MATH 369 Homework 10

Due: Tuesday April 23th, in class.

On this homework, you do not need to show your work for row reduction calculations. If you choose to use software however, remember that you will need to be able to do calculations by hand on the exam (the problems will be of manageable size).

1. Find a basis for the null space and row space of

   (a) $A = \begin{pmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 2 & 0 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{pmatrix}$

2. Find a basis for the row and column space of the matrix:

$$A = \begin{pmatrix} -1 & -4 & -7 & -3 \\ 2 & 0 & 2 & -2 \\ 2 & 3 & -4 & 1 \end{pmatrix}.$$

3. Find a subset of the vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -3 \\ 3 \\ 7 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -1 \\ 3 \\ 9 \\ 3 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} -5 \\ 3 \\ 5 \\ -1 \end{pmatrix},$$

   which form a basis for the space $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$.

4. (a) Find a 3×3 matrix whose null space is:
      i. a point,
      ii. a line,
      iii. a plane,
      iv. all of $\mathbb{R}^3$.

   (b) For each of the matrices above, find the rank.

5. Let A be a 5×7 matrix with rank 4.

   (a) What is the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$? Explain.

   (b) Does $A\mathbf{x} = \mathbf{b}$ have a solution for all vectors $\mathbf{b}$ in $\mathbb{R}^5$? Explain.