1.

$$\because 2\mathrm{n} = \mathrm{O}(\mathrm{n}^2), O(n^2) + \Theta(n^2) = \Theta(n^2)$$
$$\therefore 2\mathrm{n} + \Theta(\mathrm{n}^2) = \Theta(\mathrm{n}^2)$$

2.

假设 $\theta\big(\mathrm{g(n)}\big) \cap \mathrm{o}\big(\mathrm{g(n)}\big) \neq \emptyset$

则$\exists \mathrm{c}_1, \mathrm{c}_2, \mathrm{n}_0 \in R_+$，$s.t.$ *当*$n > n_0$*时*，$0 \leq c_1 g(n) \leq \mathrm{f(n)} \leq \mathrm{c}_2 g(n) \ldots\ldots$①

$\forall \mathrm{c} > 0, \exists \mathrm{n} > \mathrm{n}_1, s.t.$ *当*$n > n_1$*时*，$0 \leq f(n) < cg(n) \ldots\ldots$②

取 $\mathrm{N} = \max(\mathrm{n}_0, \mathrm{n}_1)$，当 $\mathrm{n} > \mathrm{N}$ 时，对$\forall$c①②式同时成立

但是，当 $\mathrm{c} < \mathrm{c}_1$ *时*，$\theta\big(\mathrm{g(n)}\big) \cap \mathrm{o}\big(\mathrm{g(n)}\big) = \emptyset$

这与假设条件矛盾

$\therefore \theta\big(\mathrm{g(n)}\big) \cap \mathrm{o}\big(\mathrm{g(n)}\big) = \emptyset$

3.

假设 $\theta\big(\mathrm{g(n)}\big) \cup \mathrm{o}\big(\mathrm{g(n)}\big) = \mathrm{O}(\mathrm{g(n)})$

即$\exists \mathrm{c}_1, \mathrm{c}_2, \mathrm{c}, \mathrm{c}_3, \mathrm{n}_0, \mathrm{n}_1, \mathrm{n}_2 \in \mathrm{R}_+$，使得 $0 \leq \mathrm{c}_1 g(n) \leq f(n) \leq c_2 g(n) \cup 0 \leq f(n) < cg(n) = 0$

$\leq f(n) \leq c_3 g(n)$*成立*

取$\mathrm{c_m} = \max\,(c_2, c_3)$

$\because$ c 的取值是任意的，当 $\mathrm{c} > \mathrm{c_m}$ *时，上式的左边与右边不成立*

$\therefore$ 假设 $\theta\big(\mathrm{g(n)}\big) \cup \mathrm{o}\big(\mathrm{g(n)}\big) \neq \mathrm{O}(\mathrm{g(n)})$

4.

令 $f(n) = \max(f(n), g(n))$

即证 $0 \le c_1(f(n)+g(n)) \le f(n) \le c_2(f(n)+g(n))$，当$\exists c_1, c_2, n_0 \in R^+$，

对$\forall n > n_0 \cdots\cdots$ ① 时都成立

取$c_1 = \frac{1}{2}, c_2 = 1$，$\because 0 \le g(n) \le f(n)$

$$\therefore 0 \le c_1(f(n)+g(n)) = \frac{1}{2}(f(n)+g(n)) \le \frac{1}{2}(f(n)+f(n)) = f(n)$$

$$\therefore c_2(f(n)+g(n)) = f(n)+g(n) \ge f(n)$$

即当$c_1 = \frac{1}{2}, c_2 = 1$ 时，对$\forall n_0 > 0$，当n_0取定时，对$\forall n > n0$，①式成立，

反过来，当 $g(n) = \max(f(n), g(n))$ 时，也能找到对应的c_1, c_2, n_0使得不等式成立

综上，有 $\max(f(n), g(n)) = \Theta(f(n)+g(n))$

5.

令 $m = \lg n, \therefore n = 2^m$

$$\therefore T(n) = 2T(\sqrt{n}) + 1 \Rightarrow T(2^m) = 2T\left(2^{\frac{m}{2}}\right) + 1$$

令 $S(m) = T(2^m), \therefore T(2^m) = 2T\left(2^{\frac{m}{2}}\right) + 1 \Rightarrow S(m) = 2S\left(\frac{m}{2}\right) + 1$

$$\therefore a = 2, b = 2, f(m) = 1$$

$$\because 1 = O(m^{\log_b a} = m)$$

$$\therefore T(n) = \theta(m) = \theta(\lg n)$$

6.

假设 $T(1) = \theta(1)$

猜测 $T(n) = O(1)$

假设 $T(k) = 1$，$k < n$

$$\therefore T(n) = \frac{n-2}{n}T(n-1) + \frac{2}{n}$$

$$= \frac{n-2}{n} + \frac{2}{n} = 1 \leq 1$$

$$\therefore \mathrm{T(n)} = \mathrm{O(1)}$$

7.

a.

g1	$2^{2^{n+1}}$
g2	2^{2^n}
g3	$\mathrm{(n+1)!}$
g4	$\mathrm{n!}$
g5	$\mathrm{e^n}$
g6	$\mathrm{n * 2n}$
g7	$\mathrm{2^n}$
g8	$\left(\frac{3}{2}\right)^{\mathrm{n}}$
g9 g10	$\mathrm{(lgn)^{lgn}}$ n^{lglgn}
g11	$\mathrm{(lgn)!}$
g12	$\mathrm{n^3}$
g13 g14	$\mathrm{n^2}$ 4^{lgn}
g15 g16	$\mathrm{lg(n!)}$ $nlgn$
g17 g18	n $\mathrm{2^{lgn}}$
g19	$(\sqrt{2})^{\mathrm{lgn}}$
g20	$2^{\sqrt{2lgn}}$
g21	$\lg^2 n$
g22	lnn
g23	$\sqrt{\mathrm{lgn}}$
g24	lnlnn
g25	$2^{\lg^* n}$
g26 g27	$\lg^* n$ $\lg^* lgn$
g28	$\lg(\lg^* n)$
g29 g30	$\mathrm{n}^{\frac{1}{lgn}}$ 1

b. $\mathrm{f(x)} = \begin{cases} 2^{2^{n+2019214540}}, n = 2k \\ 0, n = 2k+1 \end{cases}$, k=0,1,2,3…………