

MATH 369 Homework 2

Due: Thursday February 7, in class.

1. Write a short justification of each of your answers below:

- (a) If A is an augmented matrix for a system with three unknowns and five equations, what is the maximum number of leading 1's in its reduced row echelon form?
- (b) If B is an augmented matrix for a system with five unknowns and three equations, what is the maximum possible number of free parameters in the system?
- (c) If C is an augmented matrix for a system with three unknowns and five equations, then what is the minimum possible number of rows of zeros in any row echelon form of C ?

2. Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 1 \\ 3 & -4 \\ -4 & -2 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 1 & 0 \\ -3 & 4 \\ 2 & 0 \end{pmatrix}$$

For each of the expressions below either compute the result or state that it is not defined.

- (a) $A + B$,
 - (b) $B + C$,
 - (c) AB ,
 - (d) BC ,
 - (e) $AC + B$.
3. (a) Give an example of a 2×2 matrix A and a 2×2 matrix B such that $AB \neq BA$.
(b) Give an example of a 2×2 matrix A and a 2×2 matrix B such that $AB = BA$.
4. Let

$$A = \begin{pmatrix} -1 & 7 \\ 7 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 6 & -2 \\ -2 & -\frac{1}{2} \end{pmatrix}.$$

- (a) Show that $A^T = A$ and $B^T = B$.
- (b) Give a general condition that tells whether a 2×2 matrix

$$C = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

has the property that $C^T = C$.