Energy profile of lattice vibration at presence of a local drive

Huan-Kuang Wu

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Assuming the normal modes of system to be labeled by (α, q), with which the equation of motion is written as

$$[\partial_t^2 + K_\alpha(q)]r_\alpha(q,t) = 0 \tag{1}$$

The Green's function in these modes becomes

$$\tilde{G}_\alpha(q,\omega) = \frac{1}{-\omega^2 + K_\alpha(q)}, \tag{2}$$

which satisfies

$$[-\omega^2 + K_\alpha(q)]\tilde{G}_\alpha(q,\omega) = 1 \tag{3}$$

Next, the ω_0 driving force acting on the center mass at site $j = 0$ is

$$\vec{F}_{ext}(j,\omega) = 2\pi\delta(\omega - \omega_0)\delta_{j,0}(c_x\hat{R}_x + c_y\hat{R}_y) \tag{4}$$

where (R_x, R_y) is the degree of freedom for the atom at the center of a unit cell. In the eigen-basis, the force can be represented as

$$\begin{aligned} \vec{F}_{ext}(j,\omega) &= 2\pi\delta(\omega - \omega_0)\int \frac{dq}{2\pi}\sum_\alpha c_\alpha(q)\hat{\alpha}(q)e^{iqR_j} \\ \rightarrow F_{ext,\alpha}(q,\omega) &= 2\pi\delta(\omega - \omega_0)c_\alpha(q) \end{aligned} \tag{5}$$

where $c_\alpha(q) \equiv (c_x\hat{R}_x + c_y\hat{R}_y)\cdot\hat{\alpha}(q)$. Therefore, letting $\hat{\alpha}(q) = \alpha_\lambda(q)\hat{r}_\lambda$, where λ represents the coordinates of atoms (i,x) or (i,y)

$$r_\lambda(q,\omega) = 2\pi\delta(\omega - \omega_0)\sum_\alpha \frac{c_\alpha(q)\alpha_\lambda(q)}{-\omega_0^2 + K_\alpha(q)} \tag{6}$$

transforming back to real space,

$$r_\lambda(j,t) = \int \frac{d^2q}{(2\pi)^2}\sum_\alpha \frac{c_\alpha(q)\alpha_\lambda(q)}{-\omega_0^2 + K_\alpha(q)}e^{i(qj - \omega_0 t)} \tag{7}$$

Assuming the elastic energy on site j can be written as

$$U(j) = \sum_{\lambda,\lambda'} r_\lambda(j,t)\tilde{K}_{\lambda,\lambda'} r_{\lambda'}(j,t) \tag{8}$$

the time-averaged energy is

$$\langle U(j)\rangle = \frac{1}{2}\sum_{\lambda,\lambda'} r_\lambda(j,0)\tilde{K}_{\lambda,\lambda'} r_{\lambda'}(j,0) \tag{9}$$