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Robust Nonlinear Control for Automotive Active Suspension Systems

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Abstract— **To ensure a comfort ride and safe transportation for people and cargos, active suspension systems are used. An active suspension system is mainly operated according to the road conditions via controlling a hydraulic or pneumatic actuator to guarantee zero vertical acceleration and velocity at the passenger or cargo cabinet. In order to explore new control methods for comfort transportation, a robust nonlinear control scheme is proposed. In this paper, the proposed controller is designed to overcome the road disturbances and ensure a comfort ride with vehicle stability while driving. A robust feature is utilized in the proposed controller design to improve the driving performance under the unknown and uncertain parameters of the active suspension system. Preliminary simulation results are introduced to verify the proposed controller and to demonstrate the effectiveness of the design.**

I. Introduction

One of the important parts in ground vehicles is the suspension system. It provides a comfortable ride for passengers, and prolong the life of the electrical and mechanical components by isolating the movement of the vehicle body from its wheels [1]. Thus, a good suspension system reduces the body acceleration and provides a better road handling. In fact, the main objective when designing an active suspension system is to provide a comfortable ride to the passengers and maintain a sturdy contact between the wheels and the road [2].

Conventional suspension systems consist of springs, shock absorbers, and linkages that connect a vehicle to its wheels. The parameters in this type of suspension system, known as a passive suspension system, are fixed. A suspension system with higher values of stiffness and damping, known as a hard suspension, offers a better road-wheel contact, but less wheel-body isolation [3]. However, a firm wheel contact reduces the quality of ride comfort [4]. On the contrary, a suspension system with lower values of stiffness and damping, known as a soft suspension, guarantees better wheel-body isolation, but reduces the road handling [3]. Other suspension systems, known as semi-active suspension systems, have adjustable damping coefficient for the shock absorber, which provide a low power consumption, simplicity, and better performance [5]. Active suspension systems, on the other hand, integrate sensors, actuators, and controllers with passive suspension systems [4]. The performance of such suspension systems can be improved by controlling the actuators independently, to add or dissipate energy to or from the system [6].

Appling linear control scheme to reduce the vehicle vertical acceleration and improve the ride comfort will cause the wheels to break the contact with the road [2]. This conflict clearly shows that linear control strategies are insufficient to meet the objectives of comfort ride and stability on the road, hence, the need for other control strategies is essential [3]. Many control techniques have been used for active suspension systems such as sliding mode, adaptive, optimal, backstepping, neural network, fuzzy logic, and genetic algorithm controllers to mention a few.

Sliding Mode Control (SMC) is known to be applied for uncertain systems affected by unknown disturbance [2]. The unpredicted road disturbance and the nonlinearity of the active suspension system parameters make SMC a good candidate as a system controller. Thus, many researchers, such as [7] and [8], are still using SMC and working on improving the SMC performance by integrating the control design with a disturbance observer to obtain better results. Observer-based control of active suspension systems has surfaced in the past decade [9]. It utilizes the existing vehicle sensors to estimate road disturbance and then use it as an external input [10]. Different observer-based methods can be designed and analyzed in time domain such as the extended state observer and unknown input observer, and in frequency domain such as disturbance observer. The latter is widely used in the literature [10]. Intelligent nonlinear control strategies such as fuzzy logic control and neural network-based control can mimic human logic to manage the trade-offs of suspension systems. Such intelligent controllers, without the need to understand the system mathematics, have been successfully implemented and produced robust controller design [11-13] although a traditional robust control design can be implemented without any intelligent features [14]. All aforementioned control strategies attenuate the disturbance on the vehicle body, but do not completely reject it [15]. However, if the road disturbance is presented as the summation of sinusoidal functions with different frequencies (i.e., phase and amplitude), then applying an adaptive controller with backstepping procedure could cancel the disturbance on the vehicle body [15]. After all, researchers are still coming with new methods of control for active suspension systems to ensure the ride comfort while driving such as [16].

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In this paper, a robust nonlinear controller is designed to actively force the displacement of the vehicle body, with an active suspension system, to a constant vertical level under the presence of road disturbances and uncertain suspension system parameters. In fact, the proposed controller is investigated to explore new methods of control. The performance of the proposed controller was tested in simulation and demonstrated a satisfactorily performance when regulating the displacement of the vehicle body where a quarter car model is considered in this study.

II. Quarter Car Mathematical Model

A quarter car model, as shown in Fig. 1, can be developed using free body diagrams that can, in fact, visualizes forces applied to all masses in order to calculate reactions and equations of motion. From Fig. 1, the displacement dynamics of the car body and wheel, $z_s(t)$ and $z_u(t)$, respectively, can be expressed as follows

$$m_s\ddot{z}_s = -c_s\left(\dot{z}_s - \dot{z}_u\right) - k_s\left(z_s - z_u\right) + F \tag{1}$$

$$\begin{aligned} m_u\ddot{z}_u = &-c_s\left(\dot{z}_u - \dot{z}_s\right) - k_s\left(z_u - z_s\right) - c_t\left(\dot{z}_u - \dot{z}_r\right) \\ &-k_t\left(z_u - z_r\right) - F \end{aligned} \tag{2}$$

where m_s and m_u are the sprung and unsprung masses that represent the car body and wheel, respectively. The parameters $k_s, c_s, k_t,$ and c_t are the coefficients of the spring and damper that represent the suspension and tire of the quarter car model (refer to Fig. 1). The parameter F is the control force in the active suspension and $z_r(t)$ is the road disturbance that represents the road bumps and small potholes.

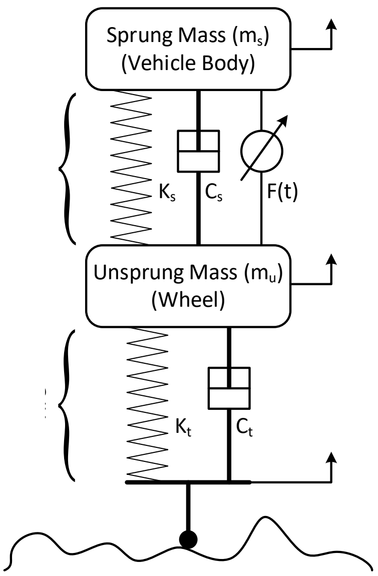

Fig. 1. A quarter car model with active suspension.

To facilitate the development of the proposed controller, the first time derivative is taken for the expression in (1) to obtain the following

$$\dddot{z}_s = G + u \tag{3}$$

where the expression in (2) was utilized and

$$G \triangleq -\beta_1\ddot{z}_s + \beta_2\dot{z}_s + \beta_3 z_s + \beta_4\dot{z}_u - \beta_5 z_u + \beta_6\dot{z}_r + \beta_7 z_r \tag{4}$$

$$u \triangleq \frac{1}{m_s}\dot{F} - \frac{c_s}{m_s m_u}F \tag{5}$$

where $\beta_1 \triangleq \frac{c_s}{m_s}$, $\beta_2 \triangleq \left(\frac{c_s^2}{m_s m_u} - \frac{k_s}{m_s}\right)$, $\beta_3 \triangleq \frac{c_s k_s}{m_s m_u}$, $\beta_4 \triangleq \left(\frac{k_s}{m_s} - \frac{c_s^2}{m_s m_u} - \frac{c_s c_t}{m_s m_u}\right)$, $\beta_5 \triangleq \left(\frac{c_s k_s}{m_s m_u} + \frac{c_s k_t}{m_s m_u}\right)$, $\beta_6 \triangleq \frac{c_s c_t}{m_s m_u}$, and $\beta_7 \triangleq \frac{c_s k_t}{m_s m_u}$.

III. Controller Formulation and Stability

A robust nonlinear controller can be designed and formulated to guarantee that the quarter car vertical displacement, velocity, and acceleration are regulated to desired values in the following sense

$$z_s \to z_{sd}, \quad \dot{z}_s \to \dot{z}_{sd}, \quad \ddot{z}_s \to \ddot{z}_{sd} \quad \text{as} \quad t \to \infty \tag{6}$$

where $z_{sd}, \dot{z}_{sd}, \ddot{z}_{sd} \in \mathbb{R}$ are desired constants. For that purpose, the following error signals are defined

$$e \triangleq z_{sd} - z_s, \quad e_1 \triangleq \dot{e} + \alpha_1 e, \quad e_2 \triangleq \dot{e}_1 + \alpha_2 e_1 \tag{7}$$

where $\alpha_1, \alpha_2 \in \mathbb{R}^+$ are constant control gains and tunable. It should be noted that when $\ddot{z}_{sd} = 0$, this means that $\dot{z}_{sd} =$ constant, and when $\dot{z}_{sd} = 0$, this means $z_{sd} =$ constant. Hence, the actual main objective of the active suspension system is to maintain the vertical acceleration and velocity of the car body, $\ddot{z}_s, \dot{z}_s$, to ZERO despite of the road conditions in order to maintain the safety and comfort of passengers and cargos in the car. Hence, the desired vertical acceleration, velocity, and displacement can be set as $\ddot{z}_{sd}, \dot{z}_{sd} = 0$ and $z_{sd} =$ constant.

A. Open-Loop Error System Dynamics

By taking the first time derivative of the error signal $e_2(t)$, introduced in (7), and by utilizing the other error signals and dynamics, introduced in (3), the open-loop error system dynamics can be written as

$$\dot{e}_2 = -G - u + \left(\alpha_1 + \alpha_2\right)e_2 - \left(\alpha_1\alpha_2 + \alpha_1^2 + \alpha_2^2\right)e_1 + \alpha_1^3 e. \tag{8}$$

It should be noted that the following inequality can be satisfied due to the linearity of the model (1) and (2)

$$G \le \rho_z\left(\|E\|\right) \tag{9}$$

where $\rho_z\left(\|E\|\right)$ is a bounding function, $E \triangleq \left[e_2, \quad e_1, \quad e\right]^T$, and $G(t)$, introduced in (4), can be rewritten as

$$G = \gamma_1 e_2 + \gamma_2 e_1 + \gamma_3 e + \beta_3 z_{sd} + \beta_4\dot{z}_u - \beta_5 z_u + \beta_6\dot{z}_r + \beta_7 z_r \tag{10}$$

where the error definitions in (7) were utilized and $\gamma_1 \triangleq \beta_1$, $\gamma_2 \triangleq -\beta_1(\alpha_1+\alpha_2)-\beta_2$, $\gamma_3 \triangleq \beta_1\alpha_1^2+\beta_2\alpha_1-\beta_3$.

B. Closed-Loop Error System Dynamics

In order to ensure that the proposed controller stabilizes the active suspension system and achieves the control objectives in (6), a Lyapunov-based stability analysis is introduced. Let $P(E,t)\in\mathbb{R}$ denotes the following non-negative function

$$P \triangleq \tfrac{1}{2}e_2^2+\tfrac{1}{2}e_1^2+\tfrac{1}{2}e^2 \tag{11}$$

where the expression in (11) can be proven to be bounded as (refer to Theorem 2.14 of [17])

$$\lambda_1\left\|E(t)\right\|^2 \le P(E,t) \le \lambda_2\left\|E(t)\right\|^2 \tag{12}$$

where $\lambda_1,\lambda_2\in\mathbb{R}^+$ are constants and $E(t)$ was introduced in (9). By taking the first-time derivative of (11), the following expression can be obtained

$$\begin{aligned}\dot{P} &= (\alpha_1+\alpha_2)e_2^2-\alpha_2e_1^2-\alpha_1e^2+\left(1-\alpha_1\alpha_2-\alpha_1^2-\alpha_2^2\right)e_2e_1\\ &\quad+\alpha_1^3e_2e+e_1e-e_2(G+u)\end{aligned} \tag{13}$$

where (7) and (8) were utilized. The control input, $u(t)$, shown in (13), can be designed as

$$u=(\alpha+\alpha_1+\alpha_2)e_2+\left(1-\alpha_1\alpha_2-\alpha_1^2-\alpha_2^2\right)e_1+\alpha_1^3e-\rho_z \tag{14}$$

where $\alpha\in\mathbb{R}^+$ is a constant control gain and ρ_z was introduced in (9). It should be noted that in order to implement the control law in (14) practically, the acceleration and velocity, $\ddot{z}_s,\dot{z}_s$, should be measured. However, in this study, they are assumed measurable. In the future work, an observer will be designed for that purpose. The expression in (13) can be further manipulated and upper bounded as

$$\dot{P}\le-\alpha\left|e_2\right|^2-\alpha_2\left|e_1\right|^2-\alpha_1\left|e\right|^2+\left|e\right|\left|e_1\right| \tag{15}$$

where the control law in (14) and the bounding expressions in (9) were utilized. It should be noted that the following inequality can be satisfied

$$\left|e_1\right|\left|e\right|\le\frac{1}{\delta}\left|e_1\right|^2+\delta\left|e\right|^2, \tag{16}$$

hence, the expression in (15) can be upper bounded as

$$\dot{P}\le-\lambda_3\left\|E\right\|^2 \tag{17}$$

where $\lambda_3\triangleq\min\left\{\alpha,\left(\alpha_2-\tfrac{1}{\delta}\right),\left(\alpha_1-\delta\right)\right\}$, $\delta\in\mathbb{R}^+$ is a constant, $\left(\alpha_2>\tfrac{1}{\delta}\right)$, and $(\alpha_1>\delta)$, hence, satisfying the globally asymptotically stability result. Equation (17) can be rewritten as

$$\dot{P}\le-\frac{\lambda_3}{\lambda_1}P \tag{18}$$

where the bounding expression in (12) was utilized. In order to compute the control force, F, introduced in (1) and (2), a Laplace transformation can be applied to obtain the following relationship

$$F(s)=\left(\frac{m_sm_u}{m_us-c_s}\right)u(s) \tag{19}$$

where the expression in (5) was utilized.

IV. Simulation Results

Numerical simulations are implemented to verify the effectiveness of the proposed robust nonlinear controller, introduced in (14) and (19), in regulating the vertical displacement of the quarter car body under various road conditions. Table 1 introduces the proposed values of the active suspension system parameters used in the numerical simulations. The controller parameter values (i.e., gains) are introduced in Table 2 where they were tuned by trial and error. To verify the proposed controller performance, two cases are introduced as listed in Table 3 and illustrated in Fig. 2. It should be noted that a harsh noise has been added to the acceleration, velocity, and displacement measurements of the car body to more challenge the proposed robust nonlinear controller.

Table 1. Quarter car model parameter values.

Parameter	Value	Unit
c_s	1000	Ns/m
c_t	60	Ns/m
k_s	18000	N/m
k_t	200000	N/m
m_s	320	kg
m_u	40	kg

Table 2. Proposed controller parameter values.

Parameter	Value
α	50
α_1	25
α_2	25
ρ_z	$e+e_1+e_2+1$

Table 3. Proposed controller parameter values.

Case	Road Disturbance
I	$z_r=0.2\sin(2\pi t)$ $\dot{z}_r=0.4\pi\cos(2\pi t)$
II	$z_r=0.01\sin(15\pi t)+0.02\sin(10\pi t+\frac{\pi}{2})+0.03\sin(\pi t)$ $\dot{z}_r=0.15\pi\cos(15\pi t)+0.2\pi\cos(10\pi t+\frac{\pi}{2})+0.03\cos(\pi t)$

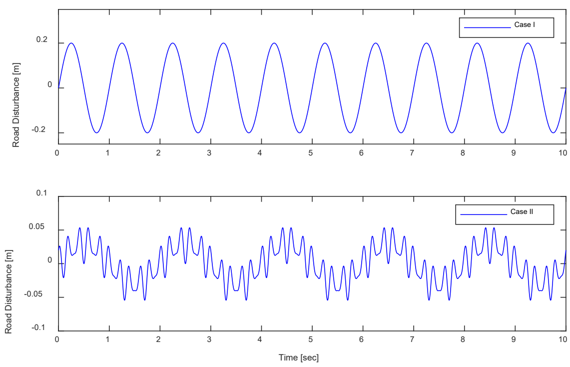

Fig. 2. The simulated road disturbance for Case I and Case II.

Fig. 3 shows the displacement responses of the sprung and unsprung masses in the active suspension system under the road disturbance described in Case I. As shown in Fig. 4, the regulation displacement error of the sprung mass fluctuates around ±3 mm although the road disturbance fluctuates ±20 cm with a relatively high frequency. That makes the sprung mass displacement error about 1.5% of the desired level that is zero displacement. The results clearly show a satisfactory performance. Fig. 5 shows the control force exerted by the active suspension actuator.

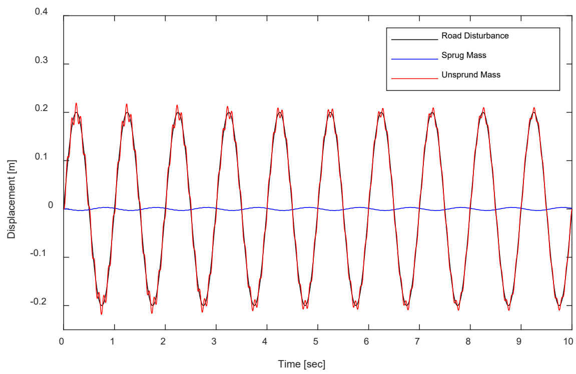

Fig. 3. The simulated displacement of the sprung and unsprung masses for Case I.

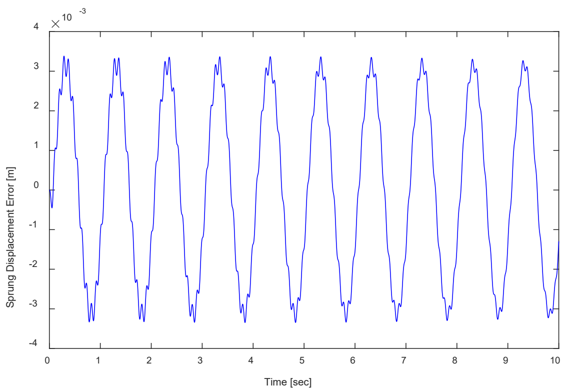

Fig. 4. The simulated regulation error of the sprung mass displacement for Case I.

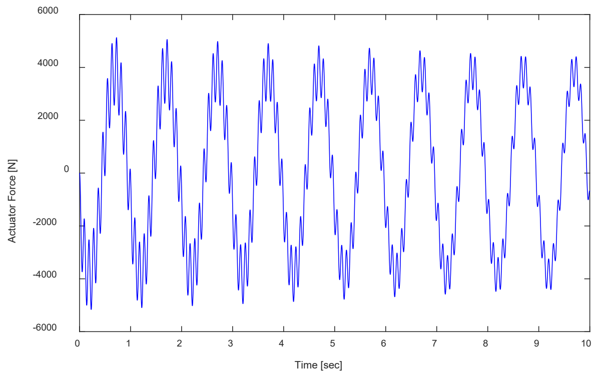

Fig. 5. The simulated control force of the active suspension system for Case I.

For the road disturbance described in Case II, the displacement responses of the sprung and unsprung masses in the active suspension system are shown in Fig. 6. The regulation displacement error of the sprung mass and the control force exerted by the active suspension system actuator are shown in Fig. 7 and Fig. 8, respectively. As shown in Fig. 7, the fluctuation of the displacement error is around ±1.5 mm when the road condition in Case II is applied that fluctuates ±5 cm with a relatively very high frequency (i.e., about 5 times the frequency in Case I). That makes the sprung displacement error about 3% of the desired level that is zero displacement. A satisfactory performance by the proposed controller is clear from Figs 6 and 7.

From Fig. 3 and Fig. 6, for both cases I and II, it is clear how well the proposed robust nonlinear controller performs while regulating the quarter car body vertical displacement under various road conditions. However, it can be seen that when the frequency of the variation in the road terrain is very rough, although the amplitude is relatively low, the regulation error becomes higher.

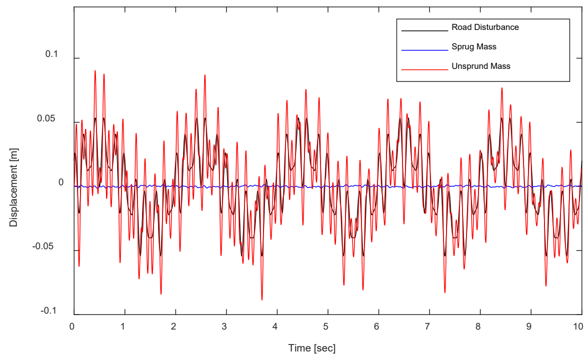

Fig. 6. The simulated displacement of the sprung and unsprung masses for Case II.

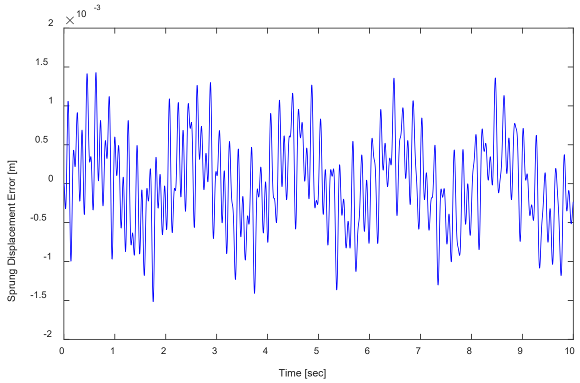

Fig. 7. The simulated regulation error of the sprung mass displacement for Case II.

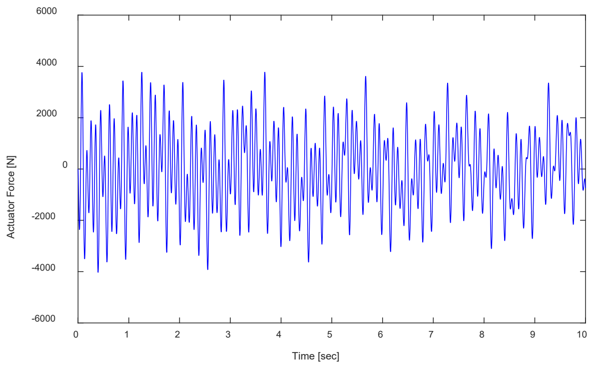

Fig. 8. The simulated control force of the active suspension system for Case II.

V. Conclusion

A robust nonlinear controller is proposed to regulate the vertical displacement of a quarter car body, via an active suspension system, to a certain level (e.g., normally zero displacement) under various road conditions and unknown system parameters. The controller design requires a partial knowledge of the active suspension system parameters, which are the masses of car body and wheel as well as the suspension damping coefficient. From the introduced numerical simulation, a satisfactory performance of the proposed controller is demonstrated when various road disturbances were applied. However, the numerical results shows that the regulation error becomes higher when the road variation frequency gets higher (i.e., disturbance gets rougher), although the amplitude is relatively low.

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