Teaching statement

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In this teaching statement I discuss a few of the defining aspects of my teaching style and philosophy. These include:

- actively encouraging and working with undergraduates to help them gain research experience before they graduate,
- an emphasis on developing those math skills that will benefit a student regardless of their eventual career.

I know from both my own experience and talking with new graduate students that participation in undergraduate research remains both a vital component for getting into a top graduate program and an important foundation for a wide variety of careers. I am actively trying to get undergraduates involved in research at Colorado State University. Currently I am working with two 4th year undergraduates on a project which uses convolutional neural networks to try to recognize very low resolution faces from the CMU-PIE (Pose, Illumination, and Expression) Face Database. As a researcher in machine learning and data science I have the somewhat unique opportunity to advise students on research problems that can be useful both on a CV geared toward a graduate position in applied math and also a CV geared toward an industry position. In these projects students can not only learn state-of-the-art machine learning and data analytics algorithms, but they can also develop complementary skills such as programming and domain-specific knowledge. I also have ideas for accessible projects for undergraduates at the intersection of representation theory and combinatorics.

Two examples of projects that I would like to have undergraduates work on in the future include:

1. *What is the geometry of the weights of a trained neural network*: Understanding why deep neural networks work so well is currently an area of great interest. By thinking of the weights of a neural network as giving a point in a high dimensional space, we can start to ask basic questions about the geometry of all such points corresponding to trained neural networks from different initializations.
2. *Proving combinatorial identities through diagrammatics*: Over the course of the last 20 years it has become clear that many fundamental objects in representation theory can be expressed via various easy to learn graphical calculuses. By manipulating diagrams based on spatial intuition one can often find new and interesting combinatorial identities. In particular, I have several questions about the graphical calculus for Heisenberg categories that would be appropriate for an undergraduate.

The reality is that the majority of students in almost any undergraduate math class will not go on to math graduate school let alone become research mathematicians. Instead they will take jobs as software developers, actuaries, data analysts, etc. One of my ongoing goals as a teacher is to try to not just teach the material for a course, but to teach it in such a way that even if a student never considers uniform continuity (for example) ever again, they will still take concrete skills away from the course that they can utilize in their future careers.

Having worked in collaboration with engineers and scientists in both academia and industry, I have personally been witness to how important it is to be able to deliver clear, coherent, concise technical arguments. This is of course an essential skill for the research mathematician but it is perhaps even more useful outside of math where one often interacts more closely with colleagues from different backgrounds. For this reason I make communication a central idea in the courses that I teach. In lower-level courses such as calculus and linear algebra for example, I put a strong emphasis on showing work in a coherent step-by-step fashion with the explicit goal that one of the writer's classmates (rather than just me) could reasonably follow the line of reasoning on the page. After incorporating these ideas into my grading scheme and discussing them frequently in class I not only notice significant improvement over the course of the semester but I also see some of the careless mistakes that were previously made because students skipped steps begin to decrease.

In upper-level courses such as real-analysis and algebra where proof-writing is a key component, communication skills can be more thoroughly developed. In this context, I make proof clarity be one of the grading criteria and have the students aim their writing style for an audience of their peers, not me. On each homework assignment I pick two problems and grade them in great detail, giving feedback on both the math and their exposition.

This is just a beginning. When I taught combinatorics at UC Davis I gave the students the option of presenting homework solutions in front of the class for extra credit. While the students that actually took this opportunity benefited from it, I had a hard time motivating all but a small minority of students to participate in this aspect of the course. In the future I would like to develop better ways of incorporating verbal communication by students into my courses. Having done technical interviews for industry positions I think that this would be a very valuable skill for my students to develop.

In the past, one could develop the skills for one's profession early in life and then effectively use those skills throughout the next three decades until retirement. With the faster and faster pace of technological change, this model of education is less and less applicable. Now, one of the greatest skills we can give students for the future is the ability to efficiently incorporate new concepts into their existing framework of knowledge on the fly.

In upper-level math courses, one of the primary skills necessary to succeed is the ability to internalize a long list of new abstract definitions, theorems, propositions, etc. As an instructor, I have recently started trying to get my students into the habit of going through the following procedure to promote internalization and retention of new ideas and concepts in math. Upon encountering a new definition/theorem/proposition X we try to:

1. Find some basic examples of X.
2. Ask why X requires various assumptions, conditions.
3. Find examples which are not X and show what exactly differentiates X. In a theorem for example, why do we need certain assumptions? What goes wrong when we don't have them?
4. Understand whether X is trying to capture some notion that we already intuitively have. If so, try to verbalize what that is.

This semester for example, I introduced the definition of a sequential compact set (a set where all sequences in the set have a convergent subsequence) to my advanced calculus course. We started by discussing some easy example sets: for example the closed interval $[0, 1]$ or the closed disc in $\mathbb{R}^2$. We then talked about how one should think about the condition that every sequence should have a convergent subsequence. This led naturally to trying to think of examples of sets that were definitely not sequentially compact such as the open interval $(0, 1)$ and the closed interval $[0, \infty)$. Finally, we discussed whether there was some basic notion that the definition of sequentially compact tries to capture. This was hard to say explicitly, but we could all agree that sequential compactness required a set to be closed and also the set couldn't "extend indefinitely". Given this discussion, by the time that we reached the big result in this particular section (that sequentially compact is equivalent to closed and bounded) the result seemed very natural and intuitive.

As mathematicians, we implicitly use some variant of the above procedure. My goal is that when students leave my class at the end of the semester, I will have nudged them toward applying their own personal variant of this procedure not only to their future math classes, but in their ultimate career setting.