Energy profile of lattice vibration at presence of a local drive

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Assuming the normal modes of system to be labeled by (α, q), with which the equation of motion is written as

$$[\partial_t^2 + K_\alpha(q)]r_\alpha(q,t) = 0 \tag{1}$$

The Green's function in these modes becomes

$$\tilde{G}_\alpha(q,\omega) = \frac{1}{-\omega^2 + K_\alpha(q)}, \tag{2}$$

which satisfies

$$[-\omega^2 + K_\alpha(q)]\tilde{G}_\alpha(q,\omega) = 1 \tag{3}$$

Next, the ω_0 driving force acting on the center mass at site $j = 0$ is

$$\vec{F}_{ext}(j,\omega) = 2\pi\delta(\omega - \omega_0)\delta_{j,0}(c_x\hat{R}_x + c_y\hat{R}_y) \tag{4}$$

where (R_x, R_y) is the degree of freedom for the atom at the center of a unit cell. In the eigen-basis, the force can be represented as

$$\begin{aligned}\vec{F}_{ext}(j,\omega) &= 2\pi\delta(\omega - \omega_0)\sum_\alpha c_\alpha\hat{r}_\alpha\int\frac{dq}{2\pi}e^{iqR_j} \\ \rightarrow F_{ext,\alpha}(q,\omega) &= 2\pi\delta(\omega - \omega_0)c_\alpha\end{aligned} \tag{5}$$

where $c_\alpha \equiv (c_x\hat{R}_x + c_y\hat{R}_y)\cdot\hat{\alpha}$. Therefore, letting $\vec{r} = r_\alpha\hat{\alpha}$,

$$r_\alpha(q,\omega) = 2\pi\delta(\omega - \omega_0)\frac{c_\alpha}{-\omega_0^2 + K_\alpha(q)} \tag{6}$$

transforming back to real space,

$$r_\alpha(j,t) = c_\alpha\int\frac{d^2q}{(2\pi)^2}\frac{e^{i(qj-\omega_0 t)}}{-\omega_0^2 + K_\alpha(q)} \tag{7}$$

Assuming the elastic energy on site j can be written as

$$U(j) = \sum_{\alpha,\beta} r_\alpha(j,t)\tilde{K}_{\alpha,\beta}r_\beta(j,t) \tag{8}$$

the time-averaged energy is

$$\langle U(j)\rangle = \frac{1}{2}\sum_{\alpha,\beta} r_\alpha(j,0)\tilde{K}_{\alpha,\beta}r_\beta(j,0) \tag{9}$$