1

$$使用数学归纳法$$

$$当k = 2\ 时$$

$$d^{(1)T}Hd^{(2)} = p^{(1)T}H\left(p^{(2)} - \frac{d^{(1)T}Hp^{(2)}}{d^{(1)T}Hd^{(1)}}d^{(1)}\right)$$
$$= p^{(1)T}H(p^{(2)} - \frac{p^{(1)T}Hp^{(2)}}{p^{(1)T}Hp^{(1)}}p^{(1)})$$
$$= p^{(1)T}Hp^{(2)} - p^{(1)T}Hp^{(2)} = 0$$

$$即d^{(1)}, d^{(2)}关于H共轭$$

$$设k < n时结论成立，即对所有不同的正整数j, t \le k < n, 有d^{(j)T}Hd^{(t)} = 0$$

$$当k = n时，有d^{(j)T}Hd^{(n)} = d^{(j)T}H\left[p^{(n)} - \sum_{i=1}^{n-1}\frac{d^{(i)T}Hp^{(n)}}{d^{(i)T}Hd^{(i)}}d^{(i)}\right](j < n)$$

$$= d^{(j)T}Hp^{(n)} - \sum_{i=1}^{n-1}\frac{d^{(i)T}Hp^{(n)}}{d^{(i)T}Hd^{(i)}}d^{(j)T}Hd^{(i)}$$
$$= d^{(j)T}Hp^{(n)} - d^{(j)T}Hp^{(n)} = 0$$

$$因此，k = n时结论成立，即d^{(1)}, d^{(2)}, \ldots, d^{(n)}关于H共轭$$

2

$$记g_i = \nabla f\left(x^{(i)}\right).由一维搜索知，g_2^T d^{(1)} = 0, 由此得到g_2 = (-2, -2, 0)^T.$$

$$根据FR共轭梯度法规定，g_1 = -d^{(1)} = (-1, 1, -2)^T, \beta_1 = \frac{\left\|g_2\right\|^2}{\left\|g_1\right\|^2} = \frac{4}{3}$$

$$则d^{(2)} = -g_2 + \beta_1 d^{(1)} = \left(\frac{10}{3}, \frac{2}{3}, \frac{8}{3}\right)^T$$

3

$$目标函数f(x) = x_1^2 + x_1x_2 + 2x_2^2 - 6x_1 - 2x_2 - 12x_3的梯度是$$

$$\nabla f(x)=\begin{bmatrix}2x_1+x_2-6\\x_1+4x_2-2\\-12\end{bmatrix},故\nabla f(\hat{x})=\begin{bmatrix}-3\\3\\-12\end{bmatrix}$$

$$在\hat{x}=(1,1,0)^T处起作用约束有$$

$$x_1+x_2+x_3=2, x\geq 0$$

$$在\hat{x}处可行方向满足条件:\ \begin{cases}d_1+d_2+d_3=0\\ d_3\geq 0\end{cases}$$

$$下降方向满足\nabla f(\hat{x})^T d<0,即-3d_1+3d_2-12d_3<0$$

$$同时满足上述三个条件的方向是\hat{x}处下降可行方向.如d=(0,-1,1)^T$$