 1) that corresponds to the node in the upper-level network. To ensure that these arcs are only used for connectivity purposes, the travel time is set to zero and the capacity is set to a very large number. The frequency of the alighting arc is also set to a very large number, whereas the frequency of the boarding arc is equal to the frequency with which the passengers are entering the transit line. Unlike the graphical representation proposed by Nguyen and Pallottino (1988), we replace the stop node with two other nodes—an alighting node s_a and a boarding node s_b—a transfer arc to connect them, a centroid that corresponds to the centroid in the upper-level network, one access arc, and one egress arc. The boarding (alighting) node is used to send (receive) passengers to (from) different transit lines and receive (send) passengers from (to) the centroid via the access (egress) link. The travel times of the access and egress links are equal to the walking times from the centroid to the transit stops, while the frequencies and capacities associated with access and egress links are very large (i.e., infinity). The transfer arc has a travel time of M, a very high frequency, and a very large capacity. Intuitively, M can be interpreted as the inconvenience cost (expressed as time-equivalent) or transfer penalty generated by a transfer, and can be calibrated from survey data. When a direct service is always preferred to a transfer service, M is set to be a large number.

There is no alighting arc, alighting node, or egress arc for a starting terminal and no boarding arc, boarding node, or access arc for an ending terminal. The consecutive bus stops of a transit line are connected by a travel arc, in which the travel time is set to be equal to the in-vehicle travel time plus the stop time at the next stop, and the stop time at each terminal is set to zero. The frequency of a travel arc is set to the frequency of the transit service, whereas its capacity is the frequency of that arc multiplied by the bus capacity. All of the general transit stops and terminals are connected through travel arcs.

Because the demand of each origin–destination (OD) pair is fixed and the flow on each link cannot be greater than that link's capacity, the total demand between an OD pair may be larger than the available capacity provided by all transit lines serving this OD pair. Hence, the lower-level formulation may not provide a feasible solution. To address this issue, a virtual link (corresponding to a walking path) with a very large capacity, a very high frequency, and a very long trip time is created to connect each OD pair. The flow on each virtual link at optimality is then equal to the unserved demand of the corresponding OD pair. In the extreme case, when the capacity between an OD pair is zero, there is still a feasible and optimal solution for that OD pair.

Based on this network representation, the transit assignment formulation proposed by Spiess and Florian (1989) can be extended to capture transfer penalty and in-vehicle congestion as follows.

$$\min_{\mathbf{v},\mathbf{w}} : z_2 = \sum_{a\in A}\sum_{e\in G_d} c_a v_a^e + \sum_{i\in Z_L}\sum_{e\in G_d} \omega_i^e, \tag{17}$$

subject to

$$v_a^e \leqslant f_a \omega_i^e \quad \text{for} \quad a \in A_i^+,\ i \in Z_L,\ e \in G_d, \tag{18}$$

$$\sum_{a\in A_i^+} v_a^e = \sum_{a\in A_i^-} v_a^e + d_i^e \quad \text{for} \quad i \in Z_L,\ e \in G_d, \tag{19}$$

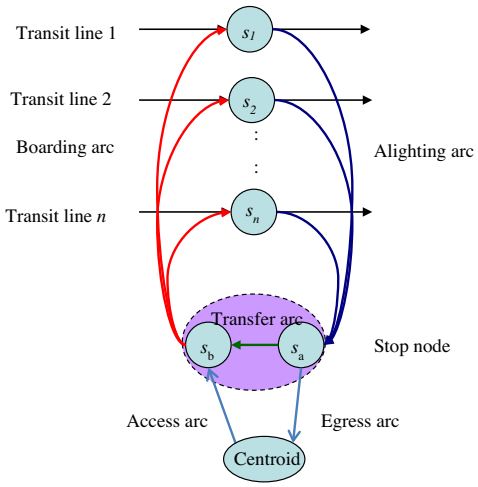


Fig. 1. A graph representation of a centroid connecting one general transit stop.

$$\sum_{e\in G_d} v_a^e \leqslant f_a k_{cap} \quad \text{for} \quad a \in A, \tag{20}$$

$$V_a^e \geqslant 0 \quad \text{for} \quad a \in A, e \in G_d, \text{ and} \tag{21}$$

$$\omega_i^e \geqslant 0 \quad \text{for} \quad i \in Z_L, \ e \in G_d. \tag{22}$$

The lower-level objective (17) is to minimize the sum of the total in-vehicle travel and stop times (i.e., the first term of the objective function) and total waiting time (i.e., the second term of the objective function). Constraint (18), which relates link flow, frequency, and waiting time, is a relaxed constraint of distributing node flows into the arcs emanating from that node. Constraint (19) is the flow conservation condition for a node. Constraint (20) ensures that the flow on each travel arc is not greater than that arc's capacity, with the capacity constraint used to model the in-vehicle congestion cost and extra delay due to passenger overloading. Constraints (21) and (22) are non-negativity conditions.

In the lower-level problem, the following points related to the proposed capacity constraint must be clarified. First, the capacity constraint must be incorporated into the lower-rather than the upper-level problem. The capacity constraint is used to model that, due to limited vehicle capacity, some passengers may not be able to board the first bus that arrives at a bus stop, and hence may experience extra delays. The capacity constraint is placed in the lower-level problem to ensure that such delays are considered in passengers' route choices. The extra delay of a passenger on a link is equal to the Lagrange multiplier associated with the capacity constraint, which appears in the equilibrium condition derived from the Karush–Kuhn–Tucker conditions of the lower-level problem. If the capacity constraint were placed in the upper-level, it would be assumed that passengers would not consider the delay in determining their route choice because the lower-level problem would be identical to the transit assignment problem proposed by Spiess and Florian (1989). This behavioral assumption is unrealistic.

The second point is related to the flow distribution. If the lower-level capacity constraint is not binding, the formulation reduces to the original strategy formulation (Spiess and Florian, 1989), where the resultant line flow is proportional to the line frequency, strictly following the assumptions that passengers arrive randomly, headway is exponentially distributed, and passengers select the first bus from a set of attractive lines that arrives at the bus stop. If the capacity constraint is binding, the flow distribution may not satisfy those assumptions because the passengers cannot board the first bus that arrives at the bus stop if it is full. In such cases, the results are approximations that are acceptable for strategic planning purposes.

The third point is related to the in-vehicle congestion cost. In the proposed model, passengers only perceive congestion costs if the capacity constraint is binding or buses are fully occupied. Otherwise, the congestion cost is neglected. However, in reality, passengers may still perceive in-vehicle congestion costs, such as the cost due to insufficient seat capacity or in-vehicle crowding, even when the capacity has not been reached. The more passengers there are inside a bus, the higher the in-vehicle congestion cost. Hence, the congestion cost should be a continuous and increasing function. In the proposed capacity constraint method, the in-vehicle congestion cost is a piecewise function, which can be addressed by developing a continuous, non-linear, and increasing in-vehicle cost function that can be linearized to reduce the problem to a linear programming problem. However, deriving this function has been left for future study.

The proposed formulation has three advantages. First, the lower-level problem is a linear programming problem and can be solved efficiently by existing algorithms. Second, it is easy for us to identify whether a particular route section is overloaded (by checking whether the corresponding Lagrange multiplier is positive) and whether the overall transit supply is sufficient (by checking virtual links carry flow)—all of which makes it easier to design appropriate improvement strategies. Third, this linear problem allows us to develop an efficient method for solving the transit network design problem.

3. Solution method

Constraint (9) is non-linear, and the decision variables are both discrete and continuous. Hence, the bi-level problem is a mixed-integer non-linear problem. It has been noted that a general network design problem is already NP-hard (Magnanti and Wong, 1984), and it is well-known that the transit route design problem is NP-hard (Zhao and Gan, 2003; Fan and Machemehl, 2004; Fan and Mumford, 2010). Our proposed problem includes the frequency setting problem and a lower-level problem that is more complicated than the general network design and the transit routing problems. Thus, our problem is also NP-hard. Given the extreme difficulty of solving NP-hard problems for exact solutions, a hybrid artificial bee colony (ABC) algorithm is proposed to solve the bi-level problem. The hybrid relies on the original ABC algorithm to solve the route design problem and incorporates a proposed iterative procedure to determine the number of passenger transfers and the optimal frequency setting. In the iterative procedure, the linear transit assignment problems (17)–(22) are solved in each iteration via the Simplex method. Then, a descent direction is obtained using the dual solutions to the transit assignment problem and used to formulate a linear integer program, which is solved to give a step size to update the frequency for the next iteration. To alleviate the computational burden of solving many transit assignment problems, a screening method based on the lower bound of the upper-level objective function is also developed. Only potentially good route design solutions are required to find the corresponding optimal frequency. However, the other solutions are kept for a neighborhood search.

3.1. Artificial bee colony (ABC) algorithm

The ABC algorithm belongs to a class of evolutionary algorithms inspired by the intelligent behavior of honey bees finding nectar sources around the hive. This class of metaheuristics has received increasing attention recently, with variations of bee algorithms proposed to solve combinatorial problems. However, in all of them, a common search strategy is applied; that is, complete or partial solutions are considered as food sources and different groups of bees try to exploit the solution space in the hope of finding good quality nectar, or high quality solutions, for the hive. They then communicate directly to inform other bees about the search space and the food sources.

In the ABC algorithm, the colony of bees is divided into employed bees, onlookers, and scouts. Employed bees are responsible for exploiting available food sources (solutions) and gathering required information. These bees also share information with onlookers, and each onlooker selects a food source near the food source chosen by one employed bee. When the source is abandoned, the employed bee becomes a scout and starts to search for a new source in the vicinity of the hive. This abandonment happens when the quality of the food source does not improve for a predetermined number of iterations.

The ABC algorithm is iterative, and starts by associating all employed bees with randomly generated food sources (solutions). In every iteration, each employed bee selects a food source in the neighborhood of the currently associated food source using a neighborhood operator, and evaluates its nectar amount (fitness) afterwards. If its nectar amount is better than that of the currently associated food source, then the employed bee keeps the new food source and discards the old one; otherwise, the employed bee retains the old food source. When all of the employed bees have finished this process, they share the nectar information for the food sources with the onlookers. Each of the onlookers then selects a food source according to a probability proportional to the nectar amount of that food source. In this study, we use the traditional roulette wheel selection method (Haupt and Haupt, 2004). Clearly, with this scheme, good food sources attract more onlookers than bad ones. After all of the onlookers have chosen their food sources, each of them selects a food source in the neighborhood of their chosen food sources (through neighborhood operators) and computes its fitness. The best food source among the particular food source of an employed bee and its neighboring food sources is the food source of the employed bee. If a solution represented by a particular food source does not improve for a predetermined number of iterations, then the food source is abandoned by its associated employed bee and the bee becomes a scout. The scout then searches randomly for a new food source. This is done by assigning a randomly generated food source (solution) to this scout. After each new food source is determined, another iteration of the ABC algorithm begins. The whole process is repeated until the termination condition is satisfied.

3.2. Overview of the hybrid artificial bee colony (ABC) algorithm

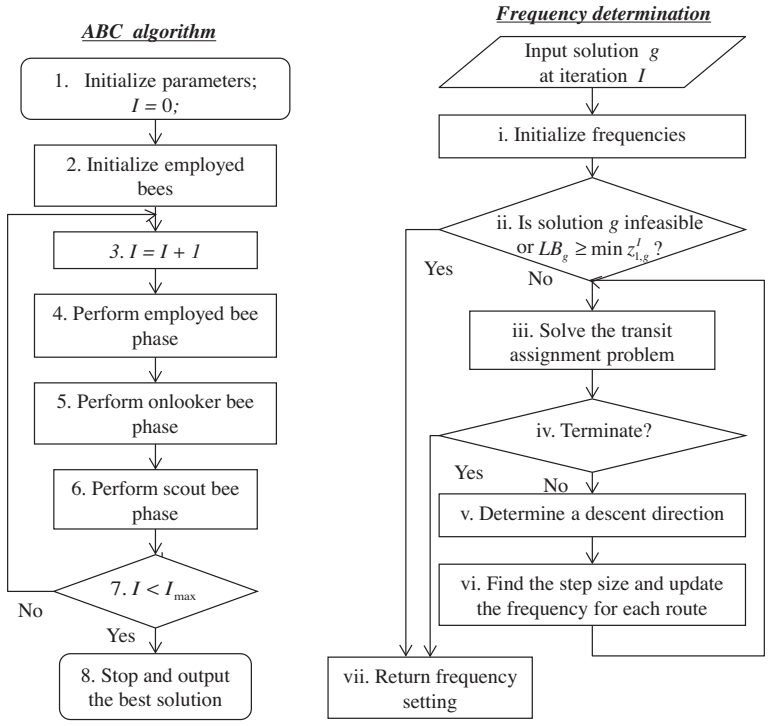
The existing ABC algorithm cannot be used directly to solve our problem because our problem is bi-level and has many constraints. Hence, a hybrid ABC algorithm is developed. The flow chart of the hybrid ABC algorithm is given in Fig. 2, which depicts the main algorithm (ABC algorithm) and the sub-algorithm (the proposed frequency determination algorithm).

3.2.1. ABC algorithm

The steps of the ABC algorithm can be described as follows.

1. Initialize the parameters, including the colony size N_c, the number of employed bees N_e, the number of onlookers N_o, the number of scouts N_s, and the predetermined number of iterations *limit*; set I, which is the counter of iterations, to be equal to zero; set the maximum number of iterations, $I_{max} = 500$.
2. Perform the initialization phase of employed bees: Generate an initial solution for each employed bee and set the limit counter for each solution to be zero.
3. Increase the number of iterations by 1, i.e., $I = I + 1$.
4. Perform the employed bee phase: Conduct a neighborhood search for each solution found by an employed bee. Evaluate the fitness of each neighbor solution. Replace the solution by its neighbor solution found by the search and set its limit counter to zero, if the latter is better. Otherwise, keep the solution of the employed bee, and increase the limit counter by 1.
5. Perform the onlooker bee phase: Perform the roulette wheel selection to determine which solution obtained by an employed bee is selected by an onlooker. Then, conduct a neighborhood search for each solution selected by an onlooker. Evaluate the fitness of each neighbor solution. Replace the solution by its neighbor solution, if the latter is better. Otherwise, keep the solution of the employed bee, and increase its limit counter by 1.
6. Perform the scout bee phase: Replace each solution that fails to improve within *limit* successive iterations by a new solution generated randomly.
7. Check the stopping criterion: If $I < I_{max}$, return to step 3.
8. Terminate and output the best solution.

For the ABC algorithm in our proposed method, the initialization phase generates a population of initial solutions by the employed bees. Afterwards, each employed bee is associated with one randomly generated solution.



(a) ABC algorithm (b) Frequency determination algorithm

Fig. 2. Flow chart of the hybrid ABC algorithm.

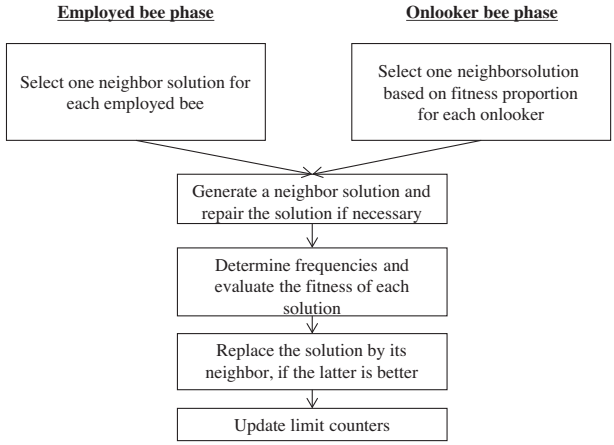


Fig. 3. Flow chart of the employed bee and onlooker bee phases.

The employed and onlooker bee phases are quite similar, as shown in Fig. 3. The only difference lies in the rule for selecting a candidate food source for a neighborhood search. In the employed bee phase, each employed bee selects its associated solution for a neighborhood search. In the onlooker bee phase, each onlooker selects a solution based on the fitness value. Hence, we expect promising solution areas to be visited and explored more frequently. Both phases require the frequency determination algorithm to determine the frequency associated with each route and evaluate the fitness value of each solution. They both conduct a greedy selection after evaluating the fitness of the neighbor solution. If the neighbor solution is better than the food source, the latter is replaced by the former and its limit counter is set to zero. Otherwise, the current solution is maintained and the limit counter is increased by 1. Finally, if all of the employed bees or onlookers complete their neighborhood searches, then the employed or onlooker bee phase is terminated.

In the scout bee phase, all of the food sources are scanned and the source that fails to improve within *limit* successive iterations is abandoned and replaced by a newly generated random solution.

3.2.2. Frequency determination algorithm

For each solution (i.e., route structure) obtained in the employed or onlooker bee phase of the ABC algorithm, the following procedure is used to determine the frequency setting:

i. Generate the initial frequencies.
ii. Decide whether to obtain an optimal frequency for each route: If LB_g (the lower bound of given solution g) is greater than $\min z^l_{1,g}$ (the minimum upper-level objective value until iteration I) or the route design is infeasible, then return the initial frequency of each route. Otherwise, proceed to the next step.
iii. Solve the lower-level transit assignment problem.
iv. If the termination criterion of the frequency determination algorithm is satisfied, then stop and return the optimal frequency setting. Otherwise, go to the next step.
v. Determine the descent direction of the lower- and upper-level objective values with respect to the frequency.
vi. Find the step size of the frequency by solving a linear integer program and update the frequency with the obtained step size, then go to step iii.

3.3. Solution generation and repairing procedures

3.3.1. Solution representation in the ABC algorithm

To search all of the possible route structures, the solution representation in the ABC algorithm should be specifically designed. Fig. 4 illustrates the representation scheme used in the ABC algorithm. One solution consists of 100 elements representing 10 routes, with 10 elements for each route. For example, the first 10 elements represent the first bus route, which starts at node 1, goes through nodes 18, 15, 10, 12, and 7, and terminates at node 25. Similarly, route 10 starts at node 16, goes through node 11, and terminates at node 27.

3.3.2. Initialization procedures

In the ABC algorithm, new solutions are generated in the initialization and scout bee phases, both of which adopt the same procedures to generate a random solution, as shown in Fig. 5.

To initialize the route elements, the following procedures are carried out sequentially. For route r, the first node is determined by randomly selecting from the available starting terminals in the study area. Then, the last node is picked from all of the available ending terminals e', the number of intermediate stops is generated, and a corresponding number of nodes is inserted between the two terminals. The probability of selecting an intermediate stop node i is determined based on passenger demand by $p_i = d_i^{e'} \Big/ \sum_{j \in Z_U} d_j^{e'}$, where p_i represents the probability of choosing node i. If there is more than one stop in a zone, stop i in that zone is randomly picked. The last step is to set the rest of the elements, if any, to zero.

3.3.3. Repairing procedures

The solution generated makes it difficult to avoid infeasibility due to the proposed random operations. Although we can add a penalty to the fitness value of an infeasible solution and leave the algorithm itself to evolve, according to our preliminary experiments, the solution quality in terms of the number of feasible solutions and the objective value in the final iteration is lower than that obtained by the algorithm with the proposed route repairing procedures. Therefore, we propose the route repairing procedures to provide better (initial) solutions. The procedures include checking zone covering, stop sequence optimization, and deleting and inserting intermediate stops.

3.3.3.1. Checking zone covering. The zone covering procedure is designed to ensure that every demand zone is visited by at least one route. Because the total number of elements in a solution (which equals the maximum number of stops multiplied by the maximum number of routes) is greater than the number of zones in the network, there must be some zones that are visited by more than one route. However, there is no guarantee that all zones are served or covered in the initialization procedure. If centroid m is not served, then node i, which is one of the candidate stops connecting to centroid m, is inserted into the selected route with the number of stops less than the maximum allowable number of stops and the least travel time increment after inserting node i. If no route can serve this centroid due to constraint (11) on the maximum number of stops,

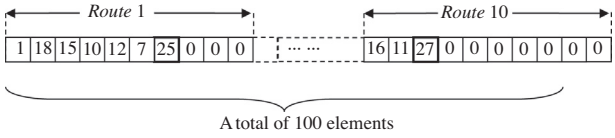


Fig. 4. Solution representation scheme.

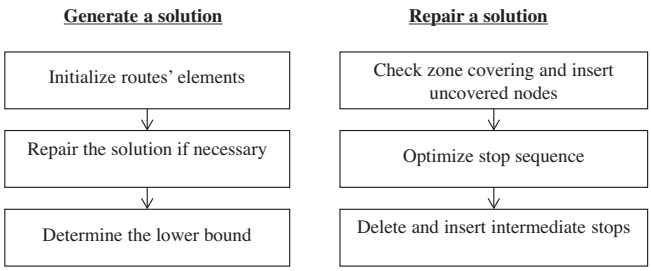


Fig. 5. Solution generation and repairing procedures.

a zone served by at least two transit lines is randomly selected and one of the stops in the zone passing by the lines is replaced by node i. These two steps guarantee that the zone-covering constraint (13) is satisfied.

3.3.3.2. Stop sequence optimization. For every generated ABC solution, a descent search heuristic is used to improve the sequence of stops on each route. The purpose of this sequence-improving process is to minimize the trip time of each route, as it does not depend on frequencies and is relatively easy to implement. The outline of the heuristic is as follows:

```
For each route in the ABC solution
Set i' = 1
  While i' ⩽ the number of intermediate stops − 1
    j' = i' + 1
    While j' ⩽ the number of intermediate stops, do
        Exchange the i'th and j'th stops
        Evaluate the trip time of the route
        If the trip time is reduced,
          then set j' = number of intermediate stops + 1, i' = 0
        else
          undo the exchange and j' = j' + 1
        endif
      endwhile
    i' = i' + 1
  endwhile
Next route in the ABC solution
```

3.3.3.3. Deleting and inserting intermediate stops. The stop sequence optimization procedure essentially rearranges the sequence of intermediate stops to form the shortest path. Nevertheless, some routes may still violate the maximum trip time constraint (12). Therefore, a stop-removal operation is conducted to eliminate nodes while ensuring the solution to satisfy the zone covering constraint. Various criteria can be used in selecting which nodes to delete, such as trip time reduction and the changes in total flow of the direct services involved after performing the node removal. Different criteria have different effects on the objective value and the algorithm performance.

We propose the following average-direct-demand $\psi_r^{ie'}$ to approximate the change in the upper-level objective value that results from removing node i from route r, which connects terminal e' directly:

$$\psi_r^{ie'} = \frac{d_i^{e'}\,\delta_r^{ie'}}{\sum_{p\neq r}\delta_p^{ie'}+1} \quad \text{for} \quad r=1 \text{ to } R_{\max},\ i\in Z_U,\ e'\in V,$$

where $\delta_p^{ie'}$ is a binary indicator variable that is equal to 1 if route p passes both nodes i and e'. $\sum_{p\neq r}\delta_p^{ie'}$ calculates the number of transit lines that provide a direct service between node i and terminal e' after removing node i from route r. Adding 1 to that number allows us to consider the case when route r heading to terminal e' originally passes node i. Hence, the denominator gives the number of transit lines that provide a direct service between node i and terminal e' before removing node i from route r. The demand $d_i^{e'}$ is obtained from the lower-level problem, and is the total flow on the boarding arcs ending at the transit stop corresponding node i in the upper-level network and heading to the transit stop corresponding to terminal

$e' \in H_e$. Overall, this average-direct-demand intends to capture the increase in the number of passenger transfers due to deleting node i. This average approximates the flow of each direct service and is determined by evenly splitting the demand between node i and terminal e' to all of the routes providing direct services for that pair of nodes. This value can be interpreted as the average increment in the number of transfer passengers when node i is deleted from route r. Therefore, a node with a smaller ratio is preferred for removal, because a smaller ratio indicates a lower average increment in the number of passengers who need to make a transfer.

To compensate for the negative effects of deleting nodes, including reducing service coverage and increasing the number of passengers who make a transfer, a reverse operation called node insertion is subsequently conducted to insert as many nodes as possible while ensuring that the resultant solution satisfies the maximum trip time constraint. The node chosen for insertion is also based on the proposed average-direct-demand, and a larger value is preferred.

3.4. Lower bound determination and fitness evaluation

Fitness is used to reflect a solution's quality and select candidate solutions for a neighborhood search. Although the reciprocal of the upper-level objective value can be used as a fitness measure for the proposed algorithm, it is cumbersome to calculate the upper-level objective for each solution because the corresponding optimal frequency must be found by the proposed frequency setting method, which involves solving the lower-level problem many times. Thus, a lower bound is calculated to determine the minimum number of passenger transfers for each solution, and then used to replace the upper-level objective value in the fitness function. Such a bound can be obtained much more quickly than the reciprocal of the upper-level objective value.

Given a route design solution, the lower bound provides the minimum number of passenger transfers, which is an optimistic estimation of the upper-level objective function value. The calculation of the lower bound is based on the assumption that each transit route has unlimited capacity. Under this condition, the passenger demand $d_i^{e'}$, $\forall i \in Z_U$, $e' \in V$, can be met without needing to make a transfer if there is any route connecting nodes i and e'. Because summing up all of the served demand provides the maximum total passenger demand without making a transfer, the minimum number of passenger transfers, or the lower bound, can be obtained by calculating the difference between the total passenger demand $\sum_{i\in Z_U\setminus V}\sum_{e'\in V} d_i^{e'}$ and the sum of all passengers not making a transfer under the assumption stated above, $\sum_{i\in Z_U\setminus V}\sum_{e'\in V}(1-NR_i^{e'})d_i^{e'}$, where $NR_i^{e'}$ equals 1 if there is no route connecting i and e', and 0 otherwise.

If the subscript g is used to denote a route design solution, then the lower bound of solution g, LB_g, can be mathematically expressed as

$$LB_g = \sum_{i\in Z_U\setminus V}\sum_{e'\in V} d_i^{e'} - \sum_{i\in Z_U\setminus V}\sum_{e'\in V}\left(1-NR_i^{e'}\right)d_i^{e'}, \tag{23}$$

where

$$NR_i^{e'} = \prod_{r=1}^{R_{max}}(1-RT_{ie'r}) \quad \text{for} \quad i \in Z_U \setminus V,\ e' \in V, \tag{24}$$

$$RT_{iwr} = X_{iwr} + \sum_{j\in Z_U\setminus V, j\neq i, j\neq w} X_{ijr}RT_{jwr} \quad \text{for} \quad i \in Z_U \setminus V,\ w \in Z_U \setminus V,\ w\neq i,\ r = 1 \text{ to } R_{max}, \tag{25}$$

RT_{ijr} = 1 if route r passes through node i and node j, and 0 otherwise.

With the lower bound of route design solution g, LB_g, the fitness of solution g, F_g, is calculated via

$$F_g = \frac{1}{LB_g + P_g}. \tag{26}$$

P_g is a penalty term for solution g and is given by

$$P_g = \alpha\sum_{r=1}^{R_{max}}\max(T_{r,g} - T_{max}, 0) + \beta\max\left(\sum_{r=1}^{R_{max}} V_{r,g} - W, 0\right), \tag{27}$$

where $V_{r,g}$ is the fleet size for route r in solution g; $T_{r,g}$ is the trip time on route r in solution g, and α and β are the penalty parameters related to the maximum trip time constraint and the fleet size constraint, respectively. The penalty method deals with infeasible solutions that cannot be repaired by any of the repairing operators. The first term in (27) penalizes the violation of constraint (12) while the second term penalizes the violation of constraint (9). Infeasible solutions are kept for neighborhood searches because it is possible for the global minima to be located close to infeasible solutions. Nevertheless, by varying the penalty parameter values, it is easy to adjust the probability of searching an infeasible solution region. When the penalty value is large, the probability is small and vice versa.

3.5. Frequency determination procedure

The following subsections describe the details of each step in the frequency setting procedure depicted in Fig. 2.

3.5.1. Step i: frequency initialization

This step is conducted to obtain the initial frequency for each route. The initial frequency for route r is calculated from

$$f_{r,g} = \frac{V_{r,g}}{2T_{r,g}}, \tag{28}$$

where $f_{r,g}$ is the frequency of route r in solution g. Given trip time $T_{r,g}$, the total fleet should be allocated carefully to meet the minimum frequency and demand conservation requirements. Thus, we propose the following procedure for allocating buses to determine the initial frequency of each route.

1. *Assign buses according to minimum frequency constraint* (10). Given the trip time on a route, the minimum number of buses required to meet the minimum frequency constraint can be determined by Eq. (28).
2. *Assign buses according to demand requirement constraint* (14). This procedure ensures that the service capacity provided for each destination e is not less than the demand ending at destination e, under the assumption that the total service capacity for the study area is not less than the total demand. In the beginning, all of the transit lines with the same ending terminals are grouped, and then two frequency values are calculated and compared for each group. One is the assigned group frequency, which is the sum of the frequencies obtained in step 1. The other is the required frequency, which is the minimum frequency required to meet the total demand for each destination group calculated by $\sum_{m\in G_s} d_m^e / k_{\text{cap}},\ \forall e \in G_d$. For each group, if the required frequency is larger than the assigned frequency, then the group frequency is insufficient and the difference between the required and assigned frequencies is added to the frequency of the route with the least trip time. The route with the least trip time is chosen because when one more bus is assigned to that route, it produces the highest increase in the frequency and the line capacity compared with other routes.
3. *Round up the fleet size and recalculate frequencies*. After the foregoing two steps, the number of buses allocated to each route is calculated and rounded up to the nearest integer. The frequency of each route is then recalculated, which usually ends with a slightly higher value than the previous result.

This procedure handles the frequency-related constraints by determining the fleet size of each route. Afterwards, if the sum of the fleet size of each route is less than or equal to the total fleet size defined by constraint (9), then the route structure simultaneously satisfies constraints (9), (10), and (14); otherwise, the route structure is infeasible and the fitness of the infeasible solution is penalized. If there are residual buses that have not been allocated to any route, they are added to the route with the least trip time because at global optimality all buses must be used.

3.5.2. Step ii: lower bound screening

After step i, we can identify whether a route structure is feasible. For infeasible solutions, it is not necessary to search for optimal frequency. For feasible solutions, only potentially good solutions proceed to obtain optimal frequency. Candidate solutions are identified by comparing the lower bound with the current best objective value. If the lower bound of a new solution is larger than the current best objective value found by the hybrid ABC algorithm, then it is impossible to determine the upper-level objective value of the new solution that is smaller than the current best objective value by adjusting its frequency and not changing the route design. In this case, the route solution cannot be globally optimal and it is redundant to carry out the frequency setting procedure. However, if the lower bound is less than the current best objective value, then obtaining a better objective value by searching optimal frequency settings is possible, and hence the solution is potentially good.

3.5.3. Step iii: solving the transit assignment problem

With an initial frequency and a feasible route structure, the lower-level transit assignment problem is solved by the Simplex method. Afterwards, both the primal and dual solutions are recorded. The primal solution indicates the number of transfer passengers and the dual solution is used, if necessary, to determine the descent direction and step size for updating frequency in later steps.

3.5.4. Step iv: termination criteria checking

The following stopping criteria are used:

Criterion (1) $z_{1,g}^k - LB_g \leqslant \varepsilon_1$ and
Criterion (2) $z_{1,g}^{k+1} - z_{1,g}^k \leqslant \varepsilon_2$,

where ε_1 and ε_2 are predefined maximum acceptable errors and $z_{1,g}^k$ is the upper-level objective value of solution g after the kth iteration. Both criteria are derived based on the definition of the lower bound, which states that the upper-level objective value $z_{1,g}^k$ cannot be reduced to a value that is smaller than the lower bound for route structure g.

Criterion (1) is used as a stopping criterion when the frequency is optimal or nearly optimal. If $z_{1,g}^{k}$ and LB_g are equal, then the frequency is optimal. If the difference is small, then the frequency is probably optimal, and is at least nearly optimal.

Criterion (2) is used when two successive objective values are close enough, which implies that the two successive solutions are probably close enough, and the latest solution is probably optimal.

3.5.5. Step v: determination of the descent direction

In this step, the descent direction of the upper-level problem with respect to frequency is determined. This descent direction is also the descent direction for the lower-level problem under the condition that the penalty parameter for transfers (i.e., M) is large enough. Hence, we can rely on the descent direction of the lower-level problem, which is derived as follows.

For the ease of presentation, we rewrite the lower-level formulation as a function of the frequencies $\mathbf{f}$ in the following vector form and omit the solution subscript g.

$$\min_{\mathbf{v},\mathbf{w}} : z_2 = h_1(\mathbf{v}(\mathbf{f})) + h_2(\mathbf{w}(\mathbf{f})), \tag{29}$$

Subject to:

$$\mathbf{g}_1(\mathbf{v}(\mathbf{f})) - \mathbf{k}(\mathbf{f}) \cdot \mathbf{m}(\mathbf{w}(\mathbf{f})) \leqslant \mathbf{0}, \tag{30}$$

$$\mathbf{g}_2(\mathbf{v}(\mathbf{f})) - \mathbf{d} = \mathbf{0}, \tag{31}$$

$$\mathbf{g}_3(\mathbf{v}(\mathbf{f})) - \mathbf{c}(\mathbf{f}) \leqslant \mathbf{0}, \tag{32}$$

$$\mathbf{v}(\mathbf{f}) \geqslant \mathbf{0}, \tag{33}$$

$$\mathbf{w}(\mathbf{f}) \geqslant \mathbf{0}, \tag{34}$$

where $\mathbf{v}(\mathbf{f})$ and $\mathbf{w}(\mathbf{f})$ represent the vectors $[v_a^e]$ and $[\omega_i^e]$, respectively, which are functions of frequencies. $h_1(\mathbf{v}(\mathbf{f})) = \sum_a \sum_e c_a v_a^e$ and $h_2(\mathbf{w}(\mathbf{f})) = \sum_i \sum_e \omega_i^e$. For constraints (30)–(32), $\mathbf{g}_1(\mathbf{v}(\mathbf{f})) = [v_a^e]$; $\mathbf{k}(\mathbf{f}) = [f_a]$; $\mathbf{m}(\mathbf{w}(\mathbf{f})) = [\omega_i^e]$; $\mathbf{g}_2(\mathbf{v}(\mathbf{f})) = \left[\sum_{a\in A_i^+} v_a^e - \sum_{a\in A_i^-} v_a^e\right]$; $\mathbf{d} = [d_i^e]$; $\mathbf{g}_3(\mathbf{v}(\mathbf{f})) = \left[\sum_e v_a^e\right]$ and $\mathbf{c}(\mathbf{f}) = [f_a k_{\text{cap}}]$. The dimensions of these matrices are not fixed, but vary with the solutions of the upper-level problem.

The descent direction is derived based on the necessary Karush–Kuhn–Tucker (KKT) conditions. At global optimality, the following conditions hold:

$$\begin{pmatrix} \nabla_{\mathbf{v}} h_1(\mathbf{v}^*(\mathbf{f})) \\ \nabla_{\mathbf{w}} h_2(\mathbf{w}^*(\mathbf{f})) \end{pmatrix} + \boldsymbol{\pi}^{\mathbf{T}} \begin{pmatrix} \nabla_{\mathbf{v}} \mathbf{g}_1(\mathbf{v}^*(\mathbf{f})) \\ (-\mathbf{k}(\mathbf{f})) \cdot \nabla_{\mathbf{w}} \mathbf{m}(\mathbf{w}^*(\mathbf{f})) \end{pmatrix} + \boldsymbol{\varphi}^{\mathbf{T}} \begin{pmatrix} \nabla_{\mathbf{v}} \mathbf{g}_2(\mathbf{v}^*(\mathbf{f})) \\ \mathbf{0} \end{pmatrix} + \boldsymbol{\mu}^{\mathbf{T}} \begin{pmatrix} \nabla_{\mathbf{v}} \mathbf{g}_3(\mathbf{v}^*(\mathbf{f})) \\ \mathbf{0} \end{pmatrix} = \mathbf{0}, \tag{35}$$

$$\begin{pmatrix} \boldsymbol{\pi} \\ \boldsymbol{\mu} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{g}_1(\mathbf{v}^*(\mathbf{f})) - \mathbf{k}(\mathbf{f}) \cdot \mathbf{m}(\mathbf{w}^*(\mathbf{f})) \\ \mathbf{g}_3(\mathbf{v}^*(\mathbf{f})) - \mathbf{c}(\mathbf{f}) \end{pmatrix} = \mathbf{0}, \tag{36}$$

$$\begin{pmatrix} \boldsymbol{\pi} \\ \boldsymbol{\mu} \end{pmatrix} \geqslant \mathbf{0}, \tag{37}$$

$$\begin{pmatrix} \mathbf{v}^*(\mathbf{f}) \\ \mathbf{w}^*(\mathbf{f}) \end{pmatrix} \geqslant \mathbf{0}, \tag{38}$$

where $\mathbf{v}^*(\mathbf{f})$ and $\mathbf{w}^*(\mathbf{f})$ stand for the optimal solutions of the lower-level problem and $\boldsymbol{\pi} = [\pi_{ia}^e]$, $\boldsymbol{\varphi} = [\varphi_i^e]$, and $\boldsymbol{\mu} = [\mu_a]$ are, respectively, the optimal multipliers for Eqs. (30)–(32). The sufficient conditions of global optimality at $(\mathbf{v}^*(\mathbf{f}), \mathbf{w}^*(\mathbf{f}))$ are also satisfied because the lower-level problem is a linear programming problem with a convex solution set (i.e., a convex problem).

To obtain the descent direction of the objective function, we form the Lagrange function L, differentiate the Lagrange function with respect to $\mathbf{f}$, and substitute $(\mathbf{v}^*(\mathbf{f}), \mathbf{w}^*(\mathbf{f}), \boldsymbol{\pi}, \boldsymbol{\varphi}, \boldsymbol{\mu})$ to the derivative to get

$$\begin{aligned} \nabla_{\mathbf{f}} L = {} & \frac{\partial \mathbf{v}^*}{\partial \mathbf{f}} \cdot \nabla_{\mathbf{v}} \mathbf{h}_1(\mathbf{v}^*(\mathbf{f})) + \frac{\partial \mathbf{w}^*}{\partial \mathbf{f}} \cdot \nabla_{\mathbf{w}} \mathbf{h}_2(\mathbf{w}^*(\mathbf{f})) \\ & + \boldsymbol{\pi}^{\mathbf{T}} \left(\frac{\partial \mathbf{v}^*}{\partial \mathbf{f}} \cdot \nabla_{\mathbf{v}} \mathbf{g}_1(\mathbf{v}^*(\mathbf{f})) - \frac{\partial \mathbf{k}(\mathbf{f})}{\partial \mathbf{f}} \cdot \mathbf{m}(\mathbf{w}^*(\mathbf{f})) - \frac{\partial \mathbf{w}^*}{\partial \mathbf{f}} (-\mathbf{k}(\mathbf{f})) \cdot \nabla_{\mathbf{w}} \mathbf{m}(\mathbf{w}^*(\mathbf{f})) \right) + \boldsymbol{\varphi}^{\mathbf{T}} \cdot \frac{\partial \mathbf{v}^*}{\partial \mathbf{f}} \cdot \nabla_{\mathbf{v}} \mathbf{g}_2(\mathbf{v}^*(\mathbf{f})) \\ & + \boldsymbol{\mu}^{\mathbf{T}} \left(\frac{\partial \mathbf{v}^*}{\partial \mathbf{f}} \cdot \nabla_{\mathbf{v}} \mathbf{g}_3(\mathbf{v}^*(\mathbf{f})) - \frac{\partial \mathbf{c}(\mathbf{f})}{\partial \mathbf{f}} \right). \end{aligned} \tag{39}$$

Rearranging equation (39), we have

$$\begin{aligned} \nabla_{\mathbf{f}} L = {} & \frac{\partial \mathbf{v}^*}{\partial \mathbf{f}} \left\{ \nabla_{\mathbf{v}} \mathbf{h}_1(\mathbf{v}^*(\mathbf{f})) + \boldsymbol{\pi}^{\mathbf{T}} \nabla_{\mathbf{v}} \mathbf{g}_1(\mathbf{v}^*(\mathbf{f})) + \boldsymbol{\varphi}^{\mathbf{T}} \nabla_{\mathbf{v}} \mathbf{g}_2(\mathbf{v}^*(\mathbf{f})) + \boldsymbol{\mu}^{\mathbf{T}} \nabla_{\mathbf{v}} \mathbf{g}_3(\mathbf{v}^*(\mathbf{f})) \right\} \\ & + \frac{\partial \mathbf{w}^*}{\partial \mathbf{f}} \left\{ \nabla_{\mathbf{w}} \mathbf{h}_2(\mathbf{w}^*(\mathbf{f})) + \boldsymbol{\pi}^{\mathbf{T}} \cdot (-\mathbf{k}(\mathbf{f})) \cdot \nabla_{\mathbf{w}} \mathbf{m}(\mathbf{w}^*(\mathbf{f})) \right\} - \boldsymbol{\pi}^{\mathbf{T}} \frac{\partial \mathbf{k}(\mathbf{f})}{\partial \mathbf{f}} \cdot \mathbf{m}(\mathbf{w}^*(\mathbf{f})) - \boldsymbol{\mu}^{\mathbf{T}} \frac{\partial \mathbf{c}(\mathbf{f})}{\partial \mathbf{f}}. \end{aligned} \tag{40}$$

Substituting equation (35) into (40), we obtain

$$\nabla_{\mathbf{f}} L = -\boldsymbol{\pi}^{\mathrm{T}} \frac{\partial \mathbf{k}(\mathbf{f})}{\partial \mathbf{f}} \cdot \mathbf{m}(\mathbf{w}^*(\mathbf{f})) - \boldsymbol{\mu}^{\mathrm{T}} \frac{\partial \mathbf{c}(\mathbf{f})}{\partial \mathbf{f}}. \tag{41}$$

Eq. (41) provides the steepest ascent direction of the Lagrange function at the current solution $(\mathbf{v}^*(\mathbf{f}), \mathbf{w}^*(\mathbf{f}), \boldsymbol{\pi}, \boldsymbol{\varphi}, \boldsymbol{\mu})$. Accordingly, $-\nabla_{\mathbf{f}} L$ is the steepest descent direction at that point. Because $\nabla_{\mathbf{f}} L = \nabla_{\mathbf{f}} z_2$ at $(\mathbf{v}^*(\mathbf{f}), \mathbf{w}^*(\mathbf{f}), \boldsymbol{\pi}, \boldsymbol{\varphi}, \boldsymbol{\mu})$, $-\nabla_{\mathbf{f}} L = -\nabla_{\mathbf{f}} z_2$ at that point. This implies that $-\nabla_{\mathbf{f}} L$ provides a descent direction of the lower-level objective function with respect to $\mathbf{f}$ at $(\mathbf{v}^*(\mathbf{f}), \mathbf{w}^*(\mathbf{f}), \boldsymbol{\pi}, \boldsymbol{\varphi}, \boldsymbol{\mu})$.

For the proposed formulation, the lower-level objective function can be decomposed into two positive and linear terms, where the second term has the coefficient M. That is,

$$z_2 = \mathbf{C}^T \mathbf{x} + M \sum_{t \in T^R} \sum_{e \in G_d} v_t^e. \tag{42}$$

$\sum_{t \in T^R} \sum_{e \in G_d} v_t^e$ is the total flow on all the transfer links, and is identical to the objective function of the upper-level problem; $\mathbf{x}$ is the vector for the rest of the decision variables of the lower-level problem; and $\mathbf{C}$ is the vector of the coefficients of $\mathbf{x}$.

Proposition 1. *When M is greater than the largest element of $\mathbf{C}$, the gradient $-\nabla_{\mathbf{f}} L$ at the current frequency solution is also a descent direction of the upper-level objective function.*

Proof. Without loss of generality, each of the current frequencies is less than infinity. Moreover, the waiting time for each transit line can only tend to zero when the frequency tends to infinity. Therefore, one can always reduce the total waiting time and, hence, the objective value of the lower-level problem by increasing the frequency of at least one transit line. Therefore, each of the elements of $-\nabla_{\mathbf{f}} L$ is always negative for the current solution, meaning that the objective value of the lower-level problem for the current frequency solution can always be reduced by increasing the frequency of at least one transit line. This implies that we can always find a descent direction, including the steepest descent direction, for the current frequency solution.

Because we only consider descent directions, we do not need to consider the constraints for the lower-level problem. Without considering the constraints of the lower-level problem, the objective value of the lower-level problem can be reduced after the current solution moves slightly along the steepest descent direction. As M is greater than the largest element of $\mathbf{C}$, it is more efficient to reduce the value of the second term than that of the first term along the descent direction. Hence, the value of the second term must be reduced along this direction. Therefore, $-\nabla_{\mathbf{f}} L$ is a descent direction to the upper-level problem. This completes the proof. □

Proposition 1 implies that it is possible to reduce the value of the upper-level objective function by reducing the value of the objective function of the lower-level problem.

3.5.6. Step vi: Step size determination and frequency updating

In addition to the descent direction given by (41), a step size must be determined to update $\mathbf{f}$. Hence, we investigate the individual component of the gradient of the Lagrange function to determine a good mathematical property that simplifies the procedure for determining the step size. The gradient of the Lagrange function is

$$\frac{\partial L}{\partial f_r} = -\sum_i \sum_a \sum_e \pi_{ia}^e \cdot \frac{\partial f_a}{\partial f_r} \cdot \omega_i^{e^*} - \sum_a \mu_a \frac{\partial [f_a k_{\mathrm{cap}}]}{\partial f_r}, \tag{43}$$

where $\frac{\partial L}{\partial f_r}$ represents the gradient with respect to the frequency of line r. The first term on the right side is defined by the dual solution of the relaxed node-flow distribution constraint (30). The second term is defined by the dual solution of the capacity constraint (32). Note that if link a is a boarding arc of transit line r, then $\frac{\partial f_a}{\partial f_r} = 1$; otherwise $\frac{\partial f_a}{\partial f_r} = 0$. Similarly, $\frac{\partial [f_a k_{\mathrm{cap}}]}{\partial f_r} = k_{\mathrm{cap}}$, if link a is a travel arc of transit line r and $\frac{\partial [f_a k_{\mathrm{cap}}]}{\partial f_r} = 0$ otherwise. Hence, $\frac{\partial L}{\partial f_r}$ is a function that is only influenced by the frequency of transit line r. Such separable property permits us to adjust the frequency of each transit line or to determine the step size Δf_r for each line r, separately.

After obtaining the descent direction for each route, the following integer linear program is proposed to determine the step size:

$$\min_{\Delta \mathbf{v}} : z_3 = \sum_r \Delta f_r \cdot \frac{\partial L}{\partial f_r}, \tag{44}$$

subject to

$$\Delta f_r = \frac{\Delta V_r}{2T_r}, \quad \text{for} \quad r = 1 \text{ to } R_{\max}, \tag{45}$$

$$\Delta f_r + f_r \geqslant f_{\min}, \quad \text{for} \quad r = 1 \text{ to } R_{\max}, \tag{46}$$

$$\sum_r (\Delta f_r + f_r) \cdot k_{\text{cap}} \cdot \delta_r^e \geqslant \sum_{m \in G_s} d_m^e, \quad \forall e \in G_d, \tag{47}$$

$$\sum_r \Delta V_r = 0, \tag{48}$$

where $\Delta \mathbf{v} = [\Delta V_r]$ and ΔV_r is the change in the fleet size of route r. Δf_r is the step size of the frequency of route r. ΔV_r and Δf_r are related through equation (45), which is derived from (28). The objective of the integer program is to minimize the increase in the value of the Lagrange function by adjusting the fleet size of each route, which is equivalent to minimizing the objective value of the lower-level problem by determining the optimal fleet allocation. Constraints (46) and (47) are derived from the upper-level constraints (10) and (14). Constraint (48) represents the fleet size conservation constraint. Because the fleet size ΔV_r is an integer decision variable, the problem becomes a linear integer programming problem. Although it is possible to adopt $[\Delta f_r]$ as a vector of continuous decision variables instead of using $\Delta \mathbf{v}$ as a vector of integer decision variables, the problem of transforming optimal continuous solutions into optimal integral solutions is even more complex and non-trivial. Hence, we adopt $\Delta \mathbf{v}$ as a vector of decision variables.

Proposition 2. *z_3 is always a non-positive number at optimality.*

Proof. It is easy to observe that the solution at the origin, $\Delta \mathbf{v} = \mathbf{0}$, is feasible, as it must satisfy all of the constraints of the integer program. Moreover, when $\Delta \mathbf{v} = \mathbf{0}$, z_3 is equal to 0. As the integer programming problem is a minimization type, the objective value of an optimal solution must not be greater than that of any feasible solution, including $\Delta \mathbf{v} = \mathbf{0}$. Hence, z_3 is always non-positive at optimality. This completes the proof. □

The implication of Proposition 2 is that the optimal allocation determined by the integer program must reduce the value of the Lagrange function and hence the value of the upper objective function of the upper-level problem, if z_3 is negative at optimality.

After solving the integer program, the iterative procedure returns to step iii with the updated frequency obtained by first obtaining Δf_r^k from ΔV_r^k using (45) and

$$f_r^k = f_r^{k-1} + \Delta f_r^k \quad \text{for} \quad r = 1 \text{ to } R_{\max}. \tag{49}$$

Here, an additional superscript k is introduced, representing the kth iteration of the frequency found in the iterative frequency determination procedure.

3.5.7. Violation of assumptions

To use the descent direction information derived from (43), we assume: (i) the optimal basis remains optimal, and (ii) M is greater than the largest coefficient in $\mathbf{C}$.

The first assumption requires that each change in frequency is within a certain allowable range; otherwise swapping between basic and non-basic variables occurs. However, in the integer program, the exact allowable range is not used. Instead, we use the feasible region defined by (10) and (14) in the upper-level problem to approximate it. As a result, the approximation creates a problem when the feasible region of the upper-level problem does not lie within the allowable range. Consequently, an inappropriate step size is found and the new frequency falls out of the allowable range. The objective value may subsequently increase, such that it takes extra iterations to reduce the objective value of the linear integer program.

There are two methods for addressing this issue. First, an additional constraint is added to limit the maximum change for each of ΔV_r^k. However, if the constraint is too tight, it leads to more iterations to obtain an optimal allocation. Therefore, a balance decision should be carefully made. Trial and error testing is more likely to provide hints on where to set the maximum change for ΔV_r^k.

Second, we use the results of our sensitivity analysis in the linear programming to derive additional constraints, which ensure that the basis remains unchanged. Without loss of generality, we rewrite the lower-level problem in the following compact form:

$$\min z_2 = \mathbf{c} \cdot \mathbf{x}, \tag{50}$$

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{51}$$

$$\mathbf{x} \geqslant \mathbf{0}, \tag{52}$$

where $\mathbf{A}$ is a matrix, $\mathbf{b}$ and $\mathbf{c}$ are column vectors, and $\mathbf{x}$ is a vector of decision variables in which the elements involve all the elements of the auxiliary variables $\mathbf{v}$ and $\mathbf{w}$.

According to the fundamental results of the sensitivity analysis, at optimality, all coefficients in row 0 of the final tableau are non-positive (for the minimization problem) and all of the right sides are non-negative; that is, for the coefficients of the variables **v** and **w** in row 0 of the final tableau, we have:

$$\mathbf{c}_B\mathbf{B}^{-1}\mathbf{A} - \mathbf{c} \leqslant \mathbf{0}, \tag{53}$$

where $\mathbf{B}^{-1}$ is the inverse of **B** and **B** is the square matrix, which contains the columns from [**A**|**I**] that correspond to the set of basic variables (in order) and $\mathbf{c}_B$ is the vector of elements in **c** that corresponds to the basis variable. For the coefficients of auxiliary variables in row 0 of the final tableau, we have

$$\mathbf{c}_B\mathbf{B}^{-1} \leqslant \mathbf{0}. \tag{54}$$

For the right sides, we have

$$\mathbf{B}^{-1}\mathbf{b} \geqslant \mathbf{0}. \tag{55}$$

Given that only the coefficients in (18) and the right side in (20) involve line frequency, only **A** and **b** involve frequency in some of the elements, and thus we only need to consider conditions (53) and (55). After revising the elements involving line frequency by adding Δf_r to them, we have a revised matrix $\mathbf{A}'$ and a revised column vector $\mathbf{b}'$, which are linear functions of $[\Delta f_r]$. We then have the following two sets of additional linear constraints for the integer program:

$$\mathbf{c}_B\mathbf{B}^{-1}\mathbf{A}' - \mathbf{c} \leqslant \mathbf{0} \text{ and} \tag{56}$$

$$\mathbf{B}^{-1}\mathbf{b}' \geqslant \mathbf{0}. \tag{57}$$

Note that both $\mathbf{b}^{-1}$ and $\mathbf{c}_B$ are known at optimality and **c** is obtained from the lower-level problem.

The second assumption is that M is greater than the largest coefficient in **C**. When this assumption is met, $-\nabla_{\mathbf{f}}L$ must be a descent direction of the upper-level objective function. Otherwise, there is no guarantee that $-\nabla_{\mathbf{f}}L$ is also a descent direction of the upper-level objective function. When $-\nabla_{\mathbf{f}}L$ is not a descent direction of the upper-level objective function, we may need to switch to using the frequency setting heuristic proposed by Szeto and Wu (2010) to determine the frequency. This heuristic is time-consuming, as it relies on solving the lower-level problem many times to determine the descent direction of each line and the optimal frequency. The details of the frequency setting heuristic are not reported here but readers can refer to Szeto and Wu (2010) for the details. Alternatively, the method proposed in this paper can be used as a heuristic to determine frequencies.

3.6. Neighbor solution generation

Due to the complexity of the problem, specific neighborhood search operators are developed to generate neighbor solutions. As each route comprises three parts—starting terminal, intermediate stops, and ending terminal—these neighborhood search operators intend to mutate all of these parts. Four operators are proposed to achieve this purpose: (a) starting terminal swap, (b) ending terminal swap, (c) intermediate stop swap, and (d) intermediate stop insertion (Fig. 6). The operations are conducted randomly in the neighborhood search phase.

The starting and ending terminal swap operators are trivial. They randomly select two routes and exchange the two starting and ending terminals, respectively. Before swapping their starting and ending terminals, the ending terminals of two candidate routes are checked to ensure that they are different to generate different solutions. For the intermediate stop swap and insertion operations, the nodes selected to perturb are based on the proposed average-direct-demand value. For instance, the node to be transferred is the one that induces the minimum average-direct-demand increment. Once a candidate node is found, a scanning procedure is carried out to check whether the selected node is in the receiving route. If so, the next best node is selected. Note that an intermediate stop deletion operator is not used here because it does not generate a better solution by itself. It is only used when the route is too long; that is, we repair the route structure because it violates the travel time constraint or the constraint on the number of stops.

4. Experiments

To investigate some of the problem's properties and the performance of the proposed solution method, a small network was created and tested, after which the proposed method was applied to solve a realistic bus network problem in Tin Shui Wai (TSW), Hong Kong. The performance of the proposed algorithm was demonstrated by comparing it with that of a GA on the Winnipeg network. For the small and TSW networks, the centroid and stop were assumed to be identical. The parameters, unless specified, were set as follows: $M = 2000$; $N_c = 100$; $N_e = 50$; $N_o = 50$; *limit* = 50; the maximum number of iterations was 500; $\alpha = \beta = 10^9$; and $\varepsilon_1 = \varepsilon_2 = 0.01$. The proposed ABC method was coded in C++ and complied with Visual Studio 2008, and the lower-level and linear integer programs were solved by CPLEX 12.4. For all of the tests, 20 runs with different initial seeds were conducted and the average performances were reported.

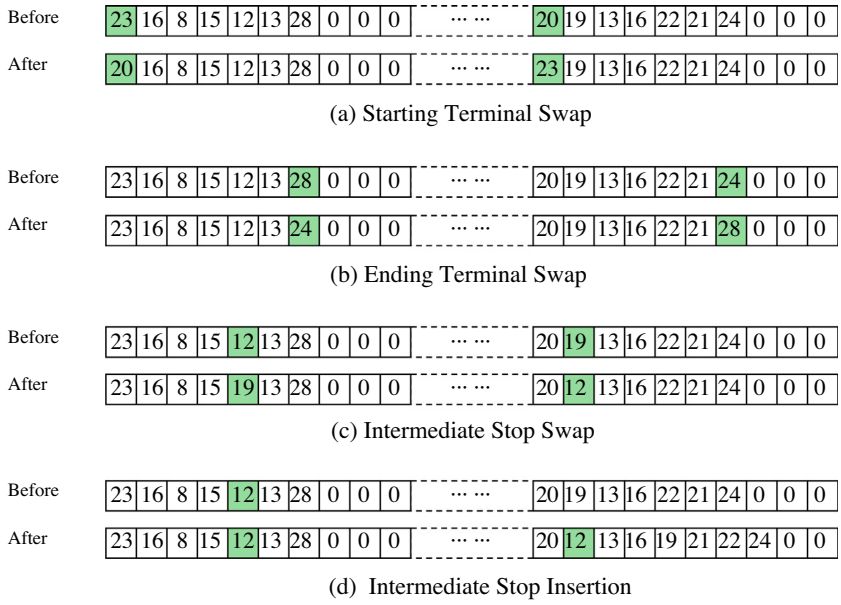


Fig. 6. Neighborhood search operations.

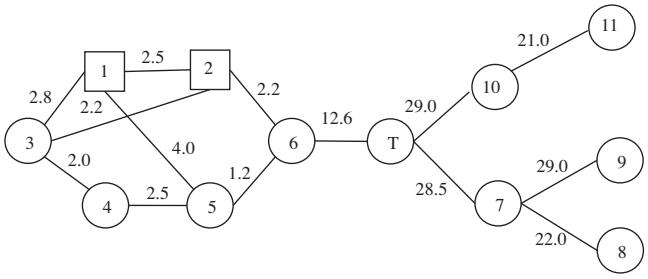


Fig. 7. Small network.

4.1. Small network experiments

Fig. 7 shows the small network that we created. The starting bus terminals were nodes 1 and 2. Nodes 7–11 were ending terminals. The study area consisted of nodes 1–6. Each zone in this example had only one stop. Hence, the passenger demand between stops equaled that between the corresponding centroids. The passenger demand is given in Appendix A. The maximum trip time was set to 23 min, the total fleet size was 60, the maximum number of intermediate stops was 3, and the maximum number of routes was 5.

4.1.1. Benchmark results with multiple solutions

To calculate the benchmark result, a brute force method (that enumerated all possible solutions) was applied. The frequency determination procedure and the lower bound screening method were incorporated to speed up the brute force method. The optimal objective value obtained was 411. From the proposed ABC algorithm, the average objective value of each run and the lowest objective value both equaled 411, implying that all of the runs successfully found the optimal objective value.

Table 1 provides the two optimal solutions, which possess different route structures and headways that are the reciprocal of the corresponding frequencies. The two structures are quite similar. The main differences are the frequency setting and the stop sequence of the fourth route. From the operator's perspective they may be significantly different in terms of other criteria such as fuel consumption, emissions, and operation cost. For illustrative purposes, the fuel cost was estimated by multiplying the frequency by the corresponding trip time. The fuel cost was HK$634.7.0 and HK$621.7 per hour for solutions 1 and 2, respectively. If the first design was chosen, a 3% greater fuel cost was spent than if the second design was chosen, implying that the operator had to select the design wisely. From the passengers' perspective, different frequencies and stop

Table 1
Optimal solutions of the small network.

Optimal solution 1			Optimal solution 2		
Stop sequence	Trip time (min)	Headway (min)	Stop sequence	Trip time (min)	Headway (min)
1, 3, 2, T, 8	22.8	13.3	1, 3, 2, T, 8	22.8	12.2
1, 3, 2, T, 9	22.8	12.4	1, 3, 2, T, 9	22.8	11.5
1, 3, 2, T, 11	22.8	13.2	1, 3, 2, T, 11	22.8	12.1
2, 5, 6, T, 10	20.2	12.3	2, 6, 5, T, 10	20.2	7.6
2, 4, T, 7	22.0	5.9	2, 4, T, 7	22.0	11.2

sequences indicated different waiting times and opportunities to find a seat. These two factors also affect passengers' route choices in reality, and can be considered in selecting one out of all optimal solutions. Although the proposed ABC algorithm only kept the best solution over iterations, it was possible to create a solution pool that contained all of the optimal solutions searched. Hence, it was not difficult to select an optimal solution that gave the best performance in the other measures.

4.1.2. Effectiveness of the lower bound screening method and hybrid ABC algorithm

To test the effectiveness of the lower bound screening method, an ABC version that did not use the screening method was developed. The other procedures were identical, with the exception of the lower bound screening method. For both versions, the infeasible solutions were identified and were not used to determine optimal frequency. Although the lower bound screening method was not used, the lower bounds were still calculated and adopted in the fitness function. The computation times are shown in Table 2. By comparing both methods, we found that the lower bound screening method reduced the computation time significantly. The computational advantage may be due to the following reasons. One is that for the version with the lower bound screening method, only potentially good solutions were required to determine optimal frequency, whereas for the version without, all of the feasible solutions had to carry out the iterative optimal frequency determination procedure. Accordingly, the total number of solutions required to determine the optimal frequencies was reduced by the lower bound screening method. The other reason is the termination criteria of the descent frequency search. For the version with the lower bound screening method, the termination criteria relied on both criteria (1) and (2), whereas for the version without the method, the termination criterion was only defined by criterion (2), which was the difference between the upper-level objective values of two successive iterations. Thus, the number of iterations required to determine optimal frequency was also reduced by the lower bound screening method. The computational advantage is likely to be even more significant in large networks with more feasible solutions and integer variables.

Table 2 also shows that the hybrid ABC algorithm with the lower bound method obtained optimal solutions much more quickly than the brute force method (which was exact), illustrating the computational efficiency of the proposed algorithm.

4.1.3. Effects of design parameters

Figs. 8–11 demonstrate the effects of various design parameters on minimizing the number of passenger transfers, including the minimum frequency f_{min}, the maximum number of intermediate stops S_{max}, the maximum fleet size W, and the maximum number of routes R_{max}. Without further specification, the default parameters were set as $f_{min} = 4.8$ buses/h, $W = 60$ buses, $S_{max} = 3$, $R_{max} = 5$, and $T_{max} = 26$ min.

4.1.3.1. Effect of minimum frequency. Minimum frequency was used to maintain a certain level of service with respect to waiting time. A higher frequency meant a higher capacity and a shorter waiting time. However, under the fleet size constraint, increasing the minimum frequency may have resulted in the deterioration of another type of service level, such as the total number of passenger transfers, as illustrated in this example. Fig. 8 depicts the effect of the minimum frequency setting, showing that a tighter minimum frequency constraint results in a higher number of passenger transfers. The frequency requirement was satisfied by cutting the trip time and reducing the number of stops visited, because the total fleet size must be fixed. Thus, some passengers received the benefit of a reduced waiting time while others bore an additional transfer cost. This finding raises an interesting equity research direction for future research. In the extreme case, when the minimum frequency was greater than 5.4 buses/h, there was no feasible solution to satisfy the fleet size constraint.

Table 2
Comparison of computation time.

	Brute force method		Hybrid ABC algorithm	
With lower bound screening	No	Yes	No	Yes
Average computation time (seconds)	8292.55	14.16	1003.80	0.20

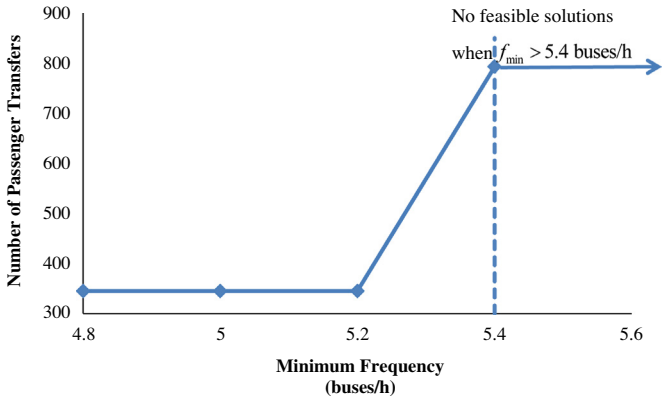


Fig. 8. Effect of minimum frequency.

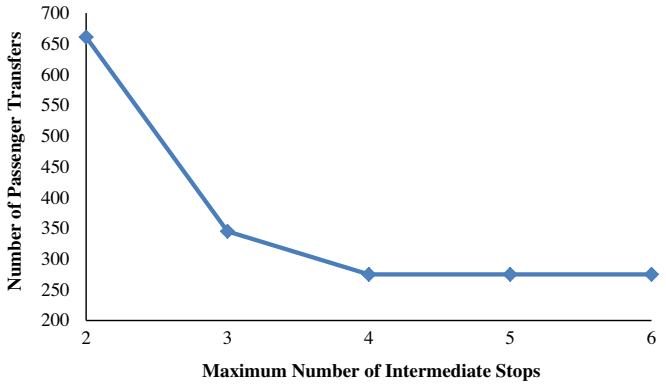


Fig. 9. Effect of the maximum number of intermediate stops.

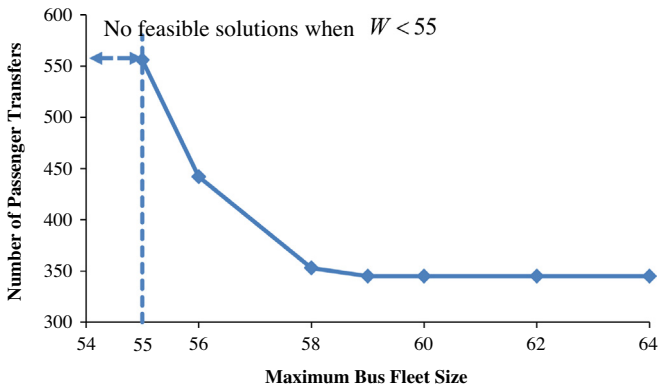


Fig. 10. Effect of the maximum bus fleet size.

4.1.3.2. Effect of the maximum number of intermediate stops. More intermediate stops may have reduced the number of transfers, at the cost of increasing the route travel time and reducing the frequency. Fig. 9 shows that the total number of transfers continued to decrease from 661 to 275 when S_{max} increased from 2 to 4. More stops could be added to the existing routes by increasing the maximum number of intermediate stops, such that the existing services could cover more demand locations and provide more direct services. However, a further increase in S_{max} did not reduce the number of transfers because no more stops could be added to the existing routes in this range of the maximum number of intermediate stops allowed. Either the maximum travel time constraint (i.e., constraint (12)) or the minimum frequency constraint (i.e., constraint (10)) instead of the constraint on the maximum number of intermediate stops (i.e., constraint (11)) was binding at optimality. Visiting

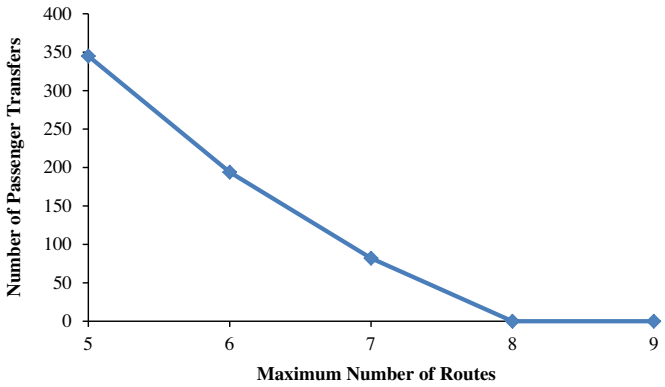


Fig. 11. Effect of the maximum number of routes.

more stops extended the round trip and total stop times. Moreover, the prolonged travel time reduced the frequency if the fleet size for the route remained unchanged. Hence, the number of stops could not be added to each route without limit. The implication of this result is that practitioners must identify which constraints are critical in improving the services because the binding constraints are different under different conditions.

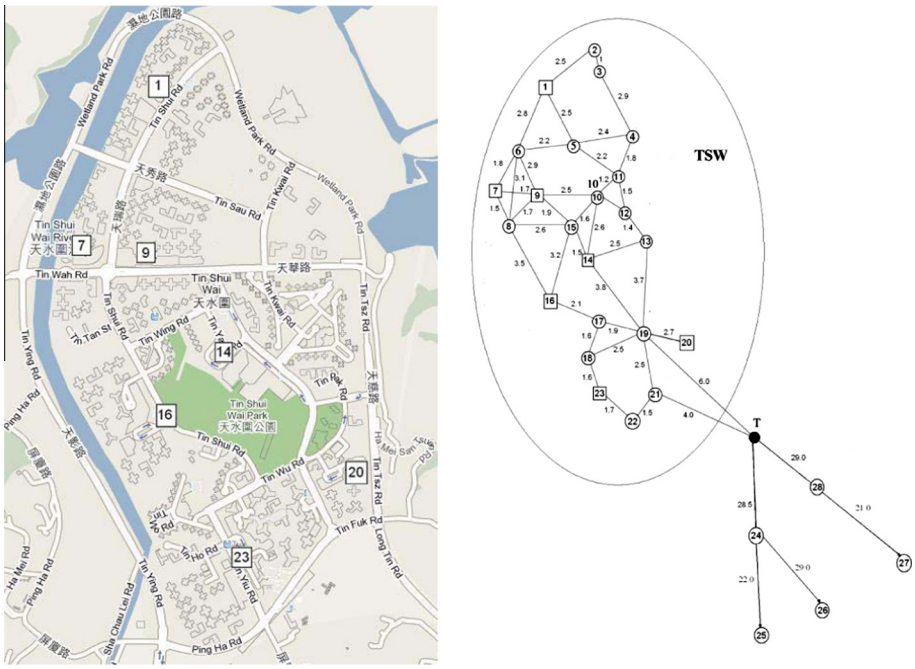
4.1.3.3. Effect of the maximum bus fleet size. The effect of the maximum bus fleet size is shown in Fig. 10. According to this figure, there was no feasible solution when $0 < W < 55$, because the minimum frequency requirement could not be satisfied, implying that the fleet size was inadequate to provide the minimum acceptable level of service. When $55 \leqslant W < 59$, the number of transfers decreased with the increase in the fleet size, because more buses were allocated to the existing routes to serve the demand between any pair of stops on the same bus route, and less demand between them required a transfer due to the insufficient capacity of the direct services. When $W \geqslant 59$, the effect of increasing the fleet size on reducing the number of transfers vanished because all of the demands between any pair of stops on the same bus route were met. Route capacity was no longer the key factor in reducing the number of transfers and the maximum fleet size constraint was no longer binding. Further reducing the number of transfers required adding more direct services.

4.1.3.4. Effect of the maximum number of routes. To illustrate the effect of the maximum number of routes, R_{max} was increased from 5 to 9. The fleet size was adjusted to 120 in this test, because according to preliminary tests no feasible solution could be found if R_{max} was greater than 6 under the default setting. The results are plotted in Fig. 11. As expected, the number of transfers decreased with an increase in R_{max}. More importantly, the number of transfers was successfully eliminated when R_{max} was equal to or greater than 8, because more routes provided more direct services and covered more nodes.

4.2. TSW network

The main study area was located in Tin Shui Wai (TSW), Hong Kong (Fig. 12a). All of the routes leave TSW through the Tai Lam Tunnel (TLT), located on the south eastern side of the area, and then continue via the highway, which is connected to urban destinations. Passengers can transfer either at the TLT station or at other nodes outside the TSW area. However, due to a lack of systematic design, the existing bus network operates in an inefficient manner, generating many transfers. Some of the bus services are routed using a low occupancy rate, which wastes resources and inconveniences passengers. However, from the passengers' perspective, ideally, there should be as many routes as possible to provide direct point-to-point services, but this is infeasible due to the relatively fixed operating cost of the operator. As with other bus operators, the operating cost was shown to be roughly proportional to the number of operating vehicles. If a single route zigzags too much and has too many stops, the travel time is long. The bus operators' main concern is then how to restructure the bus routes in TSW to reduce the number of passenger transfers without increasing the fleet size.

Fig. 12b shows the TSW network. The square nodes represent the bus terminals inside TSW, the circle nodes represent the current bus stop locations, and "T" represents the TLT bus interchange. The in-vehicle travel times (in minutes) between nodes are shown next to the corresponding links. As Fig. 12b reveals, the TSW area was divided into 23 zones, each of which had one stop or bus terminal. The stops and terminals in this area are represented by nodes 1–23 and the seven bus terminals are represented by squared nodes (Fig. 12b). All of the bus routes originating from these terminals terminated at one of the five ending terminals, nodes 24–28. The demand matrix estimated from the available data is given in Appendix B.



(a) Map of Tin Shui Wai (b) The Tin Shui Wai bus network

Fig. 12. The study network.

4.2.1. Effects of different forms of fitness functions and penalty parameter values

Because a lower bound is used to approximate the upper-level objective value, the solution quality may not be reflected accurately. Therefore, the following three different forms of fitness functions are proposed and tested:

$$F1_g = 10^{15} - LB_g - P_g,$$

$$F2_g = C_I - LB_g - P_g, \text{ and}$$

$$F3_g = \frac{1}{LB_g + P_g}.$$

$F1_g$ and $F2_g$ adopt a similar functional form in the sense that both use a constant minus the lower bound and the penalty term P_g. However, an arbitrarily selected large constant (i.e., 10^{15}) is adopted in $F1_g$, while an adaptive value, C_I, is used in $F2_g$. C_I is determined by adding a small constant (i.e., 1.0) to the maximum value of $(LB_g + P_g)$ among all of the solutions obtained in iteration I. In addition to the forms of the fitness functions, the penalty parameter values affect the probability of searching infeasible solutions. The combined effects of the form of a fitness function and the penalty parameter values are plotted in Fig. 13, assuming that the value of the penalty parameter α is equal to the value of the penalty parameter β. The y-axis represents the average upper-level objective value and the x-axis is the log penalty parameter value, demonstrating that by modifying the form of the fitness function, the performance of our algorithm significantly improves. Although the penalty values affect the performance, the effect seems to be less than that of the form of the fitness function. The best average objective value is given by $F3_g$ at $\alpha = \beta = 10^8$. This setting was adopted in the following experiments.

4.2.2. Effect of limit

The predefined number *limit* is used to determine when an employed bee becomes a scout. This value may be roughly interpreted as the sampling frequency within a solution space. A higher value means that more neighbor solutions are found and compared. In this test, *limit* was increased from 0 to 500. The average upper-level objective values are plotted in Fig. 14. When *limit* equals 0, it represents the scenario that all food sources are abandoned and regenerated in each iteration. The average objective value decreased initially and then arrived at the minimum point, when *limit* equals 150. Afterwards, the average objective value slightly increased and became varied, indicating that the average algorithm performance worsened if *limit* was too large or small. One explanation is that when *limit* was small, the promising area in the solution space could not be well exploited, whereas when *limit* was too large, many search efforts were trapped in the areas with low solution quality.

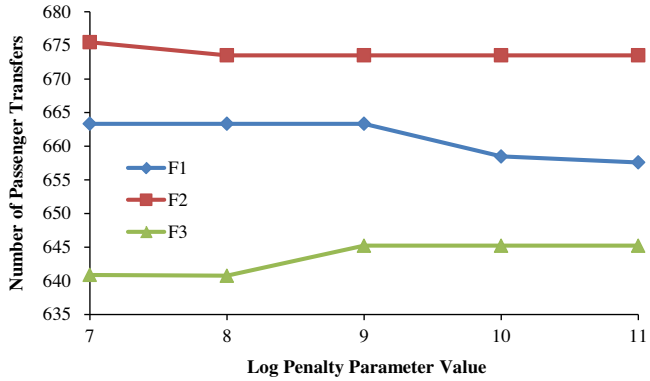


Fig. 13. Effects of various forms of fitness functions and penalty parameter values.

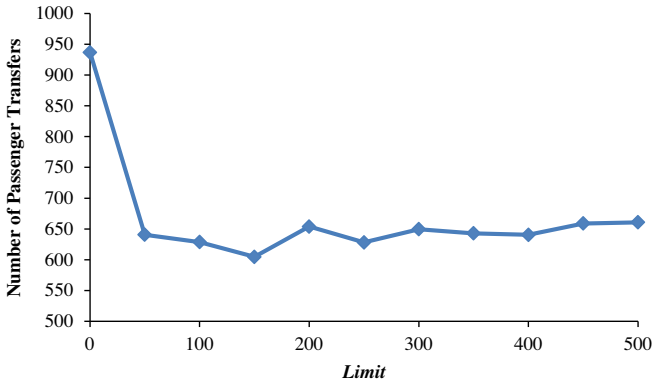


Fig. 14. Effect of *limit*.

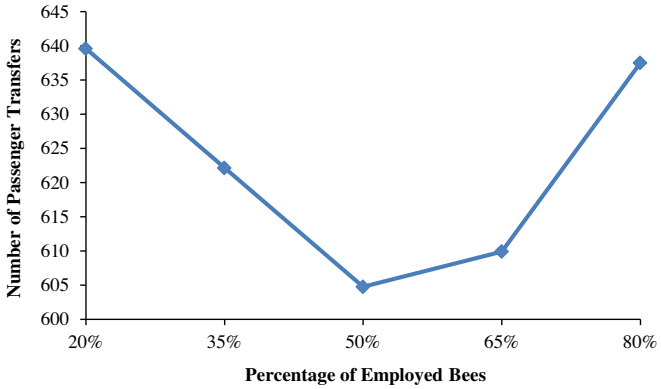


Fig. 15. Effect of colony combinations.

4.2.3. Effect of colony composition

The composition of employed bees and onlookers also affects the performance of the hybrid ABC algorithm. Given that the total number of iterations and the colony size are fixed, different percentages of employed bees result in different numbers of initial solutions and different numbers of onlookers influence the intensity of neighbor solution searching. Meanwhile, the number of employed bees also reflects the number of scouts, which controls the maximum number of new solutions generated in each iteration. Fig. 15 shows that the composition has a significant effect on the algorithm performance. In this example, when 50% of the colony is employed bees, the algorithm achieves the best average objective value. Either a higher or lower percentage prevents the further improvement of the objective value, probably because when the percentage of employed bees is low, only a few promising food sources are generated for onlookers to exploit. In contrast, if the percentage

of employed bees is high, only a few onlookers conduct neighborhood searches to exploit the solution space near promising food sources.

4.2.4. Effect of different node insertion and deletion strategies

The insertion and deletion of nodes, which is important in improving route structures, is included in the repairing procedures and the neighborhood search. We propose an average-direct-demand value, defined as the average passenger demand on the direct services, to select a node to insert or delete. To show the benefit of using the proposed measure in selecting nodes, three different strategies were compared and the results are presented in Table 3.

S1: inserting and deleting nodes based on the proposed average-direct-demand increment and decrement, respectively.
S2: inserting and deleting nodes based on the cost increment and decrement, respectively.
S3: inserting and deleting nodes based on the total direct demand increment and decrement, respectively.

Two scenarios—low and normal demand—were tested. The average upper-level objective values of 20 runs are reported in Table 3. The number in each pair of braces is the increment percentage with respect to strategy S1 in the same row and shows that the proposed strategy S1 outperformed the others in both scenarios. The higher the demand, the more notable the advantage, because the average-direct-demand took the change in the upper-level objective value due to inserting a node into consideration. In contrast, if only the total direct demand or cost change was considered, the nodes with a higher demand or shorter distance were visited more frequently, making them more likely to provide excessive services for these nodes and induce more transfers for other nodes.

4.2.5. Robustness of the obtained solution

The travel demands of the network were estimated, but the real demands may vary from day to day. To illustrate the robustness of the solution obtained by the proposed algorithm, 1000 demand matrices were generated by perturbing the estimated demand matrix and used for the evaluation. For each perturbed demand matrix, its element—the perturbed demand from node m to destination e—was randomly generated from a uniform distribution $[0.8\ d_m^e,\ 1.2\ d_m^e]$.

Table 4 compares the best solutions obtained by the hybrid ABC algorithm for the studied situation using the perturbed demand matrices. 'Std.' and 'No.' stand for 'standard deviation' and 'number', respectively. According to this table, the design solution obtained by the hybrid ABC algorithm was significantly better than the existing design, with the former reducing the number of transfers by 340% on average and successfully eliminating the unserved demand in all cases. The high unserved demand resulted from a lack of service capacity to meet the total demand of some destinations and the demand at some stops, where passengers failed to board any line. These two issues were addressed by the hybrid ABC algorithm.

The detailed solution obtained by the hybrid ABC algorithm is shown in Table 6 and the existing design is shown in Table 5. Intuitively, there are fewer stops in the proposed design, reflecting a more efficient meeting of demand. In addition, the headways of the routes are more evenly distributed for the obtained solution. Compared with the existing design, the standard deviation of headways for the best design decreased from 2.9 to 2.4 min, indicating that the difference in the level of service, in terms of frequency, among all of the routes, decreased.

4.3. Winnipeg network

To evaluate the performance of the proposed hybrid ABC algorithm, it was compared with that of a GA using the Winnipeg network obtained from Emme 3.4. The network is shown in Fig. 16(a) and comprises 154 zones, 1067 nodes, and 2995 links. The network is further divided into seven districts including one central area. In each district, 30 lines are designed for

Table 3
Comparison of the different insertion and deletion strategies.

	S1	S2	S3
50% of the Demand (low demand)	311.62	646.05 (+107.3%)	602.62 (+93.4%)
100% of the Demand (normal demand)	604.76	1313.99 (+117.3%)	1207.59 (+99.7%)

Table 4
Comparison of the solutions under random demand.

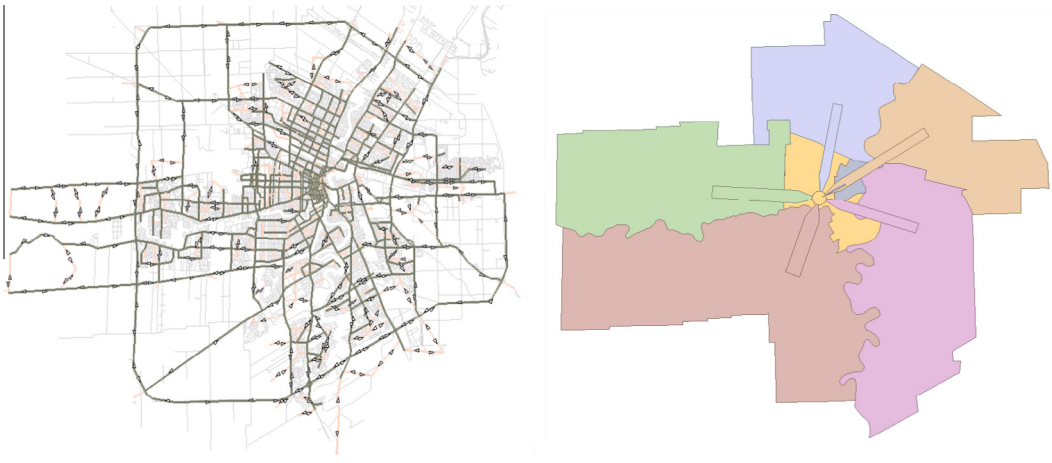
	Average No. of transfers	Std. of No. of transfers	Average unserved demand	Std. of unserved demand
Hybrid ABC algorithm	355.491	22.38	0.00	0.00
Current	1563.31	35.95	1350.85	56.18

Table 5
Existing route structures and headways.

Routes	Stop sequence	Number of buses	Headway (min)
1	20, 19, T, 25	12	9.8
2	16, 17, 18, 23, 22, 21, T, 25	17	8.1
3	16, 17, 18, 23, 22, 21, T, 24	19	5.1
4	1, 6, 9, 10, 12, 13, 19, 21,T, 25	19	8.5
5	1, 6, 5, 4, 11,12, 13, 19, T, 24	23	5.3
6	14, 15, 8, 9, 10, 12, 13, 19, T, 24	11	11.1
7	1, 6, 8, 16, 17, 18, 23, 22,21, T, 28	30	4.1
8	14, 13, 12, 10, 8, 16, 17, 18, 23, 22, T, 26	18	11.0
9	7, 6, 1, 2, 3, 4, 11, 12, 13, 19, T, 24	11	12.3
10	9, 10, 11, 5, 6, 8, 16, 17, 18, 23, 22, T, 27	16	11.3

Table 6
Best solution obtained by the proposed ABC algorithm.

Routes	Stop sequence	Number of buses	Headway (min)
1	14, 13, T, 27	11	11.6
2	16, 1, 11, 13, 21, T, 24	23	5.3
3	20, 23, 18, 16, 15, 14, T, 26	20	9.2
4	7, 5, 1, 6, 9, 8, T, 25	14	12.1
5	9, 5, 6, 8, 16, 17, 22, T, 26	15	12.2
6	7, 1, 2, 3, 4, 12, 13,T, 26	15	12.3
7	16, 8, 18, 20, 22, 23, 13, 12, T, 28	25	6.7
8	23, 18, 16, 15, 10, 12, 13, 19, T, 25	19	8.9
9	1, 2, 3, 4, 5, 7, 9, 14, 19, T, 28	14	9.8
10	16, 23, 6, 10, 11, 15, 17, 21, 19, T, 28	20	8.5



(a) Winnipeg Network (b) Districts of Winnipeg Network

Fig. 16. Winnipeg network.

commuters traveling from their home district to the central area. Each line is allowed to visit 15 stops at most within 45 min. The total fleet size is 1200 buses with a capacity of 60 passengers per bus.

For a fair comparison, the GA uses the same solution representation, initialization procedure, and frequency determination procedure as the hybrid ABC algorithm. The population size is set to be equal the colony size. Unlike the hybrid ABC algorithm, the GA requires crossover and mutation operators to generate new solutions. Nevertheless, the standard crossover and mutation operators cannot be applied directly because the proposed solution representation is different from the solution representation in the traditional GA (Haupt and Haupt, 2004). Thus, the stop crossover operator developed by Szeto and Wu (2010) was used and the proposed neighborhood operators were adopted as mutation operators.

Both algorithms ran 20 times. The computation time of each run was 300 s. The computation performances of the two algorithms is summarized in Table 7. The number within each pair of brackets in the third column is the percentage improvement of the hybrid ABC algorithm with respect to the GA in the corresponding measure. The second row shows that

Table 7
Comparison of the computation performance of the GA and hybrid ABC algorithm.

Number of passenger transfers	GA	Hybrid ABC
Average	300.4	252.6 (−15.9%)
Standard deviation	47.6	29.0 (−39.0%)
Minimum	246.0	216.0 (−12.2%)

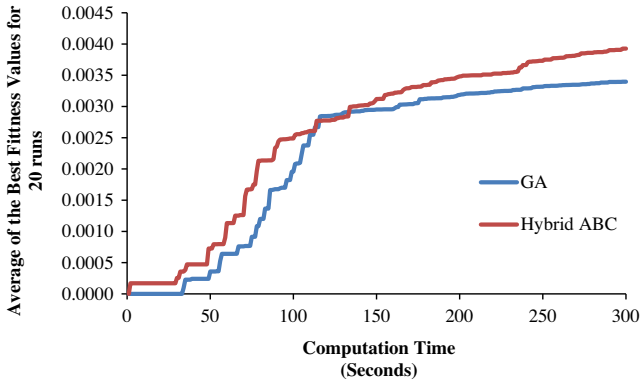


Fig. 17. Average of the best fitness values for 20 runs for the GA and hybrid ABC algorithm.

the average number of passenger transfers obtained by the hybrid ABC algorithm was lower than that of the GA. A *t*-test was also conducted to examine whether the difference in their average numbers of passenger transfers is statistically significant. The *t*-value obtained is 2.093 and hence we can conclude that the difference is significant at the 5% level and the average performance of the hybrid ABC algorithm is better at this significance level.

By comparing the standard deviations of the number of transfers in 20 runs in the third row in Table 7, it was concluded that the solution quality obtained by the hybrid ABC algorithm was much more stable. More importantly, as reflected in the last row, the best solution found by the hybrid ABC algorithm was superior to that obtained by the GA. The advantage of the hybrid ABC algorithm may be attributed to a better local search strategy, with a solution space explored by both employed bees and onlookers.

Fig. 17 shows the average of the best fitness values for 20 runs for the two algorithms during the computational process. This figure illustrates the average of the best fitness values obtained by the hybrid ABC algorithm was higher than that of GA almost throughout the computational process. This implies that the hybrid ABC algorithm could always find a better feasible solution than the GA using the same computation time. It may be because the hybrid ABC algorithm has a better local search strategy.

5. Conclusions

In this study, we proposed a bi-level model to formulate the transit network design problem. The network design and frequency settings were considered simultaneously. The number of passenger transfers was explicitly captured in the upper-level objective function, and the strict capacity constraint approach was used to handle the congestion effect in the lower-level problem. The lower-level problem is a congested transit assignment problem with the optimal strategy concept.

A hybrid ABC algorithm was developed to solve the model. The main algorithm (ABC algorithm) was used to search the solution space of route structures, while the Simplex method solved the capacity-constrained transit assignment model. A node insertion and deletion strategy based on an average-direct-demand (i.e., average passenger demand on the direct services concerned) was proposed to repair route structures, and its effectiveness was illustrated by a numerical example. A descent search method was proposed for the frequency setting and a lower bound screening method was proposed to speed up the computation. The descent direction of the lower-level problem was also a descent direction of the upper-level objective under certain conditions, and a linear integer program was formed to determine the step size for updating the frequency setting.

Numerical examples were provided to illustrate the properties of the bi-level problem and the performance of the proposed algorithm. In particular, the benchmark comparisons verified that the proposed algorithm could find optimal solutions and that the descent search method saved considerable computation time. A small experiment was also performed to show the effects of different design parameters (including minimum frequency, maximum fleet size, maximum number of routes and intermediate stops) on the objective value and the possibility of multiple design solutions.

Using the realistic TSW network study, several tests were conducted to illustrate the effects of the ABC parameters, the functional forms of the fitness function, the penalty parameter values, and the node insertion and deletion strategies, demonstrating that the solution quality can be improved by carefully adjusting the ABC parameters. We found that the best

solution obtained from the proposed algorithm was significantly better than the current design in terms of number of passenger transfers, and was more robust in terms of meeting passenger demand under demand uncertainty. Finally, the superior performance of the proposed hybrid ABC algorithm was demonstrated by comparing it with the GA in the Winnipeg network scenario.

This study opens up many research directions. First, the objective of our proposed problem – minimizing the number of transfers – was set from the perspective of passengers, which is reasonable if the operator is public and has passengers as its main consideration. However, the government may also consider the concerns of different stakeholders, including the operator. Hence, a potential future direction is to extend the proposed model to develop a multi-objective model that considers such concerns. Second, in terms of decision variables, the transit fare structure can be incorporated because it is also an important factor influencing passenger route choices. Third, the constant travel time assumption in the lower-level problem allowed us to have a linear programming problem that could be efficiently solved by existing algorithms while remaining applicable to large-sized networks, deriving the descent direction to determine the optimal frequency setting. However, it has been shown that wait/transfer times at stations are usually increasing functions of the number of passengers boarding and alighting at stations (Lam et al., 1998; Yin et al., 2004; Li et al., 2009). It would be easy to extend our proposed model to capture this phenomenon without conceptual difficulty, but it would be more difficult to develop an "efficient" solution method to solve the resultant bi-level model. This has thus been left to future study. Finally, the proposed model could be extended to consider changing demand over time using the time-dependent approach (e.g., Szeto and Lo, 2005, 2008; Lo and Szeto, 2009), time space networks (Szeto, 2013), or the day-to-day dynamic approach (Watling and Cantarella, 2013) to develop good and meaningful bus service policies that are geared toward serving new development and facilities in Hong Kong, such as columbarium facilities.

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Appendix A. Peak hourly travel demands of the small network

From/To	7	8	9	10	11	Total
1	192	148	102	94	149	685
2	100	74	78	56	102	410
3	87	77	71	46	113	394
4	96	63	49	34	85	327
5	33	24	19	15	34	125
6	19	14	14	9	23	79
Total	527	400	333	254	506	2020

Appendix B. Peak hourly travel demands of the TSW network

From/To	24	25	26	27	28	Total
1	192	148	102	94	149	685
2	54	39	38	22	54	207
3	47	40	38	27	55	207
4	33	22	21	14	30	120
5	100	74	78	56	102	410
6	87	77	71	46	113	394
7	113	76	71	46	103	409
8	100	76	71	47	117	411
9	96	63	49	34	85	327
10	33	24	19	15	34	125

(*continued on next page*)

Appendix B (*continued*)

From/To	24	25	26	27	28	Total
11	19	14	14	9	23	79
12	156	134	114	69	165	638
13	177	105	90	78	143	593
14	63	48	36	29	59	235
15	102	81	63	39	93	378
16	253	170	150	127	213	913
17	28	20	20	14	27	109
18	76	63	58	38	71	306
19	34	25	22	14	30	125
20	59	39	30	26	49	203
21	36	23	22	15	28	124
22	33	25	20	16	28	122
23	206	184	147	96	209	842
Total	2097	1570	1344	971	1980	7962

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