Reading: Ch. 13. Inventory Control (excerpts) *(Operations and Supply Chain Management: The Core)*

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CHAPTER 13

INVENTORY CONTROL

Learning Objectives:

1. Understand the different purposes for keeping inventory.
2. Understand that the type of inventory system logic that is appropriate for an item depends on the type of demand for that item.
3. Know how to calculate the appropriate order size when a one-time purchase must be made
4. Understand what the economic order quantity is and know how to calculate it.
5. Understand fixed-order quantity and fixed-time period models, including ways to determine safety stock when there is variability in demand.
6. Know why inventory turn is directly related to order quantity and safety stock.

HOSPITALS HOPE TO SAVE BY EFFECTIVE INVENTORY MANAGEMENT

Lahey Clinic is hoping to save up to $17 million over five years by acting more like a big-box retailer and automaker when it comes to managing a mundane aspect of the health-care business: medical supplies. The Burlington, Massachusetts teaching hospital's managers decided more than two years ago that they needed to eliminate the hospital's ponderous ordering and stocking bureaucracy and wring savings out of its supply chain. They studied systems deployed by Walmart and Toyota.

Now Lahey is rolling out a system that features secure supply cabinets, bar codes, and computers that keep track of each bottle of antibiotics, every syringe and intravenous bag, and all surgical masks, gowns, and latex gloves. Using thumbprint security technology, nurses open the cabinets, which resemble vending machines, and sit in every ward. Computers keep count of stock and automatically reorder from a vendor's off-site warehouse. Moreover, the system links the use of supplies to individual patients, so now the hospital knows exactly what it is spending on every type of illness and surgical procedure. In an emergency, nurses and doctors can override the system, open the entire supply cabinet, and grab anything they need quickly. However, the day-to-day goal is to squeeze waste and excess out of the supply chain, said Dr. Sanford R. Kurtz, chief operating officer at Lahey.

"The hospital represents a very chaotic environment for supply," Kurtz said. "Now, when the supplies are taken out, all the charges and supply information go into the purchasing system, and we're able to generate reports." Among the big challenges has been training doctors and nurses to change the way they operate. "There is a learning curve here," Kurtz said. "This is a major, major change."

But, he said, the savings to Lahey will be worth it. Beyond eliminating wasted and idle inventory, the system gives administrators a way to analyze how the hospital's staff actually uses expensive materials to treat patients, from operating rooms to outpatient clinics. "It's important to see how different physicians use different supplies to treat the same diagnosis," he said. "This gives us an opportunity to standardize."

The system is provided under a five-year contract with Cardinal Health of Dublin, Ohio, one of the nation's three largest pharmaceutical wholesale companies (Its Canadian division is located in Vaughn, Ontario.). Cardinal Health says its sophisticated supply systems can save Lahey Clinic $29 million in gross pharmaceutical and supply costs and $17 million in net reductions during the five years of the contract.

■ Source: Adapted from Christopher Rowland, "Hospitals Hope to Save by Supply Management," *Boston Globe*, April 10, 2006.

You should visualize inventory as stacks of money sitting on forklifts, on shelves, and in trucks and planes while in transit. That's what inventory is—money. For many businesses, inventory is the largest asset on the balance sheet at any given time, even though it is often not very liquid. It is a good idea to try to get your inventory down as far as possible.

A few years ago, Heineken, the Netherlands beer company, figured it could save a whole bunch of money on inventory-in-transit if it could just shorten the forecasting lead time. It expected two things to happen. First, it expected to reduce the need for inventory in the pipeline, thereby cutting down the amount of money devoted to inventory itself. Second, it figured that with a shorter forecasting time, forecasts would be more accurate, reducing emergencies and waste. The Heineken system, called HOPS, cut overall inventory in the system from 16 to 18 weeks to 4 to 6 weeks—a huge drop in time, and a big gain in cash. Forecasts were more accurate, and there was another benefit, too.

Heineken found that its salespeople were suddenly more productive. That's because they were not dealing with all those calls to check on inventory, or solve bad forecasting problems, or change orders that were already in process. Instead, they could concentrate on good customer service and helping distributors do better. It was a "win" all the way around.

The key here involves doing things that decrease your inventory order cycle time and increase the accuracy of your forecast. Look for ways to use automated systems and electronic communication to substitute the rapid movement of electrons for the cumbersome movement of masses of atoms.

The economic benefit from inventory reduction is evident from the following statistics: A commonly accepted annual average cost of inventory in Canada is 20 percent of its value.[1]

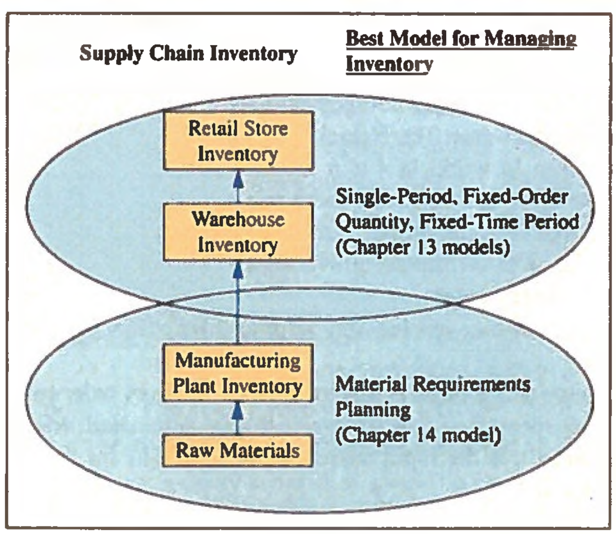
For example, if a firm carries an average inventory of $20 million, it costs the firm more than $4 million per year. These costs are due mainly to obsolescence, insurance, and opportunity costs. If the amount of inventory could be reduced to $10 million, for instance, the firm would save over $2 million, which goes directly to the bottom line. That is, the savings from reduced inventory results in increased profit. This cost of inventory can be much higher in the case of perishables such as food, or short-life-cycle products such as fashion or electronic goods.

This chapter and Chapter 14 present techniques designed to manage inventory in different supply chain settings. In this chapter the focus is on settings where the desire is to maintain a stock of inventory that can be delivered to the customer on demand. Good examples of where the models described in this chapter are used include retail stores, grocery stores, wholesale distributors, hospital supplies, and repair parts needed to fix or maintain equipment quickly. Situations where it is necessary to have the item "in-stock" are ideal candidates for the models described in this chapter.

Exhibit 13.1 depicts different types of supply chain inventories, such as raw materials, manufacturing plant, and warehouse inventories. In the upper echelons of the supply chain, which are supply points closer to the customer, stock is usually kept so that an item can be delivered quickly when the customer need occurs. Of course, there are many exceptions, but in general this is the case. The techniques most appropriate for these inventories assume that demand is random and cannot be predicted with great precision. In the cases of the models we describe in this chapter, we characterize demand by using a probability distribution and maintain stock so that the risk associated with stocking out is managed. For these applications, the following three models are discussed in the chapter:

1. **The single-period model.** This is used when we are making a one-time purchase of an item. An example might be purchasing T-shirts to sell at a one-time sporting event.

EXHIBIT 13.1 Supply Chain Inventories



2. **The fixed-order-quantity model.** This is used when we want to maintain an item "in-stock," and when we resupply the item, a certain number of units must be ordered each time. Inventory for the item is monitored until it gets down to a level where the risk of stocking out is great enough that we are compelled to order.
3. **The fixed-time period model.** This is similar to the fixed-order-quantity model; it is used when the item should be in-stock and ready to use. In this case, rather than monitoring the inventory level and ordering when the level gets down to a critical quantity, the item is ordered at certain intervals of time, for example, every Friday morning. This is often convenient when a group of items are ordered together. An example is the delivery of different types of bread to a grocery store. The bakery supplier may have 10 or more products stocked in a store. Rather than delivering each product individually at different times, it is much more efficient to deliver all 10 together at the same time and on the same schedule.

In this chapter, we want to show not only the mathematics associated with effective inventory control but also the "art" of managing inventory. Ensuring accuracy in inventory records is essential to running an efficient inventory control process. Techniques such as ABC analysis and cycle counting are essential to the actual management of the system since they focus attention on the high-value items and ensure the quality of the transactions that affect the tracking of inventory levels.

DEFINITION OF INVENTORY

Inventory is the stock of any item or resource used in an organization. An *inventory system* is the set of policies and controls that monitor levels of inventory and determine what levels should be maintained, when stock should be replenished, and how large orders should be.

By convention, *manufacturing inventory* generally refers to items that contribute to or become part of a firm's product output. Manufacturing inventory is typically classified into *raw materials, finished products, component parts, supplies,* and *work-in-process.* In services, *inventory* generally refers to the tangible goods to be sold and the supplies necessary to administer the service.

The basic purpose of inventory analysis in manufacturing and stockkeeping services is to specify (1) when items should be ordered and (2) how large the order should be. Many firms are tending to enter into longer-term relationships with vendors to supply their needs for perhaps the entire year. This changes the "when" and "how many to order" to "when" and "how many to deliver."

PURPOSES OF INVENTORY

All firms (including JIT operations) keep a supply of inventory for the following reasons:

1. **To maintain independence of operations.** A supply of materials at a work centre allows that centre flexibility in operations. For example, because there are costs for making each new production set-up, this inventory allows management to reduce the number of set-ups.

   Independence of workstations is desirable on assembly lines as well. The time that it takes to do identical operations will naturally vary from one unit to the next.

Therefore, it is desirable to have a cushion of several parts within the workstation so that shorter performance times can compensate for longer performance times. This way, the average output can be fairly stable.

2. **To meet variation in product demand.** If the demand for the product is known precisely, it may be possible (though not necessarily economical) to produce the product to exactly meet the demand. Usually, however, demand is not completely known, and a safety or buffer stock must be maintained to absorb variation.
3. **To allow flexibility in production scheduling.** A stock of inventory relieves the pressure on the production system to get the goods out. This allows for longer lead times, which permit production planning for smoother flow and lower-cost operation through larger lot-size production. High set-up costs, for example, favour producing a larger number of units once the set-up has been made.
4. **To provide a safeguard for variation in raw material delivery time.** When material is ordered from a vendor, delays can occur for a variety of reasons: a normal variation in shipping time, a shortage of material at the vendor's plant causing backlogs, an unexpected strike at the vendor's plant or at one of the shipping companies, a lost order, or a shipment of incorrect or defective material. This applies to within-the-facility production and delivery as well.
5. **To take advantage of economic purchase order size.** There are costs to place an order: labour, phone calls, typing, postage, and so on. Therefore, the larger each order is, the fewer the orders that need be written. Also, shipping costs favour larger orders—the larger the shipment, the lower the per-unit cost.

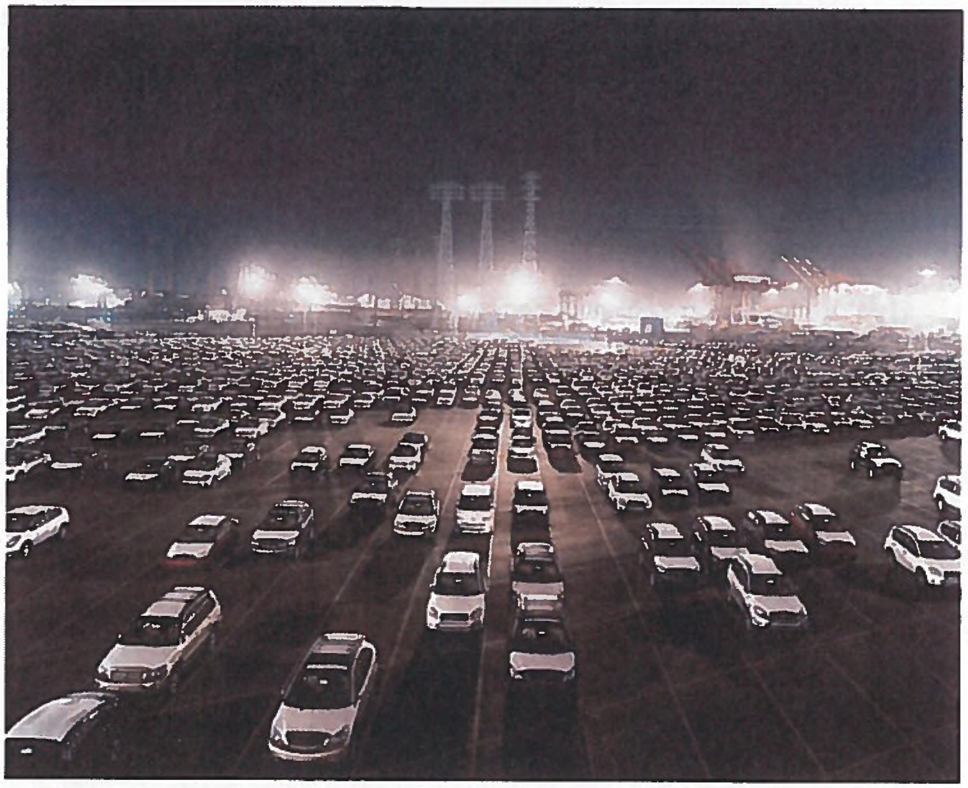
For each of the preceding reasons (especially for items 3, 4, and 5), be aware that inventory is costly and large amounts are generally undesirable. Long cycle times are caused by large amounts of inventory and are undesirable as well.

INVENTORY COSTS

In making any decision that affects inventory size, the following costs must be considered.

1. **Holding (or carrying) costs.** This broad category includes the costs for storage facilities, handling, insurance, pilferage, breakage, obsolescence, depreciation, taxes, and the opportunity cost of capital. Obviously, high holding costs tend to favour low inventory levels and frequent replenishment.
2. **Set-up (or production change) costs.** Making each different product involves obtaining the necessary materials, arranging specific equipment set-ups, filling out the required papers, appropriately charging time and materials, and moving out the previous stock of material.

   If there were no costs or loss of time in changing from one product to another, many small lots would be produced. This would reduce inventory levels, with a resulting savings in cost. One challenge today is to try to reduce these set-up costs to permit smaller lot sizes. (This is one objective of a JIT system.)
3. **Ordering (order preparation) costs.** These costs refer to the managerial and clerical costs to place the purchase order. Ordering costs include all the details, such as counting items and calculating order quantities. The costs associated with maintaining the system needed to track orders are also included in ordering costs. So this cost is similar to set-up cost except that it is related to a purchase order rather than a production order. Sometimes set-up cost and ordering cost are used interchangeably.



Toyota Priuses and other vehicles clad in protective covering await shipment to dealers at a port. In 2006, the value of the company's inventory totalled about ¥1.62 trillion and the cost of goods sold was ¥15.73 trillion. So Toyota's inventory turned over about 9.7 times per year, or roughly 38 days of inventory on hand.

4. **Shortage costs.** When the stock of an item is depleted, an order for that item must either wait until the stock is replenished or be cancelled. There is a trade-off between carrying stock to satisfy demand and the costs resulting from stockout. This balance is sometimes difficult to obtain, because it may not be possible to estimate lost profits, the effects of lost customers, or lateness penalties. Frequently, the assumed shortage cost is little more than a guess, although it is usually possible to specify a range of such costs.

Establishing the correct quantity to order from vendors or the size of lots submitted to the firm's productive facilities involves a search for the minimum total cost resulting from the combined effects of the individual costs: holding costs, set-up costs, ordering costs, and shortage costs. Of course, the timing of these orders is a critical factor that may affect inventory cost.

INDEPENDENT VERSUS DEPENDENT DEMAND

In inventory management, it is important to understand the difference between dependent and independent demand. The reason is that entire inventory systems are predicated on whether demand is derived from an end item or is related to the item itself.

Briefly, the distinction between **independent and dependent demand** is this: In independent demand, the demands for various items are unrelated to each other. For example, a workstation may produce many parts that are unrelated but meet some external demand requirement. In dependent demand, the need for any one item is a direct result of the need for some other item, usually a higher-level item of which it is a part.

In concept, dependent demand is a relatively straightforward computational problem. Needed quantities of a dependent-demand item are simply computed, based on the number needed in each higher-level item in which it is used. For example, if an automobile company plans on producing 500 cars per day, then obviously it will need 2000 wheels and tires (plus spares). The number of wheels and tires needed is *dependent* on the production levels and is not derived separately. The demand for cars, on the other hand, is *independent*—it comes from many sources external to the automobile firm and is not a part of other products; it is unrelated to the demand for other products.

Cross Functional

To determine the quantities of independent items that must be produced, firms usually turn to their sales and market research departments. They use a variety of techniques, including customer surveys, forecasting techniques, and economic and sociological trends, as we discussed in Chapter 11 on forecasting. Because independent demand is uncertain, extra units (safety stock) must be carried in inventory. This chapter presents models to determine how many units need to be ordered, and how many extra units should be carried to reduce the risk of stocking out.

INVENTORY SYSTEMS

An inventory system provides the organizational structure and the operating policies for maintaining and controlling goods to be stocked. The system is responsible for ordering and receipt of goods: timing the order placement and keeping track of what has been ordered, how much, and from whom. The system must also follow up to answer such questions as, has the supplier received the order? Has it been shipped? Are the dates correct? Are the procedures established for reordering or returning undesirable merchandise?

This section divides systems into single-period systems and multiple-period systems. The classification is based on whether the decision is just a one-time purchasing decision where the purchase is designed to cover a fixed period of time and the item will not be reordered, or if the decision involves an item that will be purchased periodically and inventory should be kept in stock to be used on demand. We begin with a look at the one-time purchasing decision and the single-period inventory model.

A Single-Period Inventory Model

Certainly, an easy example to think about is the classic single-period "newsperson" problem. For example, consider the problem that the newsperson has in deciding how many newspapers to put in the sales stand outside a hotel lobby each morning. If the newsperson does not put enough papers in the stand, some customers will not be able to purchase a paper and the newsperson will lose the profit from these sales. On the other hand, if too many papers are placed in the stand, the newsperson will have paid for papers that were not sold during the day, lowering profit for the day.

Actually, this is a very common type of problem. Consider the person selling T-shirts promoting a championship hockey or soccer game. This is especially difficult, since the person must wait to learn what teams will be playing. The shirts can then be printed with the proper team logos. Of course, the person must estimate how many people will actually want the shirts. The shirts sold prior to the game can probably be sold at a premium price, whereas those sold after the game will need to be steeply discounted.

A simple way to think about this is to consider how much risk we are willing to take for running out of inventory. Let's consider that the newsperson selling papers in the sales stand had collected data over a few months and had found that on average each Monday 90 papers were sold, with a standard deviation of 10 papers (assume that during this time the papers were purposefully overstocked in order not to run out, so the newsperson would know what "real" demand was). With these data, our newsperson could simply state a service rate he or she felt to be acceptable. For example, the newsperson might want to be 80 percent sure of not running out of papers each Monday.

Recall from your study of statistics that, assuming that the probability distribution associated with the sales of the paper is normal, then if we stocked exactly 90 papers each Monday morning, the risk of stocking out would be 50 percent, since 50 percent of the time we expect demand to be less than 90 papers and 50 percent of the time we expect demand to be greater than 90. To be 80 percent sure of not stocking out, we need to carry a few more papers. From the "cumulative standard normal distribution" table given in Appendix E, we see that we need approximately .85 standard deviation of extra papers to be 80 percent sure of not stocking out. A quick way to find the exact number of standard deviations needed for a given probability of stocking out is with the NORMSINV(probability) function in Microsoft Excel (NORMSINV(.8) = .84162). Given our result from Excel, which is more accurate than what we can get from the tables, the number of extra papers would be .84162 × 10 = 8.416, or 9 papers (there is no way to sell .4 paper!).

To make this more useful, it would be good to actually consider the potential profit and loss associated with stocking either too many or too few papers on the stand. Let's say that our newspaper person pays $.20 for each paper and sells the papers for $.50. In this case, the marginal cost associated with underestimating demand is $.30, the lost profit. Similarly, the marginal cost of overestimating demand is $.20, the cost of buying too many papers. The optimal stocking level, using marginal analysis, occurs at the point where the expected benefits derived from carrying the next unit are less than the expected costs for that unit. Keep in mind that the specific benefits and costs depend on the problem.

In symbolic terms, define

$$C_o = \text{Cost per unit of demand overestimated}$$
$$C_u = \text{Cost per unit of demand underestimated}$$

By introducing probabilities, the expected marginal cost equation becomes

$$P(C_o) \leq (1 - P)\, C_u$$

where P is the probability that the unit will not be sold and $1 - P$ is the probability of it being sold, because one or the other must occur. (The unit is sold or is not sold.)[2]

Then, solving for P, we obtain a distribution independent equation

[13.1]
$$P \leq \frac{C_u}{C_o + C_u}$$

This equation states that we should continue to increase the size of the order as long as the probability of selling what we order is equal to or less than the ratio $C_u/(C_o + C_u)$.

Returning to our newspaper problem, our cost of overestimating demand (C_o) is $.20 per paper and the cost of underestimating demand (C_u) is $.30. The probability therefore is .3/(.2 + .3) = .6. Now, we need to find the point on our demand distribution that corresponds to the cumulative probability of .6. Using the NORMSINV function to get the number of standard deviations (commonly referred to as the Z-score) of extra newspapers to carry, we get .253, which means that we should stock .253(10) = 2.53 or 3 extra papers. The total number of papers for the stand each Monday morning, therefore, should be 93 papers.

This model is very useful and, as we will see in our solved sample problem, can even be used for many service sector problems, such as the number of seats to book on a full airline flight or the number of reservations to book on a full night at a hotel.

EXAMPLE 13.1: HOTEL RESERVATIONS

A hotel near a CFL stadium always fills up on the evening before football games. History has shown that when the hotel is fully booked, the number of last-minute cancellations has a mean of 5 and a standard deviation of 3. The average room rate is $80. When the hotel is overbooked, its policy is to find a room in a nearby hotel and pay for the room for the customer. This usually costs the hotel approximately $200 since rooms booked on such late notice are expensive. How many rooms should the hotel overbook?

SOLUTION

The cost of underestimating the number of cancellations is $80 and the cost of overestimating cancellations is $200.

$$P \leq \frac{C_u}{C_o + C_u} = \frac{\$80}{\$200 + \$80} = .2857$$

Using NORMSINV(.2857) from Excel® gives a Z-score of −.56599. The negative value indicates that we should overbook by a value less than the average of 5. The actual value should be −.56599(3) = −1.69797, or 2 reservations less than 5. The hotel should overbook three reservations on the evening prior to a football game.

Another common method for analyzing this type of problem is with a discrete probability distribution found using actual data and marginal analysis. For our hotel, consider that we have collected data and our distribution of no-shows is as follows:

NUMBER OF NO-SHOWS	PROBABILITY	CUMULATIVE PROBABILITY
0	0.05	0.05
1	0.08	0.13
2	0.10	0.23
3	0.15	0.38
4	0.20	0.58
5	0.15	0.73
6	0.11	0.84
7	0.06	0.90
8	0.05	0.95
9	0.04	0.99
10	0.01	1.00

Using these data, a table showing the impact of overbooking is created. Total expected cost of each overbooking option is then calculated by multiplying each possible outcome by its probability and summing the weighted costs. The best overbooking strategy is the one with minimum cost.

		NUMBER OF RESERVATIONS OVERBOOKED										
NO-SHOWS	PROBABILITY	0	1	2	3	4	5	6	7	8	9	10
0	0.05	0	200	400	600	800	1000	1200	1400	1600	1800	2000
1	0.08	80	0	200	400	600	800	1000	1200	1400	1600	1800
2	0.1	160	80	0	200	400	600	800	1000	1200	1400	1600
3	0.15	240	160	80	0	200	400	600	800	1,000	1,200	1,400
4	0.2	320	240	160	80	0	200	400	600	800	1,000	1,200
5	0.15	400	320	240	160	80	0	200	400	600	800	1,000
6	0.11	480	400	320	240	160	80	0	200	400	600	800
7	0.06	560	480	400	320	240	160	80	0	200	400	600
8	0.05	640	560	480	400	320	240	160	80	0	200	400
9	0.04	720	640	560	480	400	320	240	160	80	0	200
10	0.01	800	720	640	560	480	400	320	240	160	80	0
Total cost		337.6	271.6	228	212.4	238.8	321.2	445.6	600.8	772.8	958.8	1,156

From the table, the minimum total cost is when three extra reservations are taken. This approach using discrete probability is useful when valid historic data are available.

Inventory Control

Single-period inventory models are useful for a wide variety of service and manufacturing applications. Consider the following:

1. **Overbooking of airline flights.** It is common for customers to cancel flight reservations for a variety of reasons. Here the cost of underestimating the number of cancellations is the revenue lost due to an empty seat on a flight. The cost of overestimating cancellations is the awards, such as free flights or cash payments, that are given to customers unable to board the flight.
2. **Ordering of fashion items.** A problem for a retailer selling fashion items is that often only a single order can be placed for the entire season. This is often caused by long lead times and the limited life of the merchandise. The cost of underestimating demand is the lost profit due to sales not made. The cost of overestimating demand is the cost that results when it is discounted.
3. **Any type of one-time order.** For example, ordering T-shirts for a sporting event, spare parts, or printing maps that become obsolete after a certain period of time.

Multiperiod Inventory Systems

There are two general types of multiperiod inventory systems: **fixed-order-quantity models** (also called the *economic order quantity,* EOQ, and ***Q*-model**) and **fixed-time-period models** (also referred to variously as the *periodic* system, *periodic review* system, *fixed-order interval* system, and ***P*-model**). Multiperiod inventory systems are designed to ensure that an item will be available on an ongoing basis throughout the year. Usually the item will be ordered multiple times throughout the year where the logic in the system dictates the actual quantity ordered and the timing of the order.

The basic distinction is that fixed-order-quantity models are "event triggered" and fixed-time-period models are "time triggered." That is, a fixed-order-quantity model initiates an order when the event of reaching a specified reorder level occurs. This event may take place at any time, depending on the demand for the items considered. In contrast, the fixed-time-period model is limited to placing orders at the end of a predetermined time period; only the passage of time triggers the model.

EXHIBIT 13.2 Fixed-Order-Quantity and Fixed-Time-Period Differences

FEATURE	Q-MODEL FIXED–ORDER QUANTITY MODEL	P-MODEL FIXED–TIME PERIOD MODEL
Order quantity	Q—constant (the same amount ordered each time)	q—variable (varies each time order is placed)
When to place order	R—when inventory position drops to the reorder level	T—when the review period arrives
Recordkeeping	Each time a withdrawal or addition is made	Counted only at review period
Size of inventory	Less than fixed–time period model	Larger than fixed–order quantity model
Time to maintain	Higher due to perpetual recordkeeping	
Type of items	Higher-priced, critical, or important items	

To use the fixed-order quantity model (which places an order when the remaining inventory drops to a predetermined order point, R), the inventory remaining must be continually monitored. Thus, the fixed-order-quantity model is a *perpetual* system, which requires that every time a withdrawal from inventory or an addition to inventory is made, records must be updated to reflect whether the reorder point has been reached. In a fixed-time-period model, counting takes place only at the review period. (We will discuss some variations of systems that combine features of both.)

Some additional differences tend to influence the choice of systems (also see Exhibit 13.2):

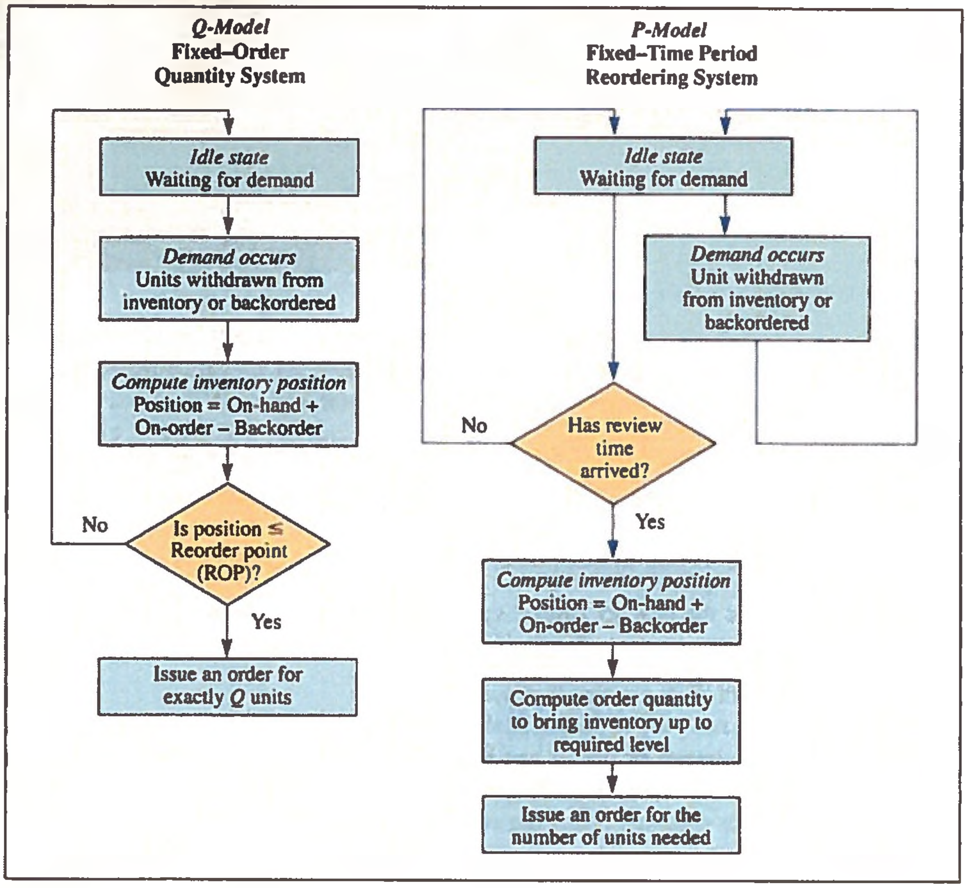
- The fixed-time-period model has a larger average inventory because it must also protect against stockout during the review period, T; the fixed-order quantity model has no review period.
- The fixed-order-quantity model favours more expensive items because average inventory is lower.
- The fixed-order-quantity model is more appropriate for important items such as critical repair parts because there is closer monitoring and therefore quicker response to potential stockout.
- The fixed-order-quantity model requires more time to maintain because every addition or withdrawal is logged.

Exhibit 13.3 shows what occurs when each of the two models is put into use and be comes an operating system. As we can see, the fixed-order-quantity system focuses on order quantities and reorder points. Procedurally, each time a unit is taken out of stock, the withdrawal is logged and the amount remaining in inventory is immediately compared to the reorder point. If it has dropped to this point, an order for Q items is placed. If it has not, the system remains in an idle state until the next withdrawal.

In the fixed-time-period system, a decision to place an order is made after the stock has been counted or reviewed. Whether an order is actually placed depends on the inventory position at that time.

Comparison for Fixed-Order-Quantity and Fixed-Time-Period Reordering Inventory Systems

EXHIBIT 13.3



FIXED-ORDER-QUANTITY MODELS

Determining Optimal Order Quantities

Fixed-order-quantity models attempt to determine the specific point, R, at which an order will be placed and the size of that order, Q. The order point, R, is always a specified number of units. An order of size Q is placed when the inventory available (currently in stock and on order) reaches the point R. **Inventory position** is defined as the on-hand plus on-order minus backordered quantities. The solution to a fixed-order-quantity model may stipulate something like this: When the inventory position drops to 36, place an order for 57 more units.

The simplest models in this category occur when all aspects of the situation are known with certainty. If the annual demand for a product is 1000 units, it is precisely 1000—not 1000 plus or minus 10 percent. The same is true for set-up costs and holding costs. Although the assumption of complete certainty is rarely valid, it provides a good basis for our coverage of inventory models.

EXHIBIT 13.4 Basic Fixed-Order-Quantity Model

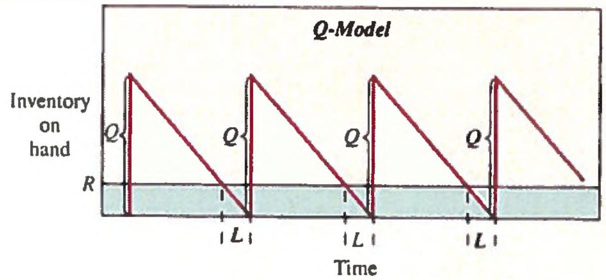


Exhibit 13.4 and the discussion about deriving the optimal order quantity are based on the following characteristics of the model. These assumptions are unrealistic, but they represent a starting point and allow us to use a simple example.

- Demand for the product is constant and uniform throughout the period.
- Lead time (time from ordering to receipt) is constant.
- Price per unit of product is constant.
- Inventory holding cost is based on average inventory.
- Ordering or set-up costs are constant.
- All demands for the product will be satisfied. (No backorders are allowed.)

The "sawtooth effect" relating Q and R in Exhibit 13.4 shows that when the inventory position drops to point R, a reorder is placed. This order is received at the end of time period L, which does not vary in this model.

In constructing any inventory model, the first step is to develop a functional relationship between the variables of interest and the measure of effectiveness. In this case, because we are concerned with cost, the following equation pertains:

$$\text{Total annual cost} = \text{Annual purchase cost} + \text{Annual ordering cost} + \text{Annual holding cost}$$

or

[13.2] $$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H$$

where

TC = Total annual cost

D = Demand (annual)

C = Cost per unit

Q = Quantity to be ordered (the optimal amount is termed the *economic order quantity*—EOQ—or Q_{opt})

S = Set-up cost or cost of placing an order

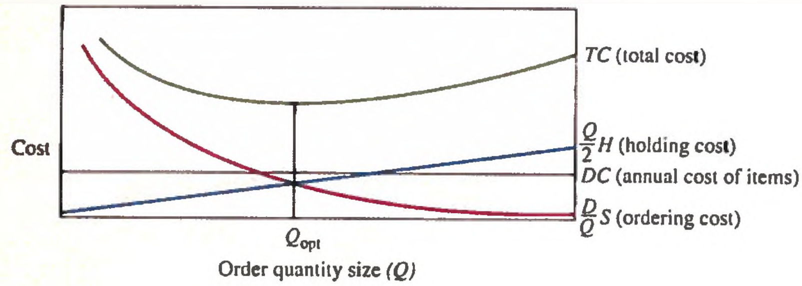
R = Reorder point (ROP)

L = Lead time

H = Annual holding and storage cost per unit of average inventory (often holding cost is taken as a percentage of the cost of the item, such as $H = iC$, where i is the percent carrying cost)

Annual Product Costs, Based on Size of the Order

EXHIBIT 13.5



On the right side of the equation, DC is the annual purchase cost for the units, $(D/Q)S$ is the annual ordering cost (the actual number of orders placed, D/Q, times the cost of each order, S), and $(Q/2)H$ is the annual holding cost (the average inventory, $Q/2$, times the cost per unit for holding and storage, H). These cost relationships are graphed in Exhibit 13.5.

The second step in model development is to find that order quantity Q_{opt} at which total cost is a minimum. In Exhibit 13.5, the total cost is minimal at the point where the slope of the curve is zero. Using calculus, we take the derivative of total cost with respect to Q and set this equal to zero. For the basic model considered here, the calculations are

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H$$

$$\frac{dTC}{dQ} = 0 + \left(\frac{-DS}{Q^2}\right) + \frac{H}{2} = 0$$

[13.3]

$$Q_{opt} = \sqrt{\frac{2DS}{H}}$$

Interactive Operations Management

Because this simple model assumes constant demand and lead time, neither safety stock nor stockout cost are necessary, and the reorder point, R, is simply

[13.4]

$$R = \bar{d}L$$

where

$\bar{d}$ = Average daily demand (stationary)
L = Lead time in days (constant)

EXAMPLE 13.2: ECONOMIC ORDER QUANTITY AND REORDER POINT

Find the economic order quantity and the reorder point, given

Annual demand (D) = 1,000 units
Average daily demand ($\bar{d}$) = 1,000/365
Ordering cost (S) = \$5 per order
Holding cost (H) = \$1.25 per unit per year
Lead time (L) = 5 days
Cost per unit (C) = \$12.50

Inventory Control

What quantity should be ordered?

SOLUTION

The optimal order quantity is

$$Q_{\text{opt}} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(1{,}000)5}{1.25}} = \sqrt{8{,}000} = 89.4 \text{ units}$$

The reorder point is

$$R = \bar{d}L = \frac{1{,}000}{365}(5) = 13.7 \text{ units}$$

Rounding to the nearest unit, the inventory policy is as follows: When the inventory position drops to 14, place an order for 89 more.

The total annual cost will be

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H$$

$$= 1{,}000(12.50) + \frac{1{,}000}{89}(5) + \frac{89}{2}(1.25)$$

$$= \$12{,}611.81$$

Note that in this example, the purchase cost of the units was not required to determine the order quantity and the reorder point because the cost was constant and unrelated to order size.

Operations and Supply Management in Practice

Determining Optimal Lot Sizes in Practice

In this chapter, we discuss inventory models based primarily on the EOQ model. Historically, EOQ-based models have been popular because of their simplicity and because they give good approximations even when the assumptions are violated to some extent. However, most practitioners would caution against using the EOQ or any other model blindly (which is what companies have often done, to the detriment of their costs and customer service).

There are many inventory models for managers to choose from. It is important to choose the inventory model that fits the situation of the item or items under consideration. Sometimes a simple modification to the EOQ can produce good results, as was the case with a Big 3 automaker and its service parts supply chain in the U.S. Furthermore, with improvements in computing power in recent years, many supply chain management and ERP software programs incorporate sophisticated methods for determining optimal order quantities in today's complex and fast-changing operational environment. These methods may be very different from EOQ-type models. Computer simulation is another method used for determining order quantities. 3M is a company that has started institutionalizing the use of sophisticated inventory modelling and computer simulation to improve its inventory management across its diverse product lines. So far, it has saved millions of dollars without compromising customer service levels.

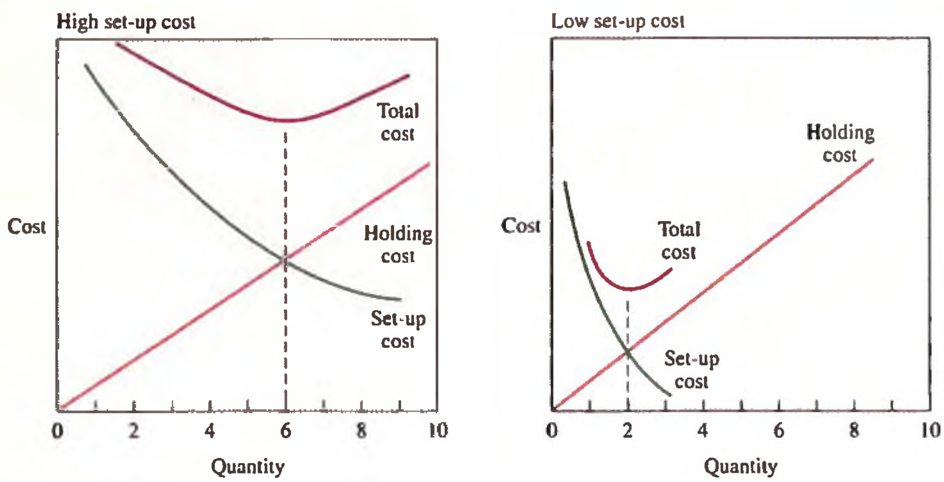
■ Sources: Dan Strike, "Reducing Inventory through Safety Stock and Lot-Size Optimization," 2003 APICS International Conference Proceedings, F10, pp. 1–9.

Alan R. Cannon and Richard E. Crandall, "The Way Things Never Were," *APICS—The Performance Advantage*, January 2004, pp. 32–36.

Chuck LaMacchia, "A New Take," *APICS—The Performance Advantage*, January 2003, pp. 20–23.

Relationship Between Lot Size and Set-up Cost

EXHIBIT 13.6

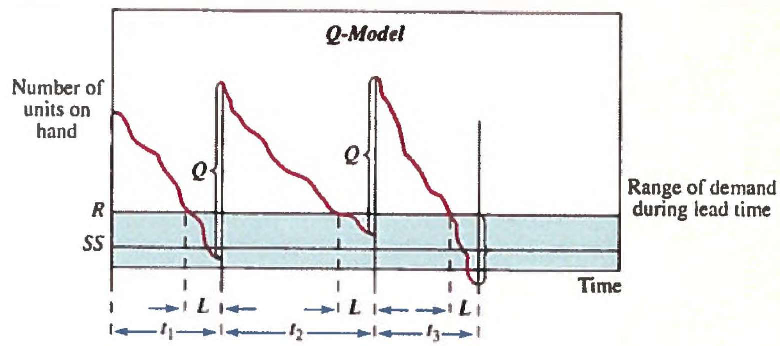


Role of Ordering Cost in Optimal Order Quantity Determination In the EOQ model (Equation 13.3), one can see that the optimal order quantity (or lot size) is related to the ordering cost (set-up cost in manufacturing). The lower this cost, the lower the optimal order quantity. Lean manufacturers were influential in showing the importance of this relationship (you may recall the Toyota example in Chapter 10 where Toyota was taking 10 minutes to change a die compared to six hours for its North American competitors). Exhibit 13.6 shows the benefits of reducing ordering cost. Not only does your optimal order quantity decrease, which leads to carrying less inventory and its associated benefits, but the total cost decreases too. In Example 13.2, if the ordering cost decreased by 50 percent to \$2.50 per order, the optimal order quantity would decrease to 63 units and the sum of the holding and ordering cost would decrease by nearly 30 percent. Further, since items are being produced in smaller batches, customers have to wait less (imagine that you are waiting in line to make photocopies; if everybody in front of you had only 10 copies to make, you would wait a lot less than if everybody had 100 copies to make). Given all these benefits of smaller lot sizes, it is no wonder that in recent years firms have been putting in great efforts to reduce ordering costs.

Fixed-Order-Quantity Model with Safety Stock

Recall that in the fixed-order-quantity model discussed thus far, the inventory is adjusted every time a sale or withdrawal is made, and when the ROP, R, is reached, an order for a fixed quantity, Q, is placed. However, in practice, the demand may not be known with certainty (hence our use of the general term $\bar{d}$ for average daily demand). As a result, the inventory depletion line will not be straight, as in Exhibit 13.4, but will be crooked, as in Exhibit 13.7. Thus, exactly when (on the time axis in Exhibit 13.7) a new order will be placed is not known. For example, because sales were slower in the second time interval (t_2), the reorder point was reached later than during t_1; i.e., $t_2 > t_1$. Note that, regardless of the inventory level, when an order is placed it will be for a fixed quantity, Q. Although

EXHIBIT 13.7 Fixed-Order-Quantity Model under Uncertainty



in Exhibits 13.4 and 13.7 the lead time L is shown as constant, it too can vary (though this issue is beyond this text).

A fixed-order-quantity system perpetually monitors the inventory level and places a new order when stock reaches some level, R. The danger of stockout in this model occurs only during the lead time, between the time an order is placed and the time it is received. As shown in Exhibit 13.7, during this lead time L, a range of demands is possible. Because of this range, sometimes when the new order arrives, we have some inventory on hand, and at other times we are actually stocked out by the time the new order arrives. This range, as shown in the exhibit, is determined either from an analysis of past demand data or from an estimate (if past data are not available).

Safety stock must therefore be maintained to provide some level of protection against stockouts. **Safety stock** can be defined as the amount of inventory carried in addition to the expected demand (so it can be applied to any inventory model, including the single period inventory model discussed earlier). For example, note that in time interval t_1, the sales level was higher than normal. As a result, some of the safety stock had to be used to satisfy demand. In fact, had there been no safety stock, a shortage would have occurred. As seen in Exhibit 13.7, R is higher than in the case in which there is no uncertainty (Exhibit 13.4) because it has to incorporate not only the expected or average demand during lead time ($\bar{d}L$), but also the safety stock.

Safety stock can be determined based on many different criteria. A common approach is for a company to simply state that a certain number of weeks of supply be kept in safety stock. It is better, though, to use an approach that captures the variability in demand.

For example, an objective may be something like "set the safety stock level so that there will only be a 5 percent chance of stocking out if demand exceeds 300 units." We call this approach to setting safety stock the probability approach.

The Probability Approach Using the probability criterion to determine safety stock is pretty simple. With the models described in this chapter, we assume that the demand over a period of time is normally distributed with a mean and a standard deviation. *Again, remember that this approach considers only the probability of running out of stock, not how many units we are short.* To determine the probability of stocking out over the time period, we can simply plot a normal distribution for the expected demand and note where the amount we have on hand lies on the curve.

Let's take a few simple examples to illustrate this. Say we expect demand to be 25 units per week and our lead time is 4 weeks. Our $\bar{d}L$ is then 25 × 4, or 100 units. Suppose we know that the standard deviation of $\bar{d}L$ is 20 units. Assume we have just placed an order for 1000 units (Q) at the beginning of the month, when we had 100 units on hand (i.e., $R = \bar{d}L = 100$ units). Since we know that that the order will be delivered at the end of the month (since $L = 4$ weeks or 1 month), this 100 units on hand should cover us for a month of sales. Ideally, in the absence of any uncertainty, this policy should work well, because, just as we deplete our inventory at the end of the month, the new order for 1000 units should be delivered.

As mentioned, in practice we may sell more or less than the $\bar{d}L$ of 100 units during the month. $\bar{d}L$ Distribution A in Exhibit 13.8 depicts the probability distribution. If our R was 100 units, i.e., we go into the month with just 100 units (no safety stock), we know that our probability of stocking out is 50 percent (the area of the curve to the left of $\bar{d}L$). In half of the lead times, we would expect demand to be greater than 100 units; in the other half, we would expect it to be less than 100 units.

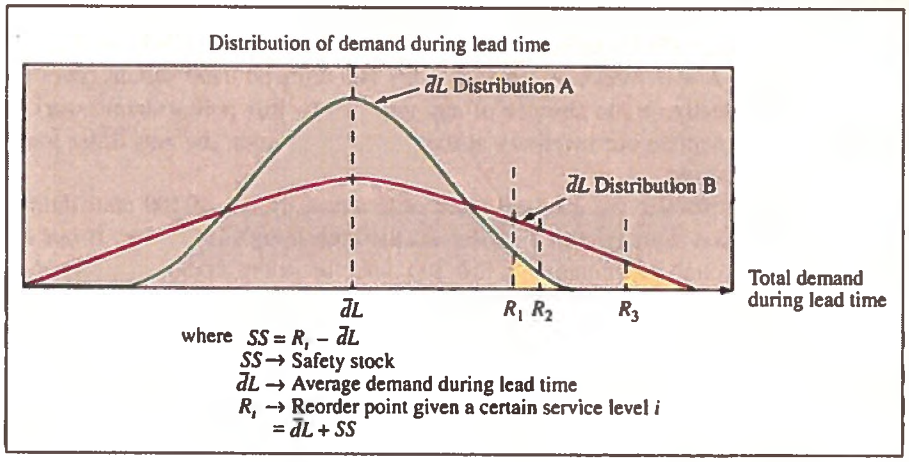
If running out this often was not acceptable, we would want to carry extra inventory to reduce this risk of stocking out. One idea might be to carry an extra 20 units of inventory for the item. In this case, we would still order 1000 units of inventory at a time, but we would schedule the delivery to arrive when we still have 20 units remaining in inventory. Thus, we would have 100 + 20, or 120 units when placing the order. This would give us that little cushion of safety stock to reduce the probability of stocking out. Let R_1 in Exhibit 13.8 represent the reorder point of 120 units in stock. Since the standard deviation associated with our demand is 20 units, we would then be carrying one standard deviation worth of safety stock. Looking at the Cumulative Standard Normal Distribution (Appendix E), online, and moving one standard deviation to the right of the mean, gives a probability of 0.84134. So we would expect approximately 84 percent of the lead times not to stock out, and 16 percent of the lead times to stock out (the area to the left of R_1 in Exhibit 13.8 represents 84 percent of the total area of $\bar{d}L$ Distribution A and the area to the right of R_1 represents 16 percent).

It is common for companies using this approach to set the probability of not stocking out at 95 percent. This means we would carry about 1.64 standard deviations of safety stock, or 33 units ($1.64 \times 20 = 32.8$) for our example. This means that our ROP would have to be 133 units ($\bar{d}L$ of 100 units plus a safety stock of 33 units). Let R_2 in $\bar{d}L$ Distribution A in Exhibit 13.8 represent this ROP (implying that 95 percent of the area under the curve is to the left of R_2, i.e., 95 percent of the lead times would not stock out). Once again, keep in mind that every time we place an order we would still be ordering 1000 units. But we would schedule the receipt so that we could expect to have 33 units in inventory when the order arrives, by placing the order when there are 133 units on hand. So the higher the level of service desired, the higher the amount of safety stock that has to be carried.

$\bar{d}L$ Distribution B in Exhibit 13.8 represents a situation where the $\bar{d}L$ has not changed from 100 units per month, but the variability of demand has increased. For example, assume that the standard deviation of this $\bar{d}L$ has increased to 30 units. Based on $\bar{d}L$ Distribution B, which has more variability than $\bar{d}L$ Distribution A, it is clear that reorder point level R_2 will not provide a 95 percent service level (if the area to the left of R_2 under $\bar{d}L$ Distribution A is 95 percent, then the area to the left of R_2 under $\bar{d}L$ Distribution B is clearly less than 95 percent). To restore the service level to 95 percent, the reorder point will have to increase to an appropriate level R_3.

Thus, the safety stock level that one should have for any given item depends on: (1) demand variation (2) the desired service level, and (3) the variability in order delivery lead time, L (though lead time variability was not discussed here, it behaves in a similar manner to demand variability). Also, as seen in Exhibit 13.7, where in time period t_3 a shortage

EXHIBIT 13.8 Safety Stock Determination



occurred even with safety stock, in practice it is difficult to achieve a 100 percent service level. The amount of safety stock required would cost too much to justify that level.

The quantity to be ordered, Q, is calculated in the usual way, considering the demand, shortage cost, ordering cost, holding cost, and so forth. The reorder point is then set to cover the expected demand during the lead time plus a safety stock determined by the desired service level. Thus, *the key difference between a fixed-order-quantity model where demand is known and one where demand is uncertain is in computing the reorder point. The order quantity is the same in both cases.* The uncertainty element is taken into account in the safety stock.

The reorder point is

$$R = \bar{d}L + z\sigma_L \qquad [13.5]$$

where

R = Reorder point in units
$\bar{d}$ = Average daily demand
L = Lead time in days (assumed to be constant)
z = Number of standard deviations for a specified service probability
σ_L = Standard deviation of usage (demand) during lead time

The term $z\sigma_L$ is the amount of safety stock. Note that if safety stock is positive, the effect is to place a reorder sooner. That is, R without safety stock is simply the average demand during the lead time. If lead time usage was expected to be 20, for example, and safety stock was computed to be 5 units, then the order would be placed sooner, when 25 units remained. The greater the safety stock, the sooner the order is placed.

Computing $\bar{d}$, σ_L, and z Demand during the replenishment lead time is really an estimate or forecast of expected use of inventory from the time an order is placed to when it is received. It may be a single number (for example, if the lead time is a month, the demand may be taken as the previous year's demand divided by 12), or it may be a summation of expected demands over the lead time (such as the sum of daily demands over a 30-day lead time). For the daily demand situation, d can be a forecast demand using any of the models in Chapter 11 on forecasting. For example, if a 30-day period was used to calculate d, then a simple average would be

[13.6]
$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n} = \frac{\sum_{i=1}^{30} d_i}{30}$$

where n is the number of days.

The standard deviation of the daily demand is

[13.7]
$$\sigma_d = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n}} = \sqrt{\frac{\sum_{i=1}^{30}(d_i - \bar{d})^2}{30}}$$

Because σ_d refers to one day, if lead time extends over k (several) days, we can use the statistical premise that the standard deviation of a series of independent occurrences is equal to the square root of the sum of the variances. That is, in general,

[13.8]
$$\sigma_L = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2}$$

For example, suppose we computed the standard deviation of demand to be 10 units per day. If our lead time to get an order is five days, the standard deviation of total demand for the five-day period, assuming each day can be considered independent, is

$$\sigma_5 = \sqrt{(10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2} = 22.36$$

Next we need to find z, the number of standard deviations of safety stock.

Suppose we wanted our probability of not stocking out during the lead time to be 0.95. The z value associated with a 95 percent probability of not stocking out is 1.64 (see Appendix E online or use the Excel NORMSINV function). Given this, safety stock is calculated as follows:

[13.9]
$$SS = z\sigma_L = 1.64 \times 22.36 = 36.67$$

We now compare two examples. The difference between them is that in the first, the variation in demand is stated in terms of standard deviation over the entire lead time, while in the second, it is stated in terms of standard deviation per day.

EXAMPLE 13.3: ECONOMIC ORDER QUANTITY

Consider an economic order quantity case where annual demand $D = 1000$ units, economic order quantity $Q = 200$ units, the desired probability of not stocking out $P = 0.95$, the standard deviation of demand during lead time units, and lead time $\sigma_L = 15$ days. Determine the reorder point. Assume that demand is over a 250-workday year.

SOLUTION

In our example, $\bar{d} = \frac{1000}{250} = 4$, and lead time is 15 days. We use the equation

$$\begin{aligned} R &= \bar{d}L + z\sigma_L \\ &= 4(15) + z(25) \end{aligned}$$

In this case z is 1.64.

Completing the solution for R, we have

$$R = 4(15) + 1.64(25) = 60 + 41 = 101 \text{ units}$$

This says that when the stock on hand gets down to 101 units, order 200 more.

EXAMPLE 13.4: ORDER QUANTITY AND REORDER POINT

Inventory Control

Daily demand for a certain product is normally distributed with a mean of 60 and standard deviation of 7. The source of supply is reliable and maintains a constant lead time of six days. The cost of placing the order is $10 and annual holding costs are $0.50 per unit. There are no stockout costs, and unfilled orders are filled as soon as the order arrives. Assume sales occur over the entire 365 days of the year. Find the order quantity and reorder point to satisfy a 95 percent probability of not stocking out during the lead time.

SOLUTION

In this problem we need to calculate the order quantity Q as well as the reorder point R.

$$\begin{aligned} \bar{d} &= 60 & S &= \$10 \\ \sigma_d &= 7 & H &= \$0.50 \\ D &= 60(365) & L &= 6 \end{aligned}$$

The optimal order quantity is

$$Q_{\text{opt}} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(60)365(10)}{0.50}} = \sqrt{876\ 000} = 936 \text{ units}$$

To compute the reorder point, we need to calculate the amount of product used during the lead time and add this to the safety stock.

The standard deviation of demand during the lead time of six days is calculated from the variance of the individual days. Because each day's demand is independent.[3]

$$\sigma_L = \sqrt{\sum_{i=1}^{L} \sigma_d^2} = \sqrt{6(7)^2} = 17.15$$

Once again, z is 1.64.

$$R = \bar{d}L + z\sigma_L = 60(6) + 1.64(17.15) = 388 \text{ units}$$

To summarize the policy derived in this example, an order for 936 units is placed whenever the number of units remaining in inventory drops to 388.

FIXED-TIME-PERIOD MODELS

In a fixed-time-period system, inventory is counted only at particular times, such as every week or every month. Counting inventory and placing orders periodically is desirable in certain situations, such as when vendors make routine visits to customers and take orders for their complete line of products, or when buyers want to combine orders to save transportation costs. Other firms operate on a fixed time period to facilitate planning their inventory count; for example, Distributor X calls every two weeks and employees know that all Distributor X's product must be counted.

Fixed-time-period models generate order quantities that vary from period to period, depending on the usage rates. These generally require a higher level of safety stock than a fixed-order-quantity system. The fixed-order-quantity system assumes continual tracking of inventory on hand, with an order immediately placed when the reorder point is reached. In contrast, the standard fixed-time-period models assume that inventory is counted only at the time specified for review. It is possible that some large demand will draw the stock down to zero right after an order is placed. This condition could go unnoticed until the next review period. Then the new order, when placed, still takes time to arrive. Thus, it is possible to be out of stock throughout the entire review period, T, and order lead time, L. Safety stock, therefore, must protect against stockouts during the review period itself as well as during the lead time from order placement to order receipt.

Thumbs Up Foods of Calgary, a manufacturer of packaged Indian food, follows a fixed-time-period production model, in general, in which an item such as a samosa might be made only once a week. Orders have to be placed by the day before the samosas are made (although extra quantities are made to cater to drop-in customers). One reason Thumbs Up follows the fixed-time-period model is that it doesn't have the physical space to set up a separate line for each product it manufactures (where it is easy to start production if the reorder point is reached). It therefore uses the same line with different fixtures. Changing fixtures entails a significant set-up activity, which makes it difficult to respond when a reorder point is reached randomly (as is usually the case in practice with a fixed-order-quantity model).

From the planning perspective of both its customers and itself, it is better for Thumbs Up to use the fixed-time-period model. Customers know that orders have to be placed on a fixed day of the week; Thumbs Up can plan ahead knowing that it makes certain items only on certain days. As mentioned, the disadvantage of the fixed-time-period model is that, because the inventory is replenished only after a fixed time interval, there is a greater chance of running out of stock than with a fixed-order-quantity model. For example, if Thumbs Up runs out of samosas midweek, customers might have to wait for the next run of samosas to be done at the end of the week. If Thumbs Up had used the fixed-order-quantity model, it could have started a new batch of samosas as soon as the reorder point was reached before midweek, thus preventing a stockout.

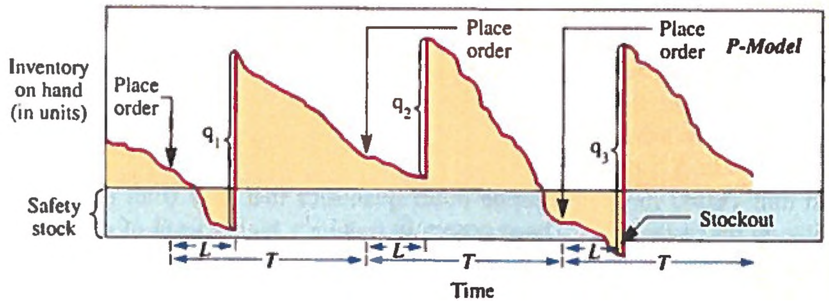
Fixed-Time-Period Model with Safety Stock

In a fixed-time period system, reorders are placed at the time of review (T), and the safety stock that must be reordered is

[13.10] $$\text{Safety stock} = z\sigma_{T+L}$$

Exhibit 13.9 shows a fixed-time-period system with a review cycle of T and a constant lead time of L. In this case, demand is randomly distributed about a mean $\bar{d}$. The quantity to order, q, is

EXHIBIT 13.9 Fixed-Time-Period Inventory Model



$$\text{[13.11]} \quad \begin{array}{ccccccc} \text{Order quantity} & = & \begin{array}{c}\text{Average demand}\\ \text{over the}\\ \text{vulnerable period}\end{array} & + & \begin{array}{c}\text{Safety}\\ \text{stock}\end{array} & - & \begin{array}{c}\text{Inventory currently}\\ \text{on hand (plus on}\\ \text{order, if any)}\end{array} \\ q & = & \bar{d}(T+L) & + & z\sigma_{T+L} & - & I \end{array}$$

where

q = Quantity to be ordered
T = The number of days between reviews
L = Lead time in days (time between placing an order and receiving it)
$\bar{d}$ = Forecast average daily demand
z = Number of standard deviations for a specified service probability
σ_{T+L} = Standard deviation of demand over the review and lead time
I = Current inventory level (includes items on order)

Note: The demand, lead time, review period, and so forth can be any time units such as days, weeks, or years so long as they are consistent throughout the equation.

In this model, demand $\bar{d}$ can be forecast and revised each review period if desired or the yearly average may be used if appropriate. We assume that demand is normally distributed.

The value of z is dependent on the probability of stocking out and can be found using Appendix E or by using the Excel® NORMSINV function.

Thus, with regard to safety stock levels, in a fixed-time-period model, in addition to the three factors (demand variability, variability in lead time, and service level), the length of the interval is also a factor. So, if a firm such as Thumbs Up made samosas only once weekly, it would have to carry more safety stock than if it made samosas daily.

As discussed in Exhibit 13.2 and from Exhibits 13.7 and 13.9, it can be seen that the fixed-order-quantity period and fixed-time-period models respond differently to demand level changes over time. In the fixed-order-quantity model (Exhibit 13.7), when demand changes, the amount ordered (Q) remains the same, but the frequency at which this amount is ordered changes (t_1, t_2 etc). In the fixed-time-period model (Exhibit 13.9), when demand changes, the frequency of ordering does not change (the time between orders is always T), but the amount of each order changes (q_1, q_2 etc).

EXAMPLE 13.5: QUANTITY TO ORDER

Daily demand for a product is 10 units with a standard deviation of 3 units. The review period is 30 days, and lead time is 14 days. Management has set a policy of satisfying 98 percent of demand from items in stock. At the beginning of this review period, there are 150 units in inventory.

How many units should be ordered?

Inventory Control

SOLUTION

The quantity to order is

$$q = \bar{d}(T + L) + z\sigma_{T+L} - I$$
$$= 10(30 + 14) + z\sigma_{T+L} - 150$$

Before we can complete the solution, we need to find σ_{T+L} and z. To find σ_{T+L}, we use the notion, as before, that the standard deviation of a sequence of independent random variables equals the square root of the sum of the variances. Therefore, the standard deviation during the period $T + L$ is the square root of the sum of the variances for each day:

[13.12]

$$\sigma_{T+L} = \sqrt{\sum_{d=1}^{T+L} \sigma_d^2}$$

Because each day is independent and is constant,

$$\sigma_{T+L} = \sqrt{(T + L)\sigma_d^2} = \sqrt{(30 + 14)(3)^2} = 19.90$$

The z value for $P = 0.98$ is 2.05.

The quantity to order, then, is

$$q = \bar{d}(T + L) + z\sigma_{T+L} - I = 10(30 + 14) + 2.05(19.90) - 150 = 331 \text{ units}$$

To ensure a 98 percent probability of not stocking out, order 331 units at this review period.

INVENTORY CONTROL AND SUPPLY CHAIN MANAGEMENT

It is important for managers to realize that how they run items using inventory control logic relates directly to the financial performance of the firm. A key measure that relates to company performance is inventory turn. Recall that inventory turn is calculated as follows:

$$\text{Inventory turn} = \frac{\text{Cost of goods sold}}{\text{Average inventory value}}$$

So what is the relationship between how we manage an item and the inventory turn for that item? Let us simplify things and consider just the inventory turn for an individual item or a group of items. First, if we look at the numerator, the cost of goods sold for an individual item relates directly to the expected yearly demand (D) for the item. Given a cost per unit (C) for the item, the cost of goods sold is just D times C. Recall that this is the same as what was used in our EOQ equation. Next, consider average inventory value. Recall from EOQ that the average inventory is $Q/2$, which is true if we assume that demand is constant. When we bring uncertainty into the equation, safety stock is needed to manage the risk created by demand variability. The fixed-order-quantity model and fixed-time-period model both have equations for calculating the safety stock required for a given probability of stocking out. In both models, we assume that when going through an order cycle, half the time we need to use the safety stock and half the time we do not. So, on average, we expect the safety stock (*SS*) to be on hand. Given this, the average inventory is equal to the following:

[13.13]
$$\text{Average inventory value} = (Q/2 + SS)C$$

The inventory turn for an individual item then is

[13.14]
$$\text{Inventory turn} = \frac{DC}{(Q/2 + SS)C} = \frac{D}{Q/2 + SS}$$

EXAMPLE 13.6: AVERAGE INVENTORY CALCULATION—FIXED-ORDER-QUANTITY MODEL

Suppose the following item is being managed using a fixed-order-quantity model with safety stock.

Annual demand (D) = 1000 units
Order quantity (Q) = 300 units
Safety stock (SS) = 40 units

What are the average inventory level and inventory turn for the item?

SOLUTION

$$\text{Average inventory} = Q/2 + SS = 300/2 + 40 = 190 \text{ units}$$

$$\text{Inventory turn} = \frac{D}{Q/2 + SS} = \frac{1000}{190} = 5.263 \text{ turns per year}$$

EXAMPLE 13.7: AVERAGE INVENTORY CALCULATION—FIXED-TIME-PERIOD MODEL

Consider the following item that is being managed using a fixed-time-period model with safety stock.

Weekly demand (d) = 50 units
Review cycle (T) = 3 weeks
Safety stock (SS) = 30 units

What are the average inventory level and inventory turn for the item?

SOLUTION

Here we need to determine how many units we expect to order each cycle. If we assume that demand is fairly steady, then we would expect to order the number of units that we expect demand to be during the review cycle. This expected demand is equal to dT if we assume that there is no trend or seasonality in the demand pattern.

$$\text{Average inventory} = dT/2 + SS = 50(3)/2 + 30 = 105 \text{ units}$$

$$\text{Inventory turn} = \frac{52d}{dT/2 + SS} = \frac{50(52)}{105} = 24.8 \text{ turns per year}$$

assuming there are 52 weeks in the year.

A caution is in order, though. The formulas in this chapter try to minimize cost. Bear in mind that a firm's objective should be something like "making money"—so be sure that reducing inventory cost does, in fact, support this. Usually, correctly reducing inventory lowers cost, improves quality and performance, and enhances profit.

Key Terms

Cycle counting A physical inventory-taking technique in which inventory is counted on a frequent basis rather than once or twice a year.

Dependent demand The need for any one item is a direct result of the need for some other item, usually an item of which it is a part.

Fixed-order-quantity model (or Q-model) An inventory control model where the amount requisitioned is fixed and the actual ordering is triggered by inventory dropping to a specified level of inventory.

Fixed-time-period model (or P-model) An inventory control model that specifies inventory is ordered at the end of a predetermined time period. The interval of time between orders is fixed and the order quantity varies.

Independent demand The demands for various items are unrelated to each other.

Inventory The stock of any item or resource used in an organization.

Inventory position The amount on-hand plus on-order minus backordered quantities. In the case where inventory has been allocated for special purposes, the inventory position is reduced by these allocated amounts.

Safety stock The amount of inventory carried in addition to the expected demand.

Formula Review

Single-period model. Cumulative probability of not selling the last unit. Ratio of marginal cost of underestimating demand and marginal cost of overestimating demand.

[13.1] $$P \le \frac{C_u}{C_o + C_u}$$

Q-model. Total annual cost for an order Q, a per-unit cost C, set-up cost S, and per-unit holding cost H.

[13.2] $$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H$$

Q-model. Optimal (or economic) order quantity.

[13.3] $$Q_{\text{opt}} = \sqrt{\frac{2DS}{H}}$$

Q-model. Reorder point R based on average daily demand $\bar{d}$ and lead time L in days.

[13.4] $$R = \bar{d}L$$

Q-model. Reorder point providing a safety stock of $z\sigma_L$.

[13.5] $$R = \bar{d}L + z\sigma_L$$

Average daily demand over a period of n days.

Inventory Control

[13.6]
$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$$

Standard deviation of demand over a period of n days.

[13.7]
$$\sigma_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n}}$$

Standard deviation of total lead time demand for a series of k independent demands.

[13.8]
$$\sigma_L = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2}$$

Q-model. Safety stock calculation.

[13.9]
$$SS = z\sigma_L$$

P-model. Safety stock calculation.

[13.10]
$$SS = z\sigma_{T+L}$$

P-model. Optimal order quantity in a fixed-period system with a review period of T days and lead time of L days.

[13.11]
$$q = \bar{d}(T + L) + z\sigma_{T+L} - I$$

P-model. Standard deviation of a series of independent demands over the review period T and lead time L.

[13.12]
$$\sigma_{T+L} = \sqrt{\sum_{d=1}^{T+L} \sigma_d^2}$$

[13.13]
$$\text{Average inventory value} = (Q/2 + SS)C$$

[13.14]
$$\text{Inventory turn} = \frac{DC}{(Q/2 + SS)C} = \frac{D}{Q/2 + SS}$$

Solved Problems

Solved Problem 1

A product is priced to sell at $100 per unit, and its cost is constant at $70 per unit. Each unsold unit has a salvage value of $20. Demand is expected to range between 35 and 40 units for the period; 35 can definitely be sold and no units over 40 will be sold. The demand probabilities and the associated cumulative probability distribution (P) for this situation are shown below.

NUMBER OF UNITS DEMANDED	PROBABILITY OF THIS DEMAND	CUMULATIVE PROBABILITY
35	.10	0.10
36	.15	0.25
37	.25	0.50
38	.25	0.75
39	.15	0.90
40	.10	1.00

Inventory Control

How many units should be ordered?

SOLUTION

The cost of underestimating demand is the loss of profit, or $C_u = \$100 - \$70 = \$30$ per unit. The cost of overestimating demand is the loss incurred when the unit must be sold at salvage value, $C_o = \$70 - \$20 = \$50$.

The optimal probability of not being sold is

$$P \leq \frac{C_u}{C_o + C_u} = \frac{30}{50 + 30} = 0.375$$

From the distribution data above, this corresponds to the 37th unit.

The following is a full marginal analysis for the problem. Note that the minimum cost is when 37 units are purchased.

		NUMBER OF UNITS PURCHASED					
UNITS DEMANDED	PROBABILITY	35	36	37	38	39	40
35	0.1	0	50	100	150	200	250
36	0.15	30	0	50	100	150	200
37	0.25	60	30	0	50	100	150
38	0.25	90	60	30	0	50	100
39	0.15	120	90	60	30	0	50
40	0.1	150	120	90	60	30	0
Total cost		75	53	43	53	83	125

Solved Problem 2

Items purchased from a vendor cost \$20 each, and the forecast for next year's demand is 1000 units. If it costs \$5 every time an order is placed for more units and the storage cost is \$4 per unit per year, what quantity should be ordered each time?

a. What is the total ordering cost for a year?

b. What is the total storage cost for a year?

SOLUTION

The quantity to be ordered each time is

$$Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(1000)5}{4}} = 50 \text{ units}$$

a. The total ordering cost for a year is

$$\frac{D}{Q}S = \frac{1000}{50}(\$5) = \$100$$

b. The storage cost for a year is

$$\frac{Q}{2}H = \frac{50}{2}(\$4) = \$100$$

Solved Problem 3

Daily demand for a product is 120 units, with a standard deviation of 30 units. The review period is 14 days and the lead time is 7 days. At the time of review, 130 units are in stock. If only a 1 percent risk of stocking out is acceptable, how many units should be ordered?

SOLUTION

$$\sigma_{T+L} = \sqrt{(14 + 7)(30)^2} = \sqrt{18\,900} = 137.5$$
$$z = 2.33$$
$$\begin{aligned} q &= \bar{d}(T + L) + z\sigma_{T+L} - I \\ &= 120(14 + 7) + 2.33(137.5) - 130 \\ &= 2710 \text{ units} \end{aligned}$$

Inventory Control

Solved Problem 4

A company currently has 200 units of a product on hand that it orders every two weeks when the salesperson visits the premises. Demand for the product averages 20 units per day with a standard deviation of 5 units. Lead time for the product to arrive is seven days. Management has a goal of a 95 percent probability of not stocking out for this product.

The salesperson is due to come in late this afternoon when 180 units are left in stock (assuming that 20 are sold today). How many units should be ordered?

Inventory Control

SOLUTION

Given $I = 180$, $T = 14$, $L = 7$, $\bar{d} = 20$

$$\sigma_{T+L} = \sqrt{21(5)^2} = 23$$
$$z = 1.64$$
$$\begin{aligned} q &= \bar{d}(T + L) + z\sigma_{T+L} - I \\ &= 20(14 + 7) + 1.64(23) - 180 \\ q &= 278 \text{ units} \end{aligned}$$

Review and Discussion Questions

1. Distinguish between dependent and independent demand in a McDonald's restaurant, in an integrated manufacturer of personal copiers, and in a pharmaceutical supply house.
2. Distinguish between in-process inventory, safety stock inventory, and seasonal inventory.
3. Discuss the nature of the costs that affect inventory size.
4. Under which conditions would a plant manager elect to use a fixed-order-quantity model as opposed to a fixed-time-period model? What are the disadvantages of using a fixed-time-period ordering system?
5. What two basic questions must be answered by an inventory-control decision rule?
6. Discuss the assumptions that are inherent in production set-up cost, ordering cost, and carrying costs. How valid are they?
7. "The nice thing about inventory models is that you can pull one off the shelf and apply it as long as your cost estimates are accurate." Comment.
8. Which type of inventory system would you use in the following situations?
   a. Supplying your kitchen with fresh food.
   b. Obtaining a daily newspaper.
   c. Buying gas for your car.
   To which of these items do you impute the highest stockout cost?
9. Why is it desirable to classify items into groups, as the ABC classification does?

Problems

1. The local supermarket buys lettuce each day to ensure really fresh produce. Each morning any lettuce that is left from the previous day is sold to a dealer that resells it to farmers who use it to feed their animals. This week the supermarket can buy fresh lettuce for $4.00 a box. The lettuce is sold for $10.00 a box and the dealer that sells old lettuce is willing to pay $1.50 a box. Past history says that tomorrow's demand for lettuce averages 250 boxes with a standard deviation of 34 boxes. How many boxes of lettuce should the supermarket purchase tomorrow?
2. Next week, Ntini Airlines has a flight from Montreal to Windsor that will be booked to capacity. The airline knows from past history that an average of 25 customers (with a standard deviation of 15) cancel their reservation or do not show up for the flight. Revenue from a ticket on the flight is $125. If the flight is overbooked, the airline has a policy of getting the customer on the next available flight and giving the person a free round-trip ticket on a future flight. The cost of this free round-trip ticket averages $250. Ntini considers the cost of flying the plane from Montreal to Windsor a sunk cost. By how many seats should Ntini overbook the flight?
3. Sukhi Kaur's Satellite Emporium wishes to determine the best order size for its best-selling satellite dish (model TS111). Sukhi has estimated the annual demand for this model at 1000 units. His cost to carry one unit is $100 per year per unit, and he has estimated that each order costs $25 to place. Using the EOQ model, how many should Sukhi order each time?
4. Dunstreet's Department Store would like to develop an inventory ordering policy of a 95 percent probability of not stocking out. To illustrate your recommended procedure, use as an example the ordering policy for white percale sheets.

   Demand for white percale sheets is 5000 per year. The store is open 365 days per year. Every two weeks (14 days) inventory is counted and a new order is placed. It takes 10 days for the sheets to be delivered. Standard deviation of demand for the sheets is five per day. There are currently 150 sheets on hand.

   How many sheets should you order?
5. Charlie's Pizza orders all of its pepperoni, olives, anchovies, and mozzarella cheese to be shipped directly from Italy. An American distributor stops by every four weeks to take orders. Because the orders are shipped directly from Italy, they take three weeks to arrive.

   Charlie's Pizza uses an average of 68 kilograms of pepperoni each week, with a standard deviation of 14 kilograms. Charlie's prides itself on offering only the best-quality ingredients and a high level of service, so it wants to ensure a 98 percent probability of not stocking out on pepperoni.

   Assume that the sales representative just walked in the door and there are currently 227 kg of pepperoni in the walk-in cooler. How many kilograms of pepperoni would you order?
6. Given the following information, formulate an inventory management system. The item is demanded 50 weeks a year.

Item cost	$10.00	Standard deviation of	
Order cost	$250.00	weekly demand	25 per week
Annual holding cost (%)	33% of item cost	Lead time	1 week
Annual demand	25 750	Service probability	95%
Average demand	515 per week		

   a. State the order quantity and reorder point.
   b. Determine the annual holding and order costs.
   c. If a price break of $50 per order was offered for purchase quantities of over 2000, would you take advantage of it? How much would you save annually?

7. Lieutenant Commander Choudhary is planning to make his monthly (every 30 days) trek to Gamma Hydra City to pick up a supply of isolinear chips. The trip will take Choudhary about two days. Before he leaves, he calls in the order to the GHC Supply Store. He uses chips at an average rate of five per day (seven days per week) with a standard deviation of demand of one per day. He needs a 98 percent service probability. If he currently has 35 chips in inventory, how many should he order? What is the most he will ever have to order?
8. Jill's Job Shop buys two parts (Tegdiws and Widgets) for use in its production system from two different suppliers. The parts are needed throughout the entire 52-week year. Tegdiws are used at a relatively constant rate and are ordered whenever the remaining quantity drops to the reorder level. Widgets are ordered from a supplier who stops by every three weeks. Data for both products are as follows:

ITEM	TEGDIW	WIDGET
Annual demand	10 000	5000
Holding cost (% of item cost)	20%	20%
Set-up or order cost	\$150.00	\$25.00
Lead time	4 weeks	1 week
Safety stock	55 units	5 units
Item cost	\$10.00	\$2.00

   a. What is the inventory control system for Tegdiws? That is, what is the reorder quantity and what is the reorder point?
   b. What is the inventory control system for Widgets?
9. Demand for an item is 1000 units per year. Each order placed costs \$10; the annual cost to carry items in inventory is \$2 each. In what quantities should the item be ordered?
10. The annual demand for a product is 15 600 units. The weekly demand is 300 units with a standard deviation of 90 units. The cost to place an order is \$31.20, and the time from ordering to receipt is four weeks. The annual inventory carrying cost is \$0.10 per unit. Find the reorder point necessary to provide a 98 percent service probability.
    Suppose the production manager is asked to reduce the safety stock of this item by 50 percent. If she does so, what will the new service probability be?
11. Daily demand for a product is 100 units, with a standard deviation of 25 units. The review period is 10 days and the lead time is 6 days. At the time of review there are 50 units in stock. If 98 percent service probability is desired, how many units should be ordered?
12. Item X is a standard item stocked in a company's inventory of component parts. Each year the firm, on a random basis, uses about 2000 of item X, which costs \$25 each. Storage costs, which include insurance and cost of capital, amount to \$5 per unit of average inventory. Every time an order is placed for more item X, it costs \$10.
    a. Whenever item X is ordered, what should the order size be?
    b. What is the annual cost for ordering item X?
    c. What is the annual cost for storing item X?
13. Annual demand for a product is 13 000 units; weekly demand is 250 units, with a standard deviation of 40 units. The cost of placing an order is \$100, and the time from ordering to receipt is four weeks. The annual inventory carrying cost is \$0.65 per unit. To provide a 98 percent service probability, what must the reorder point be?
    Suppose the production manager is told to reduce the safety stock of this item by 100 units. If this is done, what will the new service probability be?
14. In the past, Taylor Industries has used a fixed-time period inventory system that involved taking a complete inventory count of all items each month. However, increasing labour costs are forcing Taylor Industries to examine alternative ways to reduce the amount of labour involved in inventory stockrooms, yet without increasing other costs, such as shortage costs. Here is a random sample of 20 of Taylor's items.

ITEM NUMBER	ANNUAL USAGE	ITEM NUMBER	ANNUAL USAGE
1	$ 1500	11	$13 000
2	12 000	12	600
3	2200	13	42 000
4	50 000	14	9900
5	9600	15	1200
6	750	16	10 200
7	2000	17	4000
8	11 000	18	61 000
9	800	19	3500
10	15 000	20	2900

a. What would you recommend Taylor do to cut back its labour cost? (Illustrate using an ABC plan.)

b. Item 15 is critical to continued operations. How would you recommend that it be classified?

15. Gentle Ben's Bar and Restaurant uses 5000 quart bottles of an imported wine each year. The effervescent wine costs $3 per bottle and is served only in whole bottles because it loses its bubbles quickly. Ben figures that it costs $10 each time an order is placed, and holding costs are 20 percent of the purchase price. It takes three weeks for an order to arrive. Weekly demand is 100 bottles (closed two weeks per year) with a standard deviation of 30 bottles.

    Ben would like to use an inventory system that minimizes inventory cost and will provide a 95 percent service probability.

    a. What is the economic quantity for Ben to order?

    b. At what inventory level should he place an order?

16. Retailers Warehouse (RW) is an independent supplier of household items to department stores. RW attempts to stock enough items for a 98 percent service probability.

    A stainless steel knife set is one item it stocks. Demand (2400 sets per year) is relatively stable over the entire year. Whenever new stock is ordered, a buyer must assure that numbers are correct for stock on hand and then phone in a new order. The total cost involved to place an order is about $5. RW figures that holding inventory in stock and paying for interest on borrowed capital, insurance, and so on, adds up to about $4 holding cost per unit per year.

    Analysis of the past data shows that the standard deviation of demand from retailers is about four units per day for a 365-day year. Lead time to get the order is seven days.

    a. What is the economic order quantity?

    b. What is the reorder point?

17. Daily demand for a product is 60 units with a standard deviation of 10 units. The review period is 10 days, and lead time is 2 days. At the time of review there are 100 units in stock. If 98 percent service probability is desired, how many units should be ordered?

18. Pham Drug Pharmaceuticals orders its antibiotics every two weeks (14 days) when a salesperson visits from one of the pharmaceutical companies. Tetracycline is one of its most prescribed antibiotics, with average daily demand of 2000 capsules. The standard deviation of daily demand was derived from examining prescriptions filled over the past three months and was found to be 800 capsules. It takes five days for the order to arrive. Pham Drug would like to satisfy 99 percent of the prescriptions. The salesperson just arrived, and there are currently 25 000 capsules in stock.

    How many capsules should be ordered?

19. Angkor's Silk Screening produces specialty T-shirts that are primarily sold at special events. They are trying to decide how many to produce for an upcoming event. During the event itself, which lasts one day, Angkor can sell T-shirts for $20 apiece. However, when the event ends, any unsold T-shirts are sold for $4 apiece. It costs Angkor $8 to make a specialty T-shirt. Using Angkor's estimate of demand that follows, how many T-shirts should they produce for the upcoming event?

DEMAND	PROBABILITY
300	0.05
400	0.10
500	0.40
600	0.30
700	0.10
800	0.05

20. Famous Albert prides himself on being the Cookie King of the West. Small, freshly baked cookies are the specialty of his shop. Famous Albert has asked for help to determine the number of cookies he should make each day. From an analysis of past demand he estimates demand for cookies as

DEMAND	PROBABILITY OF DEMAND
1800 dozen	0.05
2000	0.10
2200	0.20
2400	0.30
2600	0.20
2800	0.10
3000	0.05

Each dozen sells for $0.69 and costs $0.49, which includes handling and transportation. Cookies that are not sold at the end of the day are reduced to $0.29 and sold the following day as day-old merchandise.

a. Construct a table showing the profits or losses for each possible quantity.
b. What is the optimal number of cookies to make?
c. Solve this problem by using marginal analysis.

21. Sarah's Muffler Shop has one standard muffler that fits a large variety of cars. Sarah wishes to establish a reorder point system to manage inventory of this standard muffler. Use the following information to determine the best order size and the reorder point:

Annual demand	3500 mufflers	Ordering cost	$50 per order
Standard deviation of daily demand	6 mufflers per working day	Service probability	90%
Item cost	$30 per muffler	Lead time	2 working days
Annual holding cost	25% of item value	Working days	300 per year

22. Volkov Products, Inc., is having a problem trying to control inventory. There is insufficient time to devote to all its items equally. Here is a sample of some items stocked, along with the annual usage of each item expressed in dollar volume.

ITEM	ANNUAL DOLLAR USAGE	ITEM	ANNUAL DOLLAR USAGE
a	$ 7000	k	$80 000
b	1000	l	400
c	14 000	m	1100
d	2000	n	30 000
e	24 000	o	1900
f	68 000	p	800
g	17 000	q	90 000
h	900	r	12 000
i	1700	s	3000
j	2300	t	32 000

a. Can you suggest a system for allocating control time?
b. Specify where each item from the list would be placed.