

MATH 369 Homework 8

Due: Tuesday April 9th, in class.

1. Does the set of vectors

$$S = \left\{ \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \right\}$$

have the same span as the set

$$S' = \left\{ \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix}, \begin{pmatrix} 0 \\ 8 \\ 9 \end{pmatrix} \right\}?$$

Justify your answer.

2. The solutions to the equation $A\mathbf{x} = \mathbf{0}$ where

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

form a subspace. Is this subspace the trivial subspace ($\{\mathbf{0}\}$) or is it larger?

3. Determine whether the vectors are linearly independent or linearly dependent in \mathbb{R}^3 :

(a)

$$\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}.$$

(b)

$$\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}, \quad \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix}.$$

4. Remember that polynomials form a vector space. Are the polynomials $\{2 - x + 4x^2, 3 + 6x + 2x^2, 2 + 10x - 4x^2\}$ linearly independent? Explain.
5. (a) Show that for any vectors \mathbf{u} , \mathbf{v} , and \mathbf{w} in a vector space V , the vectors $\mathbf{u} - \mathbf{v}$, $\mathbf{v} - \mathbf{w}$, and $\mathbf{w} - \mathbf{u}$ form a linearly dependent set.
- (b) Find a basis for $\text{span}\{\mathbf{u} - \mathbf{v}, \mathbf{v} - \mathbf{w}, \mathbf{w} - \mathbf{u}\}$.