Proposition 1. *Transfer passenger equilibrium assignment will degenerate to equal assignment when buses trajectories are uniform in time-space diagram and they are not full-loaded.*

Proof. We assume bus m is the following bus of line r that arrives at the first common stop n_1 after its preceding bus of line r' denoted as $m^{r'}(m, n_1)$. As all buses are not full-loaded, the transfer cost function via common stop n_i to line r for transfer passengers on bus $m^{r'}(m, n_1)$ degenerates to the headway between bus $m^{r'}(m, n_1)$ and bus m.

$$\omega_{n_i}^{m^{r'}(m,n_1)} = \omega_{n_i}^{m^{r'}(m,n_1),m} = d_{m,n_i} - d_{m^{r'}(m,n_1),n_i} \tag{1}$$

The depart time of bus m from stop n_i is calculated by:

$$d_{m,n_i} = \begin{cases} a_{m,n_1} + \sum\limits_{k=1}^{i} W_{m,n_k} + \sum\limits_{k=1}^{i-1} T_{m,n_k} & \text{if } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \\ a_{m,n_1} + W_{m,n_1} & \text{if } i = 1 \end{cases} \tag{2}$$

Therefore, the transfer cost function from bus m to line r' is expressed by:

$$\omega_{n_i}^{m^{r'}(m,n_1)} = \begin{cases} a_{m,n_1} + \sum\limits_{k=1}^{i} W_{m,n_k} + \sum\limits_{k=1}^{i-1} T_{m,n_k} - d_{m^{r'}(m,n_1),n_i} & \text{if } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \\ a_{m,n_1} + W_{m,n_1} - d_{m^{r'}(m,n_1),n_i} & \text{if } i = 1. \end{cases} \tag{3}$$

The dwell time of bus m at common stop $n_i \in \mathcal{N}_{com}^{r,r'}$ is derived by:

$$\begin{aligned} W_{m,n_i} &= \max\left\{W_{m,n_i}^{A}, \min\left\{W_{m,n_i}^{B}, \frac{C_{m,n_i}}{\beta}\right\}\right\} \\ &= \max\left\{W_{m,n_i}^{A}, W_{m,n_i}^{B}\right\} \\ &= \max\left\{\frac{A_{m,n_i}}{\alpha}, W_{m,n_i}^{B}\right\} \end{aligned} \tag{4}$$

where

$$\begin{aligned} W_{m,n_i}^{B} &= \frac{\left(a_{m,n_i} - d_{m^{r,r'}(m,n_i),n_i}\right)\cdot\left(\Lambda_{n_i}^{r} + \Lambda_{n_i}^{r,r'}\right) + L_{m^{r,r'}(m,n_i),n_i}^{r}}{\beta - \left(\Lambda_{n_i}^{r} + \Lambda_{n_i}^{r,r'}\right)} \\ &= \frac{\Lambda_{n_{i+1}}^{r,r'}\cdot I_{m,n_{i+1}}^{r,r'} + \Lambda_{n_{i+1}}^{r}\cdot I_{m,n_{i+1}}^{r} + \sum^{m'\in\mathcal{M}'(m-1,m)} P_{m'}^{trans}\cdot\alpha_{m',n_{i+1}}}{\beta - \left(\Lambda_{n_i}^{r} + \Lambda_{n_i}^{r,r'}\right)} \end{aligned} \tag{5}$$

and

$$A_{m,n_i} = p_{m,n_i,n_i} + \alpha_{m,n_i}\cdot P_m^{trans} \tag{6}$$

where $P_m^{trans} = \sum\limits_{j\in\mathcal{N}_{down}^{r'}} p_{m,n_1,j}$ denotes the total number of transfer passengers on bus m.

Denote $\mathcal{M}'(m-1,m) = \{m' | d_{m-1,n_{i+1}} < d_{m',n_{i+1}} < d_{m,n_i}, m' \in \mathcal{M}^{r'}\}$, and we get:

$$\begin{aligned} W_{m,n_{i+1}} &= \max\left\{\frac{A_{m,n_{i+1}}}{\alpha}, W_{m,n_{i+1}}^{B}\right\} \\ &= \max\left\{W_{m,n_{i+1}}^{A}\left(\alpha_{m,n_{i+1}}\right), W_{m,n_{i+1}}^{B}\left(\left[\alpha_{m',n_{i+1}}\right]^{m'\in\mathcal{M}'(m-1,m)}\right)\right\} \\ &= W_{m,n_{i+1}}\left(\alpha_{m,n_{i+1}}, \left[\alpha_{m',n_{i+1}}\right]^{m'\in\mathcal{M}'(m-1,m)}\right) \end{aligned} \tag{7}$$

The difference of cost function of two successive common stops are expressed as:

$$\begin{aligned}
&\omega_{n_{i+1}}^{m^{r'}(m,n_1)} - \omega_{n_i}^{m^{r'}(m,n_1)} \\
&= W_{m,n_{i+1}} + T_{m,n_i} - \left(T_{m^{r'}(m,n_1),n_i} + W_{m^{r'}(m,n_1),n_{i+1}}\right) \\
&= W_{m,n_{i+1}} - W_{m^{r'}(m,n_1),n_{i+1}}
\end{aligned} \tag{8}$$

If all common stops are used, the equilibrium conditions are equivalent to:

$$\begin{cases}
\omega_{n_i}^{m'} - \omega_{n_{i-1}}^{m'} = 0 & \text{for } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \\
0 \le \alpha_{n_i}^{m} \le 1 & \forall n_i \in \mathcal{N}_{com}^{r,r'} \\
\sum\limits_{n_i \in \mathcal{N}_{com}^{r,r'}} \alpha_{n_i}^{m'} = 1 &
\end{cases} \tag{9}$$

The transfer time via stop n_i is dependent on the proportions of transfer passengers choosing preceding common stops $\alpha_{n_1}^{m'}, ..., \alpha_{n_i}^{m'}$:

$$\begin{aligned}
\omega_{n_i}^{m'} &= a_{m,n_i} + \sum_{k=1}^{i} W_{m,n_k} + \sum_{k=1}^{i-1} T_{m,n_k} - d_{m',n_i} \\
&= a_{m,n_i} + \sum_{k=1}^{i-1} T_{m,n_k} - d_{m',n_i} + \\
&\sum_{k=1}^{i} \frac{\left(a_{m,n_k} - d_{m^{r,r'}(m,n_k),n_k}\right) \cdot \left(\Lambda_{n_k}^{r} + \Lambda_{n_k}^{r,r'}\right) + L_{m^{r,r'}(m,n_k),n_k}^{r}\left(\alpha_{n_k}^{m'}\right)}{b - \left(\Lambda_{n_k}^{r} + \Lambda_{n_k}^{r,r'}\right)}
\end{aligned} \tag{10}$$

That is:

$$\omega_{n_i}^{m'} = f\left(\boldsymbol{\alpha}^{m'}\right) \tag{11}$$

where $\boldsymbol{\alpha}^{m'} = \left[\alpha_{n_1}^{m'}, ..., \alpha_{n_i}^{m'}, ...\right]_{\left|\mathcal{N}_{com}^{r,r'}\right|}$.

If buses trajectories are uniform in time-space diagram, we have that:

$$\begin{cases}
\omega_{n_i}^{m'} = \omega_{n_{i-1}}^{m'} & \text{for } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \\
\text{Eq. (\textbf{??})} & \text{for } i = 2, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \\
\text{Eq. (\textbf{??})} & \\
\sum\limits_{k=1}^{\left|\mathcal{N}_{com}^{r,r'}\right|} \alpha_{n_k}^{m'} = 1 &
\end{cases} \Leftrightarrow \alpha_{n_i}^{m'} = \frac{1}{\left|\mathcal{N}_{com}^{r,r'}\right|}, \text{for } i = 1, ..., \left|\mathcal{N}_{com}^{r,r'}\right| \tag{12}$$

which means transfer passenger equilibrium assignment degenerates to equal assignment. If passengers are not provided full information about bus operation, their route choices are based on the historical experience instead of the real-time bus operation.