MATH 369 Homework 3

Due: Thursday February 14, in class.

1. Find the value of a that solves the matrix multiplication equation.

$$\begin{pmatrix} a & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{pmatrix} \begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix} = 0.$$

2. (a) If A and B are $n \times n$ matrices and $AB = \mathbf{0}$, is it always true that either $A = \mathbf{0}$ or $B = \mathbf{0}$? If it is always true, justify your answer. If not, give an example of a case where this is not true.

   (b) Find a 2×2 matrix B with $B \neq I_2$ and $B \neq \mathbf{0}$ so that

$$B^2 = B.$$

3. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 \\ c & 4 \end{pmatrix}.$$

   (a) For which values of c does A have an inverse?

   (b) For what values of c does the linear system

$$\begin{array}{rcrcl} x & + & 2y & = & 0 \\ cx & + & 4y & = & 0 \end{array}$$

   have infinitely many solutions?

4. An $n \times n$ matrix is called an *idempotent* if $A^2 = I$. Show that if A is an idempotent then $(I - A)$ is also an idempotent.

5. Let A and B be $n \times n$ matrices. It is generally NOT true that $(A + B)^{-1} = A^{-1} + B^{-1}$. Show this by finding two 2×2 matrices A, B which are each invertible and for which

$$(A + B)^{-1} \neq A^{-1} + B^{-1}.$$