Deterministic Self-Tuning Regulator (STR): [Continue]

Example: [Continuous-time system]

The process discussed in the previous two examples has the transfer fn:

$$G(s) = \frac{1}{s(s+1)} = \frac{B}{A}$$

- The process: degree 2
- Controller: degree 1

$\Rightarrow$ closed-loop: degree 3

- A_m: degree 2
- B_m: degree 0

$\Rightarrow A_o$ = degree 1 $\Rightarrow A_o = s + a_o$ "monic"

Let the desired response be specified by the transfer fn:

$$\frac{B_m(s)}{A_m(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\Rightarrow$ Charac. eqn:

$$AR + BS = A_o A_m$$

$$(s^2+s)(s+r_1) + (1)(s_o s + s_1) = (s+a_o)(s^2+2\zeta\omega_n s + \omega_n^2)$$

$\Rightarrow$ we have 4 unknowns with 3 eqns $\Rightarrow$ Assume a_o and then solve for unknowns

$$\boxed{\begin{aligned} 1+r_1 &= 2\zeta\omega_n + a_o \\ r_1 + s_o &= \omega_n^2 + 2\zeta\omega_n a_o \\ s_1 &= \omega_n^2 a_o \end{aligned}}$$

$\Rightarrow$ Assume a_o and then solve for the unknowns: r_1, s_o, s_1

Furthermore: $B = B^- B^+ \Rightarrow \boxed{B^+ = 1}, \boxed{B^- = 1}$

$B_m = B^- B_m' \Rightarrow B_m' = \frac{B_m}{B^-} = \frac{\omega_n^2}{1} \Rightarrow \boxed{B_m' = \omega_n^2} \Rightarrow \boxed{T = A_o B_m'}$

If we assumed $a_0 = 2$, $\zeta = 0.7$, and $\omega_n = 1$ rad/sec

Then,
$$r_1 = 2\zeta\omega_n + a_0 - 1 = \boxed{2.4}$$
$$s_0 = \omega_n^2 + 2\zeta\omega_n a_0 - r_1 = \boxed{1.4}$$
$$s_1 = \omega_n^2 a_0 = \boxed{2}$$
$$T = A_0 B_m' = (s + a_0)(\omega_n^2) = (s+2)(1) = \boxed{s+2}$$

STR controller:

$$\boxed{R = s + 2.4}, \quad \boxed{S = 1.4s + 2}, \quad \boxed{T = s + 2}$$

Example: [Book Question]

In sampling a continuous-time process model with $h = 1$, the following pulse transfer function is obtained:

$$H(z) = \frac{z + 1.2}{z^2 - z + 0.25}$$

The design specification states that the discrete-time closed-loop poles should correspond to the continuous-time charac. polynomial:

$$s^2 + 2s + 1$$

(a) Design a minimal-order discrete-time indirect self-tuning regulator. The controller should have integral action and give a closed-loop system having unit gain in stationary. Determine the charac. eqn that solves the design problem.

(b) Suggest a design that includes direct estimation of the controller parameters. Discuss why a well-working direct self-tuning regulator is more difficult to design this process than is an indirect self-tuning regulator.

(a)
$$H(z) = \frac{z+1.2}{z^2 - z + 0.25} = \frac{B(z)}{A(z)}$$

the desired charac. eqn: $P_d(s) = s^2 + 2s + 1 = (s+1)(s+1) = 0$

$\Rightarrow s_{1,2} = -1$

h=T=1 (sampling time)

$\Rightarrow z_{1,2} = e^{sT} = e^{(-1)(1)} = 0.3679$

$\Rightarrow P_d(z) = (z - 0.3679)^2 = \boxed{z^2 - 0.7358\,z + 0.1354}$

$= A_m(z)$

Requirements: - STR with minimal order
- Controller with integral action
- steady-state error equals to zero.

- choose B_m such that $\frac{B_m(1)}{A_m(1)} = 1$
- Refering to the Algorithm in the previous Lecture (Page 8), one of the compatibility conditions: $\deg B_m = \deg B = 1$

$\Rightarrow B_m = q - b_m$

$\Rightarrow B_m(1) = A_m(1) \quad \Rightarrow (1) - b_m = (1) - 0.7358 + 0.1354$

$\Rightarrow \boxed{b_m = 0.6}$

$\Rightarrow \boxed{B_m = q - 0.6}$

As we know that, the closed-loop system is:
$$Y(z) = \frac{BT}{\underbrace{AR + BS}_{\text{contains integrator}}} u_c$$

contains integrator $\Rightarrow R(1) = 0$

Since we have an unstable zero in the process plant, then the order of the controller should be increased:

$$\underbrace{A}_{\deg 2}\underbrace{R}_{\deg 2} + BS = \underbrace{A_o}_{\deg 2}\underbrace{A_m}_{\deg 2}$$

$$B = B^- B^+ \Rightarrow \boxed{B^+ = 1} \quad , \quad \boxed{B^- = q + 1.2}$$

$$\Rightarrow A_o = q^2 + a_{o1} q + a_{o2}$$

$$R = (q-1)(q+r)$$

$$S = s_o q^2 + s_1 q + s_2$$

$$T = \beta A_o = \beta(q^2 + a_{o1} q + a_{o2})$$

$$\beta = \frac{A_m(1)}{B(1)} = \frac{(1)^2 - 0.7358(1) + 0.1354}{(1) + 1.2} = \boxed{0.1816}$$

assume $A_o = q^2$ (two poles at origin)

$$AR + BS = A_o A_m$$

$$(q^2 - q + 0.25)(q-1)(q+r) + (q+1.2)(s_o q^2 + s_1 q + s_2) = q^2(q^2 - 0.7358 q + 0.1354)$$

$$\Rightarrow q^4 + (r - 2 + s_o) q^3 + (1.25 - 2r + s_1 + 1.2 s_o) q^2 + (1.25 r - 0.25 + s_2 + 1.25 s_1) q + (1.2 s_2 - 0.25 r) \equiv q^4 - 0.7358 q^3 + 0.1354 q^2$$

By mapping coefficients:

$$\left.\begin{aligned} 1.2 s_2 - 0.25 r &= 0 \\ 1.2 s_1 + s_2 + 1.25 r &= 0.25 \\ 1.2 s_o + s_1 - 2r &= -1.1146 \\ s_o + r &= 1.2646 \end{aligned}\right\} \begin{array}{l} 4 \text{ unknowns} \\ 4 \text{ eqns} \end{array} \Rightarrow \boxed{\begin{aligned} s_o &= 0.6212 \\ s_1 &= -0.5735 \\ s_2 &= 0.134 \\ r &= 0.6433 \end{aligned}}$$

$\Rightarrow$ Controller Parameters are:

$$R = (q-1)(q+0.6433) = q^2 - 0.3567\,q - 0.6433$$

$$S = 0.6212\,q^2 - 0.5735\,q + 0.134$$

$$T = 0.1816\,q^2$$

Check the requirements:

Closed-loop: $H(z) = \dfrac{BT}{AR+BS} = \dfrac{(z+1.2)(0.1816\,z^2)}{z^4 - 0.7355\,z^3 + 0.1353\,z^2}$

$\Rightarrow$ static Gain: $H(1) = \boxed{1}$ ※ ✓

(b) This system is non-minimum phase-system since there is an unstable zero in the process model:

$$A_o A_m\, y(t) = B^- \left[R\, u(t) + S\, y(t)\right]$$

this contains unstable zero

$$y(t) = \frac{B^-}{A_o A_m}\left[R\, u(t) + S\, y(t)\right]$$

$$= \frac{1}{A_o A_m}\left[R_1\, u(t) + S_1\, y(t)\right] \quad \text{where: } R_1 = B^- R \,,\; S_1 = B^- S$$

$\because T = A_o B_m'$, $y_m = \dfrac{B_m}{A_m} u_c$, $B_m = B^- B_m'$

and $A_o A_m y_m = A_o B_m u_c = \underbrace{A_o B^-}_{T} B_m' u_c = \underbrace{B^- T}_{T_1} u_c = T_1 u_c$

$$\Rightarrow y_m = \frac{1}{A_o A_m} T_1 u_c$$

$$e = y - y_m$$

$$= \frac{1}{A_o A_m}\left[R_1 u(t) + S_1 y(t) - T_1 u_c\right]$$

$$= \tilde{R}_1 u_f(t-d_o) + \tilde{S}_1 y_f(t-d_o) - \tilde{T}_1 u_{cf}(t-d_o)$$

now, estimate $\tilde{R}_1$, $\tilde{S}_1$, and $\tilde{T}_1$ with any recursive estimation method. Then, cancel B^- from the estimated variable and obtain R, S, T. Then, calculate the control signal:

$$\boxed{R\, u(t) = -S\, y(t) + T\, u_c(t)}$$

Estimation Process:

* $R_1 = B^- R = (q+1.2)(q-1)(q+r)$

$$= q^3 + (r+0.2)q^2 + (0.2r - 1.2)q - 1.2r$$

$$\Rightarrow \tilde{R}_1 = q^{-1} + \underbrace{(r+0.2)}_{\tilde{r}_0} q^{-2} + \underbrace{(0.2r-1.2)}_{\tilde{r}_1} q^{-3} - \underbrace{1.2r}_{\tilde{r}_2} q^{-4}$$

* $S_1 = B^- S = (q+1.2)(s_0 q^2 + s_1 q + s_2)$

$$= s_0 q^3 + (s_1 + 1.2 s_0) q^2 + (s_2 + 1.2 s_1) q + 1.2 s_2$$

$$\Rightarrow \tilde{S}_1 = \underbrace{s_0}_{\tilde{s}_0} q^{-1} + \underbrace{(s_1 + 1.2 s_0)}_{\tilde{s}_1} q^{-2} + \underbrace{(s_2 + 1.2 s_1)}_{\tilde{s}_2} q^{-3} + \underbrace{1.2 s_2}_{\tilde{s}_3} q^{-4}$$

* $T = B^- T = (q+1.2)(t_0 q^2 + t_1 q + t_2)$

$$= t_0 q^3 + (t_1 + 1.2 t_0) q^2 + (t_2 + 1.2 t_1) q + 1.2 t_2$$

$$\Rightarrow \tilde{T}_1 = \underbrace{t_0}_{\tilde{t}_0} q^{-1} + \underbrace{(t_1 + 1.2 t_0)}_{\tilde{t}_1} q^{-2} + \underbrace{(t_2 + 1.2 t_1)}_{\tilde{t}_2} q^{-3} + \underbrace{1.2 t_2}_{\tilde{t}_3} q^{-4}$$

$$\Rightarrow \theta = [\tilde{r}_0, \tilde{r}_1, \tilde{r}_2, \tilde{s}_0, \tilde{s}_1, \tilde{s}_2, \tilde{s}_3, \tilde{t}_0, \tilde{t}_1, \tilde{t}_2, \tilde{t}_3]$$

$$\varphi = [u_f(t-1) \cdots u_f(t-4) \quad y_f(t-1) \cdots y_f(t-4) \quad u_{cf}(t-1) \cdots u_{cf}(t-4)]$$

After estimation $\Rightarrow$ cancel B^- $\Rightarrow$ obtain R, S, T $\Rightarrow$ obtain controller structure

<u>note:</u> In this case:

1. It is difficult to cancel B^-.
2. Estimation process takes more estimate values and hence time.
3. Moreover, to estimate T_1, it is required presistently excited command signal u_c.

Algorithm [Simple Direct Self-tuner]:

<u>Data:</u> Given specifications in terms of A_m, B_m, and A_o and the relative degree d_o of the system.

<u>Step 1:</u> Estimate the coefficients of the polynomials R and S in the model:

$$y(t) = \frac{1}{A_o A_m} \left(R\,u(t) + S\,y(t) \right)$$

that is:

$$\boxed{y(t) = \tilde{R}\,u_f(t-d_o) + \tilde{S}\,y_f(t-t_o)}$$

by recursive least squares.

<u>Step 2:</u> Compute the control signal form:

$$\boxed{\tilde{R}\,u(t) = \tilde{T}\,u_c(t) - \tilde{S}\,y(t)}$$

where, R and S are obtained from the estimates in step 1, and:

$$\tilde{T} = \tilde{A}_o A_m(1) \quad \text{---} \quad (*)$$

with $\deg A_o = d_o - 1$. Repeat steps 1 and 2 at each sampling period.

Eqn (*) is obtained from the observation that the closed-loop transfer operator from command signal u_c to process output is:

$$\frac{TB}{AR + BS} = \frac{T b_o B^+}{b_o A_o A_m B^+} = \frac{T}{A_o A_m} = \frac{q^{d_o} A_m(1)}{A_m}$$

Example: [Direct STR with zero cancellation]

Consider the system in Example 1 (previous lecture Page 9)

we have:

* $\deg A = 2$
* $\deg B = 1$
* $\deg A_m = 2$
* $\deg A_o = d_o - 1 = 1 - 1 = 0$
* $d_o = \deg A - \deg B = 2 - 1 = 1$

$\Rightarrow$ assume $A_o = 1$

$\Rightarrow B_m = q^{d_o} A_m(1) = q\, A_m(1)$

$$\Rightarrow \frac{T}{\underset{1}{A_o} A_m} = \frac{q\, A_m(1)}{A_m} \quad \Rightarrow \boxed{T = q\, A_m(1)}$$

$\Rightarrow \deg R = \deg S = \deg T = \deg A - 1 = 2 - 1 = \boxed{1}$ or $(\deg A_o A_m - d_o)$

$$R = r_0 q + r_1 = r_0 q^{-1} + r_1 q^{-2}$$

$$S = s_0 q + s_1 = s_0 q^{-1} + s_1 q^{-2}$$

$$T = t_0 q^{-1}$$

where $t_0 = A_m(1)$ is chosen to give the correct static gain

$$\Rightarrow \quad y = \underbrace{\tilde{R}}_{r_0 q^{-1} + r_1 q^{-2}} u_f(t) + \underbrace{\tilde{S}}_{s_0 q^{-1} + s_1 q^{-2}} y_f(t)$$

$$\Rightarrow \quad y = r_0 u_f(t-1) + r_1 u_f(t-2) + s_0 y_f(t-1) + s_1 y_f(t-2)$$

where:

$$u(t) = A_o(q^{-1}) A_m(q^{-1}) u_f(t)$$

$$y(t) = A_o(q^{-1}) A_m(q^{-1}) y_f(t)$$

$$\Rightarrow \quad u(t) = u_f(t) + a_{m1} u_f(t-1) + a_{m2} u_f(t-2)$$

$$y(t) = y_f(t) + a_{m1} y_f(t-1) + a_{m2} y_f(t-2)$$

$$\Rightarrow \quad \theta = [r_0 \;\; r_1 \;\; s_0 \;\; s_1]$$

After estimation:

$$R u = T u_c - S y$$

$$(\tilde{r}_0 q^{-1} + \tilde{r}_1 q^{-2}) u = t_0 q^{-1} u_c - (\tilde{s}_0 q^{-1} + \tilde{s}_1 q^{-2}) y$$

$$\tilde{r}_0 u(t-1) + \tilde{r}_1 u(t-2) = t_0 u_c(t-1) - \tilde{s}_0 y(t-1) - \tilde{s}_1 y(t-2)$$

OR

$$\boxed{\tilde{r}_0 u(t) + \tilde{r}_1 u(t-1) = t_0 u_c(t) - \tilde{s}_0 y(t) - \tilde{s}_1 y(t-1)}$$

Example: [Book Question]

Consider the system:

$$G(s) = G_1(s)\, G_2(s)$$

where:

$$G_1(s) = \frac{b}{s+a}, \qquad G_2(s) = \frac{c}{s+d}$$

where a and b are unknown parameters and c and d are known. Construct discrete-time direct and indirect self-tuning algorithms for the partially known system.

solution:

$$G(s) = \frac{b}{s+a} \cdot \frac{c}{s+d} = \frac{bc}{s^2+(a+d)s+ad} \qquad \text{"Process Model"}$$

$$G_m(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{B_m}{A_m} \qquad \text{"Reference Model"}$$

By converting the process model and reference model to the discrete version with suitable sampling time.

$$H(z) = \frac{b_0 z + b_1}{z^2 + a_1 z + a_2} = \frac{B}{A}$$

$$H_m(z) = \frac{b_{m0} z + b_{m1}}{z^2 + a_{m1} z + a_{m2}} = \frac{B_m}{A_m}$$

ⓐ Indirect STR:

$$H(z^{-1}) = \frac{b_0 z^{-1} + b_1 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{Y}{U}$$

$$\Rightarrow y(t) = b_0 u(t-1) + b_1 u(t-2) - a_1 y(t-1) - a_2 y(t-2)$$

For estimation process:

$$\hat{\theta} = [\tilde{b}_0 \ \tilde{b}_1 \ \tilde{a}_1 \ \tilde{a}_2]^T$$

$$\varphi(t-1) = [u(t-1) \ u(t-2) \ y(t-1) \ y(t-2)]$$

By estimation of system parameters "$\hat{\theta}$", $H(z)$ will be obtained, and we can re-convert $H(z)$ to evaluate $H(s)$ by using any conversion formula.

Then, the same procedures are used: I. With zero cancellation
II. Without zero cancellation.

(b) Direct STR:

$$A_o A_m = AR' + B^- S$$

and $$y(t) = \frac{B^-}{A_o A_m} [R\,u(t) + S\,y(t)]$$

(I) B is cancelled:

$\Rightarrow$ $B^- = b_o$ (unknown, or to be estimated)

$\Rightarrow$ $\deg R = \deg S = 1$

$\Rightarrow$ $$y(t) = \frac{b_o}{A_o A_m} [R\,u(t) + S\,y(t)]$$

since b_o is unknown, or to estimated, then:

$$y(t) = R_1 u_f(t) + S_1 y_f(t)$$

where; $R_1 = b_o R$ $\leftarrow$ monic (not monic),
$S_1 = b_o S$

$$u_f(t) = \frac{1}{A_o A_m} u(t)$$
$$y_f(t) = \frac{1}{A_o A_m} y(t)$$

$\Rightarrow R_1 = r_0 q + r_1$ and $S_1 = s_0 q + s_1$

$\Rightarrow \tilde{R}_1 = r_0 q^{-1} + r_1 q^{-2}$ and $\tilde{S}_1 = s_0 q^{-1} + s_1 q^{-2}$

$\Rightarrow \theta = [r_0 \;\; r_1 \;\; s_0 \;\; s_1]^T$

$\varphi^T(t-1) = [u_f(t-1) \;\; u_f(t-2) \;\; y_f(t-1) \;\; y_f(t-2)]$

Then, $\tilde{R}_1 = b_0 R \iff \tilde{r}_0 q_1 + \tilde{r}_1 = b_0 q + r_1 b_0$

$\Rightarrow \boxed{b_0 = \tilde{r}_0}$ and $\boxed{r_1 = \dfrac{\tilde{r}_1}{b_0}}$ $\Rightarrow \boxed{R = q + r_1}$

$\tilde{S}_1 = b_0 S \iff \tilde{s}_0 q + \tilde{s}_1 = b_0 s_0 q + b_0 s_1$

$\Rightarrow \boxed{s_0 = \dfrac{\tilde{s}_0}{b_0}}$ and $\boxed{s_1 = \dfrac{\tilde{s}_1}{b_0}}$ $\Rightarrow \boxed{S = s_0 q + s_1}$

Therefore, R and S are obtained. For T,

Desired closed-loop $H(q) = \dfrac{\overset{\text{cancelled}}{B}T}{A_0 A_m} = \dfrac{B_m}{A_m}$

$\Rightarrow T = A_0 B_m = \overset{1}{A_0} A_m(1) q^{\overset{1}{d_0}} = \boxed{q A_m(1)}$

$\Rightarrow$ Controller is:

$$\boxed{R\,u(t) = -S\,y(t) + T\,u_c(t)}$$

II B is not cancelled:

$$B^- = B = b_0 q + b_1$$

$$A_o = q + a_o \quad \text{(monic)}$$

$$y(t) = \frac{B^-}{A_o A_m}\left[R\, u(t) + S\, y(t) \right]$$

since $B^- = B$ is not known, then it will be estimated.

$$\Rightarrow y(t) = R_1 u_f(t) + S_1 y_f(t)$$

where: $R_1 = B^- R$ ← monic $\qquad$ $\deg B^- = 1$

$S_1 = B^- S$ $\qquad$ $\deg R = 1$

$\deg A_o A_m = 3$

$$\Rightarrow R_1 = r_0 q^2 + r_1 q + r_2 \quad * q^{-3}$$

$$R_1 = r_0 q^{-1} + r_1 q^{-2} + r_2 q^{-3}$$

$$\Rightarrow S_1 = s_0 q^{-1} + s_1 q^{-2} + s_2 q^{-3}$$

$$\Rightarrow \hat{\theta} = [r_0 \quad r_1 \quad r_2 \quad s_0 \quad s_1 \quad s_2]^T$$

$$\varphi^T(t-1) = [u_f(t-1) \quad u_f(t-2) \quad u_f(t-3) \quad -y_f(t-1) \quad -y_f(t-2) \quad -y_f(t-3)]$$

Comments:

- In this case, the variable to be estimated is more than the previous case (B cancelled
- Since R_1 and S_1 are 2nd order, and it is required to cancel B^- in order to obtain R and S. Thus, this is difficult process as we will see now.

Assume $\tilde{R}_1$ and $\tilde{S}_1$ are estimated, then:

$$\tilde{R}_1 = \bar{B}R \iff \tilde{r}_0 q^2 + \tilde{r}_1 q + \tilde{r}_2 = (b_0 q + b_1)(q + r_1)$$

$$\tilde{r}_0 q^2 + \tilde{r}_1 q + \tilde{r}_2 = b_0 q^2 + (b_0 r_1 + b_1) q + b_1 r_1$$

$$\Rightarrow \boxed{b_0 = \tilde{r}_0} \qquad \tilde{r}_1 = b_0 r_1 + b_1$$

$$\tilde{r}_2 = b_1 r_1 \quad \Rightarrow b_1 = \frac{\tilde{r}_2}{r_1}$$

$$\Rightarrow \tilde{r}_1 = b_0 r_1 + \frac{\tilde{r}_2}{r_1}$$

$$r_1 \tilde{r}_1 = b_0 r_1^2 + \tilde{r}_2$$

$$b_0 r_1^2 - r_1 \tilde{r}_1 + \tilde{r}_2 = 0$$

This $eq\underline{n}$ to be solved for r_1

Look Difficult !!!

but, you can take the true value and calculate $b_1 = \frac{\tilde{r}_2}{r_1}$

$$\Rightarrow \boxed{R = q + r_1}$$

$$\tilde{S}_1 = \bar{B}S \iff \tilde{s}_0 q^2 + \tilde{s}_1 q + \tilde{s}_2 = (b_0 q + b_1)(s_0 q + s_1)$$

$$\tilde{s}_0 q^2 + \tilde{s}_1 q + \tilde{s}_2 = b_0 s_0 q^2 + (b_0 s_1 + b_1 s_0) q + b_1 s_1$$

$$\Rightarrow \tilde{s}_0 = \overset{\tilde{r}_0}{b_0} s_0 \quad \Rightarrow \boxed{s_0 = \frac{\tilde{s}_0}{\tilde{r}_0}}$$

$$\Rightarrow b_0 s_1 + b_1 s_0 = \tilde{s}_1 \quad \Rightarrow \boxed{s_1 = \frac{\tilde{s}_1 - b_1 s_0}{b_0}}$$

$$\Rightarrow \boxed{S = s_0 q + s_1}$$

To calculate T,

$$\frac{BT}{A_o A_m} = \beta \frac{B}{A_m} \quad \Rightarrow \quad T = \beta A_o$$

$$\beta = \frac{A_m(1)}{B(1)} \quad \text{(to have unit static)}$$

not known, or to be estimated from $\tilde{S}_1$ and $\tilde{R}_1$

Then, $B = b_0 q + b_1$

$B(1) = b_0 + b_1$ (which are estimated)

$$\Rightarrow \boxed{T = \frac{A_m(1)}{b_0 + b_1} \cdot A_o}$$