MATH 369 Homework 1

Due: Thursday January 31, in class.

1. Consider the system of equations

$$\begin{aligned} 3x + 5y &= 3 \\ 5x - 6y &= 0 \end{aligned}$$

   (a) Find all solutions of this system of equations (use any method you want).

   (b) Draw the lines described by the two equations and identify the point corresponding to your solution.

   Solution: By row reductions we get

$$\left(\begin{array}{cc|c} 3 & 5 & 3 \\ 5 & -6 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & \frac{18}{43} \\ 0 & 1 & \frac{15}{43} \end{array}\right).$$

   The solution is therefore $x = \frac{18}{43}$, $y = \frac{15}{43}$. I leave the graphing to you.

2. (a) Write down the equations for two parallel lines in $\mathbb{R}^2$.

   (b) Use the equations from (a) to write down a linear system in 2 unknowns which has no solutions.

   Solution:

   (a) One possible example is

$$y = 2x \qquad \text{and} \qquad y = 2x + 2.$$

   (b) The corresponding system is

$$\begin{array}{ccccc} -2x & + & y & = & 0 \\ -2x & + & y & = & 2 \end{array}$$

3. The following system has an infinite number of solutions:

$$\begin{array}{ccccccc} x & + & 3y & - & z & = & -4 \\ 3x & + & 9y & - & 3z & = & -12 \\ -x & - & 3y & + & z & = & 4 \end{array}$$

   Use parametric equations to describe them.

   Solution: By row reductions we get

$$\left(\begin{array}{ccc|c} 1 & 3 & -1 & -4 \\ 3 & 9 & -3 & -12 \\ -1 & -3 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & -4 \\ 3 & 9 & -3 & -12 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

   We see that in reduced row echelon form y, z are free variables and x is a leading variable. It follows then that a parametrization of the solutions is:

$$\begin{aligned} x &= -4 - 3t + r \\ y &= t \\ z &= r. \end{aligned}$$

4. Do problems #5, 6 in Section 1.2 of the textbook.

   Solution:

(a) Reducing the associated augmented matrix to reduced row echelon form we have

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right).$$

Hence there is a unique solution $x = 3, y = 1, z = 2$.

(b) Reducing the associated augmented matrix to reduced row echelon form we have

$$\left(\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array}\right).$$

It follows that one parametrization of the solutions is

$$\begin{aligned} x &= -\frac{1}{7} - \frac{3}{7}t, \\ y &= \frac{1}{7} - \frac{4}{7}t, \\ z &= t. \end{aligned}$$

5. The following augmented matrices correspond to different systems of equations. For each matrix, decide whether the corresponding system has no solutions, one solution, or infinitely many solutions. Explain your reasoning.

(a) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$

(b) $\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & 0 & 6 & -9 & 3 \\ 0 & 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

(c) $\left(\begin{array}{cccc|c} -1 & 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$

Solution:

(a) It is clear that the matrix is in reduced row echelon form and has no free variables. The unique solution is $x = 1, y = 2, z = 3, w = 1$.

(b) This augmented matrix can be put in reduced row echelon form fairly easy:

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & 0 & 6 & -9 & 3 \\ 0 & 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

It is clear that there is one free variable and no inconsistencies in the system. Hence there are infinitely many solutions. One parametrization is

$$\begin{aligned} x &= 2 - 2t \\ y &= t \\ z &= 2 \\ w &= 1. \end{aligned}$$

(c) There are no solutions to this system because the last line implies that $0 = 1$ which is clearly a contradiction.