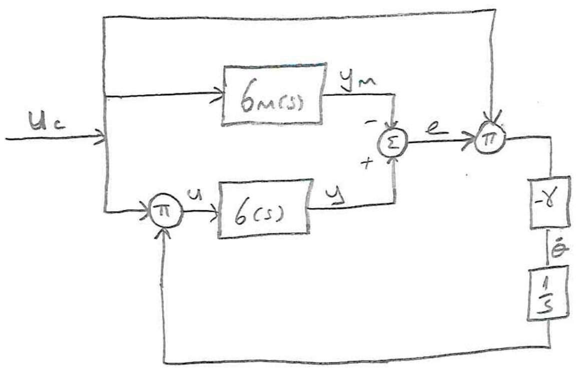
Appendix $\Rightarrow$

Lyapunov Rule $\Rightarrow \dot{\theta} = -\gamma . u_c . e$

MIT Rule $\Rightarrow \dot{\theta} = -\gamma . y_m . e$

Lyapunov Block Diagram $\Rightarrow$



$G_M(s) = \frac{2}{s+2}$

$G(s) = \frac{b}{s+a}$

$u = \theta_1 . u_c - \theta_2 . y$

Steps $\Rightarrow$

1) Process Model $\Rightarrow$ $G(s) = \frac{b}{s+a}$ $\quad \frac{Y(s)}{U(s)} = \frac{b}{s+a}$

$$Y(s).s + Y(s).a = U(s).b$$
$$\dot{y}(t) + a.y(t) = b.u(t)$$
$$\dot{y}(t) = -a.y(t) + b.u(t)$$

2) Desired Model $\Rightarrow$ $G_M(s) = \frac{2}{s+2} \Rightarrow \frac{Y_M(s)}{U_c(s)} = \frac{2}{s+2}$

$$Y_M(s).s + 2.Y_M(s) = 2.U_c(s)$$
$$\dot{y}_M(t) + 2y_M(t) = 2.u_c(t)$$
$$\dot{y}_M(t) = -2y_M(t) + 2u_c(t)$$

3) Controller $= u = \theta_1 . u_c - \theta_2 . y$

4) $\dot{y}(t) = -a.y(t) + b.\theta_1 u_c(t) - b.\theta_2 . y(t)$

$$\dot{y}(t) = y(t).(-a - b\theta_2) + u_c(t).(b.\theta_1) = \dot{y}_M(t) = -2y_M(t) + 2u_c(t)$$

$$-a - b\theta_2 = -2 \qquad 2 - a = b\theta_2 \qquad \theta_2 = \frac{2-a}{b}$$

$$b.\theta_1 = 2 \qquad \theta_1 = \frac{2}{b}$$

5) $e(t) = y(t) - y_M(t) \qquad \dot{e}(t) = \dot{y}(t) - \dot{y}_M(t)$

$$\dot{e}(t) = -a.y(t) - b.(\underbrace{\theta_1 u_c(t) - \theta_2 y(t)}_{u(t)}) + 2y_m(t) - 2u_c(t)$$

$$\dot{e}(t) = y(t)(-a - b\theta_2) + u_c(t)(-b\theta_1 - 2) + 2y_M(t) + 2y(t) - 2y(t)$$

$$\dot{e}(t) = -2(y(t) - y_M(t)) - y(t)(a + b\theta_2 - 2) + u_c(t)(b\theta_1 - 2) \quad e(t)$$

$$\dot{e}(t) = -2e - y(t)(a + b\theta_2 - 2) + u_c(t).(b\theta_1 - 2)$$

6) $V(e, \bar{\theta}_1, \bar{\theta}_2) = \frac{1}{2}e^2 + \frac{\bar{\theta}_1^2}{2\gamma} + \frac{\bar{\theta}_2^2}{2\gamma} \qquad \bar{\theta}_1 = \theta_1 - \theta_1^* \qquad \bar{\theta}_2 = \theta_2 - \theta_2^* \qquad \theta_1^* = \frac{2}{b} \qquad \theta_2^* = \frac{a}{b}$

$$\dot{V} = e.\dot{e} + \frac{\bar{\theta}_1}{\gamma}.\dot{\theta}_1 + \frac{\bar{\theta}_2}{\gamma}.\dot{\theta}_2$$

$$\dot{V} = e[-2e - y(t).(a - b\theta_2 - 2) + u_c(t)(b\theta_1 - 2)] + \frac{(\theta_1 - \frac{2}{b})}{\gamma}.\dot{\theta}_1 + \frac{(\theta_2 - \frac{a}{b})}{\gamma}.\dot{\theta}_2$$

$$\dot{V} = e[-2e - y(t)(a - b\theta_2 - 2) + u_c(t).(b\theta_1 - 2)] + \frac{\theta_1 b - 2}{b.\gamma}.\dot{\theta}_1 + \frac{\theta_2 b - a}{b.\gamma}\dot{\theta}_2$$

$$\dot{V} = -2e^2 - e.y(t).(a - b\theta_2 - 2) + e.u_c(t)(b\theta_1 - 2) + \frac{\theta_1 b - 2}{b\gamma}\dot{\theta}_1 + \frac{\theta_2 . b - a}{b\gamma}.\dot{\theta}_2$$

$$\dot{V} = -2e^2 + (b\theta_2 + 2 - a).\underbrace{(\frac{\dot{\theta}_2}{\gamma} - e.y(t))}_{0} + (b.\theta_1 - 2).\underbrace{(\frac{1}{\gamma}\dot{\theta}_1 + e.u_c(t))}_{0}$$

7) Adjustment Rules $\Rightarrow$

$\dot{\theta}_1 = -\gamma \cdot e \cdot u_c(t)$

$\dot{\theta}_2 = \gamma \cdot e \cdot y(t)$

$\dot{V} = -2e^2$