MATH 369 Homework 7

Due: Tuesday March 26, in class.

1. Determine which of the following are subspaces of $\mathbb{R}^3$. If they are a subspace, state this (no work required). If they are not, explain why:

   (a) All vectors of the form $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ with $b = a + c$.

   (b) All vectors of the form $\begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix}$.

   Solution:

   (a) This is a subspace.

   (b) This is not a subspace. Observe that
   $$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$
   is in the set. But,
   $$2\mathbf{v} = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$$
   is not.

2. Determine which of the following are subspaces of the set of 2×2 matrices. If they are, state this (no work required). If they are not, explain why:

   (a) The set of all 2×2 matrices A such that $Tr(A) = 0$.

   (b) The set of all 2×2 matrices A such that $\det(A) = 0$.

   Solution:

   (a) This is a subspace.

   (b) This is not a subspace. Observe that
   $$\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
   are in the set because their determinants are zero but
   $$A + B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
   is not because its determinant is 1.

3. Which of the following are linear combinations of
   $$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \qquad \text{and} \qquad \mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

   (a) $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$

(b) $\begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Solution:

(a) This is not a linear combination of $\mathbf{u}$ and $\mathbf{v}$. For us to have

$$\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = k_1 \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} + k_2 \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

we must have $k_2 = 2$ (to get a 2 in the first entry). Then we need $k_1 = 2$ to get a 2 in the second entry. But this gives us 6 in the third entry instead of 2.

(b) This is not a linear combination of $\mathbf{u}$ and $\mathbf{v}$. A similar argument to above can be used.

(c) This is a linear combination of $\mathbf{u}$ and $\mathbf{v}$, just set both k_1 and k_2 equal to 0.