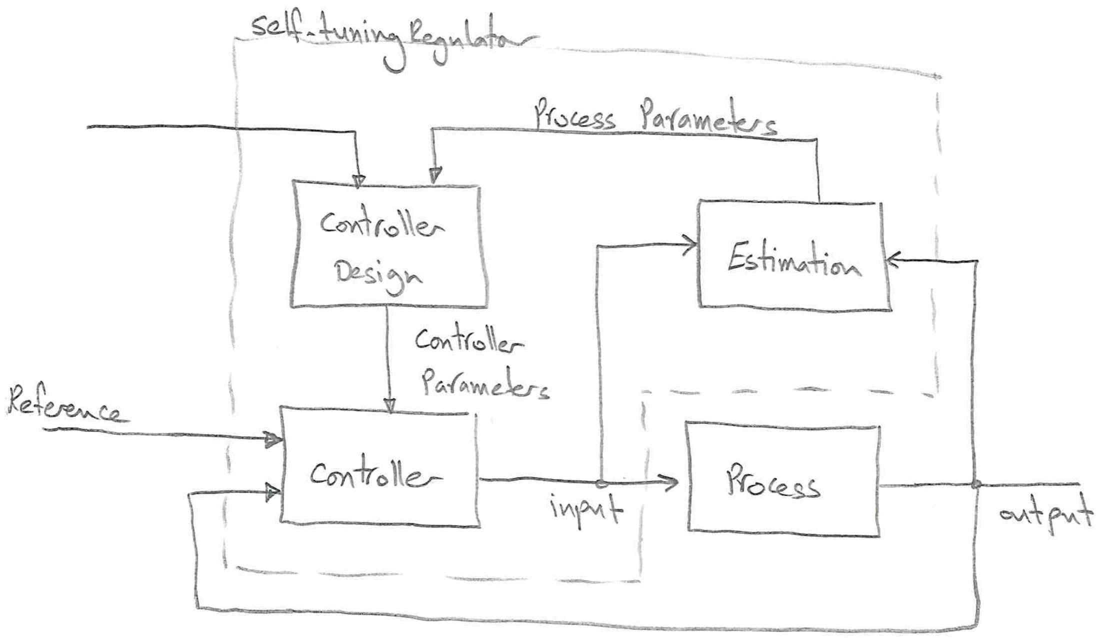
Deterministic Self-Tunnig Regulators (STR):

* Development of a control system involves many tasks such as modeling, design of a control Law, implementation, and validation.

* The self-tuning regulator (STR) attemps to automate several of these tasks.



" Block Diagram of a self-tuning regulator "

* This is illustrated in the above figure which shows a block diagram of a process with a self-tuning regulator.

* It is assumed that the structure of a process model is specified. Parameters of the model are estimated on-line, and the block labeled "Estimation" gives an estimate of the process parameters. This block is a recursive estimator of the type discussed in Lecture 2 and 3.

* The block labeled " Controller Design" contains computations that are required to perform a design of a controller with a specified method and

few design parameters that can be chosen externally.

* The block labeled "Controller" is an implementation of the controller whose parameters are obtained from the control design.

* The controller show in the figure above is a very rich structure. The choice of model structure and it's parameterization are important issues for self-tuning regulators.

* A straight-forward approach is to estimate the parameters of the transfer fn of the process. This gives an indirect adaptive algorithm. The controller parameters are not updated directly, but rather indirectly via the estimation of the process model.

* Pole Placement Design:

The idea here is to determine a controller that gives desired closed-loop poles. In addition, it is required that the system follows command signals in a specific mannar.

Process Model:

It is assumed that the process is described by the single-input, single output (SISO) system:

$$\boxed{Ay(t) = B\left[u(t) + v(t)\right]} \quad \text{—} \; (1)$$

where: y is the output

u is the input of the process

v is a disturbance.

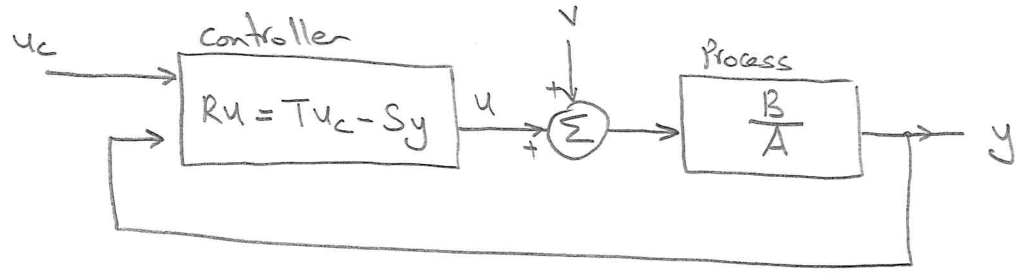
A, B are polynomials in either the differential operator, $p = d/dt$

$\deg A = n \quad , \deg B = n - d_0$

A general linear controller can be described by:

$$\boxed{R\,u(t) = T\,u_c(t) - S\,y(t)} \quad — (2)$$

where, R, S and T are polynomials. A bock diagram of the closed-loop system is show below.



Then, the closed-loop is:

$$\boxed{y(t) = \frac{BT}{AR+BS}\,u_c(t) + \frac{BR}{AR+BS}\,v(t)} \quad — (3)$$

The closed-loop characteristic polynomial is thus:

$$\boxed{AR + BS = A_c = A_m} \quad — (4)$$

The key idea of the design method is to specify the desired closed-loop charac. polynomial A_c. The polynomials R and S can be then solved from eqn (4).

⇒ note that in the design process, we consider polynomial A_c to be a design parameter that is chosen to give desired properties to the closed-loop system.

⇒ Eqn (4) always has solutions if the polynomials A and B do not have common factors.

Model following:

Eqn 4 determines only the polynomials R and S. Other conditions must be introduced to also determine the polynomial T in the controller. To do this, we will require that the respone from the command signal u_c to the output be described by the dynamics:

$$\boxed{A_m y(t) = B_m u_c(t)} \quad \text{— (5)}$$

Model Reference

Then, the following condition must hold:

$$\boxed{\frac{BT}{\underbrace{AR + BS}_{A_c}} = \frac{BT}{A_c} = \frac{B_m}{A_m}} \quad \text{— (6)}$$

⇒ The consequences of the model-following condition will now be explored.

Eqn 6 implies that there are cancellations of factors of BT and A_c.

Factor the B polynomial as:

$$\boxed{B = B^+ B^-} \quad \text{— (7)}$$

B^+: is a monic polynomial whose zeros are stable and so well damped that they can be canceled by the controller.

B^-: corresponds to unstable or poorly damped factors that can not be canceled. It thus follows that B^- must be a factor of B_m.

Hence,

$$\boxed{B_m = B^- B'_m} \quad \text{— (8)}$$

Since B^+ is canceled, it must be a factor of A_c.

Furthermore, it follows from eqn 6 that A_m must also be a factor of A_c.

The closed-loop charac. polynomial thus has the form:

$$\boxed{A_c = A_o A_m B^+} \quad — (9)$$

since B^+ is a factor of B and A_c, it follows from (4) that it also divides R. Hence:

$$A\underset{\substack{\text{should be}\\ R'B^+}}{R} + \underset{B^+B^-}{B}S = A_c = A_o A_m B^+$$

$$\boxed{R = R' B^+} \quad — (10)$$

then, the charac. polynomial is reduced to be:

$$A R' B^+ + B^+ B^- S = A_o A_m B^+$$

$$[AR' + B^- S]\, B^+ = A_o A_m B^+$$

$$\Rightarrow \boxed{AR' + B^- S = A_o A_m = A_c'} \quad — (11)$$

From eqn (7), (8) and (9) into eqn (6) gives:

$$\frac{BT}{AR + BS} = \frac{BT}{A_c} = \frac{B^+ B^- T}{A_o A_m B^+} = \frac{B_m}{A_m} = \frac{B^- B_m'}{A_m}$$

$$\Rightarrow \boxed{T = A_o B_m'} \quad — (12)$$

Causality Conditions:

To obtain a controller that is causal in the discrete-time case or proper in the continuous-time case, we must impose the conditions:

$$R\,u(t) = T\,u_c(t) - S\,y(t)$$

$$u(t) = \frac{T}{R}\,u_c(t) - \frac{S}{R}\,y(t)$$

$$\boxed{\deg R \geq \deg T}\ , \quad \boxed{\deg R \geq \deg S} \quad — (13)$$

Eqn (4) has many solutions because if R° and S° are solutions, then so are:

$$\boxed{\begin{aligned} R &= R^{\circ} + QB \\ S &= S^{\circ} - QA \end{aligned}} \quad — (14)$$

where Q is an arbitrary polynomial.

Since, there are many solutions, we may select the solution that gives a controller of lowest degree. We call this the minimum-degree solutions.

Since $\boxed{\deg A > \deg B}$, the term of highest order on the left-hand side of Eqn (4) is AR. Hence,

$$\boxed{\deg R = \deg A_c - \deg A}$$

Because of Eqn (14) there is always a solution such that $\boxed{\deg S < \deg A = n}$

$\Rightarrow$ We can thus always find a solution in which the degree of S is at most

$$\boxed{\deg S \leq \deg A - 1}$$ minimum degree solution

$$\deg S \leq \deg R$$

$$\deg A - 1 \leq \deg A_c - \deg A$$

$$\Rightarrow \boxed{\deg A_c \geq 2 \deg A - 1}$$

It follows from eqn (12) that the condition $\deg T \leq \deg R$ implies that

$$\boxed{\deg A_m - \deg B_m' \geq \deg A - \deg B^+}$$

Adding $\deg B^-$ to both sides, we find that this is equivalent to

$$\boxed{\deg A_m - \deg B_m \geq d_o}$$

This means that, in the discrete-time case the time delay of the model must be at least as large as the time delay of the process, which is a very natural condition.

Summerizing, we find that the causality conditions eqn (13) can be written as:

$$\boxed{\begin{aligned} &\deg A_c \geq \deg A - 1 \\ &\deg A_m - \deg B_m \geq \deg A - \deg B = d_o \end{aligned}} \quad \text{—} \quad (15)$$

Causality Condition

This implies that polynomials R, S, and T should have the same degrees.

Algorithm: Minimum-degree pole placement (MDPP)

- Data: Polynomial A, B
- specifications: Polynomials A_m, B_m, and A_o.
- Compatibility Conditions:

$$\boxed{\begin{aligned} \deg A_m &= \deg A \\ \deg B_m &= \deg B \\ \deg A_o &= \deg A - \deg B^+ - 1 \\ B_m &= B^- B_m' \end{aligned}}$$

step 1: Factor B as $B = B^+ B^-$, where B^+ is monic.

step 2: Find the solution R' and S with $\boxed{\deg S < \deg A}$ from

$$\boxed{AR' + B^- S = A_o A_m}$$

step 3: Form $\boxed{R = R' B^+}$ and $\boxed{T = A_o B_m'}$ and compute the control signal from the control law:

$$\boxed{Ru = Tu_c - Sy}$$

Example: [Model following with zero Cancellation]

Consider the continuous-time process described by the transfer fn

$$G(s) = \frac{1}{s(s+1)}$$

- This can be regarded as a normalize model for a motor. The pulse transfer operator for the sampling period $h = 0.5$ sec is:

$$H(q) = \frac{B(q)}{A(q)} = \frac{b_0 q + b_1}{q^2 + a_1 q + a_2} = \frac{0.1065 q + 0.0902}{q^2 - 1.6065 q + 0.6065}$$

we have $\boxed{\deg A = 2}$ and $\boxed{\deg B = 1}$.

- The design procedure thus gives a first-order controller, and the closed-loop system will be of third order.

- The sampled data system has a zero in -0.84 and poles in 1 and 0.61.

$$H(q) = \frac{0.1065 q + 0.0902}{q^2 - 1.6065 q + 0.6065} = \frac{0.1065 (q + 0.847)}{(z - 1)(z - 0.61)}$$

- Let the desired closed-loop system be:

$$\frac{B_m(q)}{A_m(q)} = \frac{b_{m0} q}{q^2 + a_{m1} q + a_{m2}} = \frac{\overset{A_m(1)}{0.1761} q}{q^2 - 1.3205 q + 0.4966}$$

→ This corresponds to a natural freq. of 1 rad/sec and a relative damping of 0.7.

$$s_{1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2} = -(0.7)(1) \pm j (1) \sqrt{1 - 0.7^2} = -0.7 \pm j\, 0.7141$$

$$z_{1,2} = \exp(sT) = \exp[(-0.7 \pm j\, 0.7141)(\overset{T}{0.5})] = 0.6602 \pm j\, 0.2463$$

→ Parameter b_{mo} is chosen so that the static gain is unity.

As I mentioned before that: $\deg A = 2$, $\deg B = 1$

⇒ Degree of controller $= \boxed{1}$ ⇒ closed-loop charac. eqn will be of degree $\boxed{3}$, but since we have zero cancellation, then degree $\boxed{2}$.

$\deg A_c = 2\deg A - 1$

$$B = B^- B^+ \Rightarrow \boxed{B^- = b_o = 0.1065}$$

$$B^+ = \frac{B}{b_o} = \frac{0.1065\,q + 0.092}{0.1065}$$

$$\Rightarrow \boxed{B^+ = q + 0.847} \quad \text{monic}$$

$$B_m = B^- B_m' \Rightarrow B_m' = \frac{B_m}{B^-} = \frac{0.1761\,q}{0.1065} \Rightarrow \boxed{B_m' = 1.6535\,q}$$

$$AR' + B^- S = A_o A_m$$

$$AR' + b_o S = A_o A_m$$

$$R = R' B^+ \Rightarrow \deg R' = 0$$

deg 1, deg 0, deg 1

⇒ since B^+ is monic, assume $R' = 1$

Also, $\deg A_o = \deg A - \deg B - 1 = \boxed{0}$ ⇒ assume $A_o = 1$

⇒ $R = r_o q + r_1$, $S = s_o q + s_1$, $T = t_o q + t_1$

since $R = R' B^+ = (1)(q + 0.847) = q + 0.847$ ⇒ $\boxed{R = q + 0.847}$

look the same zero as B^+ to be cancelled.

$$T = A_o B_m' = (1)(1.6535 q) \Rightarrow \boxed{T = 1.6535 q}$$

So, just S is remained and we can find it by the charac. eqn:

$$\cancel{A'} R' + b_o S = \cancel{A_o'} A_m$$

$$q^2 - 1.6065 q + 0.6065 + 0.1065(s_o q + s_1) = q^2 - 1.3205 q + 0.4966$$

$\Rightarrow$ if you solved it, $\boxed{s_o = 2.6854}$, $\boxed{s_1 = -1.0319}$

then, $\boxed{S = 2.6854 q - 1.0319}$

Direct STR controller is:

$$\boxed{\begin{aligned} R &= q + 0.847 \\ S &= 2.6854 q - 1.0319 \\ T &= 1.6535 q \end{aligned}}$$

Summery of the solution for the direct STR controller with zero cancellation:

- $\deg A_o = \deg A - \deg B^+ - 1$
- $B_m = A_m(1) q^{d_o}$ oR $B_m = B^- B_m'$
- $B^- = b_o$
- $B^+ = \frac{B}{b_o}$ (to be monic)
- $T = \frac{A_m(1)\, q^{d_o}}{b_o}$ or $T = A_o B_m'$
- $A_c = B^+ A_c^-$
- charac. eqn: $\boxed{A R' + b_o S = A_o A_m}$

Example: [Model following without zero cancellation]

Rules:

- $B^+ = 1$ (since no zero cancellation)
- $B^- = B$
- $B_m = \beta B$ (since same zero as closed-loop)
- $\beta = \frac{A_m(1)}{B(1)}$
- $\deg A_o = \deg A - \deg B^+ - 1$
- $T = \beta A_o$
- $A_c = A_o A_m$
- charac. eqn $\Rightarrow$ $\boxed{AR + BS = A_o A_m}$

now, consider the process of the previous example. Here, the closed-loop will have degree $\boxed{3}$ since no zero cancellation : $\deg A_c = 2 \deg A - 1$

$$\deg A_c = (2)(2) - 1 = \boxed{3}$$

$$\Rightarrow \deg A_o = \deg A - \deg B^+ - 1 = 2 - 0 - 1 = \boxed{1}$$

$$\boxed{B^+ = 1} \quad , \quad B^- = B \quad \Rightarrow \boxed{B^- = 0.1065\,q + 0.0902}$$

The model must have the same zero as the process:

$$\Rightarrow B_m = \beta B \quad , \quad \beta = \frac{A_m(1)}{B(1)} \quad \Rightarrow \boxed{\beta = 0.8953}$$

$$AR + BS = A_o A_m$$

monic $(q + r_1)$ monic $(q + a_o)$

$$(q^2 - 1.6065q + 0.6065)(q + r_1) + (0.1065q + 0.0902)(s_0 q + s_1)$$

$$= (q^2 - 1.3205q + 0.4966)(q + a_0)$$

assume $a_0 = 1$, then you have 3 unknowns (r_1, s_0, s_1) with 3 eqns.

By solving them, $\boxed{r_1 = 0.9799}$, $\boxed{s_0 = 2.8739}$, $\boxed{s_1 = -1.083}$

$$\Rightarrow T = \beta A_0 = 0.8953(q+1) \Rightarrow \boxed{T = 0.8953q + 0.8953}$$

$$R = q + r_1 \Rightarrow \boxed{R = q + 0.9799}$$

$$S = s_0 q + s_1 \Rightarrow \boxed{S = 2.8739q - 1.083}$$

If we choose $a_0 = 0 \Rightarrow \boxed{r_1 = 0.1111}$ $\boxed{s_0 = 1.6422}$, $\boxed{s_1 = -0.7471}$

$\boxed{t_0 = 0.8951}$

If we want to see the steady state Error (S.S.E):

$$\frac{B(1)T(1)}{A(1)R(1) + B(1)S(1)} = \frac{T(1)}{S(1)}$$

since it has integration

$\Rightarrow A(1) = 0$

$$T(1) = A_0(1)B_m'(1) = A_0(1)\frac{A_m(1)}{B(1)}$$

$$\Rightarrow \frac{T(1)}{S(1)} = \frac{A_0(1)A_m(1)}{B(1)S(1)} = \boxed{1}$$ no steady state error