Practice Exam 1

1. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that
$$T\left(\begin{pmatrix} 2 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 8 \\ -2 \end{pmatrix} \qquad \text{and} \qquad T\left(\begin{pmatrix} 0 \\ 3 \end{pmatrix}\right) = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$
. What is the value of
$$T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)?$$

2. Use the definition of a linear transformation to explain why the function $F : \mathbb{R}^3 \to \mathbb{R}^2$ given by
$$F\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} 4x^2 + y - z \\ 2y + 8z \end{pmatrix}$$
is NOT linear. (You would need to use the definition to get full credit on this).

3. Calculate the value of the determinant of A where
$$A = \begin{pmatrix} 1 & -1 & 4 \\ 2 & -5 & 3 \\ 2 & 1 & 0 \end{pmatrix}.$$

4. Find the value of x which minimizes the determinant of the matrix:
$$\begin{pmatrix} x & 1 & 0 \\ 0 & x & 6 \\ 0 & 4 & 2 \end{pmatrix}.$$

5. Suppose that for two vectors $\mathbf{u}$, $\mathbf{v}$ in $\mathbb{R}^n$, we know that $||\mathbf{u}|| = \sqrt{6}$, $||\mathbf{v}|| = \sqrt{24}$, and $\mathbf{u} \cdot \mathbf{v} = 12$. What is the angle between $\mathbf{u}$ and $\mathbf{v}$?

6. Find real values x_1 and x_2 so that the vector
$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$
is a unit vector AND orthogonal to
$$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

7. Recall that the set V of all real valued functions $f : \mathbb{R} \to \mathbb{R}$ with the usual addition of functions and multiplication of functions by scalars is a vector space. Consider the subset W
$$V = \{ \text{ functions } f : \mathbb{R} \to \mathbb{R} \text{ with } f(0) = 1\}$$
Is W a subspace of V?

8. Show that the collection of all vectors
$$V = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{with } 2x_1 + 3x_2 + x_3 = 1 \right\}$$
with the usual scalar multiplication and vector addition is NOT a vector space.