Practice Exam 3

1. True or false: If A is a 10×15 matrix and the row space of A has dimension has 7, then the nullity of A is 3.

2. Consider the set of vectors:

$$\mathbf{v}_1 = \begin{pmatrix} -3 \\ -9 \\ 0 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}, \mathbf{v}_4 = \begin{pmatrix} -7 \\ -8 \\ -2 \end{pmatrix}.$$

   (a) Find a basis for $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$ as the row space of a matrix.

   (b) Find a **subset** of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ that form a basis for $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$.

   (c) Are the bases you found in (a) and (b) bases for $\mathbb{R}^3$. Why or why not?

3. Let A be the matrix

$$A = \begin{pmatrix} 1 & 2 & 5 & 3 \\ 2 & 0 & 2 & 2 \\ 3 & 1 & 5 & 6 \end{pmatrix}$$

   (a) What is a basis for the row space of A?

   (b) What is a basis for the column space of A?

   (c) What is a basis for the null space of A?

   (d) What is the dimension for the null space of A^T?

   (e) What is the rank and nullity of A?

4. Suppose that A is the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & -3 & 6 \\ 0 & 1 & -1 \end{pmatrix}.$$

   (a) Check that $\det(A) \neq 0$.

   (b) Explain why this means that the vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{v}_1 = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \mathbf{v}_1 = \begin{pmatrix} 0 \\ 6 \\ -1 \end{pmatrix}$$

   are linearly independent.

   (c) Explain why this means that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ span $\mathbb{R}^3$.

   (d) Does $\det(A) \neq 0$ also imply that the row vectors of A are a basis for $\mathbb{R}^3$.

5. Let A be an $m \times n$ matrix, and $\mathbf{b}$ a vector in $\mathbb{R}^m$. Suppose that $\mathbf{v}_1$ and $\mathbf{v}_2$ are both solutions to the equation $A\mathbf{v} = \mathbf{b}$. What fundamental matrix space of A does $\mathbf{v}_1 - \mathbf{v}_2$ belong to? Explain.