

## MATH 369 Homework 6

**Due:** Thursday March 14, in class.

1. Choose  $t$  so that the vectors:

$$\mathbf{v} = \begin{pmatrix} 10 \\ -2 \\ -1 \\ 3 \\ t \end{pmatrix} \quad \text{and} \quad \mathbf{w} = \begin{pmatrix} 2 \\ -3 \\ 4 \\ 5 \\ 1 \end{pmatrix}$$

are orthogonal.

2. Suppose that  $\mathbf{v}$  is orthogonal to both  $\mathbf{u}_1$  and  $\mathbf{u}_2$  in  $\mathbb{R}^n$ . Show that  $\mathbf{v}$  is orthogonal to  $k_1\mathbf{u}_1 + k_2\mathbf{u}_2$  for any  $k_1, k_2 \in \mathbb{R}$ .
3. Decompose the vector

$$\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

into a sum  $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$  where  $\mathbf{w}_1$  has the same direction as

$$\mathbf{z} = \begin{pmatrix} 4 \\ -4 \\ 2 \\ -2 \end{pmatrix}$$

and  $\mathbf{w}_2$  is orthogonal to  $\mathbf{z}$ .

4. For each of the sets  $V$  described below with specified addition and scalar multiplication operation, state whether  $V$  is a vector space or not. If it is not a vector space, explain at least one axiom that it violates. If it is a vector space, justify this by showing that the 10 axioms all hold.
- (a)
- The set:  $V$  is the set of polynomials with real coefficients.
  - Addition operation: the standard addition of polynomials.
  - Scalar multiplication: the standard multiplication of a polynomial by a real number.
- (b)
- The set:  $V$  is the set of vectors in  $\mathbb{R}^2$  taking the form

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \text{where } x_2 > 0.$$

- Addition operation: the standard addition of vectors in  $\mathbb{R}^2$ .
  - Scalar multiplication: the standard multiplication of vectors in  $\mathbb{R}^2$  by a scalar.
- (c)
- The set:  $V$  is the set of vectors in  $\mathbb{R}^2$  taking the form

$$\mathbf{x} = \begin{pmatrix} 2t \\ t \end{pmatrix}.$$

- Addition operation: the standard addition of vectors in  $\mathbb{R}^2$ .
  - Scalar multiplication: the standard multiplication of vectors in  $\mathbb{R}^2$  by a scalar.
- (d)
- The set:  $V$  is all  $2 \times 2$  matrices.
  - Addition operation: the standard addition of matrices of size  $2 \times 2$ .
  - Scalar multiplication: the standard multiplication of  $2 \times 2$  matrices by a scalar.
- (e)
- The set:  $V$  is all vectors  $v$  in  $\mathbb{R}^3$  such that  $\|v\| = 1$  (that is, all points on the unit-sphere).
  - Addition operation: the standard addition of vectors in  $\mathbb{R}^3$ .
  - Scalar multiplication: the standard multiplication of vectors in  $\mathbb{R}^3$  by a scalar.