1

$$\mathrm{f(x)}\text{的梯度为}\nabla \mathrm{f(x)} = \begin{bmatrix} 2(6+\mathrm{x}_1+\mathrm{x}_2)+2(2-3\mathrm{x}_1-3\mathrm{x}_2-\mathrm{x}_1\mathrm{x}_2)(-3-\mathrm{x}_2) \\ 2(6+\mathrm{x}_1+\mathrm{x}_2)+2(2-3\mathrm{x}_1-3\mathrm{x}_2-\mathrm{x}_1\mathrm{x}_2)(-3-\mathrm{x}_1) \end{bmatrix}$$

$$\text{将点}\hat{x}\text{代入，得最速下降方向}\left(\text{负梯度方向}\right)d = \begin{bmatrix} -344 \\ 56 \end{bmatrix}$$

$$\mathrm{f(x)}\text{的 Hessian 矩阵为}\begin{bmatrix} 2+2(-3-x_2)(-3-x_2) & 16+12x_1+12x_2+4x_1x_2 \\ 16+12x_1+12x_2+4x_1x_2 & 2+2(-3-x_1)(-3-x_1) \end{bmatrix}$$

$$\therefore \nabla^2 f(\hat{x}) = \begin{bmatrix} 164 & -56 \\ -56 & 4 \end{bmatrix}$$

$$\therefore \nabla^2 f(\hat{x})^{-1} = -\frac{1}{2480}\begin{bmatrix} 4 & 56 \\ 56 & 164 \end{bmatrix}$$

$$\therefore \text{牛顿方向}\bar{d} = \nabla^2 f(\hat{x})^{-1} d = -\frac{1}{2480}\begin{bmatrix} 1760 \\ -10080 \end{bmatrix}$$

2

(1)

$$\nabla \mathrm{f}\left(\mathrm{x}^{(1)}\right) = \mathrm{Ax}^{(1)} + \mathrm{b}$$

$$\text{将}\mathrm{x}^{(1)} = \bar{x} + up\text{代入，得}$$

$$\nabla \mathrm{f}\left(\mathrm{x}^{(1)}\right) = \mathrm{A}(\bar{x} + \mathrm{up}) + b = A\bar{x} + uAp + b$$

$$\because \bar{x}\text{是极小点，} \quad \therefore A\bar{x} + b = 0$$

$$\because \mathrm{p}\text{ 是矩阵 A 关于 }\lambda\text{ 的特征向量，}\quad \therefore \mathrm{Ap} = \lambda\mathrm{p}$$

$$\nabla \mathrm{f}\left(\mathrm{x}^{(1)}\right) = \mathrm{u}\lambda\mathrm{p}$$

(2)

$$\text{由题意，知}\mathrm{x}^{(1)} - \bar{x} = \mu p$$

$$\text{这说明当 d = p 的时候，取步长为} -\mu\text{，则可以一步从}\mathrm{x}^{(1)}\text{到达}\bar{x}$$

$$\text{此时}\nabla \mathrm{f}\left(\mathrm{x}^{(1)}\right) = \mu\lambda\mathrm{p}\text{，则当步长为} -\frac{1}{\lambda}\text{时，可从}x^{(1)}\text{一步到达}\bar{x}$$

3

$$\forall p^{(i)}, p^{(j)} \in p^{(k)}, k = 1,2,\dots,n, i \neq j$$

$$设\lambda^{j}是A的特征值,其特征向量为p^{(j)}$$

$$\therefore Ap^{(j)} = \lambda^{(j)} p^{(j)}$$
$$p^{(i)^T} Ap^{(j)} = p^{(i)^T} \lambda^{(j)} p^{(j)} = 0$$

$$\therefore A的n个互相正交的特征向量关于A共轭$$

4

(1)

$$\because 非零向量p^{(1)},\dots,p^{(n)} \in E^n关于矩阵A是共轭的$$

$$\therefore 根据引理,有p^{(1)},\dots,p^{(n)}是线性无关的向量组$$

$$\therefore p^{(1)},\dots,p^{(n)}是R^n空间的一个基$$

$$\forall x \in R^n, 有x = \sum \lambda_i p^{(i)} \dots\dots 1$$

$$等式两边同乘p^{(i)^T}A, 得p^{(i)^T}Ax = p^{(i)^T}A\lambda_i p^{(i)}$$

$$\therefore \lambda_i = \frac{p^{(i)^T}Ax}{p^{(i)^T}Ap^{(i)}}$$
$$代入1式,即得x = \sum \frac{p^{(i)^T}Ax}{p^{(i)^T}Ap^{(i)}} p^{(i)}, \forall x \in E^n$$

(2)

$$将A^{-1}按列分块,得A^{-1} = (\beta_1,\dots,\beta_n)$$

$$根据(1),有\beta_j = \sum \frac{p^{(i)^T}A\beta_j}{p^{(i)^T}Ap^{(i)}} p^{(i)}$$

$$\therefore A^{-1} = (\beta_1,\dots,\beta_n) = \sum_{i=1}^{n} \frac{p^{(i)}p^{(i)^T}A(\beta_1,\dots,\beta_n)}{p^{(i)^T}Ap^{(i)}} = \sum_{i=1}^{n} \frac{p^{(i)}p^{(i)^T}}{p^{(i)^T}Ap^{(i)}}$$

5

$$\text{容易看出是凸规划，}\bar{x}\text{是}KKT\text{点，则}$$

$$\begin{cases} A\bar{x} - w^T = 0 \\ w(\bar{x} - b) = 0 \\ w \geq 0 \end{cases}$$

$$\text{解方程，有}\bar{x}^T A(\bar{x} - b) = w(\bar{x} - b) = 0\text{，即}\bar{x}\text{和}\bar{x} - b\text{关于}A\text{共轭}$$