

# How to Make FCore Thread-safe

Jeremy  
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# Outline

- Thread-safety in FCore
  - Motivation: Why the original is thread-unsafe
  - First try
  - Thunk
- Working with PHOAS
  - Motivation: Make simplifier work correctly
  - Three rewrite rules
  - Implementation
- Future and related work

# Motivation

# Motivation

$$\text{(CJD-Bind1)} \quad \frac{\Gamma (y : T_1 \mapsto x_2); \Delta \vdash E : T \rightsquigarrow J \text{ in } S \quad FC, x_1, x_2, f \text{ fresh}}{\Gamma; (y : T_1) \Delta \vdash E : T \rightsquigarrow f \text{ in } S'}$$

```
S' := {  
  class FC extends Function {  
    Function x1 = this;  
    void apply() {  
      ⟨T1⟩ x2 = (⟨T1⟩) x1.arg;  
      S;  
      res = J;  
    }  
  };  
  Function f = new FC();}
```

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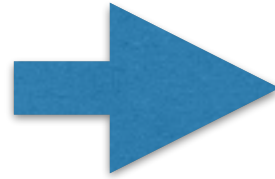
Each FC class is followed by a Function instance

# Motivation, cont.

```
sum (x : Int) : Int =  
  if x == 0  
  then 0  
  else x + sum (x - 1)
```

# Motivation, cont.

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sum (x : Int) : Int =  
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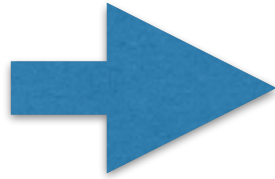


```
class Sum extends Function  
{  
  Function x1 = this;  
  public void apply ()  
  {  
    final Integer x3 = (Integer) x1.arg;  
    if (x3 == 0)  
    {  
      res = 0;  
    }  
    else  
    {  
      Function x7 = x1;  
      x7.arg = x3 - 1;  
      x7.apply();  
      final Integer x8 = (Integer) x7.res;  
      res = x3 + x8;  
    }  
  }  
}  
Function sum = new Sum();
```



# Motivation, cont.

```
sum (x : Int) : Int =  
  if x == 0  
  then 0  
  else x + sum (x - 1)
```



```
class Sum extends Function  
{  
  Function x1 = this;  
  public void apply ()  
  {  
    final Integer x3 = (Integer) x1.arg;  
    if (x3 == 0)  
    {  
      res = 0;  
    }  
    else  
    {  
      Function x7 = x1;  
      x7.arg = x3 - 1;  
      x7.apply();  
      final Integer x8 = (Integer) x7.res;  
      res = x3 + x8;  
    }  
  }  
}  
Function sum = new Sum();
```

What if shared by multiple threads?

# Motivation, cont.

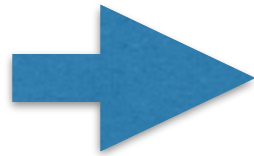
sum 600

sum 500

# Motivation, cont.

Sequential code

sum 600  
sum 500

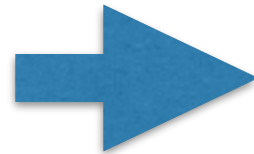


```
sum.arg = 600L;  
sum.apply();  
System.out.println("sum 6000 = " + sum.res);  
sum.arg = 500L;  
sum.apply();  
System.out.println("sum 5000 = " + sum.res);
```

# Motivation, cont.

Sequential code

sum 600  
sum 500



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sum.arg = 600L;  
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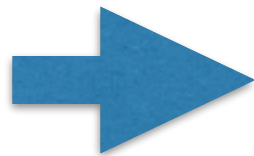
SEQUENTIAL CODE (CORRECT RESULT)

sum 600 = 180300

sum 500 = 125250

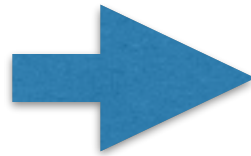
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sum 600  
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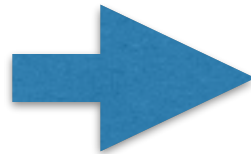
Thread-unsafe code

```
Thread thread1 = new Thread() {  
    public void run() {  
        sum.arg = 600L;  
        sum.apply();  
        res1.res = (Long) sum.res;  
    }  
};  
  
Thread thread2 = new Thread() {  
    public void run() {  
        sum.arg = 500L;  
        sum.apply();  
        res2.res = (Long) sum.res;  
    }  
};
```

# Motivation, cont.

## Thread-unsafe code

sum 600  
sum 500



```
Thread thread1 = new Thread() {  
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Thread thread2 = new Thread() {  
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        sum.arg = 500L;  
        sum.apply();  
        res2.res = (Long) sum.res;  
    }  
};
```

THREAD-UNSAFE CODE (SOMETIMES GIVES INCORRECT RESULT)  
sum 600 = 186945  
sum 500 = 34848

# The Problem ...

- The *Function* instance representing *sum* is shared by multiple threads
- There will be a race condition and results of function invocations may be incorrect
- Depending on the interleaving of two threads, one might overwrite some fields of the other
- JVM is inherently a current platform

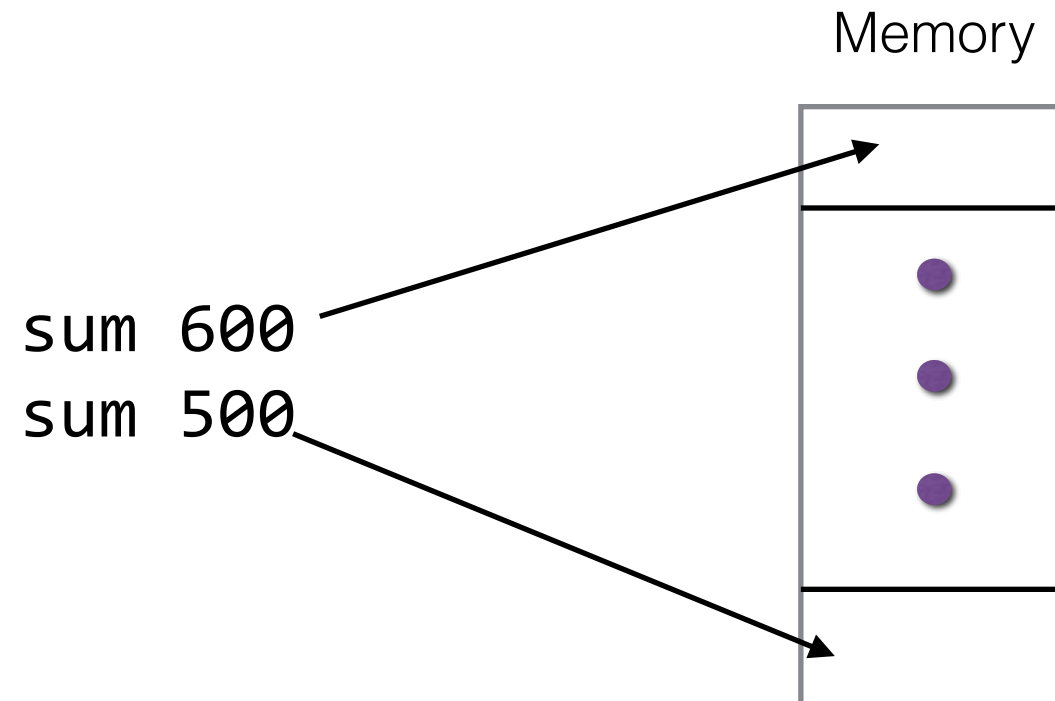


# First try

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# First try, cont.

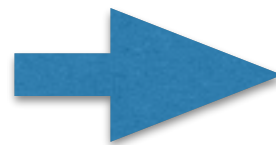
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    public void run() {  
        sum.arg = 500L;  
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    }  
};
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# First try, cont.

## Thread-unsafe code

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};
```



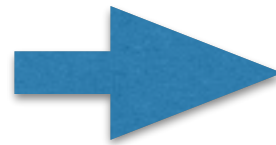
## Thread-safe code

```
thread1 = new Thread() {  
    public void run() {  
        Function sum1 = new Sum();  
        sum1.arg = 6000L;  
        sum1.apply();  
        res1.res = (Long) sum1.res;  
    }  
};  
  
thread2 = new Thread() {  
    public void run() {  
        Function sum2 = new Sum();  
        sum2.arg = 5000L;  
        sum2.apply();  
        res2.res = (Long) sum2.res;  
    }  
};
```

# First try, cont.

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    public void run() {  
        sum.arg = 500L;  
        sum.apply();  
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    }  
};
```



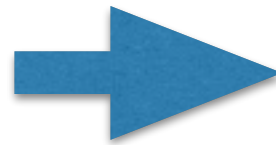
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    }  
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        Function sum2 = new Sum();  
        sum2.arg = 5000L;  
        sum2.apply();  
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    }  
};
```

# First try, cont.

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    }  
};
```

THREAD-SAFE CODE (ALWAYS GIVES CORRECT RESULT)  
sum 600 = 180300  
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# Unfortunately ...

- The previous example only involves first order functions.
- For higher-order functions, we are out of luck.

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corresponding Java code  
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int apply (Function f, int x) { ... }
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corresponding Java code  
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```
int apply (Function f, int x) { ... }
```

same instance in  
each thread

# Contribution

- We present a new approach to make FCore thread-safe
- We implement a Thunk interface to delay memory allocation

# Caveat

- Thread safety is a complicated issue in read world applications
- Writing thread-safe code is a “black art”, since you can’t reproduce all possible interactions between threads
- There are two approaches: to avoid shared state or use some sort of synchronization

# First, let's clarify ...

- In our case, what are the exact threading scenarios we are concerned about?
- How a programmer can use the generated Java code in a multithreading setting?

# User case

- For the moment, our language doesn't support some sort of parallel operator yet
- A programmer will import some code generated by the compiler
- What kind of code (Class? Interface? Object?) we provide to programmers is the key to the thread-safety problem in our domain

# Avoid shared state

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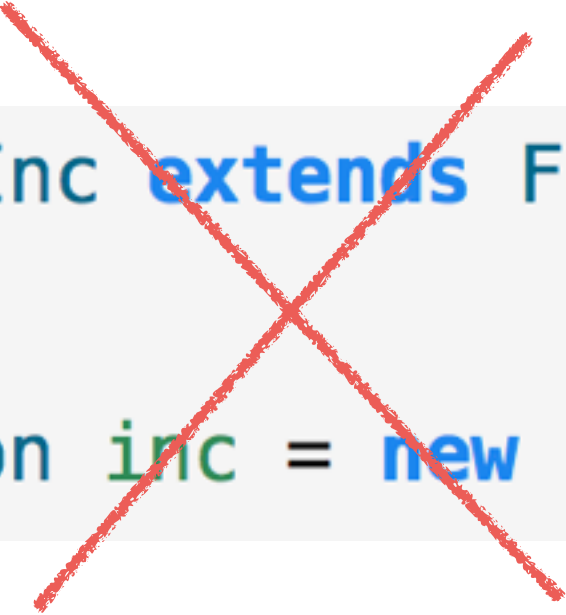
```
class Inc extends Function  
{...}
```

```
Function inc = new Inc();
```

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Function inc = new Inc();
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`inc (x : Int) = x + 1`

```
class Inc extends Function  
{...}
```

```
Thunk inc = () -> new Inc();
```

# Thunk interface

- Inspired by the concept of “thunk” in programming languages: a zero-argument function that delays the computation

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```
public interface Thunk {  
    public Function compute();  
}
```

# How it works

- With the *Thunk* interface, we can wrap the code of object allocation inside the *compute* method in order to delay memory allocation, e.g.

```
Thunk inc = () -> new Inc();
```

- In a multi-threading scenario, even if a *Thunk* instance is shared by multiple threads, each invocation of *compute* will result in a separate object instance

```
public interface Thunk {  
    public Function compute();  
}
```

# How it works, cont.

- What a programmer gets now is a *Thunk* instance. Now he/she can make a thread-safe call.

```
inc (inc 0)
```



# How it works, cont.

- What a programmer gets now is a *Thunk* instance. Now he/she can make a thread-safe call.

Thread safe

inc (inc 0)



```
Function inc1 = inc.compute();  
inc1.arg = 0;  
inc1.apply();  
Function inc2 = inc.compute();  
inc2.arg = inc1.res;  
inc2.apply();
```

# How it works, cont.

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Thread safe

inc (inc 0)



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`twice ≡ (λf:Int→Int).(λx:Int).f (f x)`

# How it works, cont.

- The Thunk interface also works with higher order functions

$\text{twice} \equiv (\lambda f:\text{Int} \rightarrow \text{Int}).(\lambda x:\text{Int}).f (f x)$

```
interface Twice {  
  default int twice (Thunk f, int x) {  
    Function fun1 = f.compute();  
    fun1.arg = (Integer) x;  
    fun1.apply();  
    Integer res1 = (Integer) fun1.res;  
    Function fun2 = f.compute();  
    fun2.arg = (Integer) res1;  
    fun2.apply();  
    Integer res2 = (Integer) fun2.res;  
    return res2;  
  }  
}
```

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    fun2.apply();  
    Integer res2 = (Integer) fun2.res;  
    return res2;  
  }  
}
```

# How it works, cont.

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    Function fun2 = f.compute();  
    fun2.arg = (Integer) res1;  
    fun2.apply();  
    Integer res2 = (Integer) fun2.res;  
    return res2;  
  }  
}
```

force the programmer  
to pass Thunk instance

# Conclusion & future work

- The Thunk interface works well with System F binders and let expressions.
- At the moment, we are implementing a modified version to work with TCE
- We leave a full implementation for future work.



# Second Part (QA)

# Problem

- Not long before, simplifier will produce code with superfluous identity functions and lambda wrappings.
- Semantically, the generated code is correct. Practically, it is slow due to the generation of extra closures.

# Problem, cont.

$\lambda(f : \text{Int} \rightarrow \text{Int}). \lambda(x : \text{Int}). f\ x$

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 $\quad (\lambda(z : \text{Int}). z)$   
 $\quad\quad (x\ ((\lambda(d : \text{Int}). d)\ y)))\ g)$

# Let's do simplification by hand!

```
λ(f : (Int -> Int) -> Int). λ(g : Int -> Int).  
  f ((λ(x : Int -> Int). λ(y : Int).  
      (λ(z : Int). z)  
      (x ((λ(d : Int). d) y))) g)
```

# Problem, cont.

```
λ(f : (Int -> Int) -> Int). λ(g : Int -> Int).  
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```

*peval*



```
\ (f : (Int -> Int) -> Int).  
  \ (g : Int -> Int).  
    f  
      let x = g  
      in  
        \ (y : Int).  
          let z = x (let d = y in d)  
          in z
```

# Problem, cont.

```
λ(f : (Int -> Int) -> Int). λ(g : Int -> Int).  
  f ((λ(x : Int -> Int). λ(y : Int).  
      (λ(z : Int). z)  
      (x ((λ(d : Int). d) y))) g)
```

*peval*



```
\ (f : (Int -> Int) -> Int).  
  \ (g : Int -> Int).
```

f

let x = g

in

```
\ (y : Int).
```

let z = x (let d = y in d)

in z



```
λ(f : (Int -> Int) -> Int).  
  λ(g : Int -> Int).  
    f g
```

# Contribution

- We propose three rewrite rules to make simplifier work correctly
- We presents a PHOAS-based implementation of the rewriting process

# Rewrite rules

- We need to perform rewriting on the core AST
- There are 3 basic patterns

# Rule 1

$\text{let } x = y \text{ in } e \quad \sim > \quad [x \rightarrow y] e$

(where  $y$  is a variable)



# Rule 2

let  $x = e$  in  $x \quad \sim > \quad e$

# Rule 3

$$\lambda x. y \ x \quad \sim > \quad y$$

(where  $y$  is a variable)

# Rule 3, cont.

$$\lambda x. y \ x \quad \sim > \quad y$$

(where  $y$  is a variable)

- The condition ( $y$  is a variable) is important!
- In general, this is not a valid transformation in call-by-value

# Rule 3, cont.

$$\lambda x. y \ x \quad \rightsquigarrow \quad y$$

(where  $y$  is a variable)

- The condition ( $y$  is a variable) is important!
- In general, this is not a valid transformation in call-by-value

$$(\lambda f . 1) \ (\lambda x . \text{infinite } x) \quad ==> \quad 1$$

$$(\lambda f . 1) \ \text{infinite} \quad ==> \quad \text{does not terminate}$$

# Let's do rewriting by hand!

```
\ (f : (Int -> Int) -> Int).  
  \ (g : Int -> Int).  
    f  
      let x = g  
      in  
        \ (y : Int).  
          let z = x (let d = y in d)  
          in z
```

1.  $\text{let } x = y \text{ in } e \sim> [x \rightarrow y] e$  (where  $y$  is a variable)
2.  $\text{let } x = e \text{ in } x \sim> e$
3.  $\lambda x. y x \sim> y$  (where  $y$  is a variable)

# PHOAS

- In the back-end, we represent ASTs with Parametric Higher Order Abstract Syntax (PHOAS)
- The function space of the meta language is used to encode the binders of the object language
- It has benefits of avoiding common issues of alpha-equivalence and name-capturing

# PHOAS, cont.

```
data Expr t e
  = Var Src.ReaderId e
  | Lit Src.Lit

-- Binders we have:  $\lambda$ , fix, letrec, and  $\Lambda$ 
  | Lam Src.ReaderId (Type t) (e -> Expr t e)
  | Fix Src.ReaderId Src.ReaderId
      (e -> e -> Expr t e)
      (Type t)  -- t1
      (Type t)  -- t
  | Let Src.ReaderId (Expr t e) (e -> Expr t e)
```

# Implementation

- It turns out, implementing Rule 1 is fairly easy, while for Rule 2 and 3, it's a bit involved
- The difficulty arises because of the abstract type arguments

`data Expr t e`

- They should not be instantiated to concrete types when constructing ASTs



# Implementation, cont.

```
-- Rule 1: let x = y in e                                => [x -> y] e    (where y is a variable)
rewrite1 :: Expr t e -> Expr t e
rewrite1 (Let _ (Var _ n) f) = rewrite1 (f n)
rewrite1 e = mapExpr rewrite1 e
```

# Implementation, cont.

Rule 2:  $\text{let } x = e \text{ in } x \quad \sim > \quad e$

- The job is to recognise this pattern. We can't just pattern match on it directly
- We need to distinguish different variables
- One way is to instantiate the type argument  $e$  to `Int`
- The requirement is, after transformation, the type argument should still be abstract

# Implementation, cont.

Expr t Int



Expr t e

# Implementation, cont.

Expr t Int  Expr t e

- The idea is simple. Use a map to record mappings of Int and e

Map Int e

# Implementation, cont.

`Expr t Int`  `Expr t e`

- The idea is simple. Use a map to record mappings of `Int` and `e`

`Map Int e`

`rewrite2 :: Map Int e -> Expr t Int -> Int -> Expr t e`

# Implementation, cont.

`Expr t Int`  `Expr t e`

- The idea is simple. Use a map to record mappings of `Int` and `e`

`Map Int e`

`rewrite2 :: Map Int e -> Expr t Int -> Int -> Expr t e`



different  
variables

# Implementation, cont.

`Expr t Int`  `Expr t e`

- The idea is simple. Use a map to record mappings of Int and e

`Map Int e`

`rewrite2 :: Map Int e -> Expr t Int -> Int -> Expr t e`

different  
variables

a counter





# Implementation, cont.

Expr t Int  Expr t e

# Implementation, cont.

Expr t Int  Expr t e

```
case expr of
  Let n e f ->
    case f num of
      Var _ num' ->
        if num == num'
        then rewrite2 env e num
        else Let n
              (rewrite2 env e num)
              (\b ->
                rewrite2 (Map.insert num b env)
                  (f num)
                  (num + 1))
      - ->
        Let n
          (rewrite2 env e num)
          (\b ->
            rewrite2 (Map.insert num b env)
              (f num)
              (num + 1))
```

# Future & related work

# Future & related work

- Related work
  - Heavily inspired by the paper *Functional Programming with Structured Graphs*
  - Using PHOAS to define cyclic structures (e.g. graphs), easy to observe and manipulate sharing and cycles

# Future & related work

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- Future work
  - The approach needs several iterations of rewritings
  - We need structured equality on ASTs (too time consuming)

Thanks you!