Practice Exam 1

1. Consider the linear system:
$$\begin{array}{rcrcrcl} x & + & 3y & - & z & = & 1 \\ 4x & + & 2y & + & z & = & 2 \\ 5x & - & y & - & z & = & 3. \end{array}$$
This system has exactly one solution. Find it in two ways:
   (a) By putting the corresponding matrix into reduced row echelon form.
   (b) By finding the inverse of the corresponding coefficient matrix.

2. (a) Find a value for a such that the augmented matrix
$$\begin{array}{rcrcrcl} 3x & + & 2y & + & z & = & 0 \\ 6x & + & y & + & 4z & = & 0 \\ 9x & + & 5y & + & az & = & 0 \end{array}$$
has exactly one free variable when put into reduced row echelon form.
   (b) For the value of a you found above, write out parametric equations that give all solutions to the system.

3. Solve the matrix equation
$$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -9 \\ -21 \end{pmatrix}$$
for x and y. Use any method that you like.

4. Let
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 0 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 2 & 2 \\ 0 & -1 \end{pmatrix} \quad C = \begin{pmatrix} -1 & 4 \\ 0 & -2 \end{pmatrix}.$$
Calculate:
   (a) $AB^T + C$ if it is defined. If it is not, state why.
   (b) $tr(BA^T + C)$ if it is defined. If it is not, state why.

5. A matrix A is called *orthogonal* if $A^{-1} = A^T$. Show that the matrix
$$A = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{pmatrix}$$
is orthogonal.

6. Consider the matrix
$$A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}.$$
Find
$$\mathbf{u} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \quad \mathbf{u}' = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}.$$
such that
$$A\mathbf{u} = A\mathbf{u}' = \mathbf{0}$$
and $\mathbf{u} \neq \mathbf{u}'$.