{A probabilistic multipath traffic assignment algorithm which obviates pathenumeration}

1971

> IN MOST widely used traffic assignment models, all trips between a fixed origin and destination are assigned to the links constituting a single shortest connecting path.$ This latter tech- nique has been designated ��all-or-nothing�� assignment. Due to the effects of trip volumes on travel time and the tripmaker's non-deterministic choice function on route selection, all-or-nothing assignment is known to contradict actual trip behavior, and the link volumes these traffic assignment models output are sometimes inaccurate to the point of compromis- ing the transportation planner's design decisions.

{Some developments in equilibrium traffic assignment}

1980

> A network optimization problem is formulated which yields a probabilistic equilibrated traffic assignment incorporating congestion effects and which as a special case, reduces to a user optimized equilibrium solution. In the resulting model, path choice is determined by a logit formula in which path costs are functions of the assigned flows. The article also demonstrates the similarity between some fixed demand incremental methods of traffic assignment and the minimization problem associated with computing the user equilibrium assignment. {\textcopyright} 1980, All rights reserved.

{Traffic Equilibrium and Variational Inequalities}

1980

> We consider the general traffic equilibrium network model where the travel cost on each link of the transportation network may depend on the flow on this as well as other links of the network. The model has been designed in order to handle situations where there is interaction between traffic on different links (e.g., two-way streets, intersections) or between different modes of transportation on the same link. For this model, we use the techniques of the theory of variational inequalities to establish existence of a traffic equilibrium pattern, to design an algorithm for the construction of this pattern and to derive estimates on the speed of convergence of the algorithm.

{The convergence of diagonalization algorithms for asymmetric network equilibrium problems��}

1982

> We provide a sufficient condition for the convergence of diagonalization algorithms for equilibrium traffic assignment problems with asymmetric Jacobian matrix B(v) of the link user cost mapping s(v) of the flow v. When , where is the diagonal of B(v) and v is the equilibrium flow, we demonstrate a local convergence theorem for nonlinear cost functions. The implication of this result for practical applications of the model are outlined.

{The Frank-Wolfe algorithm for equilibrium traffic assignment viewed as a variational inequality}

1987

> We show that the standard Frank-Wolfe algorithm used to solve the Wardrop Equilibrium traffic assignment problem has a natural interpretation in terms of variational inequalities and suggest possible analagous algorithms which could be applied to a more general class of network. {\textcopyright} 1987.

{Algorithms for solving fisk's stochastic traffic assignment model}

1991

> In this, the logit-based stochastic traffic assignment model is explored. Two new improved algorithms are presented for solving this type of stochastic assignment problem. The major improvement achieved in these algorithms is that the step length in each iteration of the search process is optimized instead of using fixed step lengths as in the existing method of successive averages (MSA). {\textcopyright} 1991.

{Stochastic user equlibrium assignment in network with queues}

1995

> A stochastic user equilibrium assignment algorithm is presented for steady state store-and-forward networks. The links of the network have constant travel times and the links or nodes have finite capacities. When capacity is reached, delay sufficient to match demand to the available capacity is generated. It has been shown by others that the equilibrium assignment in networks of this kind is the solution to a particular linear programming problem. By adding an entropy term to the objective function, a convex nonlinear programming problem is formed which yields a stochastic user equilibrium assignment. For the case of link constraints, it is proven that the Lagrange multipliers of both the linear and the non-linear programming problems give the equilibrium delays in the network. The requirements for uniqueness are investigated. Iterative algorithms are formulated for solving the nonlinear programming problem with either link or node constraints and convergence is proven. For networks where path enumeration is likely to be a problem, a column generation technique is proposed. An illustrative example is presented.

{Alternatives Assignment To Dial ' S Logit}

1995

> Two logit assignment methods for transportation networks are proposed as alternatives to Dial's algorithm. While retaining the absence of a need for the enumeration of paths, they dispense with both forward and backward passes. They, therefore, do not require minimum node-to-node cost information beforehand. Both methods admit loops and paths that are otherwise inefficient in the Dial sense, which can arise in practice as a result of driver searching behaviour. The first method considers a finite number of paths and the second method an infinite number of paths in the presence of loops. The absence of any efficiency constraint on the set of feasible paths makes the algorithms attractive for use in stochastic user equilibrium methods or in the approximation of a user equilibrium assignment through stochastic user equilibrium methods. The similarity of the structure of one of the proposed algorithms with that of the Floyd-Warshall shortest path algorithm would allow the two to be combined.

{Maintaining a topological order under edge insertions}

1996

> A topological order of the vertices of a directed acyclic graph G = (V, E) is any total order ord such that if (x, y) ⁄⁄ E, then x precedes y in ord. In this paper we consider the dynamic version of this problem, and provide simple algorithms and data structures achieving O(n) amortized time per edge insertion starting from an empty graph, which favorably compares to the trivial O(m + n) time bound per operation obtained applying the off-line algorithm. The additional space requirement, beside the representation of the graph itself, is O(n). Experimental results show that our algorithm performs in practice orders of magnitude faster than the off-line algorithm.

{Information provision in road transport with elastic demand: A welfare economic approach}

1996

> This paper analyses the welfare impacts of providing different types of information to a group of potential road users. Two groups of drivers are considered: informed and uninformed ones and three kinds of information are dealt with: perfect, imperfect and no information. The link travel cost functions are assumed to be stochastic and the information concerns these random fluctuations. The analysis is limited to a one and two link network and it is further assumed that the actors in the model base their decision-making on rational expectations. Under the assumptions that demand and link travel cost functions are linear and that the population of travellers consists of a homogeneous group except for their respective willingness-to-pay for making a trip, it is found that both the provision of perfect and imperfect information leads to a strict Pareto improvement. Furthermore, the analysis reveals that the more perfect the information, the more efficient the use of the transport network. Finally, the analysis concerning the two link network (two routes running in parallel) shows that beneficial route split effects only exist when (1) the number of informed drivers is relatively small and (2) the stochastic shocks in the link travel cost function are relatively large. In these cases, the benefits to informed drivers are substantially larger than when only mode split effects take place.

{Cyclic flows, Markov process and stochastic traffic assignment}

1996

> Dial's stochastic assignment algorithm restricts the assignment path set to "efficient path." As a result, it sometimes produces the unrealistic flow pattern that no flow is loaded on some paths where many vehicles are running in reality. To remove the drawback of Dial's algorithm, this paper presents the LOGIT type assignment that does not restrict the assignment paths. We first show the theoretical relation between the proposed model and Sasaki's assignment model through Markov process. This analysis makes it clear that the proposed assignment model can be calculated by some matrix operations. Next, we propose an efficient algorithm that does not require the matrix operation nor path enumeration over a network. The algorithm solves an equivalent program based on the entropy decomposition derived from the Markov property of LOGIT model. Finally, it is shown that the proposed approach can be easily extended to the flow dependent case (i.e. stochastic equilibrium assignment). Copyright {\textcopyright} 1996 Elsevier Science Ltd.

{Asymmetric problems and stochastic process models of traffic assignment}

1996

> There is a spectrum of asymmetric assignment problems to which existing results on uniqueness of equilibrium do not apply. Moreover, multiple equilibria may be seen to exist in a number of simple examples of real-life phenomena, including interactions at priority junctions, responsive traffic signals, multiple user classes, and multi-modal choices. In contrast, recent asymptotic results on the stochastic process approach to traffic assignment establish the existence of a unique, stationary, joint probability distribution of flows under mild conditions, that include problems with multiple equilibria. In studying the simple examples mentioned above, this approach is seen to be a powerful tool in suggesting the relative, asymptotic attractiveness of alternative equilibrium solutions. It is seen that the stationary distribution may have multiple peaks, approximated by the stable equilibria, or a unimodal shape in cases where one of the equilibria dominates. It is seen, however, that the convergence to stationarity may be extremely slow. In Monte Carlo simulations of the process, this gives rise to different types of pseudo-stable behaviour (flows varying in an apparently stable manner, with a mean close to one of the equilibria) for a given problem, and this may prevail for long periods. The starting conditions and random number seed are seen to affect the type of pseudo-stable behaviour over long, but finite, time horizons. The frequency of transitions between these types of behaviour (equivalently, the average sojourn in a locally attractive, pseudo-stable set of states) is seen to be affected by behavioural parameters of the model. Recommendations are given for the application of stochastic process models, in the light of these issues. Copyright {\textcopyright} 1996 Elsevier Science Ltd.

{A general fixed point approach to multimode multi user equilibrium Assignment with elastic demand}

1997

> This paper presents a fixed-point formulation of multi-mode multi-user equilibrium assignment with elastic demand. Users of different classes may have different behavioral characteristics as well as sets of available routes and modes. They may also behave according to different deterministic and/or probabilistic choice models with different utility specifications. Demand elasticity is dealt with without using the inverse of demand function; in addition, the mode choice can be explicitly dealt with. Conditions for existence and uniqueness of solutions are stated, which generalize and extend those in the literature. A general framework for solution algorithms is also developed, and a simple new algorithm is proposed to solve asymmetric (stochastic) multi-mode multi-user equilibrium with elastic demand.

{Curbing the computational difficulty of the logit equilibrium assignment model}

1997

> In the past, research in traffic assignment modeling has been directed primarily towards improving the deterministic model. Alternative, more behavioral principles were thought to be too demanding computationally. This paper presents two mathematical contributions that enable one to solve a logit assignment model with flow-dependent travel times at a reduced cost. First, a convergence test for

> Fisk's minimization program is introduced, based on a duality gap principle. Second, a new definition of Dial's STOCH fixedtime logit assignment procedure is given, in which the set of available paths is defined only once and the computations are re-interpreted. A numerical experiment indicates that these tools make the logit assignment model very competitive compared to the procedures conventionally used for solving the deterministic model.

{Decomposition of Path Choice Entropy in General Transport Networks}

1997

> This paper shows that the LOGIT type stochastic assignment/stochastic user equilibrium assignment can be represented as an optimization problem with only link variables. The conventional entropy function defined by path flows in the objective can be decomposed into a function consisting only of link flows. The idea of the decomposed formulation is derived from a consideration of the most likely link flow patterns over a network. Then the equivalence of the decomposed formulation to LOGIT assignment is proved by using the Markov properties that underlie Dial's algorithm. Through the analyses, some useful properties of the entropy function and its conjugate dual function (expected minimum cost function) have been derived. Finally, it is discussed that the derived results have a potential impact on the development of efficient algorithms for the stochastic user equilibrium assignment.

{A study on logit assignment which excludes all cyclic flows}

1998

> Sequences from the ribosomal nuclear internal transcribed spacers (ITS) have been widely used to infer evolutionary hypotheses across a broad range of living organisms. Intraspecific sequence variation is assumed to be absent or negliable in most species, but few detailed studies have been conducted to assess the apportionment of ITS sequence variation within and between plant populations. Buxus balearica was chosen as a model species to assess the levels of infraspecific and intragenomic ITS variation in rare and endangered species occurring in disjunct populations around the Mediterranean basin. Intragenomic polymorphic sites were detected for western and eastern accessions of B. balearica and in two accessions of the sister species B. sempervirens. Overall, 19 different ribotypes were found in B. balearica after sequencing 48 clones, whereas 15 ribotypes were detected in 19 clones of B. sempervirens. The integrity and secondary structure stability of the ribosomal sequences suggest that they are not pseudogenes. The high number of ribotypes recovered through cloning suggested that some sequences could be chimeric or generated in vivo by partial homogenization through gene conversion or unequal crossing-over. Average sequence divergence among B. balearica clones was 0.768%, and the most divergent sequences differed by 1.62%. Available evidence does not suggest that B. balearica paralogues have been obtained from other extant Buxus species through interspecific hybridization. The presence of several ribosomal sequences in box implies that the molecular forces driving the concerted evolution of this multigene family are not fully operational in this genus. Phylogenetic analyses of cloned ITS sequences from B. balearica displayed very poor resolution and only two clades received moderate bootstrap support. Despite the marked intragenomic sequence divergence found, ribosomal data suggest a clear phylogeographic split in B. balearica between western and eastern accessions. The distinct, nonchimeric sequences that are postulated as being present in each biogeographic group suggest that box populations from Anatolia (eastern Mediterranean) are relict.

{Algorithms for logit-based stochastic user equilibrium assignment}

1998

> The paper proposes an efficient algorithm for determining the stochastic user equilibrium solution for logit-based loading. The commonly used Method of Successive Averages typically has a very slow convergence rate. The new algorithm described here uses Williams' result [Williams, (1977)On the formation of travel demand models and economic evaluation measures of user benefit. Environment and Planning 9A(3), 285 C344] which enables the expected value of the perceived travel costs Srs to be readily calculated for any flow vector x. This enables the value of the Sheffi and Powell, 1982objective function [Sheffi, Y. and Powell, W. B. (1982) An algorithm for the equilibrium assignment problem with random link times. Networks 12(2), 191 C207], and its gradient in any specified search direction, to be calculated. It is then shown how, at each iteration, an optimal step length along the search direction can be easily estimated, rather than using the pre-set step lengths, thus giving much faster convergence. The basic algorithm uses the standard search direction (towards the auxiliary solution). In addition the performance of two further versions of the algorithm are investigated, both of which use an optimal step length but alternative search directions, based on the Davidon CFletcher CPowell function minimisation method. The first is an unconstrained and the second a constrained version. Comparisons are made of all three versions of the algorithm, using a number of test networks ranging from a simple three-link network to one with almost 3000 links. It is found that for all but the smallest network the version using the standard search direction gives the fastest rate of convergence. Extensions to allow for multiple user classes and elastic demand are also possible.

{Some observations on stochastic user equilibrium and system optimum of traffic assignment}

2000

> Traffic assignment models can be classified according to the behavioral assumption governing route choice. The deterministic user equilibrium (UE), stochastic user equilibrium (SUE) and system optimum (SO) models have been studied extensively in the literature. The relationship between the UE solution and the SO solution for a given network is well known, as is the relationship between UE and SUE. The question that arises concerns the relationship between SUE and (deterministic) SO. The flow pattern obtained from the SO solution serves as a yardstick for comparison with the flow patterns obtained from the UE and SUE solutions. The investigation examines whether the stochastic equilibrium is `closer' than the deterministic user equilibrium to the system optimum. This paper compares the performance of the different solutions for simple networks. The comparison is made by evaluating the relative difference in total system times for UE and SUE solutions with respect to the SO solution. This paper also presents an extension of previous results to show that the Braess' paradox can occur for certain ranges of demand volumes in the case of stochastic equilibrium and non-linear cost functions.

{A non-compensatory choice model incorporating attribute cutoffs}

2001

> This research proposes an extension to the traditional compensatory utility maximization framework which has guided most theoretical and statistical work in choice modeling applications, including those

> in transportation demand estimation work. Attribute cutoffs are incorporated into the decision problem formulation; it is then argued on extant empirical evidence that individuals may view these constraints as "soft". This leads to the formulation of a penalized utility function that allows for constraint violation, but at a cost to the overall evaluation of the good. The proposed model is able to represent fully compensatory, conjunctive and disjunctive choice strategies, as well as combinations thereof. The properties of the proposed theoretical model are examined and discussed. From the theoretical framework, statistical models of choice behavior are easily derived; in their simplest forms, these models can be estimated using existing software. A Stated Preference choice experiment is analyzed using the proposed model, which is found to be highly consistent with observed choices and superior to a structural two-stage choice set formation model. {\textcopyright} 2001 Elsevier Science Ltd. All right reserved.

{A model of route perception in urban road networks}

2002

> ATIS and new technologies are attracting increasing attention towards understanding and modeling the behavior underlying drivers' route choice. The new approach to route choice modeling is also having a significant impact on network assignment models, traditionally based on simple hypotheses of route choice. Models based on explicit route enumeration, allowing more realistic behavioral assumptions, were recently proposed E. Cascetta, F. Russo, A. Vitetta [Preprint of the Eighth IFAC Symposium on Transportation Systems, Chania, Greece, 1997]. Random utility has been the standard theoretical framework for explicit models of route choice C.F. Daganzo, Y. Sheffi [Transp. Sci. 1] (1982) 253-274], M. Ben-Akiva, M.J. Bergman, A.J. Daly, R. Ramaswamy [Proceedings of the Ninth International Symposium on Transportation and Traffic Theory, VNU Science Press, 1984], E. Cascetta, A. Nuzzolo, F. Russo, A. Vitetta [Proceedings of the ISTTT Conference, Lyon, France, 1996]. Random utility models are based on two conceptual steps: identification of available alternatives (choice set) and choice from a given choice set (specification of systematic utility and functional form). The first step is particularly relevant in route choice where several paths are, in principle, available in the network, and many empirical studies M. Ben-Akiva, M.J. Bergman, A.J. Daly, R. Ramaswamy [loc. cit.], E. Cascetta, A. Nuzzolo, F. Russo, A. Vitetta [loc. cit.], F. Russo, A. Vitetta [Proceedings of the Seventh WCTR, Sydney, Australia, 1996], R.G. Golledge [Resource Paper, Preprint 8 IATBR, Austin, Texas, 1997] seem to suggest that only a subset of these are actually perceived by trip makers, i.e., belong to their choice set. While in the literature there are papers dealing with the analysis of route perception in networks from the cognitive point of view R.G. Golledge [Resource Paper, Preprint 8 IATBR, Austin, Texas, 1997], most operational models of paths availability/perception in connection with network assignment are implicit and/or indirect. In other words, models of route perception (enumeration) are seldom explicitly specified and, explicit or implicit, are calibrated on indirect information, which is both disaggregated (routes chosen by a sample of drivers) and aggregated (measured flows). This study proposes an operational model explicitly simulating route perception by drivers in an urban network and presents some calibration results based on a sample of route stated as available/perceived by students and university workers in the city of Reggio Calabria. The results seem to confirm that few routes are actually perceived as feasible alternatives and that topological, level of service and socio-economic attributes influence users' perception. The model could be integrated within a route choice simulation procedure in network assignment models. {\textcopyright} 2002 Published by Elsevier Science Ltd.

{Quasi-variational inequality formulation of the multiclass dynamic traffic assignment problem}

2003

> We consider the extension of a single user-class macroscopic dynamic traffic assignment model to include multiple user-classes. The distinction between user-classes is typically based on vehicle characteristics, e.g. cars and trucks. Interactions between the user-classes sharing the same road infrastructure are taken into account. To deal with various different asymmetries that may occur, such as interuser-class interaction, interspatial and intertemporal asymmetries, the model is specified as a (quasi) variational inequality problem. A nested modified projection method is proposed to solve the assignment problem. The solution of the problem depends heavily on the choice of some very important input: the multiclass link travel time functions. Under mild restrictions there exists a solution, which needs however not be unique. A case study illustrates the model. ?? 2003 Elsevier Science Ltd. All rights reserved.

{Sensitivity Analysis for Stochastic User Equilibrium Network Flows��A Dual Approach}

2003

> Recently, extensive studies have been conducted on the computational methods of sensitivity analysis for the Wardropian equilibrium modeling of traffic networks and their applications. But the same problems in the context of the stochastic user equilibrium modeling seem not to have been addressed. In this paper, we present a method for sensitivity analysis for network flows at stochastic user equilibrium. Our method is developed from a dual formulation of the stochastic user efile:///C:/Users/Owner/Downloads/Advanced transportation planning/ScienceDirect_articles_18Dec2017_07-07-07.685/Semi-dynamic traffic assignment model/A SEMI-DYNAMIC TRAFFIC ASSIGNMENT MODEL WITH ENDOGENOUS.pdfquilibrium analysis. By adopting Dial's algorithm for stochastic traffic assignment, we are able to formulate a computationally efficient link-based algorithm for the sensitivity analysis. Since the Wardropian equilibrium in a traffic network is an extreme case of stochastic user equilibrium with θ �� ��, θ being a dispersion parameter in the expected utility function for stochastic route choice, the method presented here can also be used for sensitivity analysis of the Wardropian equilibrium by setting θ large enough.

{Convergence of Traffic Assignments: How Much is Enough?}

2004

> Daily traffic assignments to a large-scale road network are described for build and no-build scenarios to evaluate the addition of two proposed ramps between I-295 and SR-42 in the New Jersey part of the Delaware Valley region. The road network consists of 39,800 links connecting 1,510 zones. The user-equilibrium traffic-assignment problem was solved with a new algorithm called origin-based assignment (OBA), which can achieve highly converged solutions with reasonable computing effort. Following a description of the user-equilibrium traffic-assignment problem and the OBA algorithm, the stability of link-flow differences between the two scenarios in the vicinity of the proposed ramps are examined over a broad range of assignment convergence levels. Then, link-flow differences over this

> range of convergence levels are compared to link-flow differences between two very highly converged solutions. Examination of the findings reveals, in the writers' view, that a relative gap of 0.01% (0.0001) is required to ensure that the traffic assignments are sufficiently converged to achieve link-flow stability. These convergence levels are then interpreted in terms of the number of Frank-Wolfe iterations needed to achieve comparable relative gaps as well as the computational effort required.

{Route choice models used in the stochastic user equilibrium problem: A review}

2004

> Several route choice models are reviewed in the context of the stochastic user equilibrium problem. The traffic assignment problem has been extensively studied in the literature. Several models were developed focusing mainly on the solution of the link flow pattern for congested urban areas. The behavioural assumption governing route choice, which is the essential part of any traffic assignment model, received relatively much less attention. The core of any traffic assignment method is the route choice model. In the wellknown deterministic case, a simple choice model is assumed in which drivers choose their best route. The assumption of perfect knowledge of travel costs has been long considered inadequate to explain travel behaviour. Consequently, probabilistic route choice models were developed in which drivers were assumed to minimize their perceived costs given a set of routes. The objective of the paper is to review the different route choice models used to solve the traffic assignment problem. Focus is on the different model structures. The paper connects some of the route choice models proposed long ago, such as the logit and probit models, with recently developed models. It discusses several extensions to the simple logit model, as well as the choice set generation problem and the incorporation of the models in the assignment problem. {\textcopyright} 2004 Taylor and Francis Ltd.

{Modeling competitive multi-modal transit services: a nested logit approach}

2004

> In metropolitan areas where multi-modal trips are common, modeling the combined-mode choices of travelers, and the strategic interactions between the private service operators are important issues. This study developed a novel network approach, designated as state-augmented multi-modal (SAM) network, to explicitly consider transfer behaviors and non-linear fare structures. To overcome the independence of irrelevant alternatives (IIA) assumption associated with the standard logit approach, we integrated the SAM network with the nested logit (NL) approach. Specifically, we developed a three-level NL choice model to deal with the complex and inter-related decisions in a multi-modal network: the first level focuses on combined-mode choice, the second on transfer location choice, and the third on route choice. Using this NL SAM network as a platform, we examined the effect of fare competition on company profitability as well as on overall network congestion. A case study of the ground transportation system connecting the Hong Kong International Airport to the downtown area is provided to illustrate the approach.

{Contributions to the logit assignment model To cite this version : HAL Id : hal-00348417}

2005

> Abstract : In the past, research in traffic assignment modeling has been directed primarily towards the deterministic model. Alternative, more behavioral principles were thought to be too demanding computationally. This paper presents two mathematical contributions that enable one to solve a logit assignment model with flow-dependent travel times at a reduced cost. First, a convergence test for Fisk's minimization program is introduced, based on a duality gap principle. Second, a new definition of Dial's STOCH fixed-time logit assignment procedure is given, in which the set of available paths is defined only once and the computations are re-interpreted. A numerical experiment indicates that these tools make the logit assignment model very competitive compared to the procedures conventionally used to solve the deterministic model.

{Modelling route choice behaviour in multi-modal transport networks}

2005

> The paper presents new findings on the influence of multi-modal trip attributes on the quality and competitiveness of inter-urban multi-modal train alternatives. The analysis covers the entire trip from origin to destination, including access and egress legs to and from the train network. The focus is on preferences for different feeder modes, railway station types and train service types as well as on the relative influence of time elements and transfer penalties. Data from dedicated surveys are used including individual objective choice sets of 235 multi-modal homebound trips in which train is the main transport mode. The observed trips have origins and destinations within the Rotterdam-Dordrecht region in The Netherlands with an average total trip time of 50 minutes. Hierarchical Nested Logit models are estimated to take account of unobserved similarities between alternatives at the home-end and the activity-end of the trip respectively, resulting in two-level nesting structures which differentiate between intercity (IC) and non-intercity railway station types at the upper level and between transit and private access modes at the lower level. In order to reflect the multi-dimensional structure of the data a more advanced so-called Multi-Nested GEV model according to the Principles of Differentiation has been estimated which significantly improves the explanatory power and stresses the importance of the home-end of the multi-modal trip. {\textcopyright} Springer 2005.

{Investigating path-based solution algorithms to the stochastic user equilibrium problem}

2005

> This paper focuses on path-based solution algorithms to the stochastic user equilibrium (SUE) and investigates their convergence properties. Two general optimization methods are adapted to solve the logit SUE problem. First, a method that closely follows the Gradient Projection (GP) algorithm developed for the deterministic problem is derived. While this method is very efficient for the deterministic user equilibrium problem, we use a simple example to illustrate why it is not suitable for the SUE problem. Next, a different variant of gradient projection, which exploits special characteristics of the SUE solution, is presented. In this method the projection is on the linear manifold of active constraints. The algorithms are applied to solve simple networks. The examples are used to compare the convergence properties of the algorithms with a path-based variant of the Method of Successive Averages (MSA) and with the Disaggregate Simplicial Decomposition (DSD) algorithm. {\textcopyright} 2004 Elsevier Ltd. All rights reserved.

{On the equivalence between elimination-by-aspects and generalised extreme value models of choice behaviour}

2006

Elimination-by-aspects and generalised extreme value offer competing paradigms for the representation of a common behaviour, that of individual discrete choice. Observing certain consistencies in their mathematical structure, several eminent authors have commented on the degree of equivalence between the two paradigms. Most contributions to this debate have, however, been less than definitive. More fundamentally, the contributions lack consensus. We advance the debate by establishing formal mathematical conditions under which three-alternative tree models from the two paradigms are exactly equivalent. We then extend our analysis to consider more general models, showing that equivalence can be established for general tree models, but not for cross-nested models. {\textcopyright} 2006 Elsevier Inc. All rights reserved.

{A reliability-based stochastic traffic assignment model for network with multiple user classes under uncertainty in demand}

2006

This paper presents a novel reliability-based stochastic user equilibrium traffic assignment model in view of the day-to-day demand fluctuations for multiclass transportation networks. In the model, each class of travelers has a different safety margin for on-time arrival in response to the stochastic travel times raised from demand variations. Travelers' perception errors on travel time are also considered in the model. This model is formulated as an equivalent variational inequality problem, which is solved by the proposed heuristic solution algorithm. Numerical examples are presented to illustrate the applications of the proposed model and the efficiency of solution algorithm.

{Solving the dynamic network user equilibrium problem with state-dependent time shifts}

2006

In this paper we consider the infinite dimensional variational inequality (VI) formulation of dynamic user equilibrium (DUE) put forward by Friesz et al. (1993) [A variational inequality formulation of the dynamic network user equilibrium problem. Operations Research 41, 179-191] as well as the differential variational inequality (DVI) version reported in Friesz et al. (2001) [Dynamic network user equilibrium with state-dependent time lags. Networks and Spatial Economics 1, 319-347]. We show how the theory of optimal control and the theory of infinite dimensional variational inequalities may be combined to create a simple and effective fixed point algorithm for calculating DUE network flows that are solutions of both formulations. A numerical example is provided. {\textcopyright} 2005 Elsevier Ltd. All rights reserved.

{Effects of Choice Set Size and Route Choice Models on Path-Based Traffic Assignment}

2008

> Few of the recently developed route choice models have actually been applied in traffic assignment problems. This paper discusses the implementation of selected route choice models in stochastic user equilibrium algorithms. The focus of the paper is on path-based assignment, which is essential in the implementation of route choice models. The paper analyzes the effect of choice set size and selected choice models on problem convergence, running time and selected results. The results presented in the paper indicate that for real-size networks, generation of a large number of alternative routes is needed. Furthermore, convergence properties greatly improve if the generated routes are sufficiently disjointed.

{Markovian traffic equilibrium}

2008

> We analyze an equilibrium model for traffic networks based on stochastic dynamic programming. In this model passengers move towards their destinations by a sequential process of arc selection based on a discrete choice model at every intermediate node in their trip. Route selection is the outcome of this sequential process while network flows correspond to the invariant measures of the underlying Markov chains. The approach may handle different discrete choice models at every node, including the possibility of mixing deterministic and stochastic distribution rules. It can also be used over a multi-modal network in order to model the simultaneous selection of mode and route, as well as to treat the case of elastic demands. We establish the existence of a unique equilibrium, which is characterized as the solution of an unconstrained strictly convex minimization problem of low dimension. We report some numerical experiences comparing the performance of the method of successive averages (MSA) and Newton's method on one small and one large network, providing a formal convergence proof for MSA.

{Methodological transferability in route choice modeling}

2009

> The search for the shortest path constitutes the common practice in actual traffic studies, as this simplistic route choice model enables the universal implementation of traffic assignment and simulation procedures to every network configuration. The literature illustrates the large efforts in trying to move forward from this simplistic approach, the limited attempts in modeling route choice behavior from revealed preference data, and the nonexistent endeavor in investigating the transferability of more realistic path generation techniques and route choice models. This paper introduces a test to analyze the transferability of path generation techniques that is based on a newly defined efficiency index for the evaluation of their "cost-effectiveness". Then, equality of model estimates is tested to examine the transferability of route choice models, based on a methodology normally used in the estimation of models with mixed data (typically revealed and stated preference data) and on an existing transferability test statistic commonly used in mode choice modeling. Lastly, an experiment is presented to illustrate the implementation of the transferability tests, based on revealed preference data from two different case studies. Experiment results show that path generation techniques are totally transferable at the model specification level and partially transferable at the model parameter level, and that transferability is generally verified when parameters optimized for a larger network are successfully applied to a smaller network. Experiment results also show that not all route choice models are transferable at the model specification level, and none are transferable at the model parameter level. {\textcopyright} 2008 Elsevier Ltd. All rights reserved.

{An improved Dial's algorithm for logit-based traffic assignment within a directed acyclic network}

2010

Dial's algorithm is one of the most effective and popular procedures for a logit-type stochastic traffic assignment, as it does not require path enumeration over a network. However, a fundamental problem associated with the algorithm is its simple definition of 'efficient paths', which sometimes produces unrealistic flow patterns. In this paper, an improved algorithm based on the route extension coefficient is proposed in order to circumvent this problem, in which 'efficient paths' simultaneously consider link travel cost and minimum travel cost. Path enumeration is still not required and a similar computing efficiency with the original algorithm is guaranteed. A limitation of the algorithm is that it can only be applied to a directed acyclic network because a topological sorting algorithm is used to decide the order of the sequential calculation. A numerical example based on the Beijing subway network illustrates the effectiveness of the proposed algorithm. It is found that it is able to exclude most unrealistic paths, but include all reasonable paths when compared with path enumeration and the original Dial's algorithm.

{A class of bush-based algorithms for the traffic assignment problem}

2010

This paper studies a class of bush-based algorithms (BA) for the user equilibrium (UE) traffic assignment problem, which promise to produce highly precise solutions by exploiting acyclicity of UE flows. Each of the two building blocks of BA, namely the construction of acyclic sub-networks (bush) and the solution of restricted master problems (RMP), is examined and further developed. Four Newton-type algorithms for solving RMP, which can be broadly categorized as route flow and origin flow based, are presented, of which one is newly developed in this paper. Similarities and differences between these algorithms, as well as the relevant implementation issues are discussed in great details. A comprehensive numerical study is conducted using both real and randomly generated networks, which reveals that the relative performance of the algorithms is consistent with the analysis. In particular, the results suggest that swapping flows between shortest and longest route segments consistently outperforms other RMP solution techniques. {\textcopyright} 2009 Elsevier Ltd. All rights reserved.

{Stability of user-equilibrium route flow solutions for the traffic assignment problem}

2010

This paper studies stability of user-equilibrium (UE) route flow solutions with respect to inputs to a traffic assignment problem, namely the travel demand and parameters in the link cost function. It shows, under certain continuity and strict monotonicity assumptions on the link cost function, that the UE link flow is a continuous function of the inputs, that the set of UE route flows is a continuous multifunction of the inputs, and that the UE route flow selected to maximize an objective function with certain properties is a continuous function of the inputs. The maximum entropy UE route flow is an example of the last. On the other hand, a UE route flow arbitrarily generated in a standard traffic

> assignment procedure may not bear such continuity property, as demonstrated by an example in this paper. {\textcopyright} 2009 Elsevier Ltd. All rights reserved.

{Stochastic user equilibrium for route choice model based on random regret minimization}

2012

> A static stochastic user equilibrium (SUE) problem was formulated: the mode of random regret minimization (RRM) was used for route choices. The RRM approach assumes that individuals minimize anticipated regret, rather than maximize expected utility, when choosing from alternative routes. The cost function for the RRM model is not separable, and so a variational inequality approach was adopted to formulate the problem. A path-based algorithm was applied to solve the RRM-SUE problem with the method of successive averages. Implementation of the algorithm in a real-world network is illustrated, and the trade-offs and differences between the proposed model and the SUE based on random utility models is discussed.

{Incremental Cycle Detection, Topological Ordering, and Strong Component Maintenance}

2012

> We present two online algorithms for maintaining a topological order of a directed n-vertex acyclic graph as arcs are added, and detecting a cycle when one is created. Our first algorithm handles m arc additions in O(m 3/2) time. For sparse graphs (m/n = O(1)), this bound improves the best previous bound by a logarithmic factor, and is tight to within a constant factor among algorithms satisfying a natural locality property. Our second algorithm handles an arbitrary sequence of arc additions in O(n 5/2) time. For sufficiently dense graphs, this bound improves the best previous bound by a polynomial factor. Our bound may be far from tight: we show that the algorithm can take Ω(n 2 2 ﬀ 2 lgn) time by relating its performance to a generalization of the k-levels problem of combinatorial geometry. A completely different algorithm running in θ(n2 log n) time was given recently by Bender, Fineman, and Gilbert. We extend both of our algorithms to the maintenance of strong components, without affecting the asymptotic time bounds. {\textcopyright} 2012 ACM.

{A self-adaptive gradient projection algorithm for the nonadditive traffic equilibrium problem}

2012

> Gradient projection (GP) algorithm has been shown as an efficient algorithm for solving the traditional traffic equilibrium problem with additive route costs. Recently, GP has been extended to solve the nonadditive traffic equilibrium problem (NaTEP), in which the cost incurred on each route is not just a simple sum of the link costs on that route. However, choosing an appropriate stepsize, which is not known a priori, is a critical issue in GP for solving the NaTEP. Inappropriate selection of the stepsize can significantly increase the computational burden, or even deteriorate the convergence. In this paper, a self-adaptive gradient projection (SAGP) algorithm is proposed. The self-adaptive scheme has the ability to automatically adjust the stepsize according to the information derived from previous iterations. Furthermore, the SAGP algorithm still retains the efficient flow update strategy that only

> requires a simple projection onto the nonnegative orthant. Numerical results are also provided to illustrate the efficiency and robustness of the proposed algorithm. {\textcopyright} Published by Elsevier Ltd.

{Mathematical models and computational algorithms for probit-based asymmetric stochastic user equilibrium problem with elastic demand}

2012

> This article addresses model development and computational algorithm design for the probit-based asymmetric stochastic user equilibrium (SUE) problem with elastic demand. Two variational inequality (VI) models are first proposed for the SUE problem and then existence and uniqueness of their solutions are examined. These two VI models are, in reality, built by means of a probit-based stochastic network loading (SNL) map. Since there is no computational procedure available for calculating the SNL map, we thus propose a two-stage Monte Carlo simulation-based method to estimate the SNL map. To compromise computational time with accuracy in the estimation, a lower bound of sample size required by the Monte Carlo simulation is also investigated. Based on these two VI models and Monte Carlo simulation-based method, we design two hybrid prediction-correction (PC) - cost averaging (CA) algorithms for solving the SUE problem. Finally, two numerical examples are carried out to assess performance of the proposed algorithms. {\textcopyright} 2012 Copyright Taylor and Francis Group, LLC.

{C-logit stochastic user equilibrium model: formulations and solution algorithm}

2012

> This article considers the stochastic user equilibrium (SUE) problem with the route choice model based on the C-logit function. The C-logit model has a simple closed-form analytical probability expression and requires relatively lower calibration efforts and represents a more realistic route choice behaviour compared with the multinomial logit model. This article proposes two versions of the C-logit SUE model that captures the route similarity using different attributes in the commonality factors. The two versions differ with respect to the independence assumption between cost and flow. The corresponding stochastic traffic equilibrium models are called the length-based and congestion-based C-logit SUE models, respectively. To formulate the length-based C-logit SUE model, an equivalent mathematical programming formulation is proposed. For the congestion-based C-logit SUE model, we provide two equivalent variational inequality formulations. To solve the proposed formulations, a new self-adaptive gradient projection algorithm is developed. The proposed formulations and new solution algorithm are tested in two well-known networks. Numerical results demonstrate the validity of the formulations and solution algorithm. {\textcopyright} 2011 Copyright Taylor and Francis Group, LLC.

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{Stochastic traffic assignment, Lagrangian dual, and unconstrained convex optimization}

2012

> In this paper, traffic assignment problems with stochastic travel cost perceptions are reformulated and investigated in a new unconstrained nonlinear programming formulation. The objective function of the unconstrained formulation consists of two terms, in which the first term specifies the routing principle of the target problem through a satisfaction function and the sum of the first and second terms denotes the system cost or optimization objective. This formulation proves to be the Lagrangian dual of a generic primal formulation proposed by Maher et al. (2005) for the stochastic system-optimal problem. The primal-dual modeling framework presents such a common functional form that can accommodate a wide range of different traffic assignment problems. Our particular attention is given to the dual formulation in that its unconstrained feature opens the door of applying unconstrained optimization algorithms for its embraced traffic assignment problems. Numerical examples are provided to support the insights and facts derived from applying the primal and dual formulations to model stochastic system-optimal and user-equilibrium problems and justify the conjugate relationship between the primal and dual models. {\textcopyright} 2012 Elsevier Ltd.

{A link based network route choice model with unrestricted choice set}

2013

> This paper considers the path choice problem, formulating and discussing an econometric random utility model for the choice of path in a network with no restriction on the choice set. Starting from a dynamic specification of link choices we show that it is equivalent to a static model of the multinomial logit form but with infinitely many alternatives. The model can be consistently estimated and used for prediction in a computationally efficient way. Similarly to the path size logit model, we propose an attribute called link size that corrects utilities of overlapping paths but that is link additive. The model is applied to data recording path choices in a network with more than 3000 nodes and 7000 links. {\textcopyright} 2013 Elsevier Ltd.

{Unconstrained weibit stochastic user equilibrium model with extensions}

2013

This study provides an unconstrained minimization program as an alternative formulation for the multinomial weibit (MNW) stochastic user equilibrium (SUE) model that explicitly considers the heterogeneous perception variances with respect to different trip lengths under congested conditions. Qualitative properties of the unconstrained minimization program are given to establish the equivalency and uniqueness of the MNW-SUE solution. The advantage of the unconstrained minimization programming formulation is that it allows the development of a link-based algorithm, which obviates path storage and enumeration. The methodological contributions lie in the derivation of the expected perceived travel cost (or the satisfaction function) that enables the development of an unconstrained MNW-SUE minimization program and a link-based stochastic loading mechanism combined with recent advances in line search strategies in the link-based algorithm. Numerical examples are also provided to illustrate the features of the MNW-SUE model and the link-based algorithm along with several extensions for future research. {\textcopyright} 2013.

{Asymmetric stochastic user equilibrium problem with elastic demand and link capacity constraints}

2014

This paper focuses on model development and algorithm design for the general stochastic user equilibrium (SUE) problem with elastic demand, asymmetric link travel time functions and link capacity constraints. It first defines the generalised SUE conditions using generalised link travel time. An equivalent variational inequality (VI) model for these generalised SUE conditions is then developed and it is rigorously proven to be monotone and uniform Lipschitz-continuous. These two properties of the proposed VI model ensure the global convergence of the self-adaptive prediction-correction algorithm incorporating cost averaging method as a solution algorithm. Finally, a numerical example is utilised to assess the performance of the proposed VI model and solution algorithm. {\textcopyright} 2013 {\textcopyright} 2013 Hong Kong Society for Transportation Studies Limited.

{On the stochastic network equilibrium with heterogeneous choice inertia}

2014

As an alternative effort for quantifying recurrent traffic dynamics caused by network variations and analyzing the impact on the network performance from information provision, we describe in this paper a new equilibrium modeling scheme for stochastic networks with a finite number of states, which takes into account the behavioral inertia. A finite-dimensional variational inequality model is formulated to describe the cross-state equilibrium conditions among heterogeneous travelers with different inertial degrees and knowledge structures. Our model allows for traveler's partial understanding and inertial effect in perceiving varying network conditions and provides a different perspective (from existing stochastic and Markovian network equilibrium approaches) to describe traffic flow variations across multiple network scenarios. A disaggregate simplicial decomposition algorithm is suggested to solve the variational inequality problem. Numerical results from a few stochastic network examples demonstrate the validity and effectiveness of our methodology in modeling the inertia phenomenon within route choice behavior and the efficacy of using traveler information systems to eliminate the inertia effect. ?? 2014 Elsevier Ltd.

{Solving the logit-based stochastic user equilibrium problem with elastic demand based on the extended traffic network model}

2014

This paper proposes a novel extended traffic network model to solve the logit-based stochastic user equilibrium (SUE) problem with elastic demand. In this model, an extended traffic network is established by properly adding dummy nodes and links to the original traffic network. Based on the extended traffic network, the logit-based SUE problem with elastic demand is transformed to the SUE problem with fixed demand. Such problem is then further converted to a linearly constrained convex programming and addressed by a predictor-corrector interior point algorithm with polynomial complexity. A numerical example is provided to compare the proposed model with the method of successive averages (MSA). The numerical results indicate that the proposed model is more efficient and has a better convergence than the MSA. {\textcopyright} 2014 Elsevier B.V. All rights reserved.

{Braess paradox under the boundedly rational user equilibria}

2014

The Braess paradox and its variants have been studied under the perfectly rational behavior assumption. However, when the perfect rationality assumption is relaxed to bounded rationality, which assumes that travelers can take any route whose travel cost is within an 'indifference band' of the shortest path cost, it remains unclear under what conditions the Braess paradox occurs. This paper fills this gap by exploring relationships between the occurrence of the Braess paradox and the indifference band as well as the demand level in the setting of the boundedly rational user equilibrium (BRUE). The definition of the Braess paradox is extended based on planners' risk-taking attitudes, i.e., risk-averse, risk-prone and risk-neutral, due to the non-uniqueness of BRUE. The paradox occurrence conditions under different risk-taking attitudes are investigated using the classical Braess network and compared with those under the user equilibrium. Then we generalize the paradox conditions to simple and ordinary grid networks with regular Bureau of Public Roads (BPR) link performance functions. The impact of the link cost congestion sensitivity along with the indfference band on the occurrence of the Braess paradox is also studied. {\textcopyright} 2014 Elsevier Ltd.

{Variational inequality model for cordon-based congestion pricing under side constrained stochastic user equilibrium conditions}

2014

A major objective of the practical implemented cordon-based congestion pricing schemes is to maintain the traffic conditions within the cordon area, which is rarely considered in most of the existing studies. Thus, this paper addresses the optimal toll charge pattern that can restrict the total inbound flow of each cordon to a predetermined threshold. The toll charges on all the entry links of one cordon are required to be identical, for the ease of implementation and users' recognition. The users' route choice behaviour is assumed to follow stochastic user equilibrium (SUE) with asymmetric link travel time functions. It is shown that such an optimal toll charge pattern can be attained by solving a SUE problem with side constraints. A variational inequality (VI) model is first proposed for the optimal toll pattern, where the monotone property of this model is rigorously proved. Then, a convergent self-adaptive prediction and correction method can be adopted for solving the VI model. It is shown that

> when used in practice, the solution method only needs traffic counts on entry links of each cordon. {\textcopyright} 2013 {\textcopyright} 2013 Hong Kong Society for Transportation Studies Limited.

{Formulating the within-day dynamic stochastic traffic assignment problem from a Bayesian perspective}

2014

> This study proposes a formulation of the within-day dynamic stochastic traffic assignment problem. Considering the stochastic nature of route choice behavior, we treat the solution to the assignment problem as the conditional joint distribution of route traffic, given that the network is in dynamic stochastic user equilibrium. We acquire the conditional joint probability distribution using Bayes' theorem. A Metropolis-Hastings sampling scheme is developed to estimate the characteristics (e.g., mean and variance) of the route traffic. The proposed formulation has no special requirements for the traffic flow models and user behavior models, and so is easily implemented. {\textcopyright} 2013 Elsevier Ltd.

{Stochastic user equilibrium with equilibrated choice sets: Part II - Solving the restricted SUE for the logit family}

2015

> We propose a new class of path-based solution algorithms to solve the Restricted Stochastic User Equilibrium (RSUE), as introduced in Watling et al. (2015). The class allows a flexible specification of how the choice sets are systematically grown by considering congestion effects and how the flows are allocated among routes. The specification allows adapting traditional path-based stochastic user equilibrium flow allocation methods (originally designed for pre-specified choice sets) to the generic solution algorithm. We also propose a cost transformation function and show that by using this we can, for certain Logit-type choice models, modify existing path-based Deterministic User Equilibrium solution methods to compute RSUE solutions. The transformation function also leads to a two-part relative gap measure for consistently monitoring convergence to a RSUE solution. Numerical tests are reported on two real-life cases, in which we explore convergence patterns and choice set composition and size, for alternative specifications of the RSUE model and solution algorithm.

{A nested recursive logit model for route choice analysis}

2015

> We propose a route choice model that relaxes the independence from irrelevant alternatives property of the logit model by allowing scale parameters to be link specific. Similar to the recursive logit (RL) model proposed by Fosgerau et al. (2013), the choice of path is modeled as a sequence of link choices and the model does not require any sampling of choice sets. Furthermore, the model can be consistently estimated and efficiently used for prediction.A key challenge lies in the computation of the value functions, i.e. the expected maximum utility from any position in the network to a destination. The value functions are the solution to a system of non-linear equations. We propose an iterative method with dynamic accuracy that allows to efficiently solve these systems.We report estimation results and a cross-validation study for a real network. The results show that the NRL model yields

> sensible parameter estimates and the fit is significantly better than the RL model. Moreover, the NRL model outperforms the RL model in terms of prediction.

{A framework for and empirical study of algorithms for traffic assignment}

2015

> Traffic congestion is an issue in most cities worldwide. Transportation engineers and urban planners develop various traffic management projects in order to solve this issue. One way to evaluate such projects is traffic assignment (TA). The goal of TA is to predict the behaviour of road users for a given period of time (morning and evening peaks, for example). Once such a model is created, it can be used to analyse the usage of a road network and to predict the impact of implementing a potential project. The most commonly used TA model is known as user equilibrium, which is based on the assumption that all drivers minimise their travel time or generalised cost. In this study, we consider the static deterministic user equilibrium TA model. The constant growth of road networks and the need of highly precise solutions (required for select link analysis, network design, etc.) motivate researchers to propose numerous methods to solve this problem. Our study aims to provide a recommendation on what methods are more suitable depending on available computational resources, time and requirements on the solution. In order to achieve this goal, we implement a flexible software framework that maximises the usage of common code and, hence, ensures comparison of algorithms on common ground. In order to identify similarities and differences of the methods, we analyse groups of algorithms that are based on common principles. In addition, we implement and compare several different methods for solving sub-problems and discuss issues related to accumulated numerical errors that might occur when highly accurate solutions are required.

{Stochastic user equilibrium with equilibrated choice sets: Part I - Model formulations under alternative distributions and restrictions}

2015

> The aim of this paper is to remove the known limitations of Deterministic and Stochastic User Equilibrium (DUE and SUE), namely that only routes with the minimum cost are used in DUE, and that all permitted routes are used in SUE regardless of their costs. We achieve this by combining the advantages of the two principles, namely the definition of unused routes in DUE and of mis-perception in SUE, such that the resulting choice sets of used routes are equilibrated. Two model families are formulated to address this issue: the first is a general version of SUE permitting bounded and discrete error distributions; the second is a Restricted SUE model with an additional constraint that must be satisfied for unused paths. The overall advantage of these model families consists in their ability to combine the unused routes with the use of random utility models for used routes, without the need to pre-specify the choice set. We present model specifications within these families, show illustrative examples, evaluate their relative merits, and identify key directions for further research.

{A method of integrating correlation structures for a generalized recursive route choice model}

2016

> We propose a way to estimate a generalized recursive route choice model. The model generalizes other existing recursive models in the literature, i.e., (Fosgerau et al., 2013b; Mai et al., 2015c), while being more flexible since it allows the choice at each stage to be any member of the network multivariate extreme value (network MEV) model (Daly and Bierlaire, 2006). The estimation of the generalized model requires defining a contraction mapping and performing contraction iterations to solve the Bellman's equation. Given the fact that the contraction mapping is defined based on the choice probability generating functions (CPGF) (Fosgerau et al., 2013b) generated by the network MEV models, and these CPGFs are complicated, the generalized model becomes difficult to estimate. We deal with this challenge by proposing a novel method where the network of correlation structures and the structure parameters given by the network MEV models are integrated into the transport network. The approach allows to simplify the contraction mapping and to make the estimation practical on real data. We apply the new method on real data by proposing a recursive cross-nested logit (RCNL) model, a member of the generalized model, where the choice model at each stage is a cross-nested logit. We report estimation results and a prediction study based on a real network. The results show that the RCNL model performs significantly better than the other recursive models in fit and prediction.

{Solving a Dynamic User Equilibrium model based on splitting rates with Gradient Projection algorithms}

2016

> This article shows how Gradient Projection (GP) algorithms are capable of solving with high precision a Dynamic User Equilibrium (UE) model based on Splitting Rates, i.e. turning movements fractions by destination. Dynamic Traffic Assignment (DTA) is formulated as a Variational Inequality problem defined on temporal profiles of arc conditional probabilities that express a sequence of deterministic route choices taken at nodes by road users directed toward each destination. Congestion is represented through a macroscopic traffic model capable to reproduce a range of phenomena having increasing complexity, from links with bottleneck to intersections with spillback. Different time discretizations, from few seconds to few minutes, are also possible, which allows a range of applications from planning to operation. This assignment model, which is fully link based, is proved to be equivalent to a path based formulation. It also allows for the computation of a handy gap function for analyzing convergence to equilibrium. Numerical experiments on test networks are presented, showing that the proposed GP algorithms converge to dynamic equilibrium in a reasonable number of iterations, outperforming the Method of Successive Averages (MSA).

{Incorporating Driver Behaviors in Network Design Problems: Challenges and Opportunities}

2016

> The goal of a network design problem (NDP) is to make optimal decisions to achieve a certain objective such as minimizing total travel time or maximizing tolls collected in the network. A critical component to NDP is how travelers make their route choices. Researchers in transportation have adopted human decision theories to describe more accurate route choice behaviors. In this paper, we review the NDP with various route choice models: the random utility model (RUM), random regret-minimization (RRM) model, bounded rationality (BR), cumulative prospect theory (CPT), the fuzzy logic model (FLM) and dynamic learning models. Moreover, we identify challenges in applying behavioral route choice models to NDP and opportunities for future research.

{Solving the combined modal split and traffic assignment problem with two types of transit impedance function}

2017

The gradient projection (GP) algorithm has been shown as a successful path-based algorithm for solving various traffic assignment problems. In this paper, the GP algorithm is adapted for solving the combined modal split and traffic assignment (CMSTA) problem, which can be viewed as an elastic demand traffic equilibrium problem (EDTEP) with two modes. Using the excess-demand formulation of EDTEP, the CMSTA problem is reformulated and solved by a modified GP algorithm. Numerical results based on a real bi-modal network in the city of Winnipeg, Canada are provided to demonstrate the efficiency and robustness of the modified path-based GP algorithm for solving the CMSTA problem. In addition, the CMSTA problem is investigated with two types of impedance function for the transit mode and with different degrees of dispersion for the modal split function. The computational results show the modified GP algorithm outperforms the classical Evan's algorithm for both types of transit impedance function, and it can be as efficient as the original GP algorithm for solving the traffic assignment problem with fixed demand.

{A decomposition approach to the static traffic assignment problem}

2017

a b s t r a c t This paper describes a spatial parallelization scheme for the static traffic assignment prob-lem. In this scheme, which we term a decomposition approach to the static traffic assign-ment problem (DSTAP), the network is divided into smaller networks, and the algorithm alternates between equilibrating these networks as subproblems, and master iterations us-ing a simplified version of the full network. The simplified network used for the master iterations is based on linearizations to the equilibrium solution for each subnetwork ob-tained using sensitivity analysis techniques. We prove that the DSTAP method converges to the equilibrium solution on the full network, and demonstrate computational savings of 35–70% on the Austin network. Natural applications of this method are statewide or national assignment problems, or cities with rivers or other geographic features where subnetworks can be easily defined.

{A link-based mean-excess traffic equilibrium model under uncertainty}

2017

Traffic equilibrium models under uncertainty characterize travelers' route choice behaviors under travel time variability. In this paper, we develop a link-based mean-excess traffic equilibrium (L-METE) model by integrating the sub-additivity property and complete travel time variability characterization of mean-excess travel time (METT), and the computationally tractable additive route cost structure of the conventional user equilibrium (UE) problem. Compared to the majority of relevant models formulated in the route domain, the link-based modeling has two desirable features on modeling flexibility and algorithmic development. First, it avoids the normal route travel time distribution assumption (uniformly imposed for all routes) that inherits from the Central Limit Theorem in most route-based models, permitting the use of any suitable link travel time distributions from empirical studies. Second, the additive route cost structure makes the L-METE model solvable by readily adapting existing UE algorithms without the need of storing/enumerating routes while avoiding the computationally

> demanding nonadditive shortest path problem and route flow allocations in route-based models, which is a significant benefit for large-scale network applications under uncertainty.

{Stochastic user equilibrium with a bounded choice model}

2018

> Stochastic User Equilibrium (SUE) models allow the representation of the perceptual and preferential differences that exist when drivers compare alternative routes through a transportation network. However, as an effect of the used choice models, conventional applications of SUE are based on the assumption that all available routes have a positive probability of being chosen, however unattractive. In this paper, a novel choice model, the Bounded Choice Model (BCM), is presented along with network conditions for a corresponding Bounded SUE. The model integrates an exogenously-defined bound on the random utility of the set of paths that are used at equilibrium, within a Random Utility Theory (RUT) framework. The model predicts which routes are used and unused (the choice sets are equilibrated), while still ensuring that the distribution of flows on used routes accords to a Discrete Choice Model. Importantly, conditions to guarantee existence and uniqueness of the Bounded SUE are shown. Also, a corresponding solution algorithm is proposed and numerical results are reported by applying this to the Sioux Falls network.

{New Formulations of the Stochastic User Equilibrium with Logit Route Choice as an Extension of the Deterministic Model}

2018

> This paper addresses the stochastic user equilibrium (SUE) in the case where the route choice is the multinomial logit model (MNL). Our main finding is that MNL SUE can be formulated and solved as an immediate extension of the deterministic user equilibrium (DUE) through a particular application of Wardrop's first principle. The latter states, in general, that at equilibrium, the cost of all used routes is equal and not higher than those of unused routes. The extension is achieved by applying this statement to the ��perceived cost�� of a choice alternative, which is defined here as its generalized cost plus the logarithm of its choice probability multiplied by the logit parameter. Thus, substituting in DUE models the generalized costs with the perceived costs allows to easily adapt to MNL SUE the existing formulations and algorithms for DUE, as well as to manage a smooth transition of the route choice model from stochastic to deterministic by reducing the logit parameter down to zero. Particular consideration is devoted to the interpretation of the numerical solution as a restricted logit model, where only sufficiently good alternatives receive a positive probability. A family of MNL SUE models is then presented ranging from nonlinear optimization to variational inequalities and fixed-point problems, with both explicit and implicit path enumeration. A range of numerical tests is presented with the aim of assessing the continuity of the model results for decreasing logit parameter and proving the applicability of the proposed approach to real size networks, with particular emphasis on the performance and convergence of the methods.

{New formulations of the stochastic user equilibrium with logit route choice as an extension of the deterministic model}

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{Logit-based transit assignment: Approach-based formulation and paradox revisit}

2018

This paper proposes an approach-based transit assignment model under the assumption of logit-based stochastic user equilibrium (SUE) with fixed demand. This model is proven to have a unique solution. A cost-averaging version of the self-regulated averaging method (SRAM) is developed to solve the proposed approach-based SUE transit assignment problem. It is proven that the algorithm converges to the model solution. Numerical examples with discussions are presented to investigate the model properties, a paradoxical phenomenon due to the stochastic nature of the model, capacity paradox, and the performance of the proposed algorithm. The sensitivity analysis of different model and algorithm parameters are performed. A performance comparison between the cost-averaging SRAM, the flow-averaging SRAM, and the method of successive averages is made. The proposed methodology is demonstrated to be able to solve the Winnipeg transit network.

{Combined multinomial logit modal split and paired combinatorial logit traffic assignment model}

2018

{\textcopyright} 2018 Hong Kong Society for Transportation Studies Limited To better address the route overlap problem of the multinomial logit model used in combined modal split and traffic assignment models in the literature, this study proposes a combined multinomial logit modal split and paired combinatorial logit traffic assignment (MNL�CPCL) model. The PCL model can account for the route overlap problem using a similarity index for each pair of routes in the network. It requires significantly fewer parameters to be calibrated using real-world data. Thereby, it circumvents parameter estimation issues associated with a cross-nested logit model. An equivalent mathematical programming problem is developed for the MNL�CPCL model. Further, an analytical model is developed for sensitivity analysis of the MNL�CPCL model. Several applications of the proposed MNL�CPCL model are

> demonstrated using a numerical example by leveraging the results of sensitivity analysis. The study insights can assist decision-makers to design more effective strategies to promote ��go-green' travel modes and reduce network congestion.

{Stochastic user equilibrium with a bounded choice model}

2018

> Stochastic User Equilibrium (SUE) models allow the representation of the perceptual and preferential differences that exist when drivers compare alternative routes through a transportation network. However, as an effect of the used choice models, conventional applications of SUE are based on the assumption that all available routes have a positive probability of being chosen, however unattractive. In this paper, a novel choice model, the Bounded Choice Model (BCM), is presented along with network conditions for a corresponding Bounded SUE. The model integrates an exogenously-defined bound on the random utility of the set of paths that are used at equilibrium, within a Random Utility Theory (RUT) framework. The model predicts which routes are used and unused (the choice sets are equilibrated), while still ensuring that the distribution of flows on used routes accords to a Discrete Choice Model. Importantly, conditions to guarantee existence and uniqueness of the Bounded SUE are shown. Also, a corresponding solution algorithm is proposed and numerical results are reported by applying this to the Sioux Falls network.

{An informed user equilibrium dynamic traffic assignment problem in a multiple origin-destination stochastic network}

2018

> We develop in this paper a comprehensive linear mathematical framework to study the benefit of real-time information and the impact of resulting user adaptive route choice behaviours on network performance. The framework formulates the information-based stochastic user equilibrium (ISUE) dynamic traffic assignment (DTA) problem for a multiple origin-destination (OD) network. Using the framework, we prove the linkage between the user equilibrium (UE) and system optimal (SO) solutions underpinned by the first-in-first-out (FIFO) principle. This important property then enables us to develop an incremental loading method to obtain the ISUE solutions efficiently by solving a sequence of linear programs. Moreover, the proposed method is more scalable that avoids a huge enumeration of paths in large-scale networks as done in path-based methods of the existing literature on this topic. We show via numerical examples the impact of information on both route choices and network performance, and demonstrate the significant improvements in the obtained ISUE solution both in terms of accuracy and computational complexity.

{Traffic assignment paradox incorporating congestion and stochastic perceived error simultaneously}

2019

> This paper analyses the effects of congestion and stochastic perceived error in stochastic traffic assignment paradox, by the measure of both actual and perceived travel cost. Two different circumstances are studied: improving an existing link and adding a new link. It is found that different congestion cost functions and perceived error levels will significantly affect the road condition and the

> demand level under which paradox happens. Moreover, how the interaction between traffic demands of different O-D pairs affects the occurrence of traffic paradox is illustrated by a two O-D pairs' network. Besides, a counter-intuitive phenomenon when less stochastic perceived error yet increases the average travel cost (information paradox) is also discussed. The results of this paper help to understand the interactional impact of congestion and stochastic perceived error, and give some new insights to traffic paradox.

{An Alternative Stochastic User Equilibrium Formulation Based on Regret Theory}

2019

> Traffic assignment is the last stage of the classical transportation planning process in which, the travel demand of each O﷿CD pair is allocated to the network links, and links' flows are estimated. To solve such a problem, some assumptions should be made about travellers' decision-making behavior. One of the most popular approaches in this regard is the deterministic assignment that assumes all drivers are fully informed about the condition of the network and they always select the best (usually the shortest) route. These assumptions do not thoroughly match to the reality. To deal with this problem, the concept of stochastic user equilibrium has been introduced. The conventional stochastic user equilibrium assignments models are typically based on random utility theory. In this paper an alternative approach for stochastic user equilibrium assignment called random regret theory has been used in which a random regret-minimization (RRM) model is developed. RRM considers the regret of an option just with respect to outperformed options and furthermore does not lead to a closed-form stochastic user equilibrium (SUE) model, though based on that a formulation of SUE is proposed in a variational inequality form. In this study the definition of regret is modified and based on that a closed form SUE model is developed. This model is examined by two network examples.

{The mathematical foundations of dynamic user equilibrium}

2019

> This paper is pedagogic in nature, meant to provide researchers a single reference for learning how to apply the emerging literature on differential variational inequalities to the study of dynamic traffic assignment problems that are Cournot-like noncooperative games. The paper is presented in a style that makes it accessible to the widest possible audience. In particular, we apply the theory of differential variational inequalities (DVIs) to the dynamic user equilibrium (DUE) problem. We first show that there is a variational inequality whose necessary conditions describe a DUE. We restate the flow conservation constraint associated with each origin-destination pair as a first-order two-point boundary value problem, thereby leading to a DVI representation of DUE; then we employ Pontryagin-type necessary conditions to show that any DVI solution is a DUE. We also show that the DVI formulation leads directly to a fixed-point algorithm. We explain the fixed-point algorithm by showing the calculations intrinsic to each of its steps when applied to simple examples.

{Prism-based path set restriction for solving Markovian traffic assignment problem}

2019

> This paper deals with a main limitation of Markovian traffic assignment (MTA) models: when the network includes cyclic structures and the link costs are small enough, the fact that the MTA models assign traffic flows to all feasible paths causes computational challenges. This study addresses this issue by proposing a method of restricting path set based on the concept of choice-based prism. To achieve a prism-based path set, a novel network description and the constraints are introduced. These are flexible and compatible with the MTA operation, meaning that our method does not extinguish the mathematical and computational advantages of the MTA models. Our network description allows the expected minimum cost to take a value specific for each state, or a pair of node and choice-stage. This enables us to perform traffic assignment with a simple solution procedure, regardless of parameter setting or network structure. The numerical experiments show that our method solves the computational challenges of the MTA models and that the incorporation of the prism constraints does not increase computational effort required, or rather, it is computationally efficient even when considering the correlation among path utilities.

{Regret-based multi-objective route choice models and stochastic user equilibrium: a non-compensatory approach}

2020

> Since regret theory has been introduced to model travellers' route choices, various random regret minimization (RRM) models have been developed for the choice situation of multiple alternatives and multiple attributes. There are two approaches dealing with multi-objective optimisations: one is to combine different attributes into a single additive one and the other is to consider each attribute separately. The existing RRM models adopt the first approach to measure regret. However, travellers might not always trade off attributes in such a compensatory way. In this paper, under the assumption that travellers might consider attributes separately, we develop two new regret-based stochastic user equilibrium (SUE) models by incorporating the RRM model and the hybrid RUM-RRM model into a non-compensatory multi-objective framework. The majority of the second approaches dealing with multiple objectives generally provide a feasible solution set caused by conflicts among objectives. Different from that, the two new models provide probabilistic choice to each route, based on which a single SUE path flow pattern would be attained. Meanwhile, the compromise effect which is widely seen in consumer behaviour can be explained by the two new models. The equivalent variational inequality problems for the proposed models and a path-based algorithm using the method of successive averages have been given. Numerical examples are further conducted to illustrate the properties of the proposed models and the effectiveness of the algorithm.

{Optimization Models and Methods for Equilibrium Traffic Assignment}

2020

> {\ldots} Due to such results, the developed approach seems to be promising. The main idea of this method is based on the first principle of Wardrop as follows: if we define the journey time of a {\ldots}

{Aggregation, disaggregation and decomposition methods in traffic assignment: historical perspectives and new trends}

2020

> In this study we provide a comprehensive review of the existing literature on (dis)aggregation and decomposition methods in traffic assignment and classify them based on their characteristics. The study takes on two different perspectives. First, we explore existing methods and relate them to one or more traffic assignment components. It is found that there exists a clear separation between a demand modelling point of view, i.e., travel demand and (geographical) zoning on the one hand, and supply modelling-oriented methods, i.e. network topology and network loading, on the other. Further, we explore the existing literature on the interface between demand and supply, i.e., connector and centroid placement which is to be considered a special type of aggregation. It is found this aspect of traffic assignment has received relatively little attention in this context, even though it is shown to be of significant impact on modelling results. The second perspective in this study places the discussed aggregation methodologies in the broader perspective of clustering procedures. We do not necessarily explore clustering methods as such but mainly look at the classification of different types of clustering methods which can be projected onto the traffic assignment domain and aggregation procedures in particular. It is shown that most existing methods can be classified as supervised –C or classification based –C clustering procedures while relatively few studies explore other known approaches such as semi-supervised or unsupervised clustering techniques. Lastly, we discuss how aggregation techniques could be deployed to construct multi-scale modelling environments. There is however a lack of methodology to construct such models consistently. Findings are presented via an objective classification framework for existing (dis)aggregation and decomposition methods.

{A path-based equilibrium model for ridesharing matching}

2020

> This paper examines how the ridesharing program will reshape the spatial distribution of traffic congestion. With ridesharing services, travelers make multi-dimensional decisions concerning mode choices, route choices, and matching decisions. We build a path-based equilibrium model to describe the decision-making of travelers in the presence of the ridesharing program and thereby to forecast the stable congestion distribution in the long run. Specifically, our model can explicitly track the path information of the matched drivers and riders by solving the proposed equilibrium model. The model is formulated as variational inequalities due to the non-separable cost functions and asymmetric link interactions. We first demonstrate the non-uniqueness of mode-specific link flows and derive the condition for the uniqueness of link flows. We then derive sufficient conditions for a matching failure. A matching failure occurs if the lowest total inconvenience cost of a pair of rider and driver exceeds a threshold. In this situation, matching between the ridesharing supply (drivers) and demand (riders) cannot be established by adjusting the compensation scheme. Moreover, we derive the sufficient and necessary conditions for the existence of equilibrium with a positive ridesharing ridership. These conditions help operators gain insights into how to appropriately design compensation schemes. We reformulate the variational inequalities as the equivalent mixed complementarity problem, and propose a heuristic routing algorithm to identify the feasible routing paths for shared rides that interest both ridesharing drivers and riders. Finally, Our numerical experiments show that ridesharing is effective in relieving the overall congestion but leads to high congestion on some links. In the scenarios where the demand into the central business district (CBD) is high, the commuters entering CBD are all better off with ridesharing service. In contrast, the commuters traveling to certain non-CBD areas are worse off. Our experiments also reveal various path choice behaviors of ridesharing participants. The solo drivers always use the paths with short travel times, while the ridesharing drivers tend to use paths with short travel distances. By considering a commission to the platform, we

> demonstrate the regime in which increasing the commission rate improves the platform's revenue without harming the social welfare.

{A faster path-based algorithm with Barzilai-Borwein step size for solving stochastic traffic equilibrium models}

2021

> Step size determination (also known as line search) is an important component in effective algorithmic development for solving the traffic assignment problem. In this paper, we explore a novel step size determination scheme, the Barzilai-Borwein (BB) step size, and adapt it for solving the stochastic user equilibrium (SUE) problem. The BB step size is a special step size determination scheme incorporated into the gradient method to enhance its computational efficiency. It is motivated by the Newton-type methods, but it does not need to explicitly compute the second-order derivative. We apply the BB step size in a path-based traffic assignment algorithm to solve two well-known SUE models: the multinomial logit (MNL) and cross-nested logit (CNL) SUE models. Numerical experiments are conducted on two real transportation networks to demonstrate the computational efficiency and robustness of the BB step size. The results show that the BB step size outperforms the current step size strategies, i.e., the Armijo rule and the self-regulated averaging scheme.