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Fuzzy control of active suspension system based on quarter car model

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ABSTRACT

In this paper, the performance of model-free intelligent Fuzzy Controller analyzed with the model-based conventional controller concerning body displacement and body acceleration. The development of a mathematical model for active suspension is quite difficult which increases the difficulty of designing a model-based controller for active suspension. So, the model-free intelligent adaptive fuzzy controller has been proposed for active suspension. The Fuzzy Controller predicts and controls the performance of active suspension better than a conventional controller. The results obtained from the simulation of the road profile show that the proposed fuzzy control performs better than the conventional controller in terms of body displacement and body acceleration.

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1. Introduction

Various components play a vital role in vehicle handling, cornering, and comfort capabilities; one of those components is the vehicle's suspension, which separates the vehicle body from the road bumps and vibrations. Since ride comfort, handling, driving capability, and safety are important factors in the automotive industry, designing and developing a finer quality suspension system that can tackle rough road surface conditions is important. A vehicle suspension system that is absolute should have the capability to reduce the displacement of sprung mass and acceleration and provide sufficient deflection in suspension to maintain tire-terrain contact. Since the late 1970s, various types of suspension systems: passive, semi-active [1–4], and active [5], have been developed for automotive industry. In recent days more attention has been paid to an active system because active suspensions meet the performance, comfort, and safety requirements that the customers demand. On the other hand, an active suspension system is not yet cost-effective. The system is built-in high-end cars and supercars but is still expensive for the general public road cars. Various automotive companies have developed various suspension systems over the years. Over the year, suspension systems have been developed from passive to semi-active systems; the recent trend in suspensions is towards active [6] and adaptive suspensions [7–8]. Active suspensions are developed by various automobile companies using different types of controllers to give out the best suspension performance and the best ride comfort. The automotive industry is one of the world's highly competitive industries where manufacturers work on new methods to bring out the best performance. The suspension system is one system that is highly focused in the industry. Suspension systems before the introduction of active suspension were tuned for certain road surfaces 20 years ago and had to be manually modified for off-road and track situations.

The introduction of active suspension has changed the way customers look at a certain vehicle. A road car suspension system can be set to track conditions just by changing the suspension modes in the car. Various manufacturers use different controllers. PID(Pro portional-Integral-Derivative) controller [9], LQR(Linear Quadratic Regulator) Controller [10], Fuzzy controller [11], LQG(Linear–Quad ratic–Gaussian) Controller [12], LPV(Linear parameter-varying control) Controller [13]. It is difficult to identify the mathematical model of active suspensions as they exhibit complex and non-linear characteristics. As a result, designing a model-based controller to control an active suspension system is impracticable. Fuzzy logic controllers (FLC) are based on fuzzy sets and are used to improve the control performance of active suspension systems. They are created without using system models for development and simulation.

Lotfi Zadeh introduced Fuzzy logic in his seminar paper fuzzy set theory in 1965. Lotfi A. Zadeh elaborated his idea of fuzzy logic in the year 1973and introduced a concept called "linguistic variables" [14]. The fuzzy logic for active ride comfort of the quarter car model was introduced by Ro et al[15]. The strategy for

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controlling an automobile suspension system using fuzzy logic was employed by Yester, Mcfall, Cherry, and Jones[16]. Huang and Chao proposed a fuzzy control scheme that removes the tire deformation from the control variable for improving the control performance using a grey predictor[17]. However, the design of a traditional fuzzy controller depends on the rules established in the fuzzy rules bank by an expert or an experienced operator.

Moreover, fuzzy has its limitations; the controller demands regular updates to provide consistent performance. Therefore, the fuzzy logic controller depends on human knowledge and expertise. Hac [18], in the late 1980s, for a vehicle active suspension system, introduced a classical form of the adaptive scheme. This marked the beginning of an adaptive control scheme, where feedback gains were varied by changing the power spectral density (PSD) of terrain roughness obtained by processing measurement data. Gordon et al [19] compared non-linear controllers and linear quadratic gaussian (LQG) for active suspension.

The paper focuses on the Fuzzy logic controller suspension system and the results that an FLC suspension system produces on different road surface profiles and compares it to the results of the passive system and the PID controller system on the same road profiles. The paper also compares the passive suspension system used in almost all cars to the FLC suspension model and gives a detailed look into the gains that the FLC suspension model offers. This paper found that the FLC suspension system offers better performance and results than the generic PID suspension system model, primarily used in luxury cars and modern supercars. The reason to opt for the FLC model is that PID has a few drawbacks like poor control performance and significant time delays overcome by the FLC model. Furthermore, it looks into the advantages and comprehensive options an active suspension system offers based on road setups.

On the other hand, the passive model fails to provide the active model's results based on performance and overall ride quality. The controller for an active suspension system was designed using a quarter car model, which consists of one-fourth of the vehicle mass, a spring, and a damper linked between the wheel's mass. The quarter car model is a simplified two-degree-of-freedom model of an automobile that has been used to assess the vertical motion of the car body and wheel for active suspension. The quarter car model will be utilized to depict the heave motion of one-fourth of the vehicle mass. The rest of the paper is divided into four sections: The mathematical model of the quarter car model with the governing equation is stated in Chapter 2. In chapter 3, you will construct an intelligent controller using fuzzy logic. The simulation results for various road profiles are shown in Chapter 4. The work's conclusion will be compared with the passive suspension system and the PID controller-based active suspension system[20].

2. Design of suspension system

2.1. Mathematical representation

A Quarter Car model is a car's mass, damper, and spring, taken as one-fourth of the total value. In addition, the vehicle has two D.O.F, which are used to measure the vertical motion of the wheel and the vehicle body for active suspension.

The main objective of using a quarter car model in suspension analysis is that it can get the characteristics of an entire car and is very simple. The quarter car suspension model is shown in "Fig. 1,". The arithmetical addition of the vertical forces on the sprung, unsprung masses is used to find the equation for model motions. Let "M" represent sprung mass; unsprung mass represents the tire and axles by m. The suspension is formed by placing the variable-force generating element, shock absorber, and spring

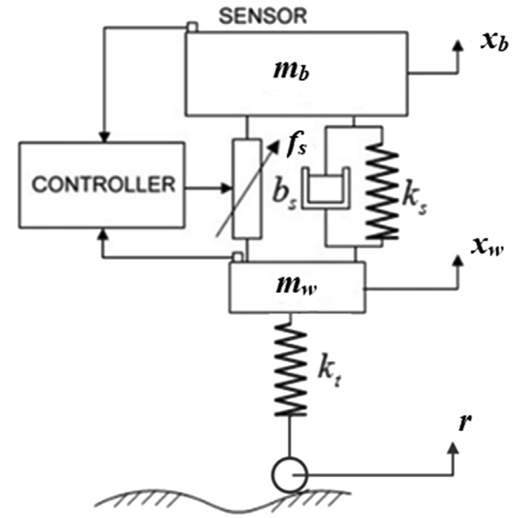

Fig. 1. Quarter car suspension Diagram.

in the middle of the sprung and unsprung masses. Obtaining a state-space illustration of a quarter car model is the main objective of mathematical modeling. The state variable can illustrate the vertical movement of wheels and the vehicle body. The linear spring of the tire is represented by stiffness Kt. The active control force 'u,' which is kept parallel with the damper 'Ca' and passive spring Ka constitutes the car's suspension system. Time is linearly dependent and expected to have no non-linear effects on the system. Description of the state variables is shown in "Table 1″.

The following are the state variables;

$$\mathbf{X}_1 = \mathbf{X}_s - \mathbf{X}_w$$
$$\mathbf{X}_2 = \mathbf{X}_s$$
$$\mathbf{X}_3 = \mathbf{X}_w - r$$
$$\mathbf{X}_4 = \dot{\mathbf{X}}_w$$

The sprung mass of the system will be 290 Kg, while the unsprung mass will be 58 Kg. The damping coefficient of the damper will be 1001 N/m/s, and the stiffness of the damper will be 16,813 N/m. Let the stiffness of the tire and the actuation force be 190,000 N/m and 10,000 N, respectively.

2.2. System description

Apart from the actuator, and while the suspension travel is below its physiological limit, the dynamics of the suspension system are specified using the linear equations (1) and (2).

$$m_b x_b + b_s(x_b - x_w) + k_s(x_b - x_w) = f_s \quad (1)$$

$$m_w x_w + b_s(x_w - x_b) + k_s(x_x - x_b) + k_t(x_w - r) = -f_s \quad (2)$$

Wherein $\ddot{x}_b$ implies frame acceleration; $\ddot{x}_W$ denotes wheel acceleration; $\dot{x}$ denotes wheel speed; fs denotes actuator force. The sector

Table 1
State Variable.

Notations	Meaning
$\dot{X}_w$	Wheel Velocity
$\mathbf{X}_w - \mathbf{r}$	Wheel Deflection
$\ddot{X}_s$	Car Body Acceleration
$\dot{\mathbf{X}}_v$	Car Body Velocity
$\mathbf{X}_s - \mathbf{X}_w$	Suspension Control

car version's motion equations (3) for Active suspension may be expressed in state-space form as follows:

$$\begin{bmatrix} \dot{X}_b \\ \ddot{X}_b \\ \dot{X}_w \\ \ddot{X}_w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_s}{M_b} & \frac{-b_s}{M_b} & K_s & M_s \\ 0 & 0 & 0 & 1 \\ \frac{M_s}{M_w} & \frac{b_s}{M_w} & \frac{-K_t-K_s}{M_w} & \frac{-b_s}{M_w} \end{bmatrix} \begin{bmatrix} X_b \\ \dot{X}_b \\ X_w \\ \dot{X}_w \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{f_s}{0} & \frac{M_s}{M_b} \\ 0 & 0 \\ M_w & M_w \end{bmatrix} \begin{bmatrix} r \\ u \end{bmatrix} \tag{3}$$

In-state space, the system's output is represented as equation (4).

$$\begin{bmatrix} X_b \\ X_b - X_w \\ X_w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ \frac{-K_s}{M_w} & \frac{b_s}{M_w} & \frac{-K_t-K_s}{M_w} & \frac{-b_s}{M_w} \end{bmatrix} \begin{bmatrix} X_b \\ \dot{X}_b \\ X_w \\ \dot{X}_w \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_t}{M_w} & \frac{-f_s}{M_w} \end{bmatrix} \begin{bmatrix} r \\ u \end{bmatrix} \tag{4}$$

3. Intelligent controller design

The objective of this study is that intelligent control of the system which is to be controlled does not ought to be bolt modeled. Simulations show that the intelligent controllers have better performance than the classical controllers. The model of the system to be managed is developed using the intelligent control system described in this study. The designer specifies the system behavior, and the IC system models the system abstractly. According to this theory, human thinking is imprecise, non-computable, and non-binary. There may be no black-and-white solutions in some cases, but grey is a good reminder.

3.1. Control problem

Ensuring the comfort and safety of the passengers inside the vehicle and making sure the lifetime of the components used in the suspension stays intact, under the application of disturbances on the road, for a more extended period are the main objectives of the control objective. The amount of vertical body acceleration felt by the person in the car determines the car's level of comfort. Avoiding the suspension system components to hit the rattle-space limits increases the lifetime of those components. The main objectives of the components are that under the influence of road disturbance, the range of acceleration of the car body must be reduced. The suspension travel limits do not increase under the addition of typical road disturbances concerning the open-loop system.

Zr will represent the road simulation to check the described control system, represented as the Eqs. (5)–(7).

$$Z_{r1} = \begin{cases} -0.1, & 0 \le t \le 10 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

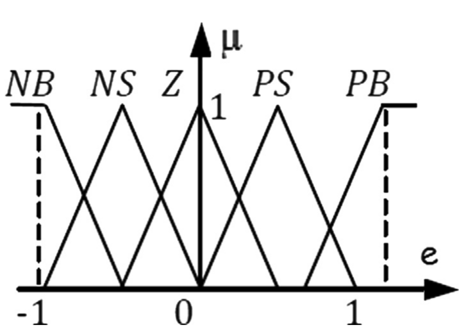

Fig. 2. Membership function for error.

$$Z_{r2} = \begin{cases} 0.1(1-\cos(2\pi t)), & 1 \le t \le 2 \\ 0.4(1-\cos(2\pi t)), & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

$$Z_{r3} = \sum_{i=0}^{N} \sqrt{\Delta n 2^k} 10^{-3} (\frac{n_0}{i\Delta n}) \cos(2\pi i \Delta n x + \varphi_i) \tag{7}$$

The abscissa variable is denoted by x which changes from 0 to L.

$$\begin{aligned} L &= 250m \\ \Delta n &= \tfrac{1}{L} \\ n_{\max} &= \tfrac{1}{B} \\ N &= \tfrac{n_{\max}}{\Delta n} = \tfrac{L}{B} \end{aligned}$$

The constant value is denoted by K, determined by the ISO road profile classification. Concerning the class A to class H profiles, the numbers rising from 3 to 9 are studied, n_0 = 0.1 cycles/m; φ_i random phase angle that follows a uniform probability distribution spans between the range of 0-2π.

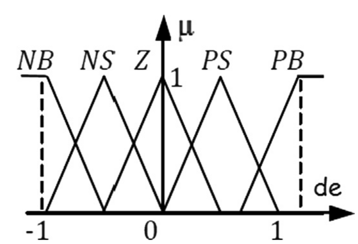

Fig. 3. Membership function for change in error.

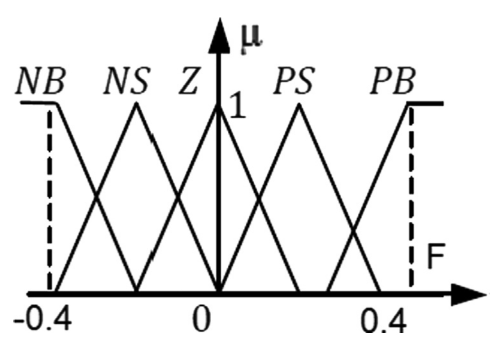

Fig. 4. Membership function for actuator force.

Table 2
Fuzzy Rule Table.

C/ce	NB	NM	ZE	PM	PB
NB	NB	NB	NM	NM	Z
NM	NB	NM	NM	ZE	PM
ZE	NM	NM	Z	PM	PM
PM	NM	ZR	PM	PM	PB
PB	ZR	PM	PM	PB	PB

[a]NB: Negative Big
[b]NM: Negative Medium
[c]ZE: Zero
[d]PM: Positive Medium
[e]PB: Positive Big
[f]C: Error
[g]Ce: Change in error.

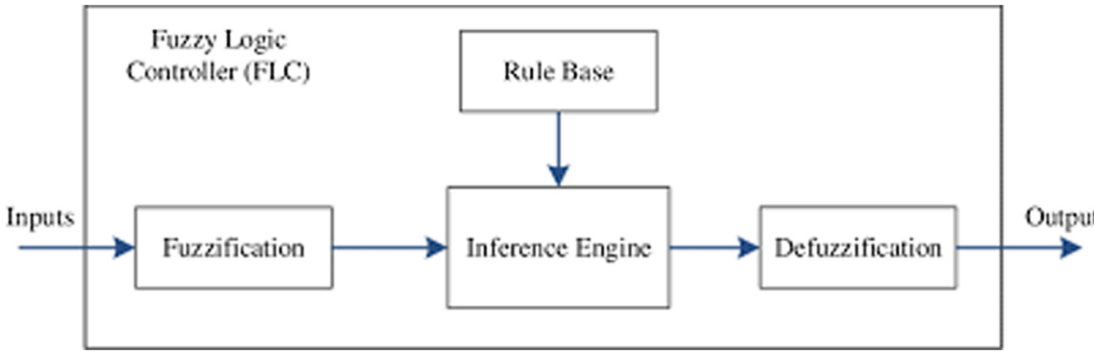

Fig. 5. Circuit diagram of Fuzzy logic control.

3.2. Fuzzy controller

Fuzzy logic is a numerical framework for evaluating analog input values in logical variables with continuous values between 0 and 1, as opposed to classical or digital logic, which works with discrete values of 1 or 0. In other words, fuzzy logic is influenced by both the condition of the input and the pace at which it changes.

Lotfi Zadeh finds fuzzy logic. In the early day, fuzzy logic was used in the copy machine. But due to its decision-making capabilities and also for real-time, Non-linearity behavior. Afterward, it has included in the active suspension system. A Fuzzy logic con-

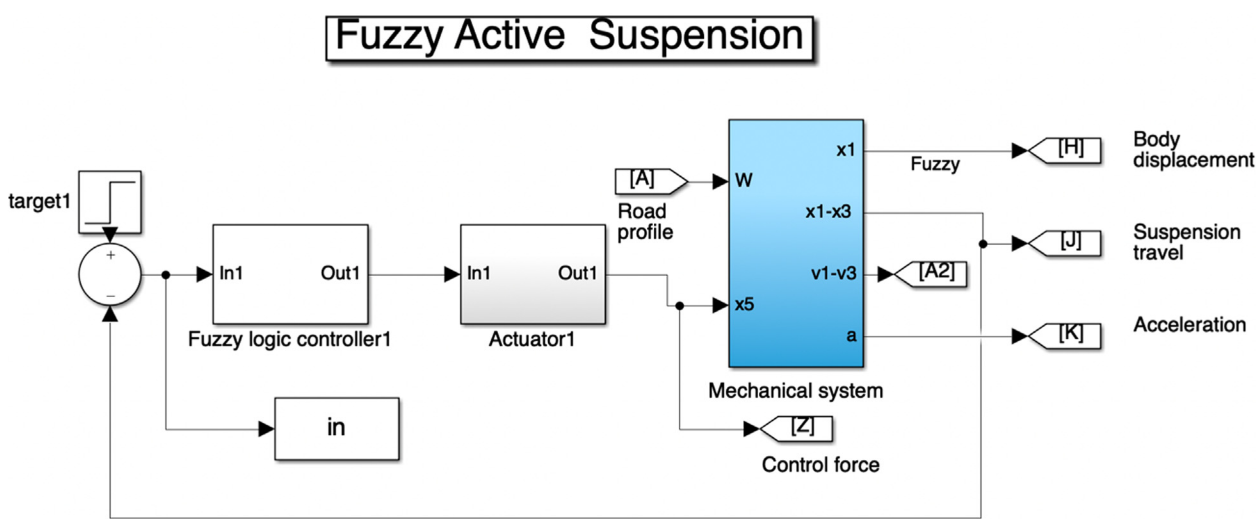

Fig. 6. Model representation of fuzzy controlled active suspension.

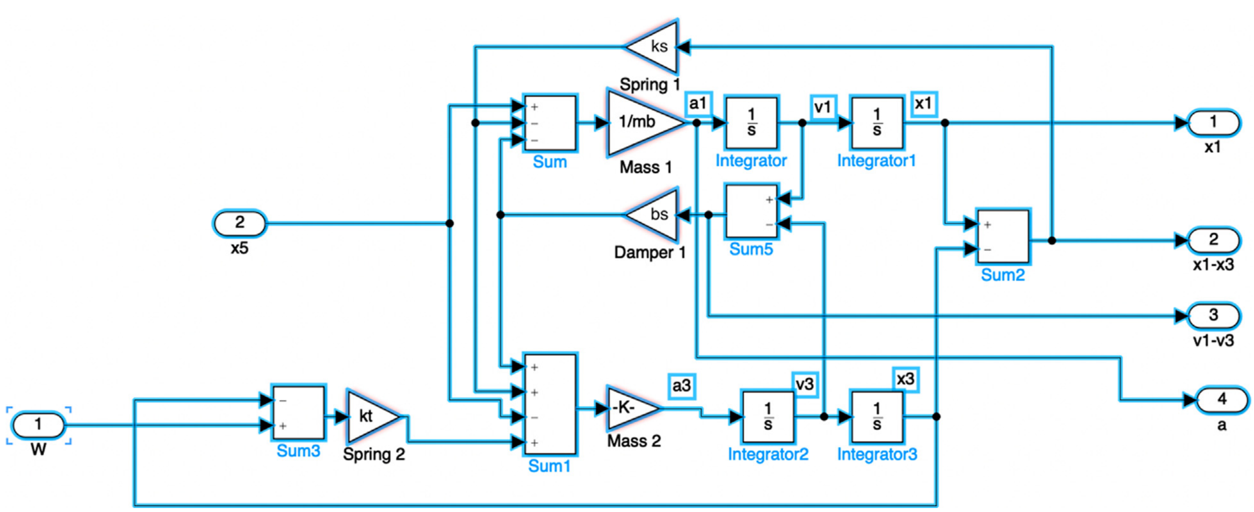

Fig. 7. Mathematical model of quarter car mechanical suspension system.

troller consists of two input ports and one output port. The input port is used to get a value of the body error (C), and the other port is used to get a change in the error of the suspension (Ce). The output port gives the value of how much force F does the actuator produces to neglect the disturbance.

Basically, inside the fuzzy controller, there are three stages first one is fuzzification, the second stage is the fuzzy inference machine, and the final stage represents defuzzification. The stage converts the given values (crisp Numbers) into fuzzy values with the help of the membership function. After converting the values by fuzzification, those values are sent to the fuzzy inference. At that stage, the values are computed according to the rule base and database. As a consequence, for every rule, one fuzzy subset is allocated for every independent variable. Inference procedures such as MIN or PRODUCT are frequently employed. The output membership function in MIN inference is trimmed over at a height proportional to the estimated level of truth of the governing premise (fuzzy logic AND). "Figs. 2-4" demonstrate a possible set of membership functions for the indicated variables of the active suspension system represented by a fuzzy set. A traditional interpretation of Mamdani was utilized as the basis for the rule. The rules are written in an "If-Then" structure, with the "If" side referred to as the conditions and the "Then" side as the judgment. The following "Table 2" with fuzzy words is the rule base utilized in the quarter car system.

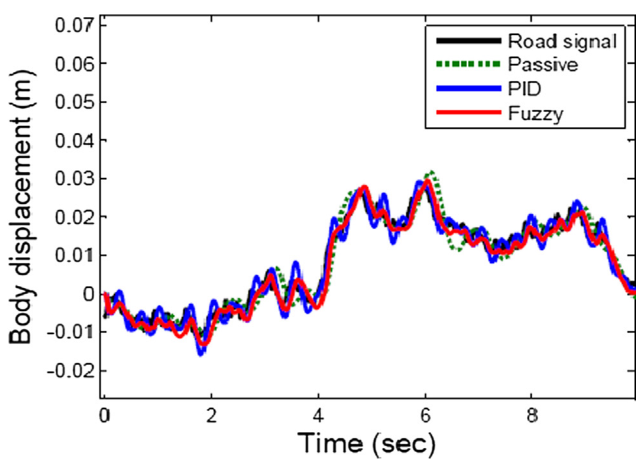

Fig. 8. Response of Body acceleration.

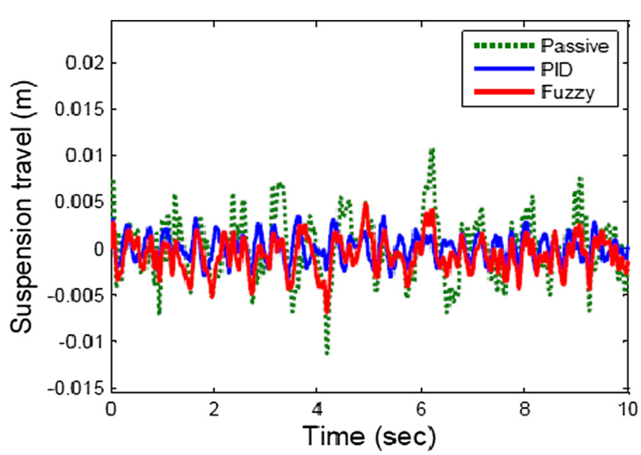

Fig. 9. Response of Suspension Travel.

The processed output is in the fuzzy values, but the actuator understands only digital output or analog output. So, defuzzification comes to play to convert the fuzzy value to an actual number. The centroid method is the one commonly used technique in defuzzification. The following is an equation (8) for the centroid defuzzification methodology:

$$Z_{COG} = \frac{\int_Z \mu_A(Z)Z\mathrm{d}z}{\int_Z \mu_A(Z)\mathrm{d}z} \tag{8}$$

Where Z_{COG} is a crisp output, $\mu_A(Z)$ is the aggregated membership function and z is the output variable. However, like all techniques, this method also has some limitations that it cannot compute complex membership functions. "Fig. 5" show the circuit diagram of the fuzzy controller.

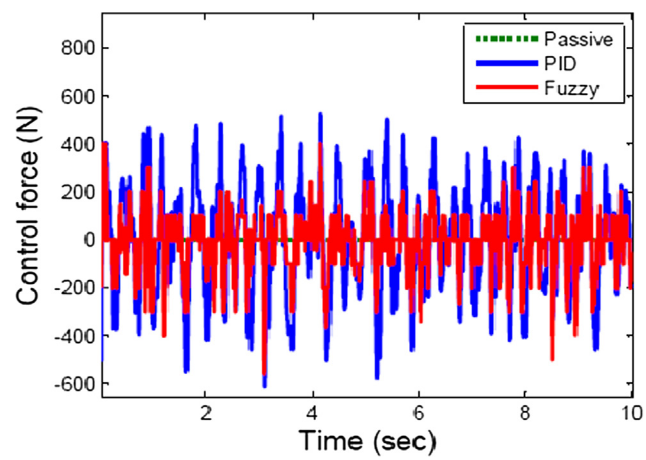

Fig. 10. Response of Control force.

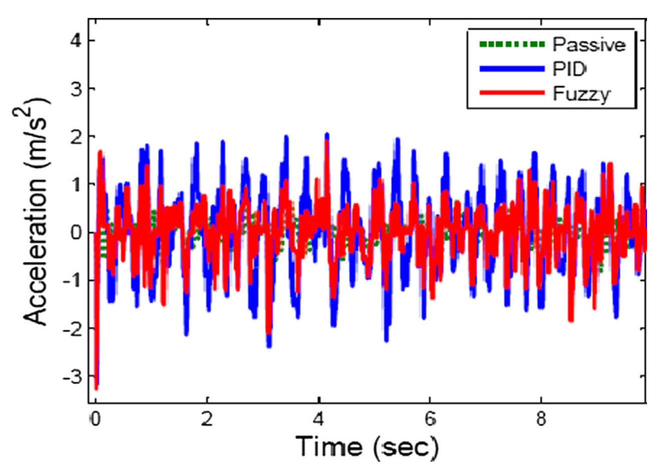

Fig. 11. Response of Body acceleration.

Table 3
Root Mean Square Values of Road Profiles Zr.

Road profile	Zr		
Parameter	Passive	PID	Fuzzy
Body displacement (m)	0.08	0.02	0.01
Suspension Travel (m)	0.03	0.02	0.01
Control force (KN)	0	0.4	0.2
Body Acceleration (m/s^2)	0.31	0.10	0.06

4. Simulation

4.1. Modelling of system

The research goal discussed in this paper was to show how to use fuzzy logic to operate a continuously dampening vehicle suspension system. When road disturbances from the smooth road and actual road roughness occur, monitor that the Model representation of the Active suspension system is built-in MATLAB software. This model representation consists of a mechanical design of the quarter car suspension system as seen before, a fuzzy controller, the mathematical model of the road profile, and a mathematical model of the actuator; all these models are combined using the Simulink as shown in "Fig. 6", for ease understanding and also for simulation purpose. The mathematical model of the quarter car suspension system is shown in "Fig. 7". By using the Fuzzy controller in the Active suspension model, the ride comfort is improved by reducing the body acceleration induced by the car body. The model and controller utilized in the study are also described, and the vehicle reaction outcomes were acquired from a variety of road input scenarios. In the end, MATLAB simulations are used to compare fuzzy-based active suspension control with (PID) control-based dynamic suspension control and passive suspension.

4.2. Simulation of system

Several simulations are used to demonstrate the performance of the Fuzzy controller system in this section. Different road profiles produce different simulation results, as shown in "Figs. 8-11". For comparison, other results of the vehicle model with PID controller are also provided. Because PID is one of the most widely used control methods, it provides fast responses, better system stability, and low steady-state errors within set limits.

A quarter car model-based active suspension is used to demonstrate the suggested fuzzy controller. For passive suspension, fuzzy controller, and PID controller, each representative variable's Root Mean Square (RMS) value is calculated. RMS readings are an excellent method to judge how well suspension settings work. As demonstrated by the RMS value of body displacement and acceleration, the suspension system improves the ride quality and decreases the amount of vibration experienced by passengers.

From Table 3, it is clear that a 50% reduction in body displacement and suspension travel can be achieved by Fuzzy controlled

Fig. 12. Prototype of suspension test rig.

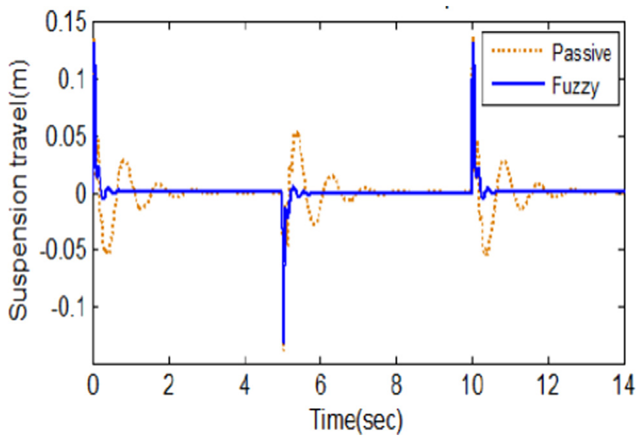

Fig. 13. Response of suspension travel.

active suspension compared to PID controlled Active suspension system. Similarly, body acceleration of 40% is reduced than PID controlled active suspension system. Also, there is an 88% reduction in body displacement that can be achieved by Fuzzy controlled active suspension compared to a Passive suspension system. Similarly, Suspension Travel by 66% is reduced in the Passive suspension system, and Body Acceleration by 80% is reduced in the Passive suspension system. The response of the fuzzy controller indicates that the body displacement and body acceleration are considerably less than that of passive and PID-controlled active suspension. Therefore, the fuzzy controller for random road inputs may accomplish a more significant degree of suspension parameter reduction, confirming the feasibility of the proposed fuzzy controller for active suspension.

4.3. Experimental verification

The prototype of suspension test rig is as shown in "Fig. 12". The suspension test rig is a bench-scale model that may be used to evaluate the efficiency of Fuzzy Active Suspension in real - time basis. "Fig. 13" depicts the Fuzzy controller's reaction to suspension travel. The trial is run for a total of 14 s, with the blink line representing passive suspension and the solid line representing Fuzzy controlled active suspension suspension travel. It is apparent that the Fuzzy controlled active suspension has more control over lowering suspension travel to improve a car's road-holding capacity than the passive suspension.

5. Conclusion

A quarter car model having two degrees of freedom was designed for active suspension. Because of the nonlinear impact of suspension and the unexpected behavior of the hydraulic actuator, developing a mathematical model for active suspension has been problematic. The fuzzy controller can handle a complicated mathematical model in active suspension, which can manage unknown parameters. As a result, a fuzzy controller for active suspension was designed to outperform the analytical control scheme and the PID controller active suspension model. The quarter car model's reaction demonstrates the viability of a fuzzy controller for active suspension over the PID controller. The fuzzy model also proves the massive gains achieved by an active suspension system over a passive system. The simulation process has been observed that the fuzzy controller reduces body displacement and Suspension travel by 50% over the PID controller. The experimental setup of quarter car model also confirms that fuzzy controller provide better reduction in suspension travel than Passive suspension system. In the future, an adaptation for the self-organizing fuzzy controller for the active suspension to build fuzzy rules by learning the system's dynamic behavior will be created. Using appropriate membership functions and fuzzy rules, the self-organized fuzzy controller will make building a fuzzy controller easier.

CRediT authorship contribution statement

J. Joshua Robert: Conceptualization, Methodology, Software, Writing – original draft, Supervision, Investigation. **P. Senthil Kumar:** Conceptualization, Methodology, Software, Formal analysis, Supervision, Project administration. **S. Tushar Nair:** Methodology, Visualization, Writing – original draft, Software. **D.H. Sharne Moni:** Writing – original draft, Data curation. **B. Swarneswar:** Writing – original draft, Data curation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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