CASPT 2021

Modeling Bus Bunching along a Common Line Corridor considering Passenger Transfer Behavior and Capacity Constraint

Full Paper

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Abstract Bus bunching is a longstanding problem in the real-life public transport system and degrades the efficiency and quality of the transit service. Various control schemes (e.g., holding, skipping, etc.) have been investigated to mitigate the adverse impacts of bus bunching problem. Nevertheless, existing researches inadequately capture the bus operation by simplifying some critical issues such as capacity constraint and transfer behavior in multi-line scenarios. The simplifications facilitate the formulations and analyses but may cause an unrealistic model and be impractical in reality. In this study, we formulate passenger behaviors including boarding, alighting, failing to board as well as transferring, and develop algorithms for bus trajectories and transfer passengers' routing results for a more realistic estimation of the bus operation. The results of numerical experiments examine the effects of passenger transfer behavior and bus capacity constraint on the propagation of bus bunching and demonstrate some useful insights: (a) Modeling routing of transfer passengers is considerable since the performances of bus operation with different passenger behaviors are distinct. (b) The propagation of bus bunching problem from a disturbed bus line to an undisturbed bus line via the common corridor is enhanced because of the presence of transfer passengers. (c) Appropriate smaller bus capacity vehicles may lead more regular service as bus dwell times are constrained to avoid severe bus bunching.

Keywords Bus bunching · Common line problem · Public transport · Capacity constraint · Passenger transfer.

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1 Introduction

In the last decade, more and more attention has been attracted on providing high-quality public transport services with the advocate of sustainable urban mobility. However, maintaining a reliable and efficiency bus operation is very challenging and the mismatching between the transit supply and passenger demand is commonly observed in bus operation. In fact, bus services in some metropolitan areas are criticized for being inefficient, irregular, and overcrowded. Low-quality bus services frustrate passengers and lead to a reduction in bus modal split rate especially for commuters. For example, the number of annual passenger trips of public (electric) bus in Beijing dropped from 4.84 billion (in 2013) to 3.13 billion (in 2019), while the number of motor vehicles increased from 5.44 million (in 2013) to 6.365 million (in 2019). [1] Among various existing issues in bus operation, bus bunching is a longstanding problem and has raised extensive researches.

Bus bunching states a phenomenon with schedule disruption and headway irregularity in bus operation. From the perspective of passengers, a more intuitive description of bus bunching is the incidental phenomenon that two or more buses arrive at a stop with headways significantly shorter than designed. The bus bunching problem is prevalent in real life due to two reasons: 1) The bus system suffers inevitable endogenous uncertainties and exogenous random disturbances. 2) The natural characteristic of bus operation, a delayed bus picks more passengers while its following bus picks less, also enlarges the variability of bus headway. In a conventional bus operation without countermeasures, a bus tends to pair with its preceding or following bus and can hardly recover to the designed schedule or expected headway especially in peak hours when passenger demand and service frequency are both high (Newell and Potts, 1964; Hickman, 2001; Daganzo, 2009).

Bus bunching degrades the reliability and effectiveness of transit service and concerns both passengers and transit agencies (Newell and Potts, 1964; Osuna and Newell, 1972; Hollander and Liu, 2008; Verbich et al, 2016). From the perspective of passengers, the average waiting time at stops are increased and more passengers are likely to encounter overcrowded experiences onboard. From the perspective of transit agencies, the underutilization of the pairing buses raises the operation cost to maintain the supply level, i.e. transit agencies have to provide more services or bear the risk of losing split rate.

The occurrence of bus bunching is mainly originated from the uncertainties of bus travel times on roads and bus dwell times at stops. On the one hand, bus travel times on roads are affected by some random issues such as urban traffic conditions, driver behaviors, etc. On the other hand, fluctuate passenger demands also induce the uncertainties of bus dwell times at stops (Fonzone et al, 2015). Therefore, the resolve of bus bunching problem requires understanding and modeling both bus movements and passenger behaviors within bus operation. Although several bus bunching models/bus dynamic formulations/bus trajectory algorithms were proposed in literature to capture the propagation of bus bunching, most of the existing studies focused on oversimplified scenarios in which some critical issues like passenger transfer behav-

[1] Data from the *2020 Beijing Transport Developm annual report*

ior and capacity constraint were not well addressed. In what follows, the literature in bus bunching are reviewed, and the research gap is identified.

In the light of the seminal work about bus bunching of Newell and Potts (1964), extensive researches on control schemes of encountering bus bunching have been conducted.

Earlier literature focused on developing static control schemes in plan level, such as adding slacks to schedules according to so-called static schedule-based holding schemes (Abkowitz and Lepofsky, 1990; Eberlein et al, 2001).Since the technique of intelligent transport and control system has been developing rapidly from the beginning of this century, various dynamic bus control schemes taking the advantage of real-time information have been proposed. The real-time control strategies can be generally categorized into two groups (Ibarra-Rojas et al, 2015). 1) Station strategies such as holding strategy, stop-skipping strategy and boarding limit strategy. 2) Inter-station control strategies such as coordinated speed adjustment and traffic signal priority. Among these dynamic control strategies, holding strategy is the most practical strategy in application (Cats et al, 2011; Eberlein et al, 2001) The efficiency of dynamic holding strategy has been testified in literature (Daganzo, 2009; Xuan et al, 2011; Bartholdi and Eisenstein, 2012; Argote-Cabanero et al, 2015).

A critical shortcoming of holding control is it results in longer bus dwell times and lower travel speed. And holding strategies deal the tiny bus bunching well while fails to work in severe schedule disruption. Therefore, other novel strategies were proposed to make up for insufficiency of holding strategies and outperform them in specific conditions. For example, stop-skipping strategy (Fu et al, 2003; Sun and Hickman, 2005) and substitution strategy (Petit et al, 2018, 2019) are investigated to recover from severe schedule disruption. Coordinated speed adjustment (Daganzo and Pilachowski, 2011) is based on bus dedicated lane premise to reduce the extra dwell time and maintain an acceptable operating speed.

However, most of the existing studies only investigated bus bunching in a single line scenario for facilitate of formulation and analyses, while urban corridors served by multiple bus lines is prevalent in real life. The common line issue, as a special characteristic of urban public transportation networks, induces more complex analyses of passenger behaviors (Argote-Cabanero et al, 2015). In the recent five years, several researches investigated bus bunching in multi-line scenarios (Hernández et al, 2015; Argote-Cabanero et al, 2015; Schmöcker et al, 2016; Petit et al, 2019; Seman et al, 2020). Nevertheless, either the transfer issue or/and the bus capacity constraint was/were overlooked in their work. For example, Hernández et al. (2015) developed an optimization model for holding strategy on multiline system. They considered capacity constraint but ignored transfer behavior. Schmöcker et al. (2016) investigated the effects of common stops on bus bunching with and without overtaking issue. Their results show that the presence of common stops has positive effects when overtaking is possible. They focused on the equilibrium queuing behavior but ignored the passenger transfer behavior and the capacity constraint. Petit et al. (2019) developed an optimization model to implement substitution strategy in a multiline system, but assumed that different bus lines were independent of each other for the sake of simplicity.

To best of our knowledge, there is no existing analytical model or research considers transfer behavior and capacity constraint simultaneously. It is necessary and crucial to consider multiple public transport lines with a more realistic model.

What's more, except two papers involved distributed passenger boarding behavior when bus overtaking was allowed (Schmöcker et al, 2016; Wu et al, 2017) most of the existing studies on bus bunching and control strategies ignored passenger equilibrium behavior under the control strategies. In fact, passenger behaviors affect the propagation of bus bunching and the performance of control strategies. Therefore, we focus on the transfer behavior and formulate a transfer-based equilibrium to simulate the routing result of transfer passengers. We refer literature in transit assignment (Schmöcker et al, 2016; Long et al, 2013; Szeto and Jiang, 2014; Jiang et al, 2016) and divert some methodologies and concepts such as fail-to-board probability and MSA algorithm in formulating transfer cost function and searching for equilibrium solutions.

The main contributions of this study are summarized as follows.

1. We propose a new bus bunching model considering passenger transfer behavior and capacity constraint simultaneously in a multi-line network. The impacts of transfer behavior and capacity constraint are illustrated based on the proposed model.
2. We propose transfer-based equilibrium formulations to simulate the routing results of transfer passengers. The necessity of modeling transfer passenger's routing behavior is examined in numerical experiments and inspires further discussions of control schemes considering the interaction with passenger behaviors.
3. We develop algorithms for bus trajectories and transfer routing results with a combination of event-based simulation and MSA iteration.
4. We propose a simulation-based evaluation indicator to evaluate the range of bus bunching in both temporal and spatial dimensions.

The remaining of the paper is organized as follows: Section 2 describes the problem we focused on and illustrates basic assumptions and notations. Section 3 proposes the formulation of bus dwell time which takes in to account passenger behaviors and capacity constraint. Then, we develop the algorithm for bus trajectories in an event-based simulation framework. Section 4 proposes the formulations of transfer-based equilibrium and transfer cost function. Furthermore, MSA iteration algorithm and a special realization of the transfer-based equilibrium are illustrated. Section 5 illustrates the specifications by a small example and show the properties of this model by demonstrating the effects of transfer behavior and capacity constraint. Section 6 conducts experiments to analyse the sensitivity of the bus bunching problem to different transfer parameters and capacity settings. Finally, Section 7 gives the conclusions and points out future directions.

2 Problem description

We consider a transit network containing a corridor served by two bus lines as well as the upstream and downstream bus stops besides it (i.e. Fig. 1). In this multi-line system, common line issue is embodied intuitively: routing behaviors occur to two specific types passengers in the common line scenario:

Type I) Passengers whose original and destination stops belong to different lines. For these passengers, they board before and alight after the common corridor and need to choose their interchange stops from the common stops.

Type II) Passengers whose origin and destination stops are all in the common corridor and could choose either one between two lines to their destinations.

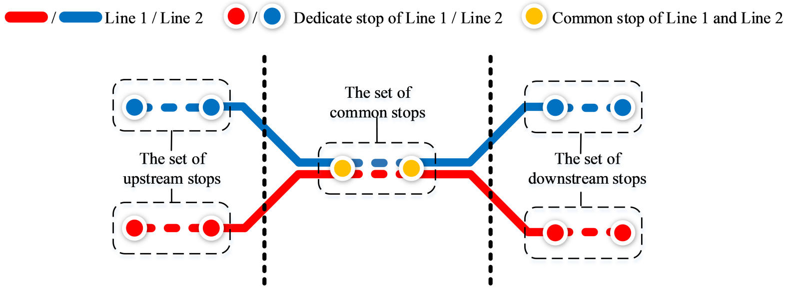


Fig. 1 Two bus lines corridor with some common line stops

It is necessary to acknowledge that the routing behavior of Type II) passengers was investigated in Schmöcker et al. (2016) by an introduction of distributed boarding equilibrium with a premise that two buses dwelling at a stop and overtaking are possible. In our study, we focus on the transfer behaviors of Type I) passengers which were not discussed in previous studies on bus bunching.

2.1 Basic assumptions

The model is developed based on these basic assumptions:

A1) Passengers avoid transfers unless necessary.

A2) A stop has only one platform for passengers to board/alight and buses are not allowed to overtake each other or skip a stop.

A3) Travel times on roads between two successive common stops are equal for buses of two lines.

A4) Buses service frequency and passenger arrival rates are constant.

A5) The time required for boarding/alighting is estimated to be proportional of the number of boarding/alighting passengers.

A6) Passengers' origin and destination stops are given.

A1) is reasonable as passengers would avoid additional transfers to save their total travel times. Therefore, Type I) passengers only transfer once and other passengers do not make any transfer once they board buses that lead to their destinations. With A2), a bus arriving at a stop occupied by another bus will queue outside the platform until the preceding bus depart. Therefore, buses follow the first-arrive-first-depart (FAFD) rule as a result which is usual for two lines sharing one platform at a common stop in real life without skipping or overtaking schemes. A3) is not a critical assumption as we focus on the passenger behaviors which mainly affect bus dwell times (Muñoz et al, 2013; Hernández et al, 2015; Schmöcker et al, 2016)(Muñoz et al., 2013; Hernández et al., 2015; Schmöcker et al., 2016). In fact, the impacts from traffic conditions could be mitigated by using dedicated bus lanes such as BRT routes. With A2) and A3), it is reasonable for Type II) passengers at common stops to prioritize the first bus among their candidate lines. As we mentioned before, bus bunching is prevalent in routes with high bus service frequency and passenger demands. A4) is realistic for a high-frequency bus route or in peak hours and is commonly adopted in existing literature (Sánchez-Martínez et al, 2016; Schmöcker et al, 2016; Wu et al, 2017). Similar estimations to A5 can be found in previous researches (Wu et al, 2017; Li et al, 2019; Wang and Sun, 2020). For a bus with two doors for boarding and alighting respectively, bus dwell time without control schemes equals the maximum value of passengers' boarding and alighting times.

2.2 Notations

The following key notations are used throughout this paper.

Table 1 Notations

Indices and sets		
r, r'	Indices of bus lines	
$\mathcal{S}^r$	The set of bus stops traversed by line r	
$\mathcal{N}_{up}^r$	The set of upstream stops with respect to the common stops of line r	
$\mathcal{N}_{down}^r$	The set of downstream stops with respect to the common stops of line r	
$\mathcal{N}_{com}^{r,r'}$	The set of common stops of line r and r'	
$\mathcal{M}^r$	The set of buses of line r	
m_k^r	The k^{th} bus of line r, and $k = 1, 2, ..., \|\mathcal{M}^r\|$	
n_k^r	The k^{th} stop of line r, and $k = 1, 2, ..., \|\mathcal{S}^r\|$	
n_k	The k^{th} stop in the common corridor, and $k = 1, 2, ..., \left\|\mathcal{N}_{com}^{r,r'}\right\|$	
m	The index of the bus, $m = m_1^r, ..., m_{\|\mathcal{M}^r\|}^r, m_1^{r'}, ..., m_{\left\|\mathcal{M}^{r'}\right\|}^{r'}$	
n	The index of the bus stop, $n = n_1^r, ..., n_{\|\mathcal{S}^r\|}^r, n_1^{r'}, ..., n_{\left\|\mathcal{S}^{r'}\right\|}^{r'}$	
$\mathcal{S}^D(n)$	The set of destination stops associated with the passengers arriving at stop n	
$\mathcal{M}^S(n)$	The set of buses serving stop n	
$m^{r,r'}(m, n)$	The index of the last bus departing from the common stop n before bus m	
$n^r(n)$	The index of the subsequent bus stop after stop n in line r	
Parameters		**Unit**
Λ_n	The arrival rate of passengers at stop n	pas/min
Λ_n^r	The arrival rate of passengers at stop n whose candidate line is line r	pas/min
$\Lambda_n^{r,r'}$	The arrival rate of passengers at stop n whose candidate lines are line r and line r'	pas/min
$\lambda_{i,j}$	The arrival rate of passengers whose origin and destination are stops i and j respectively	pas/min
H^r	The scheduled bus departure headway of line r	min
cap_m	The vehicle capacity of bus m	pas
β	The average boarding rate of passengers	pas/min
α	The average alighting rate of passengers	pas/min
$T_{m,n}$	The travel time for bus m between stop n and its subsequent stop	min

Variables		Unit
$a_{m,n}$	The arrival time of bus m at stop n	min
$W_{m,n}$	The dwell time of bus m at stop n	min
$d_{m,n}$	The departure time of bus m from stop n	min
$C_{m,n}$	The available capacity of bus m at stop n	pas
$A_{m,n}$	The number of alighting passengers of bus m at stop n	pas
$\bar{B}_{m,n}$	The number of boarding passengers of bus m at stop n	pas
$B_{m,n}$	The number of passengers who want to board bus m at stop n	pas
$p_{m,n,j}$	The number of onboard passengers whose destination is stop j when bus m arrives at stop n	pas
$P_{m,n}$	The total number of onboard passengers when bus m arrives at stop n	pas
$L_{m,n}$	The total number of leftover passengers at stop n when bus m departs	pas
$L_{m,n}^{r}$	The number of leftover passengers at stop n when bus m departs whose candidate lines is line r	pas
$L_{m,n}^{r,r'}$	The number of leftover passengers at stop n when bus m departs whose candidate lines are line r and line r'	pas
$l_{m,n,j}$	The number of leftover passengers whose destination is stop j when bus m departs from stop n.	min
$h_{m,n}^{r,r'}$	The headway between bus m and its preceding bus of either line r or line r' at stop	min
$h_{m,n}^{r}$	The headway between bus m and $m-1$ of line r at stop n	min
$\bar{\rho}_{m,n}$	The proportion of alighting transfer passengers among all transfer passengers onboard when bus m dwells at stop n.	
$\alpha_n^{m,n}$	The proportion of transfer passengers choosing stop n as a transfer stop among all transfer passengers from bus m' to bus m.	
$\omega^{m',m}$	The minimum transfer time from bus m' to the other line	min
$\omega_n^{m'}$	The transfer time from bus m' to the other line via common stop n	min
$\omega_n^{m'}$	The transfer time from bus m' to bus m common stop n	min

3 Modeling bus movement in multi-line system

Bus operation process can be broadly decomposed into running in roads and dwelling at stops. The impacts of common line issues on bus bunching are mainly rooted in passenger routing behaviors, which lead different numbers of boarding and alighting passengers and further affect bus dwell time at common stops. Therefore, before presenting the methodology for estimating bus trajectories, the main idea of this subsection is to give a more actual calculation of bus dwell times at common stops considering passenger routing behaviors and bus capacity constraint.

3.1 Passenger boarding demand

First, we identify the boarding demands of passengers at common stops. Based on A4) and A6), the total arrival rate at stop n, denoted as Λ_n (pas/min), is divided into $\lambda_{n,j}$ according to their destinations $j \in \mathcal{S}^D(n)$. $\mathcal{S}^D(n)$ denotes the set of all destinations of arriving passengers at stop n can be accessed, and then Λ_n can be obtained by:

$$\Lambda_n = \sum_{j \in \mathcal{S}^D(n)} \lambda_{n,j} \tag{1}$$

For example, for the i^{th} upstream stop of line 1 (i.e. $n = n_i^1 \in \mathcal{N}_{up}^1$) in Fig. 1, its set of destination stops is $\mathcal{S}^D\left(n_i^1\right) = \left\{n_k^1 | n_k^1 \in \mathcal{S}^1, k > i\right\} \cup \mathcal{N}_{down}^2$, which indicates that the arriving passengers contain two cases:

1) A passenger boards at the upstream stop and alight at one of the subsequent stops of the same line (i.e. $\left\{n_k^1 | n_k^1 \in \mathcal{S}^1, k > i\right\}$).
2) A passenger boards at the upstream stop and alight at one of the downstream stops of the other line (i.e. $\mathcal{N}_{down}^2$), so he/she needs to choose a common stop to make a transfer.

More generally, the arrival passengers at a common stop served by multi bus lines could be grouped according to their candidate lines. At common stop n traversed by line r and line r' (i.e. $n \in \mathcal{N}_{com}^{r,r'}$ in Fig.1), the total arrival rate of passengers Λ_n is divided into three parts:

$$\Lambda_n = \Lambda_n^r + \Lambda_n^{r'} + \Lambda_n^{r,r'} \tag{2}$$

1) Λ_n^r denotes the total arrival rate of passengers who only consider boarding a bus belonging to line r at stop n and is calculated by:

$$\Lambda_n^r = \sum_{j \in \mathcal{S}^D(n) \backslash \mathcal{S}^{r'}} \lambda_{n,j} \tag{3}$$

2) $\Lambda_n^{r'}$ denotes the total arrival rate of passengers who only consider boarding a bus belonging to line r' at stop n and is calculated by:

$$\Lambda_n^{r'} = \sum_{j \in \mathcal{S}^D(n) \backslash \mathcal{S}^{r}} \lambda_{n,j} \tag{4}$$

3) $\Lambda_n^{r,r'}$ denotes the total arrival rate of passengers that consider boarding a bus belonging to either line r or line r' at stop n, since their destination is one of the common stops, and it is calculated by:

$$\Lambda_n^{r,r'} = \sum_{j \in \mathcal{S}^D(n) \cap \mathcal{N}_{com}^{r,r'}} \lambda_{n,j} \tag{5}$$

The boarding demand of a specific bus contains all passengers whose candidate sets include this bus line. At common stop n traversed by line r and line r', the boarding demands $B_{m,n}$ of the bus m of line r (i.e. $m = m_k^r$, $k = 1, 2, 3, ..., |\mathcal{M}^r|$) are composed of $B_{m,n}^r$ and $B_{m,n}^{r,r'}$ defined as follows.

1) $B_{m,n}^r$ denotes the number of passengers whose destinations are dedicated stops of line r.
2) $B_{m,n}^{r,r'}$ denotes the number of passengers whose destination are common stops traversed by line r and line r'.

The number of boarding passengers is calculated by the sum of the two types of passengers:

$$B_{m,n} = B^r_{m,n} + B^{r,r'}_{m,n} \tag{6}$$

Due to the capacity constraint, each type of passenger demand is composed of new arriving passengers and leftover passengers who already arrived before the preceding bus departing. Denote $m^{r,r'}(m,n)$ as the index of the preceding bus departing from stop n before bus m. Mathematically, it is expressed by

$$m^{r,r'}(m,n) = \arg\max_{m'} \left\{ d_{m',n} | d_{m',n} < d_{m,n}, m' \in \mathcal{M}^r \cup \mathcal{M}^{r'} \right\} \tag{7}$$

where $d_{m,n}$, $d_{m',n}$ denote the departure time of bus m and bus m' from stop n, respectively.

The headway between buses m and $m^{r,r'}(m,n)$ at stop n is obtained by:

$$h^{r,r'}_{m,n} = d_{m,n} - d_{m^{r,r'}(m,n),n} \tag{8}$$

Thus, $B^r_{m,n}$, $B^{r,r'}_{m,n}$ and $B_{m,n}$ can be calculated by:

$$B^r_{m,n} = \Lambda^r_n \cdot h^{r,r'}_{m,n} + L^r_{m^{r,r'}(m,n),n} + L^r_{m^{r,r'}(m,n),n} \tag{9}$$

$$B^{r,r'}_{m,n} = \Lambda^{r,r'}_n \cdot h^{r,r'}_{m,n} + L^r_{m^{r,r'}(m,n),n} + L^{r,r'}_{m^{r,r'}(m,n),n} \tag{10}$$

$$B_{m,n} = \left(\Lambda^r_n + \Lambda^{r,r'}_n \right) \cdot h^{r,r'}_{m,n} + L^r_{m^{r,r'}(m,n),n} + L^{r,r'}_{m^{r,r'}(m,n),n} \tag{11}$$

where $L^r_{m^{r,r'}(m,n),n}$ and $L^{r,r'}_{m^{r,r'}(m,n),n}$ denote the numbers of leftover passengers who arrived before the last bus depart and their computations are introduced in 3.3.

The equations for common stops (i.e. Eq.(2)-(11)) are valid for dedicated stops. For example, for a dedicated stop n which only served by a single line r, the formulations are degenerated by $\Lambda^{r'}_n, \Lambda^{r,r'}_n = 0$ and for the k^{th} bus of line r (i.e. $m = m^r_k$, $k = 2, 3, ..., |\mathcal{M}^r|$), $m^{r,r'}(m,n) = m^r_{k-1} = m - 1$. Therefore, Eq.(2),Eq.(8) and Eq.(11) are degenerated into

$$\Lambda_n = \sum_{j \in \mathcal{S}^D(n)} \lambda_{n,j} = \Lambda^r_n, \tag{12}$$

$$h^r_{m,n} = d_{m,n} - d_{m-1,n} \tag{13}$$

and

$$B_{m,n} = B^r_{m,n} = \Lambda^r_n \cdot h^r_{m,n} + L^r_{m-1,n} \tag{14}$$

3.2 Number of boarding passengers under capacity constraint

The actual number of boarding passengers is constrained by the bus available capacity. At common stop n, the actual number of boarding passengers of bus m, denoted as $\overline{B}_{m,n}$, is refined by:

$$\overline{B}_{m,n} = \min\left\{C_{m,n}, B_{m,n}\right\} \tag{15}$$

where $C_{m,n}$ denote the available capacity when bus m arrives at stop n.

Therefore, the formulations of numbers of alighting and onboard passengers are necessary as $C_{m,n}$ is calculated by:

$$C_{m,n} = cap_m - P_{m,n} + A_{m,n} \tag{16}$$

where cap_m denotes the designed capacity of bus m, $P_{m,n}$ denotes the number of onboard passengers of bus m when it arrives at stop n, and $A_{m,n}$ denotes the number of alighting passengers of bus m at stop n. Alighting passengers of bus m at common stop n include the passengers whose destination is stop n and parts of transfer passengers who choose stop n as their interchange stop and $A_{m,n}$ can be calculated by:

$$A_{m,n} = p_{m,n,n} + \overline{\rho}_{m,n} \cdot \sum_{j \in \mathcal{S}^D(n) \backslash \mathcal{S}^r} p_{m,n,j} \tag{17}$$

In Eq.(17), $p_{m,n,j}$ denotes the number of onboard passengers whose destination is stop j when bus m is arriving at the stop n. $\overline{\rho}_{m,n}$ denotes the proportion of passengers transferring at stop n among all Type I) passengers on bus m and is identified by a so-called transfer-based equilibrium introduced in 4.1.

3.3 Numbers of leftover and onboard passengers

Formulations above (i.e. Eq.(11), Eq.(14) and Eq.(16)) indicate that it is necessary to identify the numbers of leftover passengers at stops as well as onboard passengers when buses depart. As the subsequent stop of stop n traversed by line r is denoted as $n^r(n)$, the numbers of leftover and onboard passengers whose destination is stop j when bus m departs from stop n (or arrives at $n^r(n)$) are denoted as $l_{m,n,j}$ and $p_{m,n^r(n),j}$, respectively. And they can be obtained by:

$$l_{m,n,j} = \begin{cases} \left(\lambda_{n,j} \cdot h_{m,n}^{r,r'} + l_{m^{r,r'}(m,n),n,j}\right) \cdot \left(1 - \frac{\overline{B}_{m,n}}{B_{m,n}}\right) & \text{if } j \in \mathcal{S}^D(n) \cap \mathcal{S}^r \\ \lambda_{n,j} \cdot h_{m,n}^{r,r'} + l_{m^{r,r'}(m,n),n,j} + \overline{\rho}_{m,n} \cdot p_{m,n,j} & \text{if } j \in \mathcal{S}^D(n) \backslash \mathcal{S}^r \end{cases} \tag{18}$$

and

$$p_{m,n^r(n),j} = \begin{cases} p_{m,n,j} + \left(\lambda_{n,j} \cdot h_{m,n}^{r,r'} + l_{m^{r,r'}(m,n),n,j}\right) \cdot \frac{\overline{B}_{m,n}}{B_{m,n}} & \text{if } j \in \mathcal{S}^D(n) \cap \mathcal{S}^r \\ \left(1 - \overline{\rho}_{m,n}\right) \cdot p_{m,n,j} & \text{if } j \in \mathcal{S}^D(n) \backslash \mathcal{S}^r \end{cases} \tag{19}$$

There are two points need to acknowledge in Eq.(18) and Eq.(19):

1) The proportion of each kind of passengers in the boarding passengers is assumed to be equal to the proportion of this kind of passengers in the accumulated queue passengers in boarding progress. In other words, the probabilities of boarding successfully of all passengers involved in a boarding process is equal to the ratio of the number of boarding passengers to the boarding demand (i.e. $\frac{\overline{B}_{m,n}}{B_{m,n}}$ in Eq.(18) and Eq.(19)).
2) The concept of *leftover passengers* is different from that in single-line models where it only refers to passengers failed to board an earlier bus due to the capacity constraint. For example, the leftover passengers at a common stop when bus m of line r departs include all passengers whose candidate set of lines doesn't contain line r.

Then, the total numbers of leftover and onboard passengers can be calculated by:

$$P_{m,n^r(n)} = \sum_{j \in \mathcal{S}^D(n)} p_{m,n^r(n),j} \tag{20}$$

and

$$L_{m,n} = \sum_{j \in \mathcal{S}^D(n)} l_{m,n,j} \tag{21}$$

To facilitate the computations of boarding demands (i.e. Eq.(9), Eq.(10) and Eq.(11)) in 3.1, the numbers of leftover passengers divided according to their candidate lines are needed:

$$L^r_{m,n} = \sum_{j \in \mathcal{S}^D(n) \backslash \mathcal{S}^{r'}} l_{m,n,j} \tag{22}$$

$$L^{r'}_{m,n} = \sum_{j \in \mathcal{S}^D(n) \backslash \mathcal{S}^{r}} l_{m,n,j} \tag{23}$$

$$L^{r,r'}_{m,n} = \sum_{j \in \mathcal{S}^D(n) \cap \mathcal{S}^r \cap \mathcal{S}^{r'}} l_{m,n,j} \tag{24}$$

3.4 Bus dwell time constrained by bus capacity and passenger alighting time

Inspired by existing work about single-line model considering bus capcaity constant (i.e. Wu et al., 2017), we derive bus dwell time by formulating the queue clearance time of passenger boarding demand and constrain it by available capacity on bus and passenger alighting time. Based on the assumption of constant passengers' arrivals and boarding rates, the boarding queue clearance time can be obtained by $\frac{\text{The initial queue length}}{\text{The queue clearance rate}}$. Particularly, at a common stop n traversed by line r and line r', the boarding queue clearance time for bus m denoted by $W^B_{m,n}$ can be obtained by:

$$W^B_{m,n} = \frac{\left(a_{m,n} - d_{m^{r,r'}(m,n),n}\right) \cdot \left(\Lambda^r_n + \Lambda^{r,r'}_n\right) + L^r_{m^{r,r'}(m,n),n} + L^{r,r'}_{m^{r,r'}(m,n),n}}{\beta - \left(\Lambda^r_n + \Lambda^{r,r'}_n\right)} \tag{25}$$

where $a_{m,n}$ denotes the arrival time of bus m at stop n and β denotes the average boarding rate of passengers. In this work, it is assumed that the boarding rate is much larger than passenger arrival rates (i.e. $\beta \gg \Lambda_n$ for all stop n).

For the simplicity of formulas, we introduce the bus service empty period referring to the period during which there is no bus dwelling at the stop. For example, when the bus service empty period at stop n ended up with the arrival of m, the length of the period, denoted by $I_{m,n}^{r,r'}$, is obtained by:

$$I_{m,n}^{r,r'} = a_{m,n} - d_{m^{r,r'}(m,n),n} \tag{26}$$

Then, the expression of boarding queue clearance time can be expressed as:

$$W_{m,n}^{B} = \frac{I_{m,n}^{r,r'} \cdot \left(\Lambda_n^r + \Lambda_n^{r,r'}\right) + L_{m^{r,r'}(m,n),n}^{r} + L_{m^{r,r'}(m,n),n}^{r,r'}}{\beta - \left(\Lambda_n^r + \Lambda_n^{r,r'}\right)} \tag{27}$$

The passenger alighting time of bus m of line r at stop n, denoted as $W_{m,n}^{A}$, can be calculated by:

$$W_{m,n}^{A} = \frac{A_{m,n}}{\alpha} \tag{28}$$

where α denotes the average alighting rate of passengers.

As a result, the actual dwell time of bus m at stop n, denoted as $W_{m,n}$, is the maximum value between the boarding and alighting time:

$$W_{m,n} = \max\left\{W_{m,n}^{A}, \min\left\{W_{m,n}^{B}, \frac{C_{m,n}}{\beta}\right\}\right\} \text{ for } m \in \mathcal{M}^1 \cup \mathcal{M}^2; n \in \mathcal{S}^1 \cup \mathcal{S}^2 \tag{29}$$

3.5 Algorithm for bus trajectories

Before conducting the calculations for bus dwell times and bus trajectories, some initialization is needed to start the algorithm.

1) Initialize the bus arrival times at the first stop of each line

We denote the arrival time at the first bus stop of the first bus of line r as t^r (for $r = 1, 2$), based on the assumption about constant bus depart frequency in A4), the arrival times of all buses of line r at the first stop can be determined by:

$$a_{m_k^r, n_1^r} = t^r + H^r \cdot (k-1) \text{ for } k = 1, 2, ..., |\mathcal{M}^r| \tag{30}$$

where H^r denotes the departure headway of line r.

2) Initialize dwell times of the first bus of each line at all stops served by it

As the first bus of each line ($m = m_1^r,$ for $r = 1, 2$) has no preceding bus, we presuppose:

$$I_{m,n}^r = H^r, \text{for } m = m_1^r; n = n_k^r; k = 1, 2, ..., |\mathcal{S}^r|; r = 1, 2 \tag{31}$$

and

$$I_{m,n}^{r,r'} = \left(\frac{1}{H^r} + \frac{1}{H^{r'}}\right)^{-1}, \text{for } m = m_1^r; n = n_k^r; k = 1, 2, ..., |\mathcal{S}^r|; r = 1, 2 \tag{32}$$

The bus dwell times of the first bus of each line at all served stops can be obtained according to Eq.(27).

With the initialization above, the pseudocode of the algorithm for simulating bus trajectories is presented in **Algorithm 1**.

Algorithm 1 Event-based algorithm for bus trajectories

Input parameters: α,β,ε,$\lambda_{i,j}$,$T_{m,n}$
Initialization: $a_{m_k^r,n_1^r}$ for $k = 1, 2, ..., |\mathcal{M}^r|$
for $\mathcal{N} \in \left\{\mathcal{N}_{up}^1, \mathcal{N}_{up}^2, \mathcal{N}_{com}^{1,2}, \mathcal{N}_{down}^1, \mathcal{N}_{down}^2\right\}$ **do**
  for $n \in \mathcal{N}$ **do**
    Get the service bus fleet of stop n: $\mathcal{M}^S(n)$
    for $m \in \mathcal{M}^S(n)$ sorted by their arrival time at stop n **do**
      Calculate the number of alighting passengers $A_{m,n}$ by Eq.(17)
      Calculate the available capacity $C_{m,n}$ by Eq.(16)
      Calculate the bus dwell time $W_{m,n}$ of bus m at stop n by Eq.(27), Eq.(29) and Eq.(28)
      Calculate the departure time of bus m from stop n by:

$$d_{m,n} = a_{m,n} + W_{m,n} \tag{33}$$

      Calculate the arrival time of bus m at the next stop by Eq.(34)
      Calculate the number of boarding passengers $B_{m,n}$ by Eq.(11)
      Calculate the number of boarding passengers $\bar{B}_{m,n}$ by Eq.(15)
      Calculate the number of onboard passengers after boarding and alighting with Eq.(19)
      Calculate the number of leftover passengers after boarding and alighting with Eq.(18)
      if n is not the last stop of its line (i.e. $n \neq n_{|\mathcal{S}^r|}^r$) **then**
        Calculate the arrival time of bus m at the next stop (without loss of generality, assume bus m belongs to line r):

$$a_{m,n^r(n)} = d_{m,n} + T_{m,n} \tag{34}$$

      end if
    end for
  end for
end for

Moreover, there is one thing worthy noting. According to A5), the subsequent bus will queue outside the platform if the bus arrived before the boarding process of its preceding bus finished (i.e. $a_{m,n} < d_{m^{r,r'}(m,n),n}$). In Eq.(27), we can notice that the bus boarding time could be zero without the minimum headway, and the subsequent bus may depart together with its ahead bus in the case of no alighting demand. Therefore, we modify the actual time arrival at platform by a minimum headway denoted as $\varepsilon > 0$:

$$a_{m,n} = \max\left\{a_{m,n}, d_{m^{r,r'}(m,n),n} + \varepsilon\right\} \text{ for } m \in \mathcal{M}^1 \cup \mathcal{M}^2; n \in \mathcal{S}^1 \cup \mathcal{S}^2 \quad (35)$$

4 Modeling transfer passenger routing

4.1 Transfer-based equilibrium

Formulas about numbers of alighting, onboard and leftover passengers (i.e. Eq.(17), Eq.(19) and Eq.(18)) are related to the probabilities for transfer passengers onboard to alight at each stop (i.e. $\bar{\rho}_{m,n}, m \in \mathcal{M}^r \cup \mathcal{M}^{r'}, n \in \mathcal{N}_{com}^{r,r'}$), which represent transfer passengers' routing choice among all candidate interchange stops.

Therefore, like approach-based equilibrium formulations (Long et al., 2013; Szeto et al., 2014; Jiang et al., 2016), we formulate a transfer-based equilibrium among all Type I) passengers on each bus to model their transfer behavior. The proportion of transfer passengers on bus m those transfer to the other line via common stop n_i are denoted as $\alpha_{n_i}^m$. And then, $\bar{\rho}_{m,n_i}$ can be written by the approach-probabilities $\alpha_{n_i}^m$ for all common stop $n_i \in \mathcal{N}_{com}^{r,r'}$ as:

$$\bar{\rho}_{m,n_i} = \begin{cases} \alpha_{n_i}^m & \text{for } i = 1, \\ \dfrac{\alpha_{n_i}^m}{1 - \sum\limits_{k=1}^{i-1} \alpha_{n_k}^m} & \text{for } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \end{cases} \quad (36)$$

Specifically, routing approaches of transfer passengers from upstream stops of line r' (i.e. stop $n \in \mathcal{N}_{up}^{r'}$) to downstream stops of line r (i.e. stop $n \in \mathcal{N}_{down}^{r}$) via common stops (i.e. stop $n_1, n_2, ..., n_{\left|\mathcal{N}_{com}^{r,r'}\right|} \in \mathcal{N}_{com}^{r,r'}$) are intuitively demonstrated by the network shown in Fig.2.

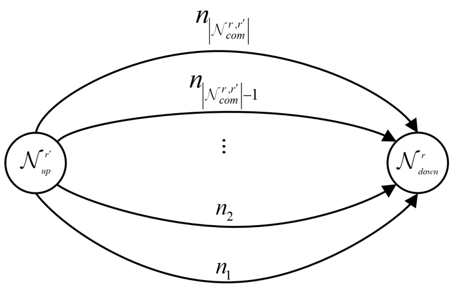


Fig. 2 Transfer from from upstream stops of line r' to downstream stops of line r via common stops

As the transfer-based equilibrium conditions are:

$$\omega_{n_i}^m \begin{cases} = \omega^m, & \text{if } \alpha_{n_i}^m > 0 \\ \geq \omega^m, & \text{if } \alpha_{n_i}^m = 0 \end{cases} \text{ for all } n_i \in \mathcal{N}_{com}^{r,r'} \quad (37)$$

where $\omega_{n_i}^m$ denotes the transfer cost for passengers on bus m those transfer via stop n_i, the transfer-based equilibrium constraints can be represented as follows:

$$\begin{cases} \left(\omega_{n_i}^m - \omega^m\right) \cdot \alpha_{n_i}^m = 0 & \forall n_i \in \mathcal{N}_{com}^{r,r'} \\ \omega_{n_i}^m - \omega^m \geq 0 & \forall n_i \in \mathcal{N}_{com}^{r,r'} \\ 0 \leq \alpha_{n_i}^m \leq 1 & \forall n_i \in \mathcal{N}_{com}^{r,r'} \\ \sum\limits_{n_i \in \mathcal{N}_{com}^{r,r'}} \alpha_{n_i}^m = 1 & \end{cases} \tag{38}$$

4.2 Transfer cost function

To implement the assignment of transfer passengers, explicit transfer cost function is necessary. Without loss of generality, we assume bus m is the following bus belonging line r that arrives at stop n_i after its preceding bus of line r' which is denoted by $m^{r'}(m, n_i)$ and can be expressed by:

$$m^{r'}(m,n) = \arg\max_{m'} \left\{ d_{m',n} | d_{m',n} < d_{m,n}, m' \in \mathcal{M}^{r'} \right\} \tag{39}$$

According to A3), bus travel times of two lines between successive common stops in the corridor are equal (i.e. $T_{m,n_i} = T_{m',n_i}, \forall m \in \mathcal{M}^r, m' \in \mathcal{M}^{r'}, n_i \in \mathcal{N}_{com}^{r,r'}$). Therefore, the arrival sequences of service buses at all common stops are as same as that at the first common stop n_1:

$$m^{r'}(m, n_i) = m^{r'}(m, n_1), \forall n_i \in \mathcal{N}_{com}^{r,r'} \tag{40}$$

Due to the capacity constraint, part of transfer passengers on bus $m^{r'}(m, n_i)$ may fail to board the following arrival bus m as shown in Fig.3.

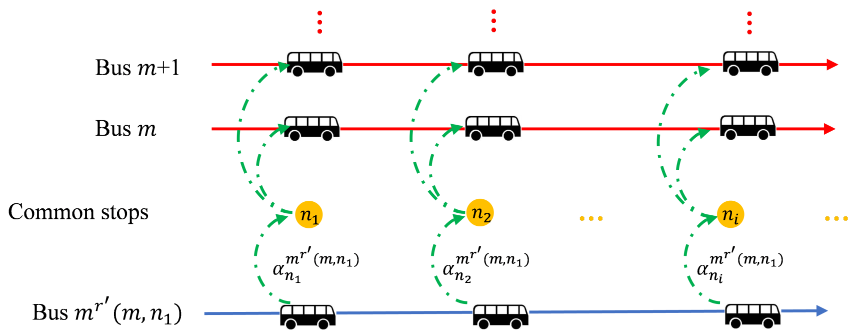


Fig. 3 Transfer passengers from bus $m^{r'}(m, n_i)$ to the other line via common stops

The expected transfer cost function at each stop involves probabilities that transfer passengers successfully board the arrival buses. For example, the expected cost at stop

n_i for transfer passengers from bus $m^{r'}(m,n_i)$ is:

$$\begin{aligned}\omega_{n_i}^{m^{r'}(m,n_i)} =& \frac{\overline{B}_{m,n_i}}{B_{m,n_i}} \cdot \omega_{n_i}^{m^{r'}(m,n_i),m} + \\ & \left(1-\frac{\overline{B}_{m,n_i}}{B_{m,n_i}}\right) \cdot \frac{\overline{B}_{m+1,n_i}}{B_{m+1,n_i}} \cdot \omega_{n_i}^{m^{r'}(m,n_i),(m+1)} + ...\end{aligned} \tag{41}$$

where $\omega_{n_i}^{m^{r'}(m,n_i),m}$, $\omega_{n_i}^{m^{r'}(m,n_i),(m+1)}$,... denote the transfer time passengers from bus $m^{r'}(m,n_i)$ to bus $m, m+1, ...$, respectively. The transfer cost between from bus $m^{r'}(m,n_i)$ to bus m is derived by the headway of the two buses:

$$\omega_{n_i}^{m^{r'}(m,n_i),m} = d_{m,n_i} - d_{m^{r'}(m,n_i),n_i} \tag{42}$$

4.3 MSA

To assign transfer passengers based on the transfer cost function proposed in 4.1, we design an MSA iteration.

Algorithm 2 MSA for transfer routing results

Step 0: Initialization
Assign the transfer passengers evenly among all common stops as an initial solution:

$$k = 0 : \alpha_{n_i}^{m}(1) = \frac{1}{\left|\mathcal{N}_{com}^{r,r'}\right|} \text{ for } i = 1, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \text{ and } m \in \mathcal{M}^r \cup \mathcal{M}^{r'} \tag{43}$$

Step 1: $k \leftarrow k+1$
Get all buses' transfer cost at each common stop by Eq.(41) in the k^{th} iteration: $\omega_{n_i}^{m'}(k)$
Step 2:
Get attached solution by all-or-nothing assignment: $\alpha_{n_i}^{m}(k)$
Calculate the new solution the average between previous solution and attached solution:

$$\begin{aligned}\alpha_{n_i}^{m}(k) \leftarrow \left(1-\frac{1}{k}\right) \cdot \alpha_{n_i}^{m}(k-1) + \frac{1}{k} \cdot \alpha_{n_i}^{m}(k) \\ \forall i = 1, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \text{ and } m \in \mathcal{M}^r \cup \mathcal{M}^{r'}\end{aligned} \tag{44}$$

if $\alpha_{n_i}^{m}(k) - \alpha_{n_i}^{m}(k-1) > \varepsilon$ **then**
  back to **Step 1**
else
  Stop.
end if

4.4 A special realization of equilibrium assignment

Based on the formulations of transfer-based equilibrium and transfer cost function, a special realization of equilibrium assignment is noticeable. According to the proposed transfer-based equilibrium, the Wardrop's first principle is fulfilled: for all common stops selected as interchange stops, transfer costs from line 1 to line 2 (or form line 2 to line 1) are equal. For a common corridor with homogeneous boarding demand and served by two symmetrical bus lines (i.e. with the same service frequency and travel times on roads), the equilibrium assignment is analytical. The explicit statement is concluded in Proposition 1 and a numerical verification based on MSA will be introduced in 5.2.

Proposition 1 *In a two-line system, equal assignment of transfer passengers among all candidate interchange stops is a special realization of equilibrium assignment when buses trajectories without disturbances are uniform in time-space diagram and all buses are not full-loaded.*

Proof See Appendix. □

5 A small network to illustrate the model

In this section, we use a small two-line transit network as an example to show the properties of the proposed model by demonstrating the effects of transfer behavior and capacity constraint on bus bunching.

5.1 System specifications

As shown in Fig.4, the example consists 10 stops divided into 5 stop sets: $\mathcal{N}_{up}^1 = \{1,2\}, \mathcal{N}_{up}^2 = \{3,4\}, \mathcal{N}_{com}^{1,2} = \{5,6\}, \mathcal{N}_{down}^1 = \{7,8\}, \mathcal{N}_{down}^2 = \{9,10\}$. The constant bus departure intervals of two bus lines are set as $H^1 = H^2 = 6$ (min), and the first bus of line 2 departs 3 min later than the first bus of line 1 (i.e. $t^1 = 0, t^2 = 3$). All stops except the two terminals have the same passenger arrival rates as 5 pas/min (i.e. $\Lambda_n = 5$ for $n \in \mathcal{S}^1 \cup \mathcal{S}^2 \setminus \{8, 10\}$). The arrival rate at each stop is divided evenly among all destination stops:

$$\lambda_{n,j} = \frac{\Lambda_n}{|\mathcal{S}^D(n)|}; \forall j \in \mathcal{S}^D(n); n \in \mathcal{S}^1 \cup \mathcal{S}^2 \setminus \left\{ n^1_{|\mathcal{S}^1|}, n^2_{|\mathcal{S}^2|} \right\} \tag{45}$$

The travel times for all buses in each link between stops are set as 3 min (i.e. $T_{m,n} = 3, m \in \mathcal{M}^1 \cup \mathcal{M}^2, n \in \mathcal{S}^1 \cup \mathcal{S}^2 \setminus \{8, 10\}$).

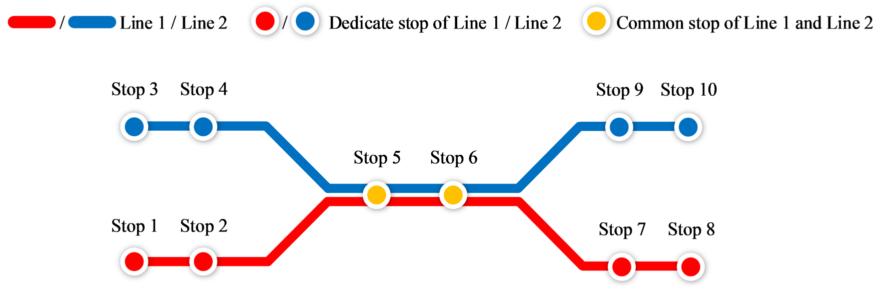
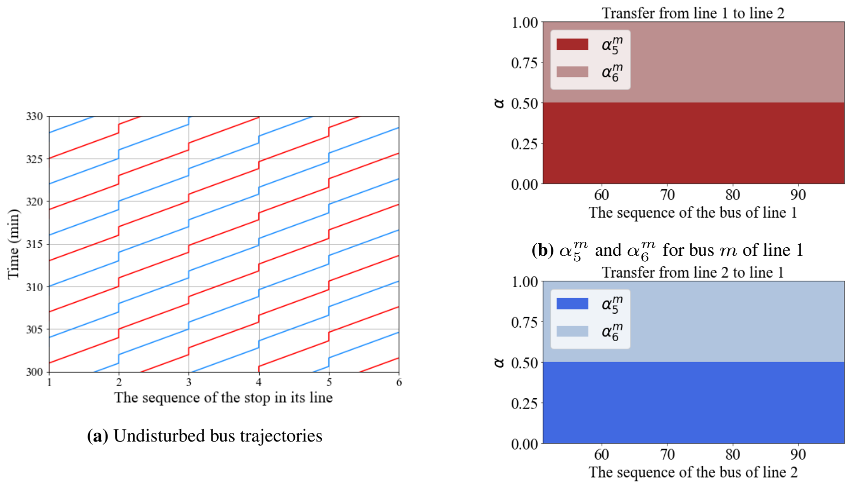


Fig. 4 An example of two-line systems with 10 stops

5.2 Effect of transfer behaviors

With the parameters above, we conduct algorithms for bus trajectories and transfer passengers' routing results. Fig.5a shows bus trajectories in time-space diagram in which horizontal axis is corresponding to the sequence of the stop in its line to facilitate the demonstration of the bus bunching phenomenon in the common corridor. We observe that buses run steadily and there is no bus full-loaded. Routing result of transfer passengers shown in Fig.5b,5c demonstrates that the equilibrium assignment degenerates to the equal assignment (i.e. $\alpha_5^m = \alpha_6^m = 0.5$ for $m \in \mathcal{M}^1 \cup \mathcal{M}^2$) as a specific example verifying Proposition 1.



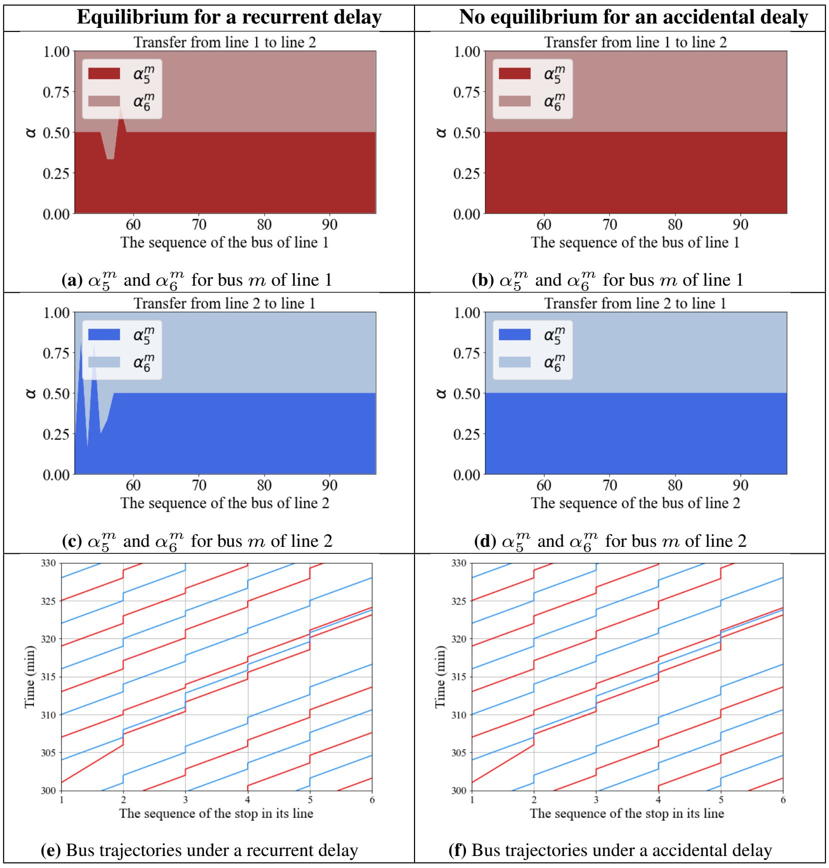
(a) Undisturbed bus trajectories

(b) α_5^m and α_6^m for bus m of line 1

(c) α_5^m and α_6^m for bus m of line 2

Fig. 5 Undisturbed bus trajectories and routing result of transfer passengers

The routing result of transfer passengers shown in Fig.5b,5c represents a routing behavior based on the expected bus operation without disturbance. If there is a recurrent delay or a long-lived control strategy in bus operation, the routing behaviors of transfer passengers tend to a new equilibrium. As a comparison, when the system encounters an accidental delay or an interim control strategy, the routing behaviors of transfer passengers onboard won't form an equilibrium as they are not informed with the operation. We introduce a 2 min travel delay occurring to one bus of line 1 between stop 1 and stop 2, and the bus dwellings at subsequent stops of this line and downstream stops of the other line could be involved in a bunching problem. As shown in Fig.6, the buses trajectories and transfer passengers' routing results are different under a recurrent delay. The deviations of bus trajectories casused by the delay is shown in Fig.7. It demonstrates that a recurrent delay may cause heavier bus bunching problem maintaining longer time and affecting more buses than transfer passengers' routing behavior bus operations makes different impacts on bus bunching comparing with that based on their expected bus operation.



(a) α_5^m and α_6^m for bus m of line 1

(b) α_5^m and α_6^m for bus m of line 1

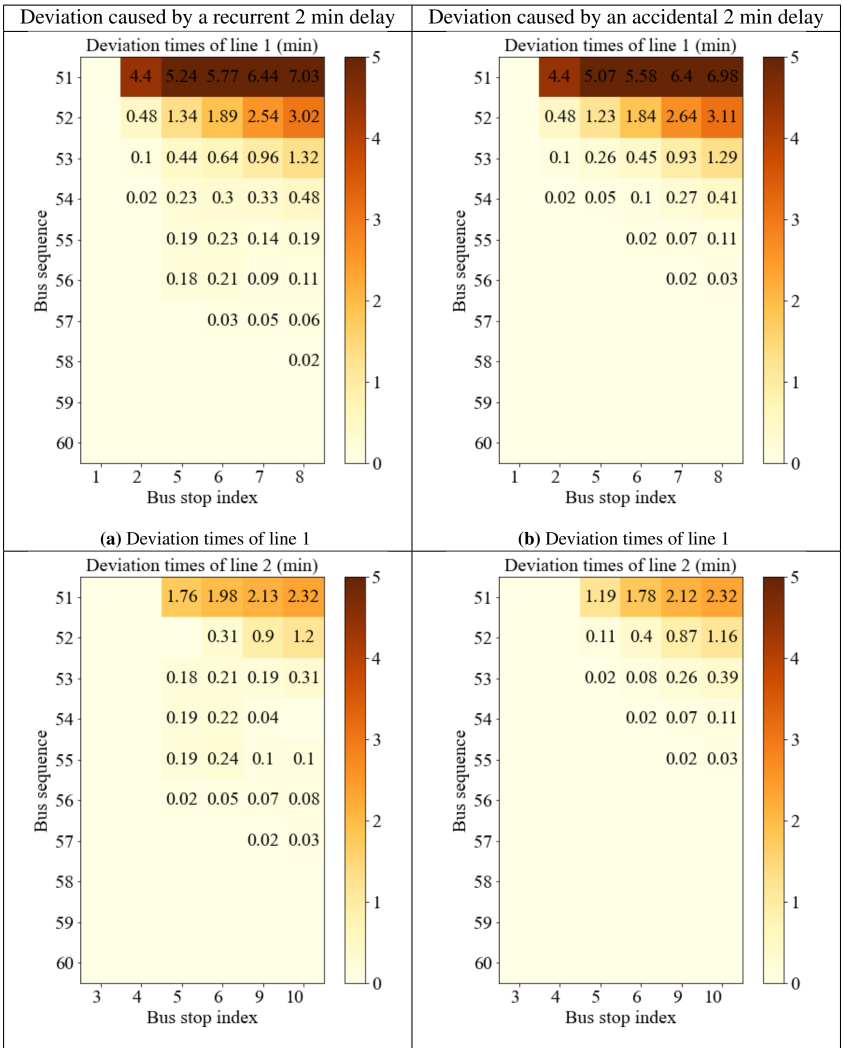
(c) α_5^m and α_6^m for bus m of line 2

(d) α_5^m and α_6^m for bus m of line 2

(e) Bus trajectories under a recurrent delay

(f) Bus trajectories under a accidental delay

Fig. 6 Different transfer routing results and bus trajectories under a recurrent delay and an accidental delay



(a) Deviation times of line 1 **(b)** Deviation times of line 1 **(c)** Deviation times of line 2 **(d)** Deviation times of line 2

Fig. 7 Different deviations of bus dwell times caused by a recurrent delay and an accidental delay

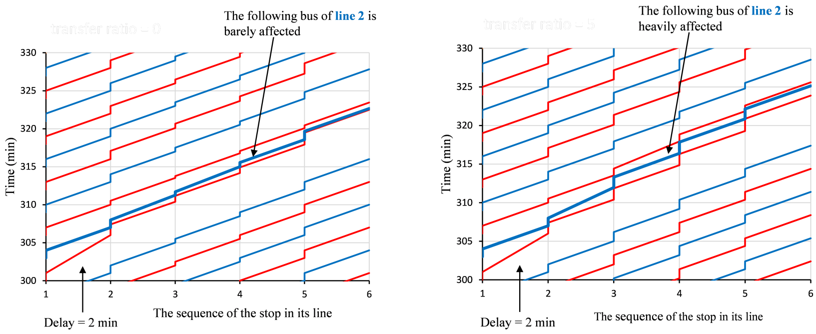
What's more, we conduct numerical experiments to confirm the effect of the presence of transfer passenger flow on bus bunching. First, we introduce a transfer parameter $\mu \geq 0$ to adjust the proportion of transfer passengers among all passenger demand. For example, let us consider the i^{th} upstream stop of line r (i.e. $n = n_i^r$). As the destination stops set of stop n is $\mathcal{S}^D(n) = \mathcal{S}^D(n_i^r) = \{n_i^r | n_k^r \in \mathcal{S}^r, k > i\} \cup \mathcal{N}_{down}^{r'}$, we calculate the passenger arrival rate with the destination stop j by

$$\lambda_{n,j} = \begin{cases} \frac{\Lambda_n}{\left|\left\{n_k^r | n_k^r \in \mathcal{S}^r, k>i\right\}\right| + \mu \cdot \left|\mathcal{N}_{down}^{r'}\right|} & j \in \{n_k^r | n_k^r \in \mathcal{S}^r, k > i\} \\ \frac{\mu \cdot \Lambda_n}{\left|\left\{n_k^r | n_k^r \in \mathcal{S}^r, k>i\right\}\right| + \mu \cdot \left|\mathcal{N}_{down}^{r'}\right|} & j \in \mathcal{N}_{down}^{r'} \end{cases} \tag{46}$$

In this way, the proportion of transfer passengers increases with μ while the total arrival rate calculated by Eq.(1) remains constant. There are two special cases:

1) when $\mu = 0$, there is no transfer passengers between two lines.
2) when $\mu = 1$, the composition of arrival passengers degenerates to the basic pattern as Eq.(45).

We set two conditions as $\mu = 0$ and $\mu = 5$ for a comparison and the bus trajectories are shown in the Fig.8a and Fig.8b. It is observed that the bus of line 2 (i.e. blue lines) following the disturbed bus of line 1 (i.e. red lines) is barely affected in the system with no transfer passengers but heavily affected in the second condition. The comparison demonstrates that the presence of transfer passenger flow enhances the propagation of bus bunching problem between bus lines.



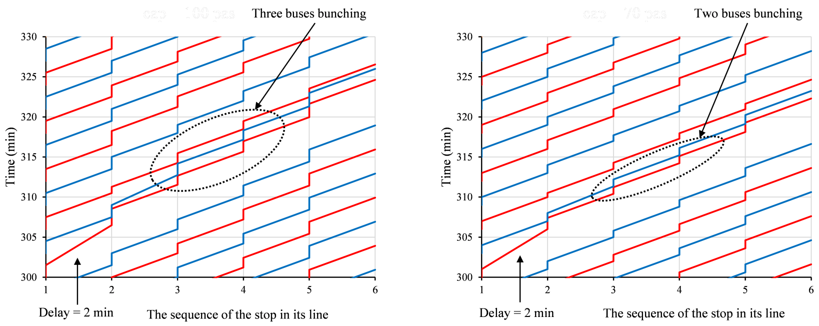
(a) Disturbed bus trajectories with $\mu = 0$

(b) Disturbed bus trajectories with $\mu = 5$

Fig. 8 Effect of different transfer parameters

5.3 Effect of capacity constraint

We devise a comparison between two different capacity settings where the bus capacity of all buses are set as 150 pas and 80 pas while other parameters specified in 5.1 remain the same. The bus trajectories obtained are shown in Fig.9a and Fig.9b. The bus trajectories demonstrate that the bus bunching phenomenon under different capacity settings are significantly different. Specifically, there are three buses bunching at the common corridor (i.e. when a bus arrives at the bus stop, the preceding bus is still at the bus stop) when the bus capacities are set as 150 pas while only two buses are observably involved when the bus capacities are set as 80 pas. As we mentioned in 3.2, the bus capacity is the upper bound of the number of onboard passengers and limits the bus dwell time at stops when a bus encounters a large boarding demand caused by bus bunching.



(a) Disturbed bus trajectories with $cap = 150$ (pas) **(b)** Disturbed bus trajectories with $cap = 80$ (pas)

Fig. 9 Effect of different capacity settings

6 Evaluation and sensitivity analyses

In this section, a set of evaluation indices are introduced to quantify the performance of bus operation affected by bus bunching.

6.1 Range of evaluation

As we trigger bus bunching by an initial disturbance, we focus on evaluating the operation of affected buses instead of the overall system. For the example shown in Fig.6e, Fig.6a and Fig.6c, we put a 2 min delay to the 51^{st} bus of line 1 (red line), and then, 56 bus arrivals or departures deviate from the undisturbed trajectories as shown in Fig.10.

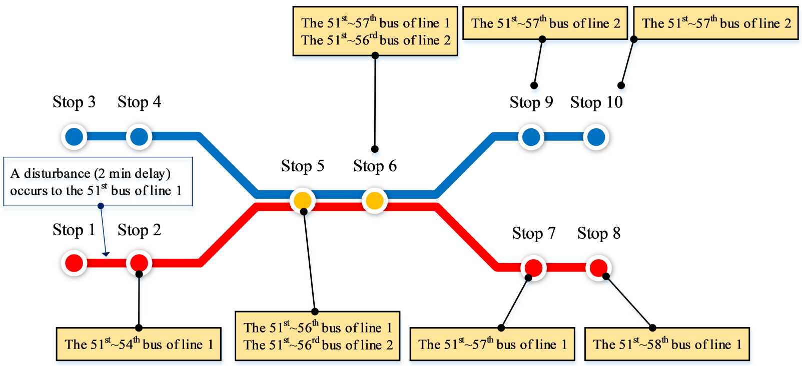


Fig. 10 A disturbance occurs to the 51^{st} bus of line 1 (red line) from stop 1 to stop 2 and subsequent buses and downstream stops are affected

The numbers of affected buses at stops reveal the temporal and spatial distribution of the effect of bus bunching to some extent. As shown in Fig.7, bus bunching problem is heavier in the downstream of the bus line and transfer passengers' routing based on real-time bus operation information makes bus bunching maintain longer time comparing with transfer passengers' routing based on historical or expected bus operation.

To facilitate the calculation of evaluation indices introduced following, we introduce a binary variable:

$$x_{m,n} = \begin{cases} 1 & \text{if the dwelling of bus } m \text{ at stop } n \text{ is affected;} \\ 0 & \text{otherwise.} \end{cases} \tag{47}$$
$$\forall m \in \mathcal{M}^1 \cup \mathcal{M}^2; n \in \mathcal{S}^1 \cup \mathcal{S}^2$$

6.2 Evaluation measurements

We adopt two evaluation indexes: the average waiting time at stops and the standard deviation of bus time headway. Passengers' waiting time at stops and headway variability are commonly used as evaluation measurements and optimization objectives in related literature (i.e. Delgado et al., 2012; Hernández et al. 2015; Liang et al., 2016; Schmöcker et al., 2016; Wu et al., 2017).

The passengers waiting for bus m of line r at common stop n can be divided into four groups:

1) The passengers arrived after the departure of bus $m-1$ who take only line r.
2) The passengers arrived before the departure of bus $m-1$ who take only line r.
3) The passengers arrived after the departure of bus $m^{r,r'}(m,n)$ who take both line r and line r'.
4) The passengers arrived before the departure of bus $m^{r,r'}(m,n)$ who take both line r and line r'.

Based on the constant arrival rates assumption in A2), the total waiting time of the four groups of passengers for bus m of line r can be estimated as $\frac{h^r_{m,n}}{2}, h^r_{m,n}, \frac{h^{r,r'}_{m,n}}{2}$ and $h^{r,r'}_{m,n}$ respectively. The total waiting time for bus m of line r at stop n can be derived as:

$$w^r_{m,n} = \frac{1}{2}\Lambda^r_n \cdot {h^r_{m,n}}^2 + \frac{1}{2}\Lambda^{r,r'}_n \cdot {h^{r,r'}_{m,n}}^2 + L^r_{m-1,n} \cdot h^r_{m,n} + L^r_{m-1,n} \cdot h^r_{m,n} \tag{48}$$

The average waiting time of passengers in study range can be calculated by:

$$\bar{w} = \frac{\sum\limits_{n \in \mathcal{S}^1 \cup \mathcal{S}^2} \sum\limits_{m \in \mathcal{M}^1 \cup \mathcal{M}^2} x_{m,n} \cdot w^r_{m,n}}{\sum\limits_{n \in \mathcal{S}^1 \cup \mathcal{S}^2} \sum\limits_{m \in \mathcal{M}^1 \cup \mathcal{M}^2} x_{m,n} \cdot \bar{B}_{m,n}} \tag{49}$$

Similarly, the average waiting time of a single line r can be calculated by:

$$\bar{w}^r = \frac{\sum\limits_{n \in \mathcal{S}^r} \sum\limits_{m \in \mathcal{M}^r} x_{m,n} \cdot w^r_{m,n}}{\sum\limits_{n \in \mathcal{S}^r} \sum\limits_{m \in \mathcal{M}^r} x_{m,n} \cdot \bar{B}_{m,n}} \tag{50}$$

The standard deviation of the time headway of affected buses of line r is calculated by

$$\sigma^r = \sqrt{\frac{\sum_{n\in\mathcal{S}^r}\sum_{m\in\mathcal{M}^r} x_{m,n}\cdot\left(h^r_{m,n}-\bar{h}^r\right)^2}{\sum_{n\in\mathcal{S}^r}\sum_{m\in\mathcal{M}^r} x_{m,n}}} \tag{51}$$

where $\bar{h}^r = \frac{\sum_{n\in\mathcal{S}^r}\sum_{m\in\mathcal{M}^r} x_{m,n}\cdot h^r_{m,n}}{\sum_{n\in\mathcal{S}^r}\sum_{m\in\mathcal{M}^r} x_{m,n}}$ denotes the mean headway of line r among affected buses.

6.3 Sensitivity analysis

Sensitive analysis is carried out to demonstrate the impacts of bus capacity and transfer flows on the bus bunching. Considering a busy system with high demands and high-frequency services, we set the network structure as $\left|\mathcal{N}^1_{up}\right| = \left|\mathcal{N}^2_{up}\right| = \left|\mathcal{N}^{1,2}_{com}\right| = \left|\mathcal{N}^1_{down}\right| = \left|\mathcal{N}^2_{down}\right| = 5$ and the passenger arrival rates as 7.5 pas/min for all stops except the two terminals. The bus departure intervals for both lines are set as 3 min (i.e. $H^1 = H^2 = 3$), and the first bus of line 2 departs 1.5 min later than the first bus of line 1 (i.e. $t^1 = 0, t^2 = 1.5$). The settings about bus travel times and disturbance are as same as the example in Section 5.

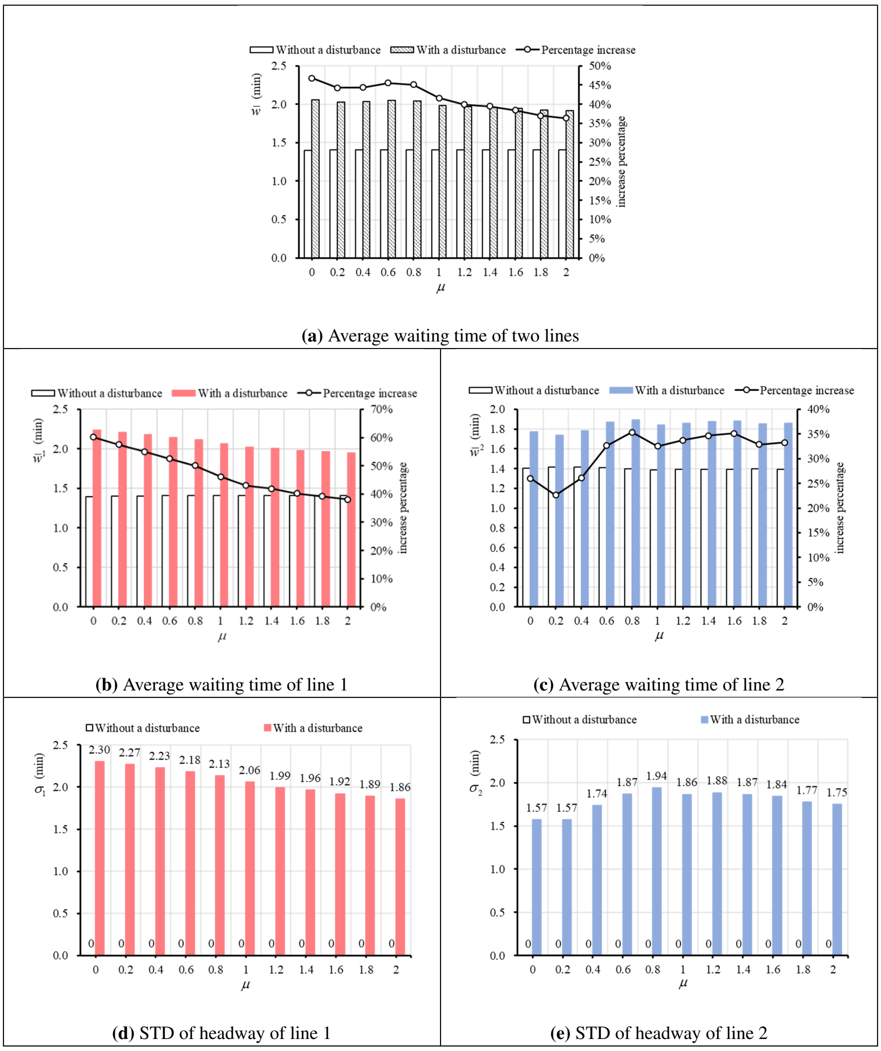
Table 2 Basic parameter settings

Basic parameter settings		**Unit**
Boarding rate	$\beta = 30$	pas/min
Alighting rate	$\alpha = 40$	pas/min
The minimum headway	$\varepsilon = 0.1$	min
Disturbance	$delay = 2$ occurs in the 51^{st} bus of line 1 in the travel link from stop 1 to stop 2	min
Network	$\|\mathcal{N}^1_{up}\|$=$\|\mathcal{N}^2_{up}\|$=$\|\mathcal{N}^{1,2}_{com}\|$=$\|\mathcal{N}^1_{down}\|$=$\|\mathcal{N}^2_{down}\|$=5	
Bus departure timetable	$H^1 = H^2 = 3, t^1 = 0, t^2 = 1.5$	min
Passenger demand	$\Lambda_n = 7.5, \forall n \in \mathcal{S}^1 \cup \mathcal{S}^2 \setminus \{20, 25\}$	pas/min
Transfer parameter	$\mu = 1$	
Travel times	$T_{m,n} = 6$, $\forall m \in \mathcal{M}^1 \cup \mathcal{M}^2, n \in \mathcal{S}^1 \cup \mathcal{S}^2 \setminus \{20, 25\}$	min
Capacity	$cap_m = 150, \forall m \in \mathcal{M}^1 \cup \mathcal{M}^2$	pas
The routing of transfer passengers	Without real-time information	

6.3.1 Sensitivity to transfer parameter

The sensitivity to transfer parameter considers the numerical experiments with the transfer parameter from 0, 0.2, up to 2 while other parameters follow the basic settings introduced in Table 2. The results shown in Fig.11 demonstrate that the impacts of transfer behaviors on bus operation change with different degrees of transfer passenger flow. As the proportion of transfer passengers increase, the effect of disturbance on line 1 tends to be alleviated judged from passengers' average waiting time at stops and bus headway variability. However, as the average waiting time and headway irregularity of line 1 both reduce significantly and monotonously, bus operation of line 2 shows complex change.

In fact, transfer passengers lead a propagation of bus bunching problem at common stops. For the initially disturbed line 1, the boarding demands of transfer passengers are from the undisturbed upstream of line 2. In other words, the impacts of bus bunching on line 1 is reduced as the effects of the disturbance are shared by both lines.



(a) Average waiting time of two lines

(b) Average waiting time of line 1

(c) Average waiting time of line 2

(d) STD of headway of line 1

(e) STD of headway of line 2

Fig. 11 The sensitivity analysis of transfer parameter

6.3.2 Sensitivity to bus capacity

In this analysis, the bus capacity is set from 120, 140 up to 200 (pas) while other parameters follow the basic settings in Table 2. According to the results shown in Fig.12, all evaluation indices remain stable when bus capacity changes in the condition without a disturbance. It demonstrates that capacity constraint does not work as all buses are not full-loaded. In the condition with a disturbance, indicators with different bus capacity settings increase. What's more, the increases of average waiting time and standard deviation of headway are larger when bus capacity setting is larger. The trend indicates that:

1) Capacity constraint works when bus bunching occurs. In other words, there exist passengers failing to board because of the underutilization of the bunching buses.
2) The impacts of bus bunching become more significant with the vehicle capacity increases, and the performance of the bus service tends to be worse.

7 Conclusions and further work

In this study, we investigate the bus bunching phenomenon by modelling bus movements and passenger behaviors in a multi-line scenario. The model is more realistic compared with the state-of-the-art works in the sense that the bus dwell times at common stops are estimated considering passenger transfer behavior and capacity constraint simultaneously. Based on the model, algorithms for bus trajectories and transfer passengers' routing result are developed. Bus dwellings at stops as well as the numbers of passengers boarding, alighting, and waiting at stops are identified. The distinct effects of bus bunching on the performance are identified by the evaluation indicators: bus bunching range, the average waiting time and the standard deviation of headway. Analyses of the results of numerical experiments reveal these conclusions:

1) The propagation of bus bunching is distinct considering transfer passenger flow and capacity constraint. This confirms the necessity of the modeling the behavior of transfer passengers and implies that motivated passenger behaviors (demand side) have the potential to cooperate with operation tactics (supply side) to alleviate the bus bunching problem.
2) The sensitivity analysis of transfer parameters shows that the increment of transfer passenger flow enhance the propagation of bus bunching phenomenon from a disturbed line to an undisturbed line.
3) The sensitivity analysis of bus capacity indicates that an appropriate reduction in the bus capacity can alleviate the effect of bus bunching problems. This verifies the applicability of boarding limit strategy or using different bus sizes to alleviate bus bunching problem in multi-line scenarios.

This study opens many research directions, such as:

1) Examine the effect of different combinations of bus service frequencies and offsets;

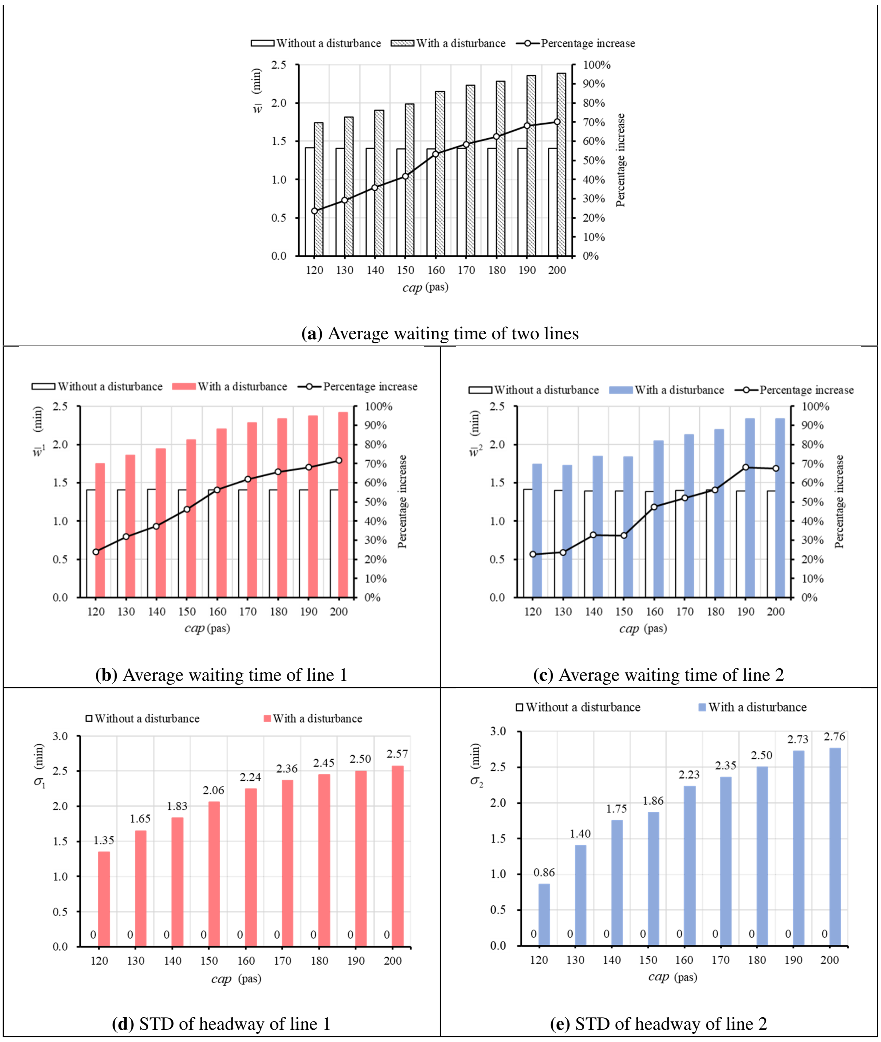


Fig. 12 The sensitivity analysis of bus capacity

2) Generalize the modeling framework for a transit corridor served by more than two transit lines;
3) Relax the assumptions of the proposed model. For example, allowing more than one platform for different bus lines to board and alight passengers at some larger transfer stops;
4) Develop a simulation-based optimization framework to optimize the control strategy (e.g. bus holding) considering common line issues and passenger equilibrium behaviors.

Appendix: Proof of proposition 1

Without loss of generality, we assume bus m is the following bus of line r that arrives at the first common stop n_1 after its preceding bus of line r' denoted as $m^{r'}(m, n_1)$. As all buses are not full-loaded, the transfer cost function via common stop n_i to line r for transfer passengers on bus $m^{r'}(m, n_1)$ degenerates to the headway between bus $m^{r'}(m, n_1)$ and bus m.

$$\omega_{n_i}^{m^{r'}(m,n_1)} = \omega_{n_i}^{m^{r'}(m,n_1),m} = d_{m,n_i} - d_{m^{r'}(m,n_1),n_i} \tag{52}$$

The depart time of bus m from stop n_i is calculated by:

$$d_{m,n_i} = \begin{cases} a_{m,n_1} + \sum\limits_{k=1}^{i} W_{m,n_k} + \sum\limits_{k=1}^{i-1} T_{m,n_k} & \text{if } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \\ a_{m,n_1} + W_{m,n_1} & \text{if } i = 1 \end{cases} \tag{53}$$

Therefore, the transfer cost function from bus m to line r' is expressed by:

$$\omega_{n_i}^{m^{r'}(m,n_1)} = \begin{cases} a_{m,n_1} + \sum\limits_{k=1}^{i} W_{m,n_k} + \sum\limits_{k=1}^{i-1} T_{m,n_k} - d_{m^{r'}(m,n_1),n_i} & \text{if } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \\ a_{m,n_1} + W_{m,n_1} - d_{m^{r'}(m,n_1),n_i} & \text{if } i = 1. \end{cases} \tag{54}$$

The dwell time of bus m at common stop $n_i \in \mathcal{N}_{com}^{r,r'}$ is derived by:

$$\begin{aligned} W_{m,n_i} &= \max\left\{W_{m,n_i}^{A}, \min\left\{W_{m,n_i}^{B}, \frac{C_{m,n_i}}{\beta}\right\}\right\} \\ &= \max\left\{W_{m,n_i}^{A}, W_{m,n_i}^{B}\right\} \\ &= \max\left\{\frac{A_{m,n_i}}{\alpha}, W_{m,n_i}^{B}\right\} \end{aligned} \tag{55}$$

where

$$\begin{aligned} W_{m,n_i}^{B} &= \frac{\left(a_{m,n_i} - d_{m^{r,r'}(m,n_i),n_i}\right) \cdot \left(\Lambda_{n_i}^{r} + \Lambda_{n_i}^{r,r'}\right) + L_{m^{r,r'}(m,n_i),n_i}^{r}}{\beta - \left(\Lambda_{n_i}^{r} + \Lambda_{n_i}^{r,r'}\right)} \\ &= \frac{\Lambda_{n_{i+1}}^{r,r'} \cdot I_{m,n_{i+1}}^{r,r'} + \Lambda_{n_{i+1}}^{r} \cdot I_{m,n_{i+1}}^{r} + \sum\limits^{m' \in \mathcal{M}'(m-1,m)} P_{m'}^{trans} \cdot \alpha_{m',n_{i+1}}}{\beta - \left(\Lambda_{n_i}^{r} + \Lambda_{n_i}^{r,r'}\right)} \end{aligned} \tag{56}$$

and

$$A_{m,n_i} = p_{m,n_i,n_i} + \alpha_{m,n_i} \cdot P_m^{trans} \tag{57}$$

where $P_m^{trans} = \sum\limits_{j \in \mathcal{N}_{down}^{r'}} p_{m,n_1,j}$ denotes the total number of transfer passengers on bus m.

Denote $\mathcal{M}'(m-1, m) = \{m' | d_{m-1,n_{i+1}} < d_{m',n_{i+1}} < d_{m,n_i}, m' \in \mathcal{M}^{r'}\}$, and we get:

$$\begin{aligned} W_{m,n_{i+1}} &= \max\left\{\frac{A_{m,n_{i+1}}}{\alpha}, W_{m,n_{i+1}}^{B}\right\} \\ &= \max\left\{W_{m,n_{i+1}}^{A}\left(\alpha_{m,n_{i+1}}\right), W_{m,n_{i+1}}^{B}\left(\left[\alpha_{m',n_{i+1}}\right]^{m' \in \mathcal{M}'(m-1,m)}\right)\right\} \\ &= W_{m,n_{i+1}}\left(\alpha_{m,n_{i+1}}, \left[\alpha_{m',n_{i+1}}\right]^{m' \in \mathcal{M}'(m-1,m)}\right) \end{aligned} \tag{58}$$

The difference of cost function of two successive common stops are expressed as:

$$\begin{aligned}
&\omega_{n_{i+1}}^{m^{r'}(m,n_1)} - \omega_{n_i}^{m^{r'}(m,n_1)} \\
&= W_{m,n_{i+1}} + T_{m,n_i} - \left(T_{m^{r'}(m,n_1),n_i} + W_{m^{r'}(m,n_1),n_{i+1}}\right) \\
&= W_{m,n_{i+1}} - W_{m^{r'}(m,n_1),n_{i+1}}
\end{aligned} \tag{59}$$

If all common stops are used, the equilibrium conditions are equivalent to:

$$\begin{cases}
\omega_{n_i}^{m'} - \omega_{n_{i-1}}^{m'} = 0 & \text{for } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \\
0 < \alpha_{n_i}^{m} \leq 1 & \forall n_i \in \mathcal{N}_{com}^{r,r'} \\
\sum\limits_{n_i \in \mathcal{N}_{com}^{r,r'}} \alpha_{n_i}^{m'} = 1 &
\end{cases} \tag{60}$$

and

$$\alpha_{n_i}^{m'} = \frac{1}{\left|\mathcal{N}_{com}^{r,r'}\right|}, \text{for } i = 1, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \tag{61}$$

is a solution of (60).

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