MATH 369 Homework 4

Due: Thursday February 28, in class.

1. Let
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}.$$
Find a parametrization of all vectors $\mathbf{b}$ such that there is an $\mathbf{x}$ for which $A\mathbf{x} = \mathbf{b}$.

2. Say whether each of the maps below is linear. Justify your reasoning using the definition of a linear map.
   (a) The map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by
   $$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x + y \\ y - z + 1 \\ x + 5y + z \end{pmatrix}.$$
   (b) The transformation $T_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ which takes a vector in $\mathbb{R}^2$ and rotates it θ degrees counterclockwise about the origin.
   (c) The transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ which sends
   $$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x + y \\ 2x^2 - y \end{pmatrix}.$$

3. Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation and that we know that
$$T(\mathbf{e}_1) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad T(\mathbf{e}_2) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \qquad T(\mathbf{e}_3) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$
Calculate the value of $T(\mathbf{v})$ where
$$\mathbf{v} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}.$$

4. (a) Calculate the determinants of the matrices:
$$A = \begin{pmatrix} 1 & 4 & -1 \\ 2 & 0 & -1 \\ 7 & -10 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$
   (b) Determine whether the matrices above are invertible.

5. Let $n = 1,000,000$. Calculate $\det(I_n)$. Justify your answer.