MATH 369 Homework 5

Due: Thursday March 7, in class.

1. (a) Calculate the determinants of the matrices:

$$A = \begin{pmatrix} 1 & 4 & -1 \\ 2 & 0 & -1 \\ 7 & -10 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

   (b) Determine whether the matrices above are invertible.

2. Let $n = 1,000,000$. Calculate $\det(I_n)$. Justify your answer using our rule for calculating determinants from class (or whatever your favorite method is).

3. Find the values of λ such that

$$A = \begin{pmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{pmatrix}$$

   has $\det(A) = 0$.

4. For which values of θ is

$$A = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   invertible?

5. (a) Calculate the norm of the vector

$$\mathbf{v} = \begin{pmatrix} 6 \\ -1 \\ 2 \\ 0 \end{pmatrix}.$$

   (b) Find a unit vector that points in the same direction as $\mathbf{v}$.

6. Give a rough explanation of what the matrix

$$A = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

   does to a vector

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

   when we multiply $A\mathbf{x}$. It may be helpful to choose a value of θ such as $\frac{\pi}{4}$ and look where A sends some easy vectors, such as $\mathbf{e}_1$ and $\mathbf{e}_2$. (Hint: this question relates to Homework 4).