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OPTIMIZING OBSERVING CAMPAIGNS FOR STRONG GRAVITATIONAL LENS TIME DELAYS

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ABSTRACT

We report on a flexible and extendable inference technique for measuring time delays between images in time-varying strong gravitational lenses. Robust time delays and meaningful estimates of the measurement uncertainties are necessary for inferring the structure and substructure properties of the lensing objects, and to determine cosmological parameters including the Hubble constant. Quasar light curves have been shown to be described well by damped random walk behavior. We build a generative model of paired quasar light curves which are offset by a constant delay and magnitude, and build an inference process using the emcee Markov Chain Monte Carlo engine. With this framework, we explore the problem of designing an observational campaign that satisfies specified thresholds for a successful measurement of short time delays at a particular precision level. We apply it to a notional *Hubble* Space Telescope lens-monitoring experiment, to determine the observing campaign characteristics, and the reproducible photometric measurement precision levels that are required to secure a greater than 90% probability of measuring a 1.5-day time delay to a combined random and systematic precision of better than one hour. We discuss the extensibility of this approach to other experimental designs.

Subject headings: gravitational lensing — cosmology: dark matter

1. INTRODUCTION

Consider a concise introductory few sentences for strong lensing and gravitational lenses, possibly accompanied by a new simple graphic showing a typical quasar lens, with a "full" light curve on top of the source-plane quasar, and the various "offset" light curves connected to each of the lensed images but each on top of the other in a set of boxes, so that they are all on the observer's time frame of reference. This should be an easy illustration to put together, would set the tone for the paper for a more general audience (thinking of graduate students for example), and would be a great graphic to use for future talks on the topic, so very much worth the effort.

Gravitational lensing time delays are a competitive tool for cosmological measurements (**?**), particularly for the Hubble constant (H_0; **?**). Indeed, there is presently tension in the value for H_0 inferred by the recent *Planck* cosmic microwave background analysis, and the best-determined values derived by state of the art modeling of gravitational lenses (e.g. **??**). Such measurements depend on detailed modeling of often complex gravitational environments, but the foundation is the time delay estimate itself, which is a quantity that depends on many details of the observational campaign timing and duration, and the associated photometric measurements of each individual image in a lens. Heroic campaigns are required (e.g. **?**), followed by analysis of the data timestream where the potential effect of degenerate or incorrect solutions are ideally identified and characterized in advance.

Time delays also have the potential of being used to infer properties of the dark matter substructure expected to be contained within lensing galaxies (**?**). Exploiting time delays in this way requires an order of magnitude higher precision than the current typical $\sim$ 1-day precision measurements, as well as robust knowledge of any systematic offset effects, and so the analysis challenge is correspondingly greater.

Therefore, the challenges in measuring robust time delays may be great, but the payoff has the potential of being great. This motivates us to develop a time delay measurement technique that can be used both for analysis of existing data, but also for the systematic study of tailored sets of observations and observational characteristics, for optimal design of campaigns, tailored to the needs of the desired scientific goals. Many techniques for *fitting* observations have been explored to date; see the discussion in **?** and **TDC1** for a detailed review. In the context of the long-standing COSMOGRAIL efforts to measure time delays in gravitational lenses using long-term ground-based campaigns, THISREFERENCE explored the influence of various relevant observational choices for time delay measurements. **Perhaps say one or two more things about this here.**

In this work, we take advantage of a predictive model for the time variations of quasar light curves, to set up a Bayesian inference framework. These are developed in Section 2, based on descriptions of simulated quasar light curve pairs for particular observational conditions. A notional experimental design is explored in detail in Section 3, to demonstrate our framework's application to achieving a desired time delay precision at a high confidence of success. We discuss the results, and extensions of this methodology to other experiments in Section 4. All code is written in python, version controlled via the github repository, and is available upon request.

2. INFERENCE ANALYSIS FRAMEWORK

In this work, we concentrate on optical, radio-quiet quasars. The variability in these types of objects has been shown to be consistent with a damped random walk process, where the fluctuations have a character-

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istic amplitude and decay time that correlate with the black hole mass and luminosity (**?**). We will call these the "light curve structure parameters" in the discussion that follows. The damped random walk parameters that describe a quasar light curve are an average magnitude $\langle m \rangle$, its short time variability σ, in units of mag day$^{-1/2}$ and its relaxation time τ in days. Given a magnitude x_i at time t_i, a point x_{i+1} at time t_{i+1} is given by

$$\begin{aligned} x_{i+1} = &\langle m \rangle \\ &+ e^{-(t_{i+1}-t_i)/\tau} \left(x_i - \langle m \rangle\right) \\ &+ \sigma e^{-(t_{i+1}-t_i)/\tau} \int_0^{t_{i+1}-t_i} e^{s/\tau} dB(s) \end{aligned} \tag{1}$$

where $dB(s)$ is a temporally uncorrelated normally distributed random variable with zero mean and variance dt. The integral over the Gaussian distributed random numbers $dB(s)$ is the "random walk" part of the model while the exponential with constant τ provides the "dampening". The first point x_1 in a randomly generated series is obtained by taking the limit of $t_1 - t_0 \to \infty$, which results in a gaussian distributed variable with mean $\langle m \rangle$ and variance $\tau\sigma^2/2$. The dependence of the quasar light curve parameters σ and τ on its black hole mass according to (**?**).

The behavior is likely driven by stochastic thermal processes in their accretion disks, but for our purposes what matters most is the model prescription, so we can use a Bayesian inference approach to connect the model parameters, $\mathbf{m}$ to the data observables $\mathbf{d}$.

$$p(\mathbf{m}|\mathbf{d}) = \frac{p(\mathbf{d}|\mathbf{m})p(\mathbf{m})}{p(\mathbf{d})}.$$

In our application $\mathbf{d}$ is a collection of observed magnitudes x_i with photometric uncertainties σ_i at times t_i. We denote the set of data values by $X = \{(t_i, x_i, \sigma_i)\}$. The model parameters $\mathbf{m}$ are the quasar parameters ($\langle m \rangle$, σ, τ). The probability distribution $p(\mathbf{d}|\mathbf{m})$ corresponding to the light curve model given by Equation **??**, is

$$p(X|\langle m \rangle, \sigma, \tau) = \prod_{i=1}^{n} \frac{\exp\left(-\frac{1}{2}\frac{(\hat{x}_i - x_i^*)^2}{\Omega_i + \sigma_i^2}\right)}{\sqrt{2\pi\left(\Omega_i + \sigma_i^2\right)}} \tag{2}$$

where

$$\begin{aligned} &x_i^* = x_i - \langle m \rangle \\ &\hat{x}_0 = 0 \\ &\Omega_0 = \frac{\tau\sigma^2}{2} \\ &\hat{x}_i = a_i \hat{x}_{i-1} + \frac{a_i \Omega_{i-1}}{\Omega_{i-1} + \sigma_{i-1}^2}\left(x_{i-1}^* - \hat{x}_{i-1}\right) \\ &\Omega_i = \Omega_0\left(1 - a_i^2\right) + a_i^2 \Omega_{i-1}\left(1 - \frac{\Omega_{i-1}}{\Omega_{i-1} + \sigma_{i-1}^2}\right), \text{and} \\ &a_i = e^{-(t_i - t_{i-1})/\tau}. \end{aligned}$$

The prior distribution of model values $p(\mathbf{m})$ is determined by the extent of exisiting measurements or theory while the probability distribution of the data $p(\mathbf{d})$ is treated as a normalization factor.

We implement the Bayesian inference approach in two ways; first to analyze a single time-stream of observations, towards estimating the structure parameters of the light curve; and second, to analyze combined pairs of ostensibly correlated light curves, to determine the combination of the structure parameters and the offset parameters that best join the light curves. The likelihood function used to reconstruct the quasar light curve damped random walk parameters is

$$\begin{aligned} &\log \mathcal{L}_K(\langle m \rangle, \sigma, \tau | X) = \\ &\quad -\frac{1}{2}\sum_{i=1}^{n}\left\{\frac{(\hat{x}_i - x_i^*)^2}{\Omega_i + \sigma_i^2} + \log\left[2\pi\left(\Omega_i + \sigma_i^2\right)\right]\right\}. \end{aligned} \tag{3}$$

The offset light curves of two images of a single intrinsic quasar's light curve are merged according to a hypothesized delay and magnitude (or multiplicative-flux) offset. In other words, given a set of light curves $X_1 = \{(t_i, x_{1,i}, \sigma_{1,i})\}$ and $X_2 = \{(t_i, x_{2,i}, \sigma_{2,i})\}$, , observed with the same time sample t_i, we map the points in X_2 to $X_2' = \{(t_i - \Delta t, x_{2,i} - \Delta m, \sigma_{2,i})\}$ such that the merged data X_1 and X_2', denoted by $M(\Delta t, \Delta m; X_1, X_2)$ is best described by a single light curve model. The likelihood for the merged lightcurves under a hypothesis $(\Delta t, \Delta m)$ and ($\langle m \rangle$, τ, and σ) is given by

$$\begin{aligned} &\mathcal{L}(\Delta t, \Delta m, \sigma, \tau, \langle m \rangle | X_1, X_2) = \\ &\mathcal{L}_K\left(\sigma, \tau, \langle m \rangle | M(\Delta t, \Delta m; X_1, X_2)\right). \end{aligned} \tag{4}$$

with a posterior probability is given by

$$\begin{aligned} &p(X_1, X_2 | \Delta t, \Delta m, \langle m \rangle, \sigma, \tau) \propto \\ &\quad \mathcal{L}(\Delta t, \Delta m, \langle m \rangle, \sigma, \tau | X_1, X_2) \times \\ &\quad p(\Delta t, \Delta m, \langle m \rangle, \sigma, \tau). \end{aligned} \tag{5}$$

The assignement of priors is flat in Δt. The parameters σ and τ are scale parameters, so their priors are given by $p(\sigma) = \sigma^{-1}$ and $p(\tau) = \tau^{-1}$. The parameter $\langle m \rangle$ and Δm are also scale parameters but as they are logarithmic, we assign them flat priors.

With the posterior probability expression, we can now use a Markov Chain Monte Carlo (MCMC) algorithm to explore the likelihood space of the model parameters, based on specified observational parameters. We use the emcee algorithm of **?**, and the triangle plotting algorithm of CITATIONHERE. In Figure 1 we show a sample generated pair of light curves drawn for the same intrinsic quasar light curve properties, with a time delay of Δt=1.5 day and a magnitude offset of Δm=0.3 magnitudes. The input quasar light curve parameters are consistent with a black hole mass and luminosity of RX J1131-1231. The triangle-plot shows the posterior likelihood distributions the delay parameters (Δt, Δm) and the light curve structure parameters ($\langle m \rangle$, σ, τ), including their marginalized distributions.

3. AN EXPERIMENTAL DESIGN

To test our approach we explore what observational design parameters would be required to achieve a greater than 90% probability of measuring a 1.5 day time delay, to a combined random and systematic precision of better than 1.0 hour. To make this exploration more concrete, we assume space-quality high resolution observations (e.g. with the *Hubble* Space Telescope; *HST*),

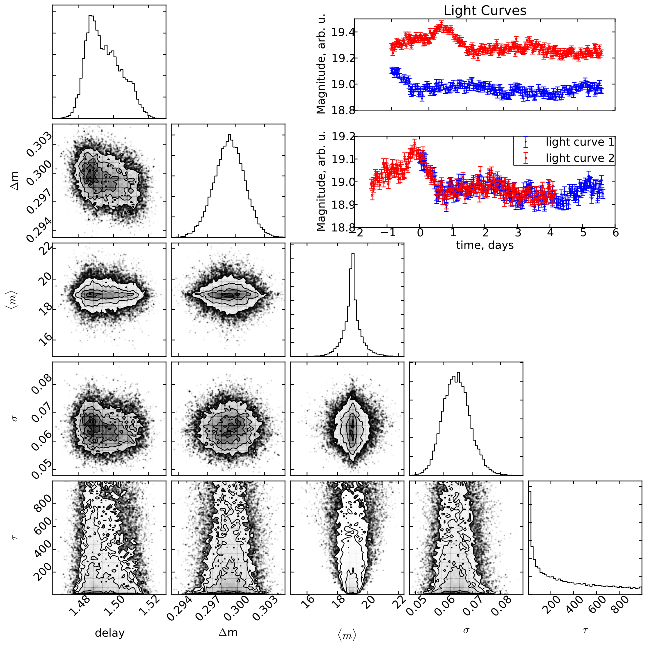

FIG. 1.— **change arb. u. to mag, also, flip order of y-axis** Markov Chain Monte Carlo (MCMC) results for an example for the parameter reconstruction of a light curve and its delayed and magnitude offset image. The quasar light curve parameters τ, σ and $\langle m \rangle$ are estimated under a hypothesis value for the delay and Δm. On the top right the quasar light curve (blue) and its delayed magnitude offset image (red) are shown with the image delay and magnitude offset corrected from the most probable values of the delay and Δm distributions. The quasar light curve parameters are $\langle m \rangle$=19.0 magnitudes, σ=0.07 mag $day^{-1/2}$ and τ=121 days. In this example, the observations were modeled with a photometric uncertainy of 0.02 magnitudes.

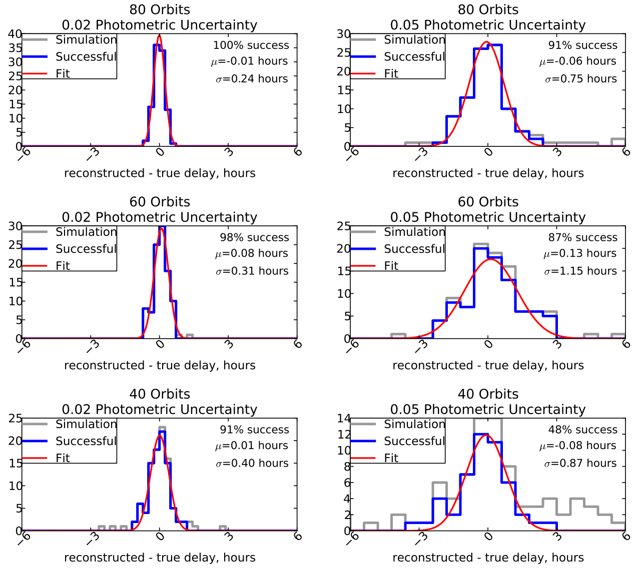

FIG. 2.— The probability of a successful measurement and resolution are shown for several examples of number of orbits and photomoetric uncertainty. The delay residuals from the simulations are shown in gray. The distribution of successful instances, with the requirements described in the text, are shown in blue. A Gaussian function is fit to the distribution of successful delay reconstructions for a convenient estimate of the final (non-outlier) time delay measurement fidelity.

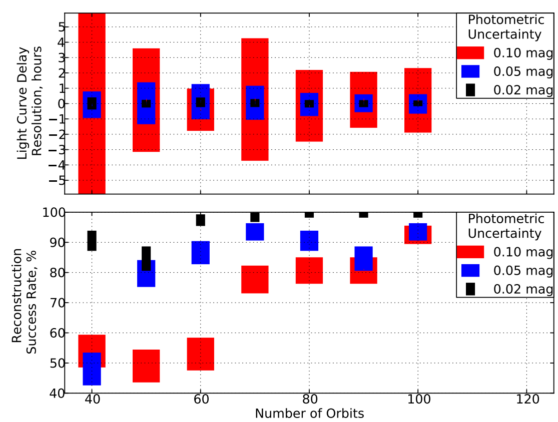

FIG. 3.— Summary of results for the delay measurement resolution (top panel) reconstruction success rate (bottom panel). Photometric uncertainties of 0.02 mag result in delay resolutions smaller than one day and are down to six hours for 80 orbits and above. Even with photometric uncertainties of 0.05 mag, delay resolutions smaller that 1 day can be achieved. The reconstruction success is 90% for more than 60 orbits and photometric uncertainties below 0.05 mag.

to confidently be able to assume completely independent non-overlapping photometric measurements of each image in our notional gravitational lens, and photometric stability that may be more robustly assumed to be stable to at least $\sim 1\%$. For the purposes of this setup, we adopt the *HST* "orbit" as a natural unit of time, which equals ~ 90 minutes.

In nature, one set of observations may accurately represent our statistical description of a process, but that set may or may not in itself contain enough information to allow a clear inference of the details of the underlying process. To test the efficacy of a planned experimental design, so we can confidently forecast the probability of success, an understanding of the likely nature of outlier measurements, and the performance as a function of the specific target values for each observational constraint, we generate a large set of simulated observations drawn from the same observational characteristics, and do the full inference analysis on each of these. The results from such a Monte Carlo exploration of an experimental setup are shown in Figure 2, for the set of observational choices shown in the same Figure.

The cuts required by for a valid reconstruction are described as follows. We fit a Gaussian curve to the distribution of 100 simulation delay results (see Figure 2). We have found that the python SciPy curve_fit method, which employs the Levenberg-Marquardt algorithm, is not driven by outliers. The standard deviation σ from the fit is used to reject reconstruction results by requiring that a reconstructed delay be within 3σ of the true delay. We also require that the uncertainty in the delay, obtained from the posterior distribution of the MCMC process (see the top left panel of Figure 1 for an example), to be within 3σ to reject poor reconstructions. We define the success as the fraction of light curve delay reconstructions surviving these cuts.

4. DISCUSSION AND CONCLUSIONS

We discuss the lessons from our analysis and the process of having designed an experiment, with an emphasis on the possible pitfalls. The way that the priors are set up, and carefully establishing enough burn-in chains in the MCMC process, are important. For a consistency check, we can analyze each image's timestream separately as well as jointly, and compare derived values for the light curve structure parameters. We have also implicitly assumed that the photometric measurement uncertainties are uncorrelated, which may not be the case for e.g. deconvolved ground-based observations. However, it is straightforward to incorporate such covariances in our analysis.

The setup in Section 3, by concentrating on a relatively short intrinsic time delay, largely avoids the effect that stochastic microlensing variations in one or both of the images would have. This is a variational component that can be included with additional parametrizations in our model, and will be the topic of future work. Isolating and characterizing the microlensing signal in long-term monitoring observations of lenses is important in its own right, because it can be used as an additional measurement of the internal smooth dark matter component in lensing galaxies (**?**), as well as a probe for the detailed structure of the quasar accretion disk (e.g. **?**).

The method developed here has been applied to the Time Delay Challenge exercise (references here; marked there as the "JPL" contributor), which emulates data similar to what is expected from the Large Synoptic Survey Telescope (LSST). As those are expected to be extremely long time-streams (extending to years), microlensing was a significant element in many of the light curves, which impacted our performance. A new "grander" time delay challenge is being discussed, with a focus on achieving absolute precision time delay measurements that are more on the order of the experimental setup of Section 3.

Final paragraphs should end with a short discussion of future plans, e.g. OMEGA, testing the damped random walk model at short time scales with Kepler, etc.

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REFERENCES