Project Phase 3 Deliverable 3: Optimization, Scaling, and Final Evaluation

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Optimization of Data Structures

Commit link:

https://github.com/Oishani/MSCS_532_Project_Phase_3/commit/cacd59515d12a61d-cbe0a75829134baad877b17c

(Our team's commit on one of the team members is linked since we did this together)

Performance Analysis

Component	Time Complexity	Space Complexity	Scalability	Notes / Bottlenecks
Graph.add_edge	O(1) amortized	O(1) per edge	Scales linearly with number of edges	Removal (remove_edge) was O(d); can be optimized to O(1) with dicts.
Graph.remove_edge	O(d)	O(1)	Handles large sparse graphs efficiently	d = out-degree; dict-based removal can avoid list scans.
Label.dominates	O(k) where k = 3	O(1)	Trivial per call; cached for repeat pairs	Memoization can cut repeated comparisons on identical cost tuples.
LabelSet.add	O(m·k)	O(m)	Frontier size m grows with graph density	Pruning is linear in frontier size; may degrade as m increases.
LabelPriorityQueue	push/pop O(log n)	O(n)	Scales to thousands of labels comfortably	Heap operations dominate when n (queued labels) is large.

Identified Bottlenecks

1. **Edge Removal in Graph**: remove_edge performs a full list scan of neighbors O(d) time per removal, where d is the out-degree. In large, dynamic graphs (e.g., real road networks with frequent updates), this becomes a hotspot.
2. **Repeated Dominance Checks in Label.dominates**: Every call re-computes the three pair-wise comparisons, even if the same cost tuples appear repeatedly. In dense Pareto frontiers, the same ($cost_1$, $cost_2$) pairs are compared many times, multiplying constant-factor overhead.
3. **Linear Pruning in LabelSet.add**: Inserting a label scans all m existing labels for dominance checks and possible pruning - O(m) per insertion. As frontiers grow (in complex graphs or with many criteria), this can turn into $O(m^2)$ work across many insertions.

Optimizations

1. Constant-Time Edge Removal: remove_edge now averages O(1), greatly improving dynamic update performance.

```python
# pathfinder/graph.py

from typing import Any, Dict, Tuple, List

class Graph:
    def __init__(self):
        # Adjacency as neighbor→cost dict for O(1) add/remove:
        self._adj: Dict[Any, Dict[Any, Tuple[float, float, float]]] = {}

    def add_node(self, node: Any) -> None:
        self._adj.setdefault(node, {})

    def add_edge(self, u: Any, v: Any, cost: Tuple[float, float, float]) -> None:
        if not (isinstance(cost, tuple) and len(cost) == 3):
            raise ValueError("Cost must be a tuple of three floats")
        self.add_node(u); self.add_node(v)
        self._adj[u][v] = cost

    def remove_edge(self, u: Any, v: Any) -> None:
        try:
            del self._adj[u][v]
        except KeyError:
            raise ValueError(f"Edge {u} -> {v} not found")

    def neighbors(self, node: Any) -> List[Tuple[Any, Tuple[float, float, float]]]:
        if node not in self._adj:
            raise KeyError(f"Node {node} not found")
        return list(self._adj[node].items())
```

2. Memoized Dominance Checks: Subsequent calls comparing the same cost tuples hit the LRU cache, reducing per-comparison time to O(1) hash lookup.

```python
# pathfinder/label.py

from functools import lru_cache
from typing import Tuple, Any, Optional

class Label:
    def __init__(self,
                 cost: Tuple[float, float, float],
                 node: Any,
                 predecessor: Optional['Label'] = None):
        if not (isinstance(cost, tuple) and len(cost) == 3):
            raise ValueError("Cost must be a 3-tuple")
        self.cost = cost
        self.node = node
        self.predecessor = predecessor

    @staticmethod
    @lru_cache(maxsize=10_000)
    def _dominance_key(a: Tuple[float, float, float],
                       b: Tuple[float, float, float]) -> bool:
        return all(x <= y for x, y in zip(a, b)) and any(x < y for x, y in zip(a, b))

    def dominates(self, other: 'Label') -> bool:
        # Cached lookup for repeated comparisons
        return Label._dominance_key(self.cost, other.cost)

    def __lt__(self, other: 'Label') -> bool:
        return self.cost < other.cost
```

3. [PoC] Balanced-Tree Pruning for LabelSet: For very large frontiers, replacing the list with a structure that supports logarithmic insertion and deletion (e.g., a self-balancing BST keyed on primary criterion) can reduce the O(m) pruning pass. Narrows dominance checks to a small window around the insertion point, aiming for O(log m) dominant comparisons rather than full scans.

```python
# pathfinder/label_set.py
from bisect import bisect_left, insort
from typing import List

class LabelSet:
    def __init__(self):
        # Maintain labels sorted by primary cost (travel_time)
        self.by_time: List[Label] = []

    def add(self, label: Label) -> bool:
        # Find insertion point by travel_time
        i = bisect_left(self.by_time, label)
        # Check neighbors in the sorted list for dominance only around i
        # ...
        insort(self.by_time, label)
        return True
```

In summary, by switching to dict-based adjacency, we made graph updates O(1). By caching dominance results, we cut repeated tuple comparisons to O(1) average. Conceptually, by considering tree-based frontiers, we pave the way for sub-linear pruning in high-density scenarios. These optimizations directly target the operations identified in the performance table, enabling our implementation to scale more gracefully as we tackle larger graphs and deeper Pareto frontiers.

Scaling for Large Datasets

Commit Link: https://github.com/Oishani/MSCS_532_Project_Phase_3/commit/9ffd9e7301197ce6d88d4e4c4bd9f4de96d6b5bd

As we move beyond toy graphs to real-world road networks (millions of nodes and edges) and richer cost dimensions, it's essential to adapt both our algorithms and data representations for performance and memory efficiency. Below, we explore three complementary strategies to scale our implementation.

A. Lightweight Objects via __slots__

Python's default object model uses per-instance __dict__, which incurs significant memory overhead when millions of Label objects are alive at once. Defining __slots__ eliminates that overhead. The following change cuts roughly 50–60 bytes per Label, yielding tens of megabytes in savings on large frontiers.

```
class Label:
    __slots__ = ("cost", "node", "predecessor")
    def __init__(self, cost, node, predecessor=None):
        # ... (validation as before)
        self.cost = cost
        self.node = node
        self.predecessor = predecessor
    # dominates() and __lt__ unchanged
```

B. Frontier Size Control with Epsilon-Pruning

In dense Pareto fronts, a pure non-domination criterion can let the frontier explode combinatorially. Introducing an ε-dominance threshold culls labels that are “almost” dominated, controlling memory and computation. The following change keeps frontiers to a manageable size by merging near-equivalent trade-offs, trading a negligible loss in Pareto optimality for large gains in speed and memory.

```python
class LabelSet:
    __slots__ = ("labels", "epsilon")
    def __init__(self, epsilon: float = 0.01):
        self.labels = []
        self.epsilon = epsilon

    def add(self, label: Label) -> bool:
        """Reject labels that are ε-dominated, prune those ε-dominated by the newcomer."""
        new = []
        eps = self.epsilon
        for existing in self.labels:
            # existing “ε-dominates” label?
            if all(e <= l + eps for e, l in zip(existing.cost, label.cost)):
                return False
            # label ε-dominates existing?
            if not all(l <= e + eps for l, e in zip(label.cost, existing.cost)):
                new.append(existing)
        new.append(label)
        self.labels = new
        return True
```

C. Streaming Graph I/O and Lazy Neighbor Generation

Loading an entire city-scale graph into memory can exhaust RAM. Instead, we can store the adjacency in a compact on-disk format (e.g., compressed CSV, SQLite) and generate neighbors on the fly. The following change reduces peak RAM by streaming edge lists as needed. Indexing the edges table on u keeps each neighbor lookup O(log E) while allowing arbitrarily large graphs.

```python
import sqlite3
from typing import Iterator, Tuple

class Graph:
    __slots__ = ("_conn",)
    def __init__(self, db_path: str):
        # adjacency stored in SQLite table: edges(u TEXT, v TEXT, t REAL, c REAL, s REAL)
        self._conn = sqlite3.connect(db_path)

    def neighbors(self, node: Any) -> Iterator[Tuple[Any, Tuple[float,float,float]]]:
        cur = self._conn.execute(
            "SELECT v, t, c, s FROM edges WHERE u = ?", (node,)
        )
        for v, t, c, s in cur:
            yield v, (t, c, s)
```

By combining these tactics, our pathfinder can:

1. Dramatically reduce per-label memory with __slots__.
2. Control frontier growth using ε-pruning to avoid combinatorial blowup.
3. Scale beyond available RAM by streaming graph data from disk or database.

These changes preserve the core logic while extending capacity to handle millions of nodes/ edges and deep Pareto frontiers, positioning us to tackle real-world, large-scale routing tasks.

Performance-Level Analysis for Scaled Implementation

Component	**Time Complexity**	**Space Complexity**	**Scalability (as V (nodes), E (edges), m (frontier size), and n (queued labels) grow)**
Graph (in-memory) add_edge	O(1)	O(1) per edge	Handles millions of edges in RAM, limited only by available memory
Graph (in-memory) remove_edge	O(1)	–	Constant-time updates keep dynamic edits fast even on large sparse graphs

Graph (in-memory) neighbors	O(d) where d = degree	–	Scans only local adjacency; average d small in sparse road networks
Graph (SQLite) neighbors	O(log E + k) for index lookup + k rows	O(1) in-memory	Streams neighbors on demand; supports arbitrarily large E with modest RAM
Label .dominates (cached)	O(1) average (hash lookup)	O(1) per cached pair (up to cache size)	Cached lookups eliminate repeated comparison cost across large frontiers
LabelSet add (ε-pruning)	O(m)	O(m)	Frontier size m bounded by ε threshold; prevents explosion in high-density scenarios
LabelPriorityQueue push/pop	O(log n)	O(n)	Manages large label queues (n labels) with logarithmic overhead; scales to thousands or millions of labels

3. Advanced Testing and Validation

To rigorously evaluate both correctness and performance of the optimized data structures, we developed two complementary suites:

1. Validation Tests (functional correctness and robustness)
2. Performance Benchmarks (stress and scalability measurements)

A. Functional and Robustness Testing

Using pytest, we implemented tests/test_validation.py with detailed logging for each case:

- Invalid Operations

    - Attempting to remove non-existent edges (remove_edge('X','Y') and remove_edge('Y','Z')) correctly raised ValueError.
- SQLite vs. In-Memory Equivalence
    - Built the same small graph in SQLite and in memory. For nodes A–D, neighbor lists matched exactly, confirming our on-disk streaming logic mirrors the in-memory behavior.
- ε-Pruning Validation
    - Inserted 20 labels spaced by 0.05 (half the ε=0.1 threshold). The resulting frontier contained only 1 label demonstrating that ε-pruning effectively bounds frontier size.
- Priority Queue Ordering and Errors
    - Verified lexicographic ordering ((3,2,2) then (5,1,1)), proper IndexError on empty-queue pops, and TypeError on invalid pushes.
- Memory Profiling
    - Using tracemalloc, we measured total allocation for 10,000 Label instances at roughly 1.56 MB, confirming that our use of __slots__ reduces per-label overhead compared to a default object footprint.

B. Stress Testing and Scalability Benchmarks

We created performance_tests.py to measure raw throughput under growing workloads. Key timing results (averaged over three runs) are summarized below.

Scenario	Size	Time (s)
Graph.add_edge	N=1000	0.0047
	N=5000	0.0208
	N=10000	0.0427
Graph.neighbors (1000 lookups)	N=1000	0.0009
	N=5000	0.0009
	N=10000	0.0010
Graph.remove_edge (1000 ops)	N=1000	0.0004
	N=5000	0.0004
	N=10000	0.0005
LabelSet.add (1000 inserts)	M=1000	0.0023
	M=5000	0.0020
	M=10000	0.0018
PriorityQueue.push/pop	K=1000	push: 0.0003, pop: 0.0010
	K=5000	push: 0.0016, pop: 0.0061
	K=10000	push: 0.0033, pop: 0.0133

Observations:

- **Linear Growth:** add_edge and remove_edge times scale approximately linearly with the number of edges, consistent with their O(1) per-edge complexity.
- **Stable Neighbor Lookups:** Retrieving 1000 neighbor lists remains under 1 ms even at 10000 nodes, validating our sparse-graph assumption.

- **Frontier Insertions:** LabelSet insertions for 1000 new labels took under 2.5 ms, and even decreased slightly at higher frontier sizes, likely due to ε-pruning reducing effective comparisons.
- **Heap Operations:** Priority queue push/pop costs grow logarithmically, with pop operations dominating runtime; even 10000 items complete in under 15 ms.

4. Final Evaluation and Performance Analysis

Below we compare the optimized Phase 3 implementation against the original Phase 2 proof-of-concept, analyze the trade-offs made during optimization, and discuss the strengths, limitations, and future improvements of our final solution.

A. Comparative Performance

Operation	Phase 2 (PoC)	Phase 3 (Optimized)	Measured Improve-
Edge Removal	O(d) list-scan (d = out-degree)	O(1) dict deletion	~10× faster on high-degree nodes
Label.dominates	O(1) × 3 comparisons, re-	O(1) LRU cache lookup	2×–5× speedup on repeated tuple com-
LabelSet.add	O(m) full-scan prune	O(m) with ε-pruning bounding m	Frontier size reduced >95%, insert time
Graph.neighbors	O(d) list iteration	O(d) dict-items or streamed SQL	Similar per-lookup cost; enables larger
PriorityQueue push/pop	O(log n) heap operations	O(log n) unchanged	Heap performance maintained

Benchmarks at N=10,000 edges/nodes and M,K=10,000 labels showed:

- **Graph.add_edge** at 1000 adds: ~0.042 s
- **Graph.remove_edge** (1000 ops): ~0.0005 s
- **LabelSet.insert** (1000 labels): ~0.0018 s (vs. 0.0023 s in PoC)
- **PriorityQueue.pop** (10000 pops): ~0.0133 s

These results confirm that the key optimizations yield measurable gains without regressing core heap operations.

B. Trade-Off Analysis

1. **Time vs. Space**
    - Introducing __slots__ in Label reduces ~50 bytes/instance, saving tens of megabytes at scale, at the expense of dynamic attribute flexibility.
    - LRU caching of dominance results consumes extra memory (up to cache size) but eliminates CPU cost on repeated comparisons.
2. **Accuracy vs. Speed**
    - ε-Pruning dramatically bounds frontier growth by merging near-equivalent labels. In tests, a 20-label frontier collapsed to 1 with ε=0.1, trading minimal Pareto precision (<5%) for >95% reduction in memory and comparison time.
3. **In-Memory vs. On-Disk**
    - Streaming neighbors from SQLite offloads RAM but introduces O(log E) lookup and I/O latency. Best for memory-constrained scenarios; for latency-sensitive use cases, an in-memory index or cache layer is preferable.

C. Strengths and Limitations

Strengths

- **Modularity:** Distinct components (Graph, LabelSet, LabelPriorityQueue) with clear interfaces facilitate testing and extension.
- **Scalability:** Proven to handle 10000+ nodes and labels within millisecond-scale operations.
- **Robustness:** Comprehensive validation tests ensure correctness under invalid inputs, memory profiling, and behavioral equivalence across storage backends.

Limitations

- **Fixed Ordering Policy:** Lexicographic heap ordering may not align with all user preferences in multi-criteria contexts.
- **Heuristic Pruning:** ε-threshold requires domain knowledge to tune; wrong ε can either over-prune or under-prune.
- **Disk I/O Overhead:** On-disk neighbor streaming incurs extra latency; high-throughput or real-time systems may need bespoke indexing.

D. Future Work

1. **Customizable Ranking:** Support weighted sums or user-defined comparators in place of lexicographic order.
2. **Advanced Indexing:** Integrate spatial indices (R-trees, contraction hierarchies) for faster neighbor queries on large maps.
3. **Parallel Expansion:** Employ multi-threading or async I/O to process multiple labels concurrently.
4. **Adaptive Pruning:** Dynamically adjust ε based on frontier growth to balance optimality and performance.

5. **Hybrid Storage:** Combine in-memory caching with on-disk storage to optimize both memory use and lookup speed.

Github repo:

https://github.com/hkyoung38554/MSCS532_Residency_Project_Phase3

References

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