

MATH 369 Homework 7

Due: Tuesday March 26, in class.

1. Determine which of the following are subspaces of \mathbb{R}^3 . If they are a subspace, state this (no work required). If they are not, explain why:

(a) All vectors of the form $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ with $b = a + c$.

(b) All vectors of the form $\begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix}$.

Solution:

(a) This is a subspace.

(b) This is not a subspace. Observe that

$$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

is in the set. But,

$$2\mathbf{v} = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$$

is not.

2. Determine which of the following are subspaces of the set of 2×2 matrices. If they are, state this (no work required). If they are not, explain why:

(a) The set of all 2×2 matrices A such that $\text{Tr}(A) = 0$.

(b) The set of all 2×2 matrices A such that $\det(A) = 0$.

Solution:

(a) This is a subspace.

(b) This is not a subspace. Observe that

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

are in the set because their determinants are zero but

$$A + B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

is not because its determinant is 1.

3. Which of the following are linear combinations of

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

(a) $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$

(b) $\begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Solution:

(a) This is a linear combination of \mathbf{u} and \mathbf{v} . We have

$$\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}.$$

(b) This is not a linear combination of \mathbf{u} and \mathbf{v} . Observe that in order to have:

$$\begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} = k_1 \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} + k_2 \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

we need to have $k_2 = 0$ (so that the first coordinate is equal to 0). Then we need to set $k_1 = -2$ in order to get the second coordinate as 4. But then the third coordinate is equal to -4 instead of 5.

(c) This is a linear combination of \mathbf{u} and \mathbf{v} , just set both k_1 and k_2 equal to 0.