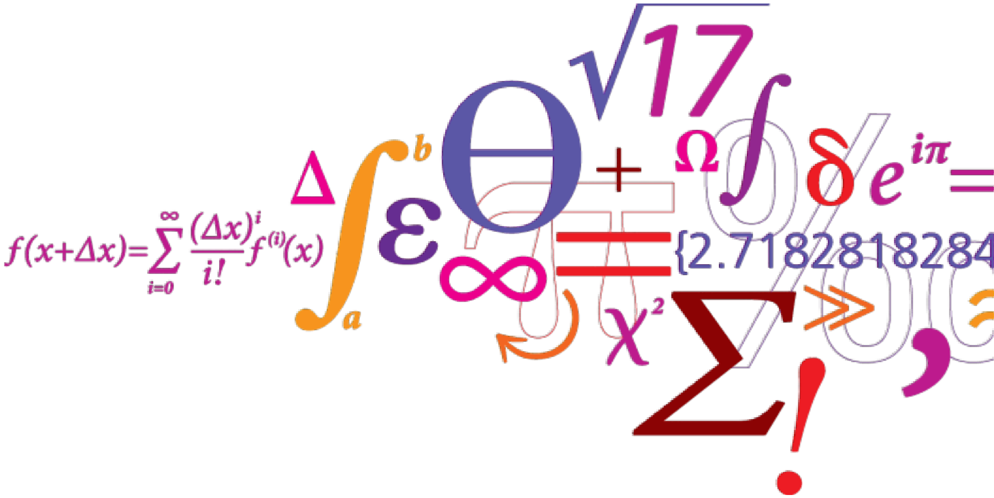
Integrated Optimisation of Public Transport System

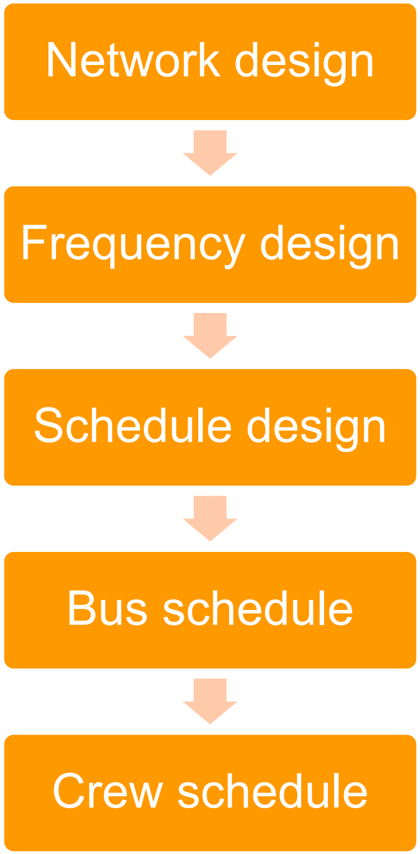
Dr. Yu Jiang
Associate Professor
Technical University of Denmark



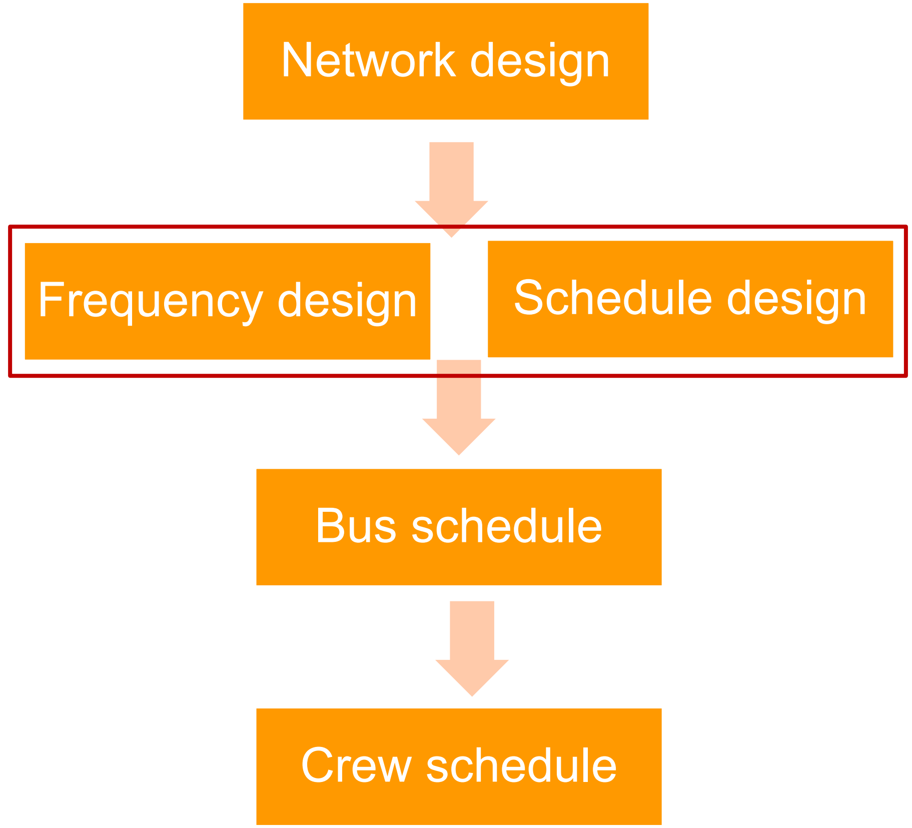
Introduction of Public Transport

1. A passenger trip could rely on both frequency-based service and schedule-based public transport service.
2. However, existing studies design frequency and schedule separately.

Most Significant Contribution:
A New Framework for Public Transport Network Design



Existing framework
since Ceder and Wilson (1987)



New framework

<u>RGC Potential Impact</u>

1. Reach: commuter, operators
2. Significance: services, industry, organisations, government

Overview of the Model

Input

Demand, Travel Time

Objective

Minimise

Total Travel Cost + Operation Cost

s.t.

Operational Constraints for

Stochastic User Equilibrium with

a Bounded Choice Set (BSUE)

Realistic Seating Constraints

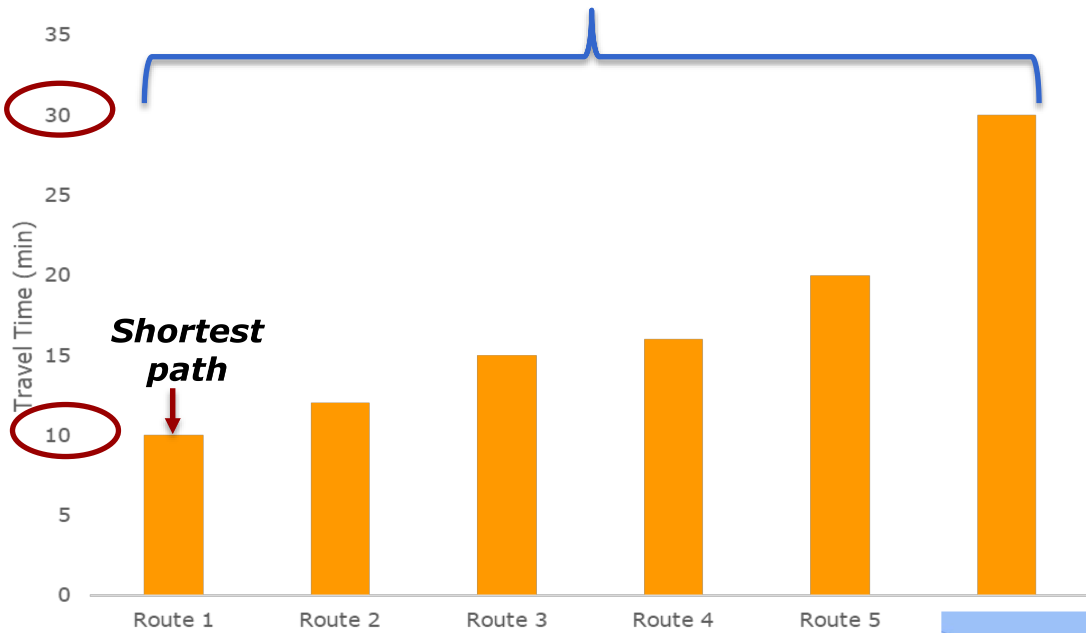
Mixed-integer linear programming formulation

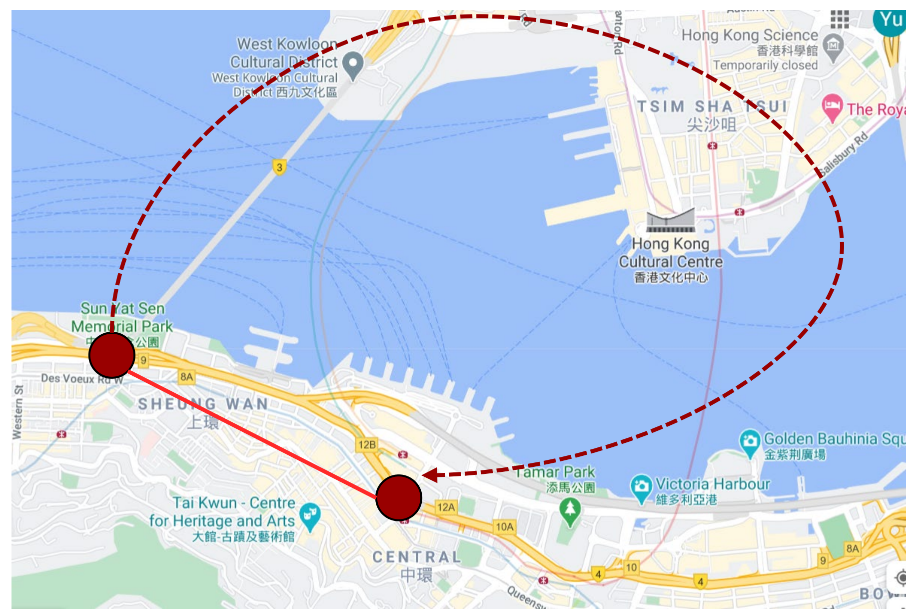
Output

Frequency, Timetable, Flow

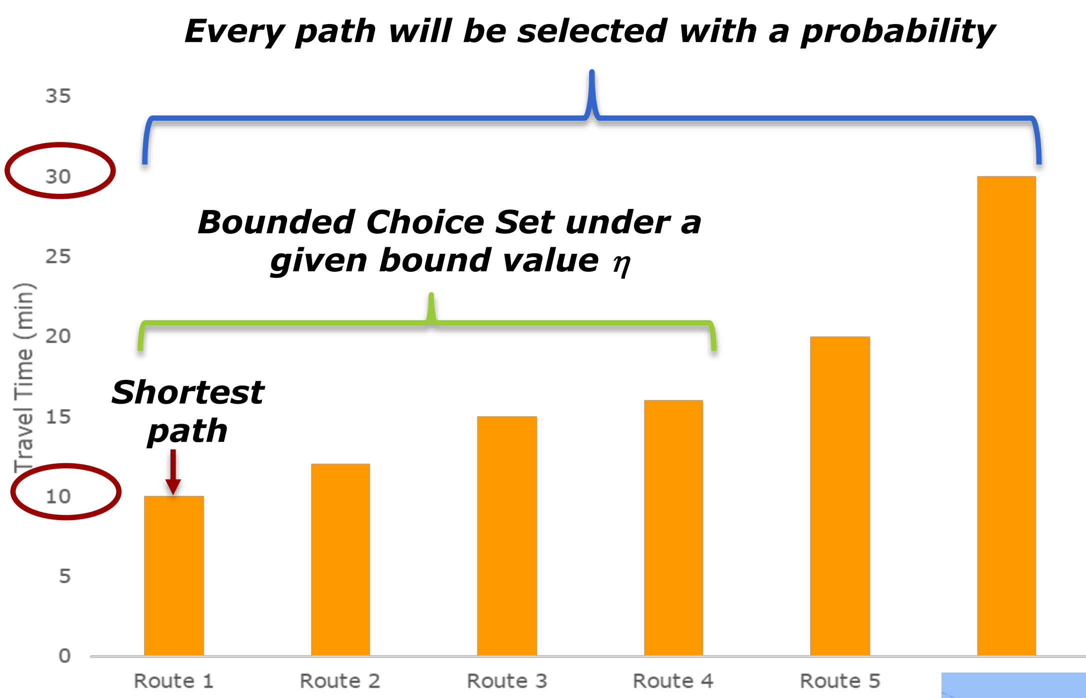
Stochastic User Equilibrium with a Bounded Choice Set (BSUE)

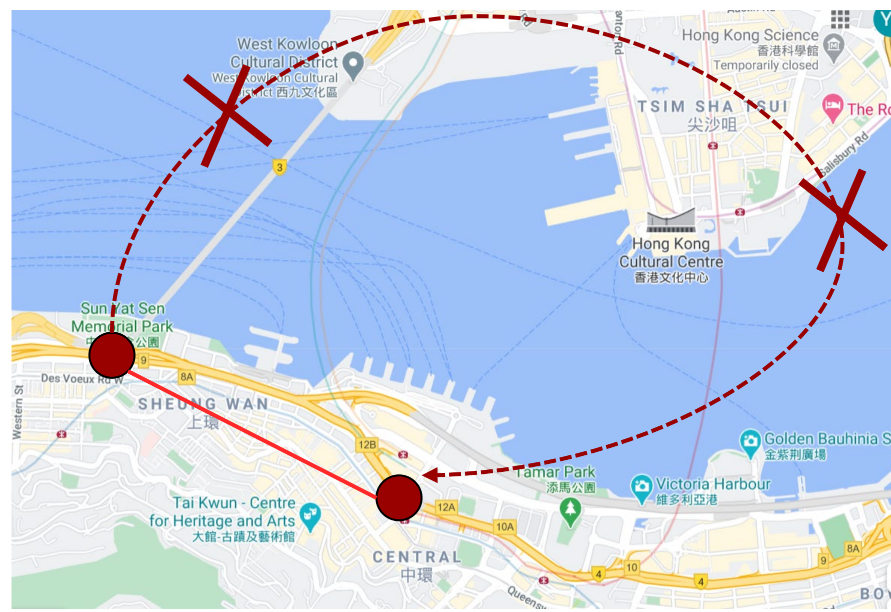
Every path will be selected with a probability





Stochastic User Equilibrium with a Bounded Choice Set (BSUE)





Formulation of BSUE

Travel cost associated with path *r*

Bounded value

$$p_r = \frac{\exp\left(-\theta\left(\pi_r - \min\left(\pi_{r''} : r'' \in \mathcal{R}_{\text{BSUE}}\right) - \eta\right)\right) - 1}{\sum_{r' \in \mathcal{R}_{\text{BSUE}}} \exp\left(-\theta\left(\pi_{r'} - \min\left(\pi_{r''} : r'' \in \mathcal{R}_{\text{BSUE}}\right) - \eta\right)\right) - 1}, \forall r \in \mathcal{R}_{\text{BSUE}}$$

Probability of selecting path *r*

Bounded choice set

Minimum travel cost in the choice set

Linearization

Step 1.

$$\frac{p_{r,\mathrm{BSUE}}}{p_{r',\mathrm{BSUE}}} = \frac{\exp\left(-\theta\left(\pi_r - \min\left\{\pi_{r''} : r'' \in \mathcal{R}_{\mathrm{BSUE}}\right\} - \eta\right)\right) - 1}{\exp\left(-\theta\left(\pi_{r'} - \min\left\{\pi_{r''} : r'' \in \mathcal{R}_{\mathrm{BSUE}}\right\} - \eta\right)\right) - 1}$$

Step 2.

Introducing auxiliary variable y_r

$$y_r = \exp\left(-\theta\left(\pi_r - \min\left\{\pi_{r''} : r'' \in \mathcal{R}_{\mathrm{BSUE}}\right\} - \eta\right)\right) - 1$$

$$\ln\left(p_{r,\mathrm{BSUE}}\right) - \ln\left(p_{r',\mathrm{BSUE}}\right) = \ln\left(y_r\right) - \ln\left(y_{r'}\right)$$

Step 3.

Introducing two more auxiliary variables

$$\chi_{r,\mathrm{BSUE}} = \ln\left(p_{r,\mathrm{BSUE}}\right) \quad z_{r,\mathrm{BSUE}} = \ln\left(y_r\right)$$

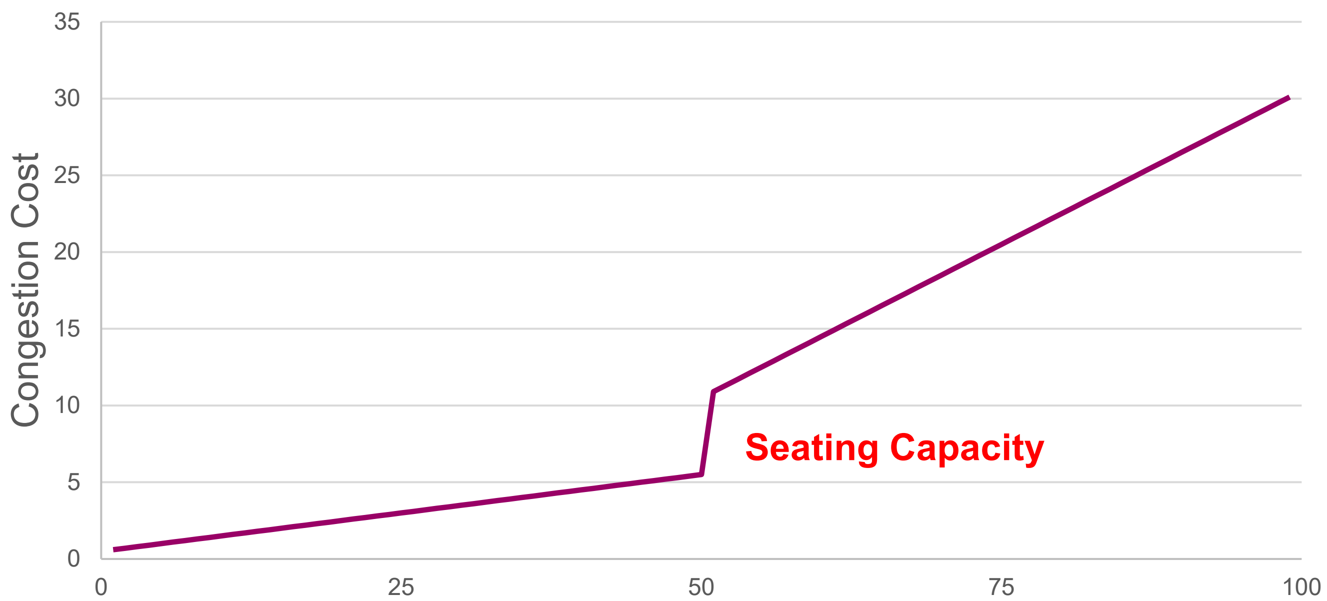
$$\chi_{r,\mathrm{BSUE}} - \chi_{r',\mathrm{BSUE}} = z_{r,\mathrm{BSUE}} - z_{r',\mathrm{BSUE}}$$

Step 4.

Finally apply the techniques to linearize the logarithmic equation

Realistic Seating Behaviour

- Seating passengers maintain seated
- Standing passengers take a seat when vehicle become uncongested
- Stepwise congestion cost



Seating Constraints

$$\Delta^{\text{congest}}_{l,(i,j)} \begin{cases} = 1, \text{ if } v^{\text{onboard}}_{l,(i,j)} \leq Cap^{\text{seat}}_{l,(i,j)} \\ = 0, \text{ if } v^{\text{onboard}}_{l,(i,j)} > Cap^{\text{seat}}_{l,(i,j)} \end{cases}$$

Binary decision variable = 1, if the number of passengers is less than the number of seat

$$\Delta^{w,\text{seat}}_{r,l,(i,j)} \begin{cases} = 1, \text{if the passenger has a seat} \\ = 0, \text{otherwise} \end{cases}$$

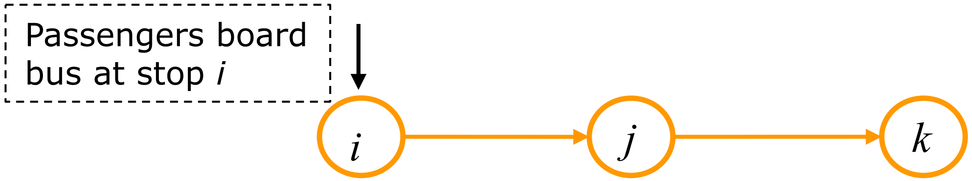
Binary decision variable = 1, if passengers can take a seat

Set of Constraints

$$\Delta^{\text{seat}}_{r,l,(i,j)} \leq \Delta^{\text{seat}}_{r,l,(j,k)} \qquad \Delta^{\text{seat}}_{r,l,(j,k)} \geq \Delta^{\text{congest}}_{l,(j,k)}$$

$$\Delta^{\text{seat}}_{r,l,(i,j)} = \Delta^{\text{congest}}_{l,(i,j)} \qquad \Delta^{\text{congest}}_{l,(j,k)} + \Delta^{\text{seat}}_{r,l,(i,j)} \geq \Delta^{\text{seat}}_{r,l,(j,k)}$$

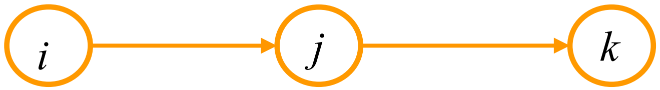
Correctness of the Constraints (I)



Scenarios		Congestion status		Seating status	
No.	Description	$\Delta^{congest}_{l,(i,j)}$	$\Delta^{congest}_{l,(j,k)}$	$\Delta^{w,seat}_{r,l,(i,j)}$	$\Delta^{w,seat}_{r,l,(j,k)}$
I	Both (i, j) (j, k) are congested, passengers do not have seat on the two links	0	0	0	0
II	Both (i, j) (j, k) are not congested, passengers stay seated on the two links	1	1	1	1
III	(i, j) is congested, passengers do not have a seat. (j, k) becomes uncongested and passengers take a seat.	0	1	0	1
IV	(i, j) is not congested and passengers take a seat. (j, k) become congested. Seating passengers main seated.	1	0	1	1

All satisfy the constraints

Correctness of the Constraints (II)

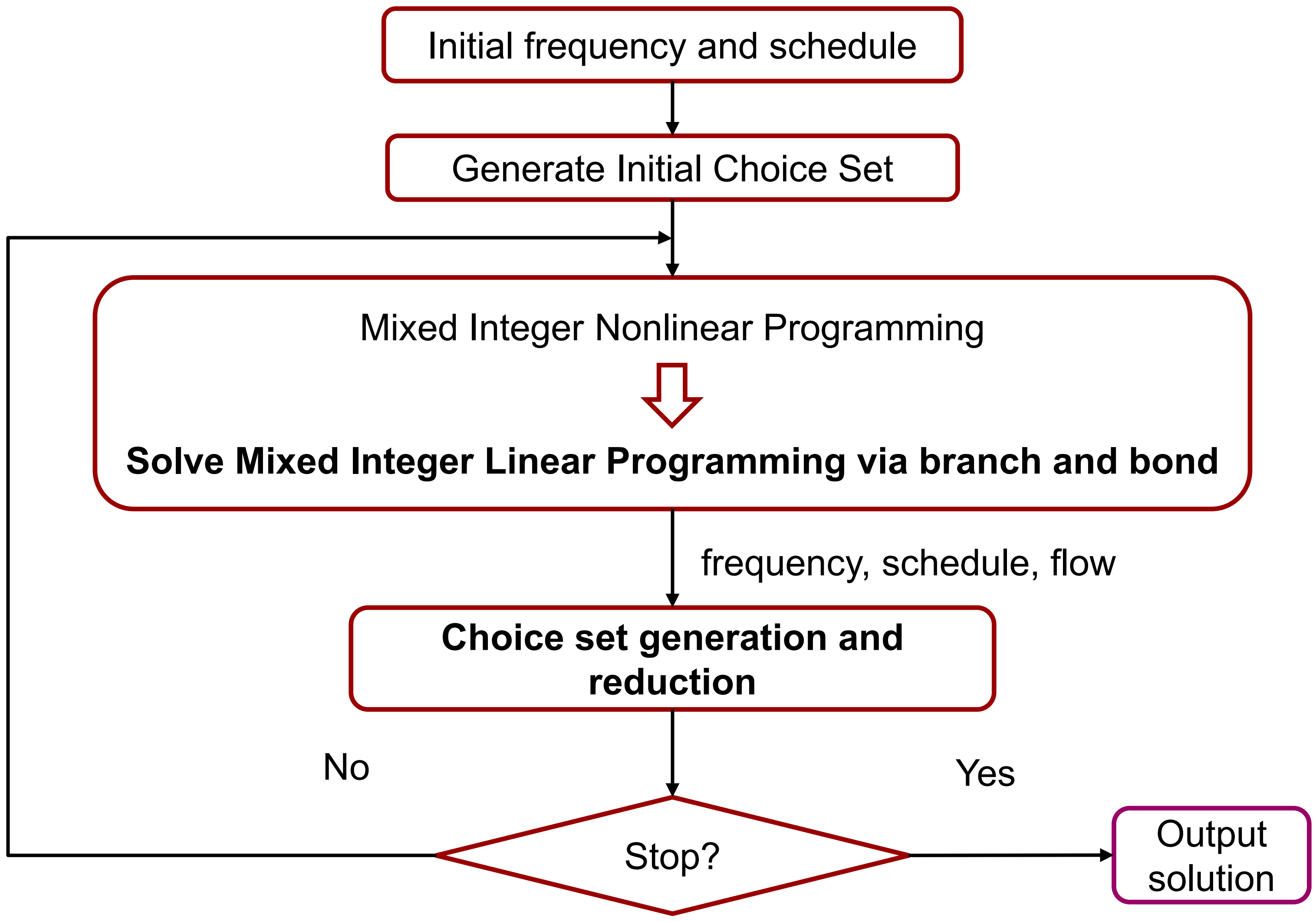


4 decision variables, 2^4 = 16 possible combinations

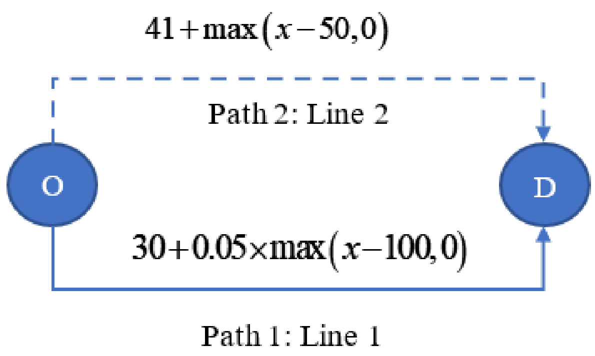
No.	$\Delta^{congest}_{l,(i,j)}$	$\Delta^{congest}_{l,(j,k)}$	$\Delta^{w,seat}_{r,l,(i,j)}$	$\Delta^{w,seat}_{r,l,(j,k)}$	Violated Constraint
1	0	0	0	0	None. Scenario I
2	0	0	0	1	$\Delta^{congest}_{l,(j,k)} + \Delta^{w,seat}_{r,l,(i,j)} \geq \Delta^{w,seat}_{r,l,(j,k)}$ $(0+0 \geq 1)$
3	0	0	1	0	$\Delta^{w,seat}_{r,l,(i,j)} \leq \Delta^{w,seat}_{r,l,(j,k)}$ $(1 \leq 0)$
4	0	0	1	1	$\Delta^{w,seat}_{r,l,(i,j)} = \Delta^{congest}_{l,(i,j)}$ $(1 = 0)$
5	0	1	0	0	$\Delta^{w,seat}_{r,l,(j,k)} \geq \Delta^{congest}_{l,(j,k)}$ $(1 \geq 0)$
6	0	1	0	1	None. Scenario III
7	0	1	1	0	$\Delta^{w,seat}_{r,l,(i,j)} \leq \Delta^{w,seat}_{r,l,(j,k)}$ $(1 \leq 0)$
8	0	1	1	1	$\Delta^{w,seat}_{r,l,(i,j)} = \Delta^{congest}_{l,(i,j)}$ $(1 = 0)$
9	1	0	0	0	$\Delta^{w,seat}_{r,l,(i,j)} = \Delta^{congest}_{l,(i,j)}$ $(0 = 1)$
10	1	0	0	1	$\Delta^{w,seat}_{r,l,(i,j)} = \Delta^{congest}_{l,(i,j)}$ $(0 = 1)$
11	1	0	1	0	$\Delta^{w,seat}_{r,l,(i,j)} \leq \Delta^{w,seat}_{r,l,(j,k)}$ $(1 \leq 0)$
12	1	0	1	1	None. Scenario IV
13	1	1	0	0	$\Delta^{w,seat}_{r,l,(i,j)} = \Delta^{congest}_{l,(i,j)}$ $(0 = 1)$
14	1	1	0	1	$\Delta^{w,seat}_{r,l,(i,j)} = \Delta^{congest}_{l,(i,j)}$ $(0 = 1)$
15	1	1	1	0	$\Delta^{w,seat}_{r,l,(i,j)} \leq \Delta^{w,seat}_{r,l,(j,k)}$ $(1 \leq 0)$
16	1	1	1	1	None. Scenario II

Except the four scenarios, all other combinations violate one or more constraints

Solution: Overview



Results:
Schedule Synchronisation "Paradox"



Demand $=150$, departure is 0

$\eta=10$, $\theta=0.15$

	Departure time	
	Before	After
Line 1	0	0
Line 2	10	0

Figure 1 Network and data for demonstrating the paradox phenomenon

Table 1 Results of the before and after study

Cases	Paths	Used Path	Path Flow	Path Cost	Total Travel Cost
Before	Path 1	Yes	150.00	32.50	4875.00
	Path 2	No	-	(51.00)	
After	Path 1	Yes	142.48	32.12	4885.34
	Path 2	Yes	7.52	41.00	

Comprehensive discussions of Paradoxes will be presented in another ***single author*** paper under preparation

A Variational Inequality Formulation for the Stochastic User Equilibrium with a Bounded Choice Set


Q&A