Q2-1) $G(s)=\frac{1}{s.(s+1)} \Rightarrow G(z)=\frac{z-1}{z}.Z\left\{\frac{G(s)}{s}\right\}=\frac{z-1}{z}.Z\left\{\frac{1}{s.s.(s+1)}\right\}=\frac{z-1}{z}.Z\left\{\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s+1}\right\}$

$$G(z)=\frac{z-1}{z}.\left[\frac{Tz}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-T}}\right]=\frac{T(z-e^{-T})-(z-1).(1-e^{-T})}{(z-1).(z-e^{-T})} \Rightarrow$$

$$G(q)=\frac{q(h-1+e^{-h})+(1-he^{-h}-e^{-h})}{q^2-q(1+e^{-h})+e^{-h}} \qquad T=0{,}3\ sec.$$

$$G(q)=\frac{0{,}04081q+0{,}03694}{q^2-1.74089q+0{,}7408} \qquad \deg(A)=2 \quad \deg(B)=1$$

Q2-2-a) the zeros of $G(q) \Rightarrow q=-0{,}90517$ the poles of $G(q) \Rightarrow q_1=-1,\ q_2=-0{,}7408$

$w_n=1$ rad/s $\zeta=0{,}7$

$S_{1,2}=-\zeta w_n \mp j w_n\sqrt{1-\zeta^2}=-0{,}7\mp j0{,}7141$

$z_{1,2}=e^{sT}=e^{(-0{,}7\mp j0{,}7141).T}=e^{-0{,}7.T}.e^{\mp j0{,}7141T}=e^{-0{,}21}.e^{\mp j0{,}2122}=e^{-0{,}21}.(\cos(0{,}2122)\mp j.\sin(0{,}2122))$

$z_{1,2}=0{,}79205\mp j0{,}17232$

Degree of Controller $\Rightarrow \deg A_c=\deg A-1=2-1=1$

$B=B^-B^+ \Rightarrow B^-=b_0=0{,}04081 \quad B^+=\frac{B}{b_0}=\frac{0{,}04081q+0{,}03694}{0{,}04081}=q+0{,}90517 \rightarrow$ it is monic.

$B_m=B^-B_m' \Rightarrow B_m'=\frac{B_m}{B^-}=\frac{0{,}17619}{0{,}04081} \Rightarrow B'_m=4{,}31519$

$AR'+B'S=A_oA_m \quad AR'+b_0S=A_oA_m \quad \underset{\deg 1}{R}=\underset{\deg 0}{R'}\underset{\deg 1}{B^+} \Rightarrow \deg R'=0$ Since B^+ is monic, $R'=1$ can be assumed

Also; $\deg A_o=\deg A-\deg B-1=0$ $A_o=1$ assumed

$R=6.q+r_1,\quad S=s_0q+s_1 \quad T=t_0q+t_1$

$R=R'B^+=1.(q+0{,}90517)=q+0{,}90517 \qquad \boxed{R=q+0{,}90517}$

$T=A_o.B_m'=1.(4.31519) \Rightarrow \boxed{T=4.31519}$

$q^2-1{,}7408q+0{,}7408+0{,}04081(s_0q+s_1)=q^2-1{,}3205q+0{,}4966$

$0{,}04081s_0-1{,}6065=1{,}3205 \qquad s_0=7{,}0080$

$0{,}04081s_1+0{,}7408=0{,}4966 \qquad s_1=-2{,}6929$ $\Bigg\}\ \boxed{S=7{,}0080q-2{,}6929}$

The model $\Rightarrow$ $\boxed{\begin{array}{l}R=q+0{,}90517\\ S=7{,}0080q-2{,}6929\\ T=4.31519\end{array}}$

Q2-2-b) $\deg A_c=(2)(2)-1=3$; This means that the closed loop will have degree of 3 since no zero cancellation.

$\deg A_o=\deg A-\deg B^+-1=2-0-1=1 \Rightarrow B^+=1,\ B^-=B \Rightarrow B^-=0{,}04081q+0{,}03694$

The model must have the same zero as the process: $B_m=\beta B,\ \beta=\frac{A_m(1)}{B(1)} \Rightarrow \beta=\frac{q^2-1.3205q+0{,}4966}{0{,}04081q+0{,}03694}$

$$\beta=\frac{1^2-1.3205.1+0{,}4966}{0{,}04081.1+0{,}03694}=2{,}26495$$

$$\underset{\substack{\text{monic}\\(q+r_1)}}{AR}+BS=\underset{\substack{\text{monic}\\(q+a_0)}}{A_o}A_m$$

Q2-2-b-(Continued)

$$(q^2-1{,}7408q+0{,}7408)(q+r_1)+(0{,}04081q+0{,}03694)(s_0.q+s_1)=(q^2-1{,}3205q+0{,}4966)(q+a_0$$

$$q^3-1{,}7408q^2+0{,}7408q+r_1.q^2-1{,}7408r_1.q+0{,}7408r_1+0{,}04081s_0.q^2+0{,}03694s_0.q+0{,}04081qs_1+0{,}03$$

$$=q^3+q^2\underbrace{(-1{,}7408+r_1+0{,}04081s_0)}_{(1)}+q\underbrace{(0{,}7408-1{,}7408r_1+0{,}03694s_0+0{,}04081s_1)}_{(2)}+\underbrace{0{,}7408r_1+0{,}0369}_{(3)}$$

$$=q^3-1{,}3205q^2+0{,}4966q+q^2a_0-1{,}3205q.a_0+0{,}4966a_0=q^3+q^2\underbrace{(-1{,}3205+a_0)}_{(1)}+q\underbrace{(0{,}4966-1{,}3205a_0)}_{(2)}+\underbrace{0{,}49}_{(3)}$$

$\Rightarrow$
- $-1{,}7408+r_1+0{,}04081s_0=-1{,}3205+a_0$
- $0{,}7408-1{,}7408r_1+0{,}03694s_0+0{,}04081s_1=0{,}4966-1{,}3205a_0$
- $0{,}74081r_1+0{,}03694s_1=0{,}4966a_0$

If $a_0=1\Rightarrow r_1+s_0.0{,}04081=1.4203$

$r_1.(-1{,}7408)+s_0.0{,}03694+s_1.0{,}04081=-1{,}5647\Rightarrow\begin{bmatrix}1 & 0{,}04081 & 0\\ -1{,}7408 & 0{,}03694 & 0{,}04081\\ 0{,}74081 & 0 & 0{,}03694\end{bmatrix}\begin{bmatrix}r_1\\ s_0\\ s_1\end{bmatrix}=\begin{bmatrix}1{,}42\\ -1{,}56\\ 0{,}49\end{bmatrix}$

$r_1.0{,}74081+0{,}03694s_1=0{,}4966$

$\underline{r_1=0{,}9811}\quad \underline{s_0=10{,}7619}\quad \underline{s_1=-6{,}2321}$

$T=\beta A_0=2{,}26495(q+1)=2{,}26495q+2{,}26495$ $\boxed{T=2{,}2649q+2{,}2649}$

$R=q+r_1\Rightarrow\boxed{R=q+0{,}9811}$

$S=s_0q+s_1=10{,}7619q-6{,}2321\Rightarrow\boxed{S=10{,}7619q-6{,}2321}$

If $a_0=0\Rightarrow\begin{bmatrix}1 & 0{,}04081 & 0\\ -1{,}7408 & 0{,}03694 & 0{,}04081\\ 0{,}74081 & 0 & 0{,}03694\end{bmatrix}\begin{bmatrix}r_1\\ s_0\\ s_1\end{bmatrix}=\begin{bmatrix}0{,}4203\\ -0{,}2442\\ 0\end{bmatrix}$

$r_1=0{,}1803\quad s_0=5{,}8808\quad s_1=-3{,}6158$

$T=\beta A_0=2{,}2645(q+1)\Rightarrow\boxed{T=2{,}2649q+2{,}2649}$

$R=q+r_1\Rightarrow\boxed{R=q+0{,}1803}$

$S=s_0q+s_1\Rightarrow\boxed{S=5{,}8808q-3{,}6158}$